SHOUT
SHARE
IT

JAN 2 - 1917

64361

PROCEEDINGS

OF THE

NATIONAL ACADEMY OF SCIENCES

OF THE UNITED STATES OF AMERICA

VOLUME 2, 1916

Smithsonian Institution
255103
National Museum

EDITORIAL BOARD

ARTHUR A. NOYES, *Chairman* EDWIN B. WILSON, *Managing Editor*
ARTHUR L. DAY, *Home Secretary* GEORGE E. HALE, *Foreign Secretary*

J. J. ABEL
W. B. CANNON
J. McK. CATTELL
J. M. CLARKE
E. G. CONKLIN
J. M. COULTER
C. B. DAVENPORT
SIMON FLEXNER
E. B. FROST
R. G. HARRISON
W. H. HOLMES
J. P. IDDINGS
R. A. MILLIKAN
E. H. MOORE
ALEXANDER SMITH

Publication Office: Williams & Wilkins Company, Baltimore
Editorial Office: Massachusetts Institute of Technology, Cambridge

Home Office of the Academy: Washington, D. C.

TABLE OF CONTENTS

PAGE

Officers and Members of the Academy, November 15, 1915 x
Report of the Annual Meeting, April, 1916 300
Award of Medals 307
Research Grants from the Trust Funds of the Academy 307, 743
Preliminary Report upon the Possibility of Controlling the Land Slides Adjacent to the Panama Canal 193
Symposium on the Exploration of the Pacific 391
Report of the Autumn Meeting, November, 1916 744
Notices of Biographical Memoirs 737
Index 751
Errata 192

NATIONAL RESEARCH COUNCIL

PAGE

Preliminary Report of the Organizing Committee to the President of the Academy 507
Report of the First Meeting of the Council 602
Reports of the Meetings of the Executive Committee 605, 738
Organization of the Research Council 607
Report of the Second Meeting of the Council 738
Scientific Research for National Defense as Illustrated by the Problems of Aeronautics (Abstract) .... *By Lieut. Col. George O. Squier, U. S. A.* 740

MATHEMATICS

PAGE

Upper Limit of the Degree of Transitivity of a Substitution Group *By G. A. Miller* 61
An Extension of Feuerbach's Theorem .... *By F. Morley* 171
Deformations of Transformations of Ribaucour .... *By L. P. Eisenhart* 173
On the Linear Dependence of Functions of Several Variables, and Certain Completely Integrable Systems of Partial Differential Equations .... *By Gabriel M. Green* 209
Point Sets and Allied Cremona Groups (Part II) .... *By Arthur B. Coble* 244
On a Theorem of Lucas .... *By M. B. Porter* 247
Interpretation of the Simplest Integral Invariant of Projective Geometry *By E. J. Wilczynski* 248
On the Foundations of Plane Analysis Situs .... *By Robert L. Moore* 270
A General Theory of Surfaces .... *By Edwin B. Wilson and C. L. E. Moore* 273
Differential Equations and Implicit Functions in Infinitely Many Variables *By William L. Hart* 309
Note on Lucas' Theorem .... *By M. B. Porter* 335
A Variable System of Sevens on Two Twisted Cubic Curves .... *By H. S. White* 337
A Proof of White's Porism .... *By A. B. Coble* 530
On Certain Asymptotic Expressions in the Theory of Linear Differential Equations .... *By W. E. Milne* 543
On Newton's Method of Approximation .... *By Henry B. Fine* 546
Point Sets and Cremona Groups (Part III) .... *By Arthur A. Coble* 575
Some Problems of Diophantine Approximation: A Remarkable Trigonometrical Series .... *By G. H. Hardy and J. E. Littleswood* 583

PAGE

Newton's Method in General Analysis .... *By Albert A. Bennett* 592

A Set of Independent Postulates for Cyclic Order, *By Edward V. Huntington* 630

Certain General Properties of Functions .... *By Henry Blumberg* 646

ASTRONOMY

PAGE

A Possible Origin for Some Spiral Nebulae .... *By George F. Becker* 1

Studies of Magnitudes in Star Clusters, I. On the Absorption of Light in Space .... *By Harlow Shapley* 12

Studies of Magnitudes in Star Clusters, II. On the Sequence of Spectral Types in Stellar Evolution .... *By Harlow Shapley* 15

On the Albedo of the Planets and Their Satellites ...... *By Henry Norris Russell* 74

The Minute Structure of the Solar Atmosphere ....
By George E. Hale and Ferdinand Ellerman 102

Monochromatic Photography of Jupiter and Saturn .... *By R. W. Wood* 109

On the Observed Rotations of a Planetary Nebula ....
By W. W. Campbell and J. H. Moore 129

A Short Period Cepheid with Variable Spectrum .... *By Harlow Shapley* 132

The Spectrum of δ Cephei .... *By Walter S. Adams and Harlow Shapley* 136

Investigations in Stellar Spectroscopy, I. A Quantitative Method of Classifying Stella Spectra .... *By Walter S. Adams* 143

Investigations in Stellar Spectroscopy, II. A Spectroscopic Method of Determining Stellar Parallaxes .... *By Walter S. Adams* 147

Investigations in Stellar Spectroscopy, III. Application of a Spectroscopic Method of Determining Stellar Distances to Stars of Measured Parallax
By Walter S. Adams 152

Investigations in Stellar Spectroscopy, IV. Spectroscopic Evidence for the Existence of Two Classes of M Type Stars .... *By Walter S. Adams* 157

Discovery of Eight Variable Stellar Spectra .... *By Harlow Shapley* 208

Systematic Motion Among Stars of the Helium Type .... *By Benjamin Boss* 214

The Situation in Regard to Rowland's Preliminary Table of Solar Spectrum Wave-Lengths .... *By Charles E. St. John* 226

Changes in the Form of the Nebula N. G. C. 2261 .... *By Edwin P. Hubble* 230

On Some Relations Between the Proper Motions, Radial Velocities, and Magnitudes of Stars of Classes B and A .... *By C. D. Perrine* 289

Asymmetry in the Proper Motions and Radial Velocities of Stars of Class B and Their Possible Relation to a Motion of Rotation. . . .*By C. D. Perrine* 292

The Pyranometer: An Instrument for Measuring Sky Radiation ....
By C. G. Abbot and L. B. Aldrich 333

The Work of the American Meteor Society in 1914 and 1915 ....
By Charles P. Olivier 372

An Apparent Dependence of the Apex and Velocity of Solar Motion, as Determined from Radial Velocities, Upon Proper Motion. .*By C. D. Perrine* 376

Preliminary Evidence of Internal Motion in the Spiral Nebula Messier 101
By A. van Maanen 386

On the Suggested Mutual Repulsion of Fraunhofer Lines ....
By Charles E. St. John 458

The Rotation and Radial Velocity of the Spiral Nebula N. G. C. 4594........
By Francis G. Pease 517

A Simple Method For Determining the Colors of the Stars ....
By Frederick H. Seares 521

Studies of Magnitudes in Star Clusters, III. The Colors of the Brighter Stars in Four Globular Systems .... *By Harlow Shapley* 525

PAGE
PRELIMINARY RESULTS ON THE COLOR OF NEBULAE..........*By Frederick H. Seares* 553
SPECTROGRAPHIC OBSERVATIONS OF RELATIVE MOTIONS IN THE PLANETARY NEBULAE *By W. W. Campbell and J. H. Moore* 566

PHYSICS AND ENGINEERING

PAGE
EXPERIMENTAL EVIDENCE FOR THE ESSENTIAL IDENTITY OF THE SELECTIVE AND NORMAL PHOTO-ELECTRIC EFFECTS............*By R. A. Millikan and W. H. Souder* 19
CONCOMITANT CHANGES IN TERRESTRIAL MAGNESIUM AND SOLAR RADIATION...... *By L. A. Bauer* 24
QUANTUM RELATIONS IN PHOTO-ELECTRIC PHENOMENA........*By R. A. Millikan* 78
THE EMISSION QUANTA OF CHARACTERISTIC X RAYS........*By David L. Webster* 90
THE ELECTROMOTIVE FORCE PRODUCED BY THE ACCELERATION OF METALS........ *By Richard C. Tolman and T. Dale Stewart* 189
THE HIGH FREQUENCY SPECTRUM OF TUNGSTEN....*By Albert W. Hull and Marion Rice* 265
DYNAMICAL STABILITY OF AEROPLANES....................*By Jerome C. Hunsaker* 278
THEORY OF AN AEROPLANE ENCOUNTERING GUSTS........*By Edwin Bidwell Wilson* 294
NOTE ON THE PHOSPHORESCENCE OF URANYL SALTS...........*By Edward L. Nichols* 328
ON THE MOBILITIES OF GAS IONS IN HIGH ELECTRIC FIELDS..*By Leonard B. Loeb* 345
THE LIGHT EXCITATION BY SLOW POSITIVE AND NEUTRAL PARTICLES.............. *By A. J. Dempster* 374
CHANNELED GRATING SPECTRA, OBTAINED IN SUCCESSIVE DIFFRACTIONS.......... *By C. Barus* 378
A NEW METHOD OF MEASURING THE ACCELERATION OF GRAVITY AT SEA............ *By Lyman J. Briggs* 399
MARINE METEOROLOGY AND THE GENERAL CIRCULATION OF THE ATMOSPHERE...... *By Charles F. Marvin* 421
THE ABSORPTION COEFFICIENTS OF SOFT X RAYS..................*By C. D. Miller* 441
AN ATTEMPT TO DETECT THE MUTUAL INFLUENCE OF NEIGHBORING LINES IN ELECTRIC FURNACE SPECTRA SHOWING ANOMALOUS DISPERSION......*By Arthus S. King* 461
INTERFEROMETER METHODS BASED ON THE CLEAVAGE OF A DIFFRACTED RAY........ *By C. Barus* 475
THE EFFECT OF AN ELECTRIC FIELD ON THE LINES OF LITHIUM AND CALCIUM........ *By Janet T. Howell* 528
THE INTERFERENCES OF SPECTRA BOTH REVERSED AND INVERTED..*By Carl Barus* 576
PATH DIFFERENCES WITHIN WHICH SPECTRUM INTERFERENCES ARE OBSERVED...... *By Carl Barus* 609
NON-REVERSED SPECTRA OF RESTRICTED COINCIDENCE ....*By Carl Barus* 614
A NEW THERMOMETER SCALE...............................*By Alexander McAdie* 670
AN IONIZATION MANOMETER ....*By O. E. Buckley* 683

CHEMISTRY

PAGE
A PECULIAR CLAY FROM NEAR THE CITY OF MEXICO..........*By E. W. Hilgard* 8
POLYPEPTIDE-HYDANTOINS.................................. *By Treat B. Johnson* 69
THE CHEMICAL ACTIVITY OF THE IONS OF HYDROCHLORIC ACID DETERMINED BY ELECTROMOTIVE FORCE MEASUREMENTS..........................*By James H. Ellis* 83
THE ABUNDANCE OF THE ELEMENTS IN RELATION TO THE HYDROGEN-HELIUM STRUCTURE OF THE ATOMS...............................*By William D. Harkins* 216
CHANGE OF THE IONIZATION OF SALTS IN ALCOHOLIC SOLVENTS WITH THE CONCENTRATION..........................*By Frederick G. Keyes and W. J. Winninghoff* 342
SYNTHESIS OF THE BASE $C_3H_4ON_2$ DERIVED FROM METHYL-AMINOMETHYL-3,4-DIHYDROXYPHENYLCARBINOL............................*By Charles A. Rouiller* 464

PAGE

DENSITY OF RADIO-LEAD FROM PURE NORWEGIAN CLEVEITE..................... *By T. W. Richards and C. Wadsworth, 3d.* 505

THE ACTION OF ALKALI IN THE PRODUCTION OF LIPOLYTICALLY ACTIVE PROTEIN.... *By K. George Falk* 557

STERIC HINDRANCE AND THE EXISTENCE OF ODD MOLECULES (FREE RADICALS).... *By Gilbert N. Lewis* 586

THE COBALTAMMINES......*By William D. Harkins, R. E. Hall and W. A. Roberts* 598

THE EQUILIBRIUM BETWEEN ACIDS AND BASES IN SEA WATER.................. *By Lawrence J. Henderson and Edwin J. Cohn* 608

ELECTRICAL CONDUCTION IN DILUTE AMALGAMS................................ *By Gilbert N. Lewis and Thomas B. Hine* 634

THE OSMOTIC PRESSURE AND LOWERING OF THE FREEZING-POINT OF MIXTURES OF SALTS WITH ONE ANOTHER AND WITH NON-ELECTROLYTES IN AQUEOUS SOLUTIONS *By William D. Hawkins, R. E. Hall, and W. A. Roberts* 642

FURTHER STUDY OF THE ATOMIC WEIGHT OF LEAD OF RADIOACTIVE ORIGIN *By Theodore W. Richards and Charles Wadsworth, 3d* 694

A REVISION OF THE ATOMIC WEIGHT OF TIN *By Gregory Paul Baxter and Howard Warner Starkweather* 718

GEOLOGY AND PALEONTOLOGY

PAGE

SUB-MARINE SOLUTION OF LIMESTONE IN RELATION TO THE MURRAY-AGASSIZ THEORY OF CORAL ATOLLS............................*By Alfred Goldsborough Mayer* 28

THE EXTENSION OF THE MONTANA PHOSPHATE DEPOSITS NORTHWARD INTO CANADA *By Frank D. Adams and W. J. Dick* 62

THE RESULTS OF INVESTIGATIONS OF THE ECOLOGY OF THE FLORIDIAN AND BAHAMAN SHOAL-WATER CORALS........................*By Thomas Wayland Vaughan* 95

CAMBRIAN TRILOBITES....................................*By Charles D. Walcott* 101

THE MECHANICS OF INTRUSION OF THE BLACK HILLS (S. D.) PRE-CAMBRIAN GRANITE *By Sidney Paige* 113

ON THE FOSSIL ALGAE OF THE PETROLEUM-YIELDING SHALES OF THE GREEN RIVER FORMATION OF COLORADO AND UTAH.....................*By Charles A. Davis* 114

BANDED GLACIAL SLATES OF PERMO-CARBONIFEROUS AGE, SHOWING POSSIBLE SEASONAL VARIATIONS IN DEPOSITION.............................*By Robert W. Sayles* 167

GEOGRAPHIC HISTORY OF THE SAN JUAN MOUNTAINS SINCE THE CLOSE OF THE MESOZOIC ERA......................*By Wallace W. Atwood and Kirtley F. Mather* 177

THE AGE OF THE MIDDLE ATLANTIC COAST UPPER CRETACEOUS DEPOSITS.......... *By W. B. Clark, E. W. Berry, and J. A. Gardner* 181

UPPER CRETACEOUS FLORAS OF THE WORLD..................*By Edward W. Berry* 185

PRELIMINARY REPORT UPON THE POSSIBILITY OF CONTROLLING THE LAND SLIDES ADJACENT TO THE PANAMA CANAL... 193

ON THE PRESENCE OF A MEDIAN EYE IN TRILOBITES........*By Rudolph Ruedemann* 234

CLIFT ISLANDS IN THE CORAL SEAS...........................*By W. M. Davis* 284

THE EXPLORATION OF THE PACIFIC...........................*By W. M. Davis* 391

THE IMPORTANCE OF GRAVITY OBSERVATIONS AT SEA ON THE PACIFIC........... .. *By John F. Hayford* 394

THE PROBLEM OF CONTINENTAL FRACTURING AND DIASTROPHISM IN OCEANICA...... *By Charles Schuchert* 407

IN RELATION TO THE EXTENT OF KNOWLEDGE CONCERNING THE OCEANOGRAPHY OF THE PACIFIC...*By G. W. Littlehales* 419

ON THE DISTRIBUTION OF PACIFIC INVERTEBRATES................*By Wm. H. Dall* 424

MID-PACIFIC LAND SNAIL FAUNAS............................*By H. A. Pilsbry* 429

PAGE
EXTINGUISHED AND RESURGENT CORAL REEFS....................By W. M. Davis 466
THE ORIGIN OF CERTAIN FIJI ATOLLS..........................By W. M. Davis 471
THE GEOLOGIC RÔLE OF PHOSPHORUS........................By Eliot Blackwelder 490
DOMINANTLY FLUVIATILE ORIGIN UNDER SEASONAL RAINFALL OF THE OLD RED SANDSTONE..By Joseph Barrell 496
THE INFLUENCE OF SILURIAN-DEVONIAN CLIMATES ON THE RISE OF AIR-BREATHING VERTEBRATES..By Joseph Barrell 499
SPHENACODON MARSH, A PERMOCARBONIFEROUS THEROMORPH REPTILE FROM NEW MEXICO..By Samuel W. Williston 650
THE ORIGIN OF VEINS OF THE ASBESTIFORM MINERALS...........By Stephen Taber 659
A NEW TEST OF THE SUBSIDENCE THEORY OF CORAL REEFS....By Reginald A. Daly 664
ON SOME ANOMALIES IN GEOGRAPHIC DISTRIBUTION OF PACIFIC COAST MOLLUSCA By William Healey Dall 700
THE EARLIEST FRESH-WATER ARTHROPODS....................By Charles Schuchert 726

MINERALOGY AND PETROLOGY

SOME MINERALS FROM THE FLUORITE-BARITE VEIN NEAR WAGON WHEEL GAP, COLORADO.................................By Esper S. Larsen and Roger C. Wells 360
THE PETROLOGY OF SOME SOUTH PACIFIC ISLANDS AND ITS SIGNIFICANCE............ By Joseph P. Iddings 413
A CONTRIBUTION TO THE PETROGRAPHY OF JAPAN..By J. P. Iddings and E. W. Morley 452
A CONTRIBUTION TO THE PETROGRAPHY OF THE PHILIPPINE ISLANDS .... By J. P. Iddings and E. W. Morley 531

BOTANY

PAGE
THE ARCHEGONIUM AND SPOROPHYTE OF *Treubia Insignis* GOEBLE.................. By Douglas Houghton Campbell 30
RECENT EXPLORATIONS IN THE CACTUS DESERTS OF SOUTH AMERICA....By J. N. Rose 73
THE NATURE OF MECHANICAL STIMULATION....................W. J. V. Osterhout 237
THE MARINE ALGAE OF THE PACIFIC..........................By W. G. Farlow 424
SOME PROBLEMS OF THE PACIFIC FLORAS................By Douglas H. Campbell 434
THE EXCRETION OF ACIDS BY ROOTS...............................By A. R. Haas 561
NEW DETERMINATIONS OF PERMEABILITY........................By S. C. Brooks 569
THE OAKS OF AMERICA.................................. By William Trelease 626
PHYSIOLOGICAL STUDIES ON RHIZOPHORA...............By Howard H. M. Bowman 685

ZOOLOGY

PAGE
ON THE EFFECTS OF FEEDING PITUITARY BODY (ANTERIOR LOBE) SUBSTANCE, AND CORPUS LUTEUM SUBSTANCE TO GROWING CHICKS............By Raymond Pearl 50
EFFECTS OF CENTRIFUGAL FORCE ON THE POLARITY OF THE EGGS OF CREPIDULA...... By Edwin G. Conklin 87
OBSERVATIONS ON AMEBA FEEDING ON INFUSORIA, AND THEIR BEARING ON THE SURFACE-TENSION THEORY........................By S. O. Mast and F. M. Root 188
ON THE EFFECT OF REMOVAL OF THE PRONEPHROS OF THE AMPHIBIAN EMBRYO...... By Ruth B. Howland 231
THE SEX OF PARTHENOGENETIC FROGS.........................By Jacques Loeb 313
THE DISTRIBUTION OF THE CHONDRIOSOMES TO THE SPERMATOZOA IN SCORPIONS...... By Edmund B. Wilson 321
THE NEUROMUSCULAR STRUCTURE OF SEA-ANEMONES..By G. H. Parker and E. G. Titus 339
DIFFERENTIAL MITOSES IN THE GERM-CELL CYCLE OF DINEUTES NIGRIOR.......... By R. W. Hegner and C. P. Russell 356

The Effectors of Sea-Anemones.......................... *By G. H. Parker* 385
Nervous Transmission in Sea-Anemones.......................... *By G. H. Parker* 437
The Responses of the Tentacles of Sea-Anemones.............. *By G. H. Parker* 438
Locomotion of Sea-Anemones.............................. *By G. H. Parker* 449
The Behavior of Sea-Anemones.............................. *By G. H. Parker* 450
A Comparison of the Rates of Regeneration from Old and from New Tissue *By Charles Zeleny* 484
The Effect of Successive Removal Upon the Rate of Regeneration.......... *By Charles Zeleny* 487
On the Immunity Coloration of Some Nudibranchs........... *By W. J. Crozier* 672
The Influence of the Marginal Sense Organs on Metabolic Activity in Cassiopia Xamachana Bigelow.............................. *By L. R. Cary* 709
Observations upon Tropical Fishes and Inferences from Their Adaptive Coloration.............................. *By W. H. Longley* 733

GENERAL BIOLOGY

An Apparent Correspondence Between the Chemistry of Igneous Magmas and of Organic Metabolism.......................... *By Henry S. Washington* 623
On Volume in Biology.......................... *By Lawrence J. Henderson* 653

GENETICS

PAGE
The Numerical Results of Diverse Systems of Breeding...... *By H. S. Jennings* 45
A Preliminary Report on Further Experiments in Inheritance and Determination of Sex.............................. *By Richard Goldschmidt* 53
On the Degree of Inbreeding Which Exists in American Jersey Cattle...... *By Raymond Pearl and S. W. Patterson* 58
Hereditary Reaction-System Relations—An Extension of Mendelian Concepts.......................... *By R. E. Clausen and T. H. Goodspeed* 240
Size Inheritance in Guinea-Pig Crosses...................... *By W. E. Castle* 252
De Vriesian Mutation in the Garden Bean, Phaseolus Vulgaris............ *By J. Arthur Harris* 317
The Effect of Parental Alcoholism (and Certain Other Drug Intoxications) Upon the Progeny in the Domestic Fowl............ *By Raymond Pearl* 380
On the Inheritance of Certain Glume Characters in the Cross *Avena Fatua* × *A. Sativa* var. Kherson.............................. *By Frank M. Surface* 478
Sex Intergrades in a Species of Crustacea................ *By Arthur M. Banta* 578
Some Effects of the Continued Administration of Alcohol to the Domestic Fowl, with Special Reference to the Progeny.......... *By Raymond Pearl* 675

PHYSIOLOGY AND PATHOLOGY

PAGE
A Theory of Nerve-Conduction.................... *By Alfred Goldsborough Mayer* 37
Studies of Ductless Glands by the Electrical Method...... *By W. B. Cannon* 319
The Relation of Myelin to the Loss of Water in the Mammalian Nervous System with Advancing Age.......................... *By Henry H. Donaldson* 350
The Processes Taking Place in the Body by Which the Number of Erythrocytes Per Unit Volume of Blood is Increased in Acute Experimental Polycythaemia.............................. *By Paul D. Lamson* 365
The Influence of Morphin Upon the Elimination of Intravenously Injected Dextrose in Dogs...................... *By I. S. Kleiner and S. J. Meltzer* 369
Further Evidence as to the Relation Between Crown Gall and Cancer...... *By Erwin F. Smith* 444

IS THERE A TEMPERATURE COEFFICIENT FOR THE DURATION OF LIFE?.............. *By Jacques Loeb and J. H. Northrop* 456
THE MECHANISM OF DIFFUSION OF ELECTROLYTES THROUGH ANIMAL MEMBRANES *By Jacques Loeb* 511
SALT ANTAGONISM IN GELATINE................................*By W. O. Fenn* 534
SIMILARITY IN THE BEHAVIOR OF PROTOPLASM AND GELATIN..........*By W. O. Fenn* 539
ON THE HYDROGEN ION CONCENTRATION OF SEA WATER, AND THE PHYSIOLOGICAL EFFECT OF THE IONS OF SEA WATER....................*By J. F. McClendon* 689
SOME INTERRELATIONS BETWEEN DIET, GROWTH, AND THE CHEMICAL COMPOSITION OF THE BODY....................*By Lafayette B. Mendel and Sarah E. Judson* 692
FURTHER STUDIES OF NERVE CONDUCTION IN CASSIOPEA, *By Alfred Goldsborough Mayer* 721

ANTHROPOLOGY

PAGE
BRIEF NOTES ON RECENT ANTHROPOLOGICAL EXPLORATIONS UNDER THE AUSPICES OF THE SMITHSONIAN INSTITUTION AND THE U. S. NATIONAL MUSEUM............ *By Aleš Hrdlička* 32
ZUÑI CULTURE SEQUENCES................................*By A. L. Kroeber* 42
ARCHAELOGICAL EXPLORATIONS AT PECOS, NEW MEXICO............*By A. V. Kidder* 119
MAN AND METALS..*By Walter Hough* 123
THE FAILURE AND REVIVAL OF THE PROCESS OF PIGMENTATION IN THE HUMAN SKIN *By A. E. Jenks* 164
THE GENETIC RELATIONS OF CERTAIN FORMS IN AMERICAN ABORIGINAL ART........ *By Clark Wissler* 224
TERMS OF RELATIONSHIP AND SOCIAL ORGANIZATION..........*By Truman Michelson* 297
NEW DATA ON THE ARCHAEOLOGY OF VENEZUELA............*By Herbert J. Spinden* 325
THE PACIFIC AS A FIELD FOR ETHNOLOGICAL AND ARCHAEOLOGICAL INVESTIGATION.... *By J. Walter Fewkes* 427
NEW EVIDENCE IN REGARD TO THE INSTABILITY OF HUMAN TYPES...*By Franz Boas* 713

PSYCHOLOGY

PERSONAL EQUATION AND STEADINESS OF JUDGMENT IN THE ESTIMATION OF THE NUMBER OF OBJECTS IN MODERATELY LARGE SAMPLES........*By J. Arthur Harris* 65
A NEW METHOD OF STUDYING IDEATIONAL AND ALLIED FORMS OF BEHAVIOR IN MAN AND OTHER ANIMALS...................................*By Robert M. Yerkes* 631
IDEATIONAL BEHAVIOR OF MONKEYS AND APES............*By Robert M. Yerkes* 639
SOME PSYCHO-PHYSIOLOGICAL PROCESSES AS AFFECTED BY ALCOHOL..*By W. R. Miles* 703

OFFICERS AND MEMBERS OF THE ACADEMY
NOVEMBER 13, 1916

OFFICERS OF THE ACADEMY

William H. Welch, *President*
Charles D. Walcott, *Vice-President*
George E. Hale, *Foreign Secretary*
Whitman Cross, *Treasurer*
Arthur L. Day, *Home Secretary*

Additional Members of the Council

E. G. Conklin
J. M. Coulter
R. H. Chittenden
A. A. Noyes
W. H. Howell
M. I. Pupin

MEMBERS OF THE ACADEMY

Abbot, Charles Greeley	*Smithsonian Institution, Washington, D. C.*
Abbot, Henry L., U. S. A.	*23 Berkeley St., Cambridge, Mass.*
Abel, John Jacob	*Johns Hopkins University, Baltimore, Md.*
Allen, J. Asaph	*American Museum of Natural History, New York City.*
Ames, Joseph S.	*Johns Hopkins University, Baltimore, Md.*
Barnard, E. E.	*Yerkes Observatory, Williams Bay, Wis.*
Barus, Carl	*Brown University, Providence, R. I.*
Baxter, Gregory Paul	*T. J. Coolidge Jr. Mem. Lab., Cambridge, Mass.*
Becker, George F.	*U. S. Geological Survey, Washington, D. C.*
Bell, A. Graham	*1331 Connecticut Ave., Washington, D. C.*
Benedict, Francis Gano	*Nutrition Laboratory, Boston, Mass.*
Bliss, Gilbert Ames	*University of Chicago, Chicago, Ill.*
Boas, Franz	*Columbia University, New York City.*
Bogert, Marston Taylor	*Columbia University, New York City.*
Bôcher, Maxime	*Harvard University, Cambridge, Mass.*
Boltwood, B. B.	*Yale University, New Haven, Conn.*
Bolza, Oskar	*Reichsgrafenstr. 10, Freiburg, Germany.*
Branner, John C.	*Stanford University, California.*
Britton, Nathaniel Lord	*New York Botanical Gardens, New York City.*
Bumstead, Henry Andrews	*Yale University, New Haven, Conn.*
Campbell. D. H.	*Stanford University, California.*
Campbell, William W.	*Lick Observatory, Mount Hamilton, California.*
Cannon, Walter Bradford	*Harvard University, Cambridge, Mass.*
Castle, William Ernest	*186 Payson Road, Belmont, Mass.*
Cattell, James McK.	*Garrison, N. Y.*
Chamberlin, Thomas C.	*University of Chicago, Chicago, Ill.*
Chandler, Charles F.	*Columbia University, New York City.*
Chittenden, Russell H.	*Sheffield Scientific School, New Haven, Conn.*
Clark, W. B.	*Johns Hopkins University, Baltimore, Md.*
Clarke, F. W.	*U. S. Geological Survey, Washington, D. C.*
Clarke, J. M.	*State Hall, Albany, N. Y.*
Comstock, George C.	*Washburn Observatory, Madison, Wis.*
Conklin, E. G.	*Princeton, N. J.*
Coulter, J. M.	*University of Chicago, Chicago, Ill.*
Councilman, William T.	*Harvard Medical School, Boston, Mass.*
Crafts, James M.	*19 Commonwealth Ave., Boston, Mass.*

Crew. Henry	*Northwestern University, Evanston, Ill.*
Cross, Whitman	*U. S. Geological Survey, Washington, D. C.*
Dall, William H.	*Smithsonian Institution, Washington, D. C.*
Dana, Edward S.	*Yale University, New Haven, Conn.*
Davenport, Charles B.	*Cold Spring Harbor, N. Y.*
Davis. William Morris	*31 Hawthorn St., Cambridge, Mass.*
Day, Arthur L.	*Geophysical Laboratory, Washington, D. C.*
Dewey, John	*Columbia University, New York City.*
Dickson, Leonard E.	*University of Chicago, Chicago, Ill.*
Donaldson, Henry Herbert	*Wistar Institute of Anatomy, Philadelphia, Pa.*
Elkin, William L.	*Yale University Observatory, New Haven, Conn.*
Farlow, W. G.	*Harvard University, Cambridge, Mass.*
Fewkes, Jesse Walter	*Bureau of American Ethnology, Washington, D. C.*
Flexner, Simon	*Rockefeller Institute, New York City.*
Folin, Otto	*Harvard Medical School, Boston, Mass.*
Franklin, Edward Curtis	*Stanford University, California.*
Frost, Edwin B	*Yerkes Observatory, Williams Bay, Wis.*
Gilbert, Grove K.	*U. S. Geological Survey, Washington, D. C.*
Gomberg, Moses	*University of Michigan, Ann Arbor, Mich.*
Gooch, Frank A.	*Yale University, New Haven, Conn.*
Goodale, George L.	*Harvard University, Cambridge, Mass.*
Hague, Arnold	*U. S. Geological Survey, Washington, D. C.*
Hale, George E.	*Solar Observatory Office, Pasadena, Cal.*
Hall, Edwin H.	*Harvard University, Cambridge, Mass.*
Hall, Granville Stanley	*Clark University, Worcester, Mass.*
Harper, R. A.	*Columbia University, New York City.*
Harrison, Ross G.	*Yale University, New Haven, Conn.*
Hastings, Charles S.	*Yale University, New Haven, Conn.*
Hayford, John F.	*Northwestern University, Evanston, Ill.*
Hillebrand, William F.	*Bureau of Standards, Washington, D. C.*
Holmes, William H.	*U. S. National Museum, Washington, D. C.*
Howard, Leland Ossian	*U. S. Dept. of Agriculture, Washington, D. C.*
Howell, William H.	*Johns Hopkins University, Baltimore, Md.*
Iddings, Joseph P.	*U. S. Geological Survey, Washington, D. C.*
Jackson, Charles L.	*6 Boylston Hall, Cambridge, Mass.*
Jennings, Herbert Spencer	*Johns Hopkins University, Baltimore, Md.*
Kemp, James F.	*Columbia University, New York City.*
Leuschner, Armin O.	*University of California, Berkeley, Cal.*
Levene, Phœbus Aaron Theodor	*Rockefeller Institute, New York City.*
Lewis, Gilbert N.	*University of California, Berkeley, Cal.*
Lillie, Frank Rattray	*University of Chicago, Chicago, Ill.*
Lindgren, Waldemar	*Massachusetts Institute of Technology, Boston, Mass.*
Loeb, Jacques	*Rockefeller Institute, New York City.*
Lusk, Graham	*Cornell University Medical College, New York City.*
Mall, Franklin P.	*Johns Hopkins University, Baltimore, Md.*
Mark, Edward L.	*109 Irving St., Cambridge, Mass.*
Mayer, Alfred Goldsborough	*Carnegie Institution, Maplewood, N. J.*
Meltzer, Samuel James	*Rockefeller Institute, New York City.*
Mendel, Lafayette B.	*18 Trumbull St., New Haven, Conn.*
Mendenhall, Thomas C.	*329 North Chestnut St., Ravenna, Ohio.*
Merriam, C. Hart	*1919 16th St., Washington, D. C.*
Merritt, Ernest	*Cornell University, Ithaca, N. Y.*
Michael, Arthur	*219 Parker St., Newton Center, Mass.*

Michelson, Albert A.	*University of Chicago, Chicago, Ill.*
Millikan, Robert Andrews	*University of Chicago, Chicago, Ill.*
Moore, Eliakim H.	*University of Chicago, Chicago, Ill.*
Morgan, T. H.	*Columbia University, New York City.*
Morley, Edward W.	*West Hartford, Conn.*
Morse, Edward S.	*Salem, Mass.*
Morse, Harmon N.	*Johns Hopkins University, Baltimore, Md.*
Moulton, F. R.	*University of Chicago, Chicago, Ill.*
Nichols, Edward L.	*Cornell University, Ithaca, N. Y.*
Nichols, Ernest F.	*Yale University, New Haven, Conn.*
Noyes, Arthur A.	*Massachusetts Institute of Technology, Boston, Mass.*
Noyes, William A.	*University of Illinois, Urbana, Ill.*
Osborn, H. F.	*American Museum of Natural History, New York City.*
Osborne, T. B.	*Agricultural Experiment Station, New Haven, Conn.*
Osgood, William Fogg	*Harvard University, Cambridge, Mass.*
Parker, George H.	*16 Berkeley St., Cambridge, Mass.*
Pearl, Raymond	*Maine Agricultural Experiment Station, Orono, Me.*
Pickering, Edward C.	*Harvard College Observatory, Cambridge, Mass.*
Pirsson, Louis V.	*41 Trumbull St., New Haven, Conn.*
Prudden, T. Mitchell	*Columbia University, New York City.*
Pumpelly, Raphael	*Gibbs Ave., Newport, R. I.*
Pupin, Michael I.	*Columbia University, New York City.*
Ransome, Frederick Leslie	*U. S. Geological Survey, Washington, D. C.*
Reid, H. Fielding	*Johns Hopkins University, Baltimore, Md.*
Remsen, Ira	*Johns Hopkins University, Baltimore, Md.*
Richards, Theodore W.	*Wolcott Gibbs Memorial Laboratory, Cambridge, Mass.*
Rosa, Edward B.	*Bureau of Standards, Washington, D. C.*
Sargent, Charles S.	*Arnold Arboretum, Jamaica Plain, Mass.*
Schlesinger, Frank	*Allegheny Observatory, Allegheny, Pa.*
Schuchert, Charles	*Yale University, New Haven, Conn.*
Scott, William B.	*Princeton University, Princeton, N. J.*
Smith, Alexander	*Columbia University, New York City.*
Smith, Edgar F.	*University of Pennsylvania, Philadelphia, Pa.*
Smith, Erwin F.	*Bureau of Plant Industry, Washington, D. C.*
Smith, Theobald	*Princeton, N. J.*
Stieglitz, Julius	*University of Chicago, Chicago, Ill.*
Story, William E.	*Clark University, Worcester, Mass.*
Thaxter, Roland	*Harvard University, Cambridge, Mass.*
Thomson, Elihu	*Swampscott, Mass.*
Trelease, William	*University of Illinois, Urbana, Ill.*
Trowbridge, John	*Harvard University, Cambridge, Mass.*
Van Hise, C. R.	*University of Wisconsin, Madison, Wis.*
Van Vleck, E. B.	*University of Wisconsin, Madison, Wis.*
Vaughan, Victor Clarence	*University of Michigan, Ann Arbor, Mich.*
Verrill, A. E.	*47 Cedar Grove Ave., New London, Conn.*
Walcott, Charles D.	*Smithsonian Institution, Washington, D. C.*
Webster, Arthur G.	*Clark University, Worcester, Mass.*
Welch, William H.	*807 St. Paul St., Baltimore, Md.*
Wells, Horace L.	*Yale University, New Haven, Conn.*
Wheeler, William M.	*Harvard University, Cambridge, Mass.*
White, David	*U. S. Geological Survey, Washington, D. C.*
White, Henry Seely	*Vassar College, Poughkeepsie, N. Y.*
Williston, Samuel Wendell	*University of Chicago, Chicago, Ill.*

Wilson, Edmund B.	*Columbia University, New York City.*
Wood, Horatio C.	*4107 Chester Ave., Philadelphia, Pa.*
Wood, Robert W.	*Johns Hopkins University, Baltimore, Md.*
Woodward, Robert S.	*Carnegie Institution, Washington, D. C.*

Smith, Sidney I.	*Yale University, New Haven, Conn.*

FOREIGN ASSOCIATES

Arrhenius, S. A.	*Nobelinstitut, Stockholm.*
Backlund, Oskar	*Astronomical Observatory, Pulkova.*
Baeyer, Adolph Ritter von	*Universität, Munich.*
Barrois, Charles	*Université, Lille.*
Brøgger, W. C.	*Universitet, Christiania.*
Crookes, Sir William	*London.*
Darboux, Gaston	*Academy of Sciences, Paris.*
Deslandres, Henri	*Astrophysical Observatory, Meudon.*
Dewar, Sir James	*University, Cambridge.*
Fischer, Emil	*Chemisches Institut der Universität, Berlin.*
Forsyth, A. R.	*Trinity College, Cambridge.*
Geikie, Sir Archibald	*Haslemere, Surrey.*
Groth, Paul von	*Universität, Munich.*
Heim, Albert	*Zürich.*
Hilbert, David	*Universität, Göttingen.*
Kapteyn, John C.	*Rijks Universiteit, Groningen.*
Klein, Felix	*Universität, Göttingen.*
Kössel, Albrecht	*Heidelberg.*
Kustner, Karl Friedrich	*Bonn.*
Lankester, Sir E. Ray	*South Kensington, London.*
Larmor, Sir Joseph	*St. Johns College, Cambridge.*
Lorentz, Hendrik Anton	*Rijks Universiteit, Leiden.*
Ostwald, Wilhelm	*Grossbothen, bei Leipzig*
Pavlov, Ivan Petrovitch	*Imperial Institute for Experimental Medicine, Petrograd.*
Penck, Albrecht	*Universität, Berlin.*
Pfeffer, Wilhelm	*Botanisches Institut der Universität, Leipzig.*
Picard, Charles Émile	*Université, Paris.*
Rayleigh, Lord	*University, Cambridge.*
Retzius, Gustav	*Högskolan, Stockholm.*
Rutherford, Sir Ernest	*University, Manchester.*
Schuster, Arthur	*Secretary of the Royal Society, London.*
Seeliger, Hugo Ritter von	*Universität, Munich.*
Thomson, Sir Joseph	*University, Cambridge.*
Volterra, Vito	*Università, Rome.*
Vries, Hugo de	*Universiteit, Amsterdam.*
Waals, Johannes D. van der	*Amsterdam.*
Waldeyer, Wilhelm	*Universität, Berlin*
Wolf, Max F. J. C.	*Heidelberg.*
Wundt, Wilhelm	*Universität, Leipzig.*

VOLUME 2 JANUARY, 1916 NUMBER 1

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES
OF THE
UNITED STATES OF AMERICA

Smithsonian Institution
JUN 24 1921
National Museum

EDITORIAL BOARD

Arthur A. Noyes, *Chairman*
Edwin B. Wilson, *Managing Editor*
Arthur L. Day, *Home Secretary*
George E. Hale, *Foreign Secretary*

J. J. Abel
W. B. Cannon
J. McK. Cattell
J. M. Clarke
E. G. Conklin
J. M. Coulter
C. B. Davenport
Simon Flexner
E. B. Frost
R. G. Harrison
W. H. Holmes
J. P. Iddings
R. A. Millikan
E. H. Moore
Alexander Smith

Publication Office: Williams & Wilkins Company, Baltimore
Editorial Office: Massachusetts Institute of Technology, Boston
Home Office of the Academy: Washington, D. C.

Entered as second-class matter at the Post Office at Baltimore, Md., under the Act of August 24, 1912

INFORMATION TO CONTRIBUTORS

THE PROCEEDINGS is the official organ of the Academy for the publication of brief accounts of important current researches of members of the Academy and of other American investigators, and for reports on the meetings and other activities of the Academy. Publication in the Proceedings will supplement, not replace, that in journals devoted to the special branches of science. The Proceedings will aim especially to secure prompt publication of original announcements of discoveries and wide circulation of the results of American research among investigators in other countries and in all branches of science.

ARTICLES should be brief, not to exceed 2500 words or 6 printed pages. Technical details of the work and long tables of data should be reserved for publication in special journals. But authors should be precise in making clear the new results and should give some record of the methods and data upon which they are based. The viewpoint should be comprehensive in giving the relation of the paper to previous publications of the author or of others, and in exhibiting, where practicable, the significance of the work for other branches of science.

MANUSCRIPTS should be prepared with a current number of the Proceedings as a model in matters of form, and should be typewritten in duplicate with double spacing, the author retaining one copy. Illustrations should be confined to text-figures of simple character, though more elaborate illustrations may be allowed in special instances to authors willing to pay for their preparation and insertion. Particular attention should be given to arranging tabular matter in a simple and concise manner.

REFERENCES to persons should include initials; citations of literature, numbered consecutively, should be placed at the end of the article; and short footnotes should be avoided. Indicate 'subject' for 'Contents.'

PAPERS by members of the Academy may be sent to Edwin Bidwell Wilson, Managing Editor, Mass. Institute of Technology, Boston, Mass. Papers by non-members should be submitted through some member.

PROOF will be sent to authors only on their request, in order to avoid delay in publication. Authors are therefore requested to make final revisions on the typewritten manuscripts. The editors cannot undertake to do more than correct obvious minor errors.

REPRINTS should be ordered at the time of submission of manuscript. They will be furnished to authors at cost, approximately as follows:

Reprints of - -	2 pp.	4 pp.	6 pp	8 pp.	Covers extra
Charge for first 100 copies	$1.00	$1.30	$2.20	$2.20	$2.00
Charge for additional 100s	.25	.45	.85	.85	.50

INFORMATION TO SUBSCRIBERS

SUBSCRIPTIONS at the rate of $5.00 per annum should be made payable to the National Academy of Sciences, and sent to Williams & Wilkins Company, Baltimore, or Arthur L. Day, Home Secretary, National Academy of Sciences, Smithsonian Institution, Washington, D. C. Single numbers. $0.50.

Copyright, 1915, by the National Academy of Sciences

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES

Volume 2 JANUARY 15, 1916 Number 1

Smithsonian Institution
JUN 24 1921
National Museum.

A POSSIBLE ORIGIN FOR SOME SPIRAL NEBULAE

By George F. Becker

UNITED STATES GEOLOGICAL SURVEY, WASHINGTON

Read before the Academy, November 17, 1915. Received, November 22, 1915

In speculations on the evolution of nebulae it has become the fashion to postulate an initial spheroid consisting exclusively of elastic fluids, this assumption lending itself most readily to exact reasoning and computation.

Kant, Herschel, and Laplace, however, did not assume gaseous nebulae. In a paper on Kant as a natural philosopher, printed in 1898, I gave an outline of his hypothesis including the following passage:[1]

> Tendencies to motions in all directions, excepting in one resultant plane, are suppressed by mutual interferences of the free particles. Most of the material accumulates at the center, in the sun, but a wide, thin disc of heterogeneous matter remains. This disc consists of discrete particles each of which has acquired such a velocity and direction as to maintain the appropriate orbital motion. . . . . Mutual attraction and adhesion, beginning at relatively massive particles, cause the agglomeration of the particles in any zone or ring to single planets or to groups of planetary bodies.

From this and other passages it appears that to this very original thinker solid particles were the most essential components of nebulae.

William Herschel's nebular hypothesis was founded upon induction and observations. Nowhere in his own works do I find so graphic a résumé as the following which Laplace gave in the *Système du Monde:*[2]

> Herschel, while observing the nebulae by means of his powerful telescopes has followed the progress of their condensation, not in a single one, for this progress could not become sensible to us until centuries had passed, but in the aggregate; as in a vast forest one traces the growth of the trees among the individuals of diverse ages which it contains. He first observed nebulous

matter spread out in diverse bodies in different parts of the heavens of which it occupies a great area. He saw in some of these bodies the nebulous matter slightly condensed about one or several slightly luminous nuclei. In other nebulae, these nuclei shone more brightly relatively to the nebulosity which surrounded them. The atmosphere of each nucleus was about to segregate by a final condensation: multiple nebulae resulted formed of brilliant nuclei very close together and each surrounded by an atmosphere: occasionally the nebulous matter condensing uniformly produced the nebulae called *planetary*. Finally a greater degree of condensation transformed all of these nebulae into stars. The nebulae classified according to this philosophic view indicate with extreme probability their future transformation into stars and the former nebulous state of the stars now in existence.

It will be observed that nebulae with a single nucleus are mentioned only as special cases of multinuclear and, therefore, heterogeneous nebulae. Herschel himself made repeated references to the character of nebulous matter. For him it was anything which could shine, and while he was unable to come to definite conclusions, he regarded it as probable that it comprised matter in all three states.

Laplace's information as to nebulae was derived almost wholly from Herschel; so far as I can ascertain he made no telescopic observations of his own on these bodies, and his ideas of the constitution of nebulae seem to have been identical with those of the great English observer. Both in the passage quoted above and in his famous note on the nebular hypothesis Laplace referred to atmospheres, but they were cloudy or dusty atmospheres not inconsistent with the presence of multiple nuclei; they were characterized by a 'numberless variety of densities' and seem to have resembled volcanic clouds. In other essays, such as that on the barometer, Laplace showed his complete mastery of Boyle's and Gay Lussac's laws and in these memoirs he frequently employs the word gas. In his note on the nebular hypothesis he never once uses this word and the very first application he made of his theory was to offer an explanation of the genesis of the Pleiades. In his time, the presence of nebulous matter in this cluster was unknown, but the photographic plate shows that it is embedded in one of the most astonishing of the great nebular masses. Laplace's selection of the Pleiades was thus a striking example of his prescience.

Much as more modern science has contributed to knowledge of the nebulae, the question of their constitution is still unsettled. That some of them contain luminous gas (helium, hydrogen, nebulium) was shown by Huggins 50 years ago, but many nebulae and among them the spirals show continuous spectra. The continuity might be due to gases at high pressures; but that gases should assume so complex a configuration as that of a spiral is difficult to understand while the continuous spectrum is more characteristic of solids than of gases, and Mr. Slipher has adduced evidence that some nebulous matter shines by reflected light. The extreme tenuity and presumably low temperature of nebulae increase the difficulty of a purely gaseous hypothesis, which is further contradicted by the abundance of multinuclear nebulae.

I infer that it is perfectly legitimate to speculate on celestial clouds composed of matter in more than one and perhaps in all three states.

Provided that there is a limit to the expansion of gas, as G. W. Hill believed, a gaseous spheroid may assume a figure of equilibrium and this is the favorite postulate among modern cosmogonists. Some of the more regular nebulae lend color to it; but the forms of the celestial clouds are as varied as those of our atmosphere and might in large part be similarly classified. Surely some of the elongated nebulae, resembling cirrus or cirro-cumulus terrestrial clouds, must eventually develop into more highly organized forms; or, inversely, some well-developed nebulae may have originated from whisps of nebulous matter, such as abound in the sky.

It is not possible at present to assign a definite origin for these nebulous streamers. The hypothesis that the whole galaxy was nebulous at a certain epoch leads to a dilemma as was long since pointed out.[3] In 1900 Arrhenius put forward his theory that energy is degraded in the solar state but raised to a higher level in the nebular state,[4] and some such 'third law' of thermodynamics seems logically inevitable. But whether or not there is a regenerative process at work among the heavenly bodies, there are at least inequalities of action and preferential movements (so suggestively discussed by Kapteyn) which would almost inevitably lead to a carding or filamentation of nebulous masses.

It has always been impossible to suppose that nebulae were devoid of internal motion, but such movements were first demonstrated in 1914 by Messrs. Buisson, Fabry, and Bourget[5] for the Orion nebula. Their result has been confirmed by Messrs. Frost and Maney,[6] who point out that this nebula must be considered as a mass writhing and seething in irregular contortions. A similar statement must be true of all nebulae.

Consider then an elongated nebula composed of heterogeneous matter and bounded by a surface which may be very irregular but not so irregular that any part of its longest principal axis falls outside of the mass. Such a nebula would bear some resemblance to a staff or baculum which may be provided with knots, knobs, and excrescences. For the sake of brevity I will characterize such a nebula as bacular. A baculum then must be possessed of energy and moment of momentum, but I shall suppose its axis of rotation to stand at an angle to its greatest principal axis of inertia.

Such a bacular nebula would pass through some of the phases familiar to readers of cosmogony; energy would be dissipated by collisions; a part of the material would subside towards a central nucleus or several excentric nuclei; excepting for some outlying cometary detritus, the mass would tend to flatten in the invariable plane; and finally orbits of great eccentricity would or might be reduced by collisions to an approximately circular form, while of course the moment of momentum would remain nearly or quite constant. The flattening of such a nebula and its rotation about the centre of inertia of the whole system are so nearly independent that the principle of superposition is applicable; the mass may be supposed to subside into its invariable plane without flexure of the axis of the baculum, and thereafter to rotate about a line perpendicular to this axis.

The question then arises how the axis of a bacular nebula would be distorted compatibly with Kepler's third law if the orbits of the component particles were sensibly circular, or if eccentricities are neglected, only a first approximation being sought.

In its simplest form Kepler's third law expresses the equality between the attraction of a heavy point on a particle moving in a circular orbit and the centrifugal force, or normal acceleration, of the particle. If ω is the angular velocity of the particle, r its distance, and M the mass of the heavy point,

$$\frac{M}{r^2} = \omega^2 r.$$

Here M may mean the mass of a system concentrated at its centre of inertia and it is easy to see that r may be replaced by the mean distance a, of a particle moving in an elliptical orbit.

The law so expressed would be valid for the solar system, so far as two bodies are concerned, were it practicable to take the centre of inertia of that system as an origin; most astronomical problems, however, involve not absolute but relative motions, and Kepler's law must then be modified by substituting $M+m$ for $1/M$.[7] For small planets m can be neglected because the sun's centre is then almost exactly at the centre of inertia of the system.

If a system consisted of a single nucleus and a great number of minute secondary particles which, at a given epoch, were arranged on an axis, and thereafter for a time τ described circular orbits, then, putting $\tau\omega=\varphi$ and $a=r$ the particles would be found on a curve

$$r^3\varphi^2=\text{constant.}$$

This is a spiral with a single well-defined point, which is a point of inflection. If r_0, φ_0 are the coordinates of this point it is easy to show that $\varphi_0^2=2/9$ so that

$$r^3\varphi^2=2r_0^3/9.$$

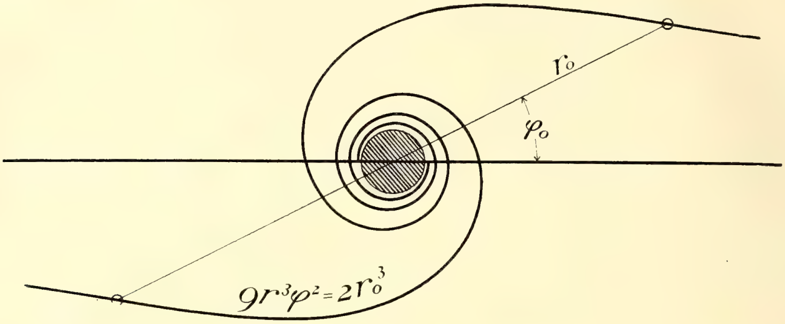
If the axial row of particles extended on both sides of the nucleus, the spiral would have two branches, each approaching the axis asymptotically from opposite sides. The length of any portion of the axis would be much extended by conversion into the spiral, the extension at any point being ds/dr, which, at the point of inflection is $\sqrt{3/2}$, while closer to the centre it is still larger.

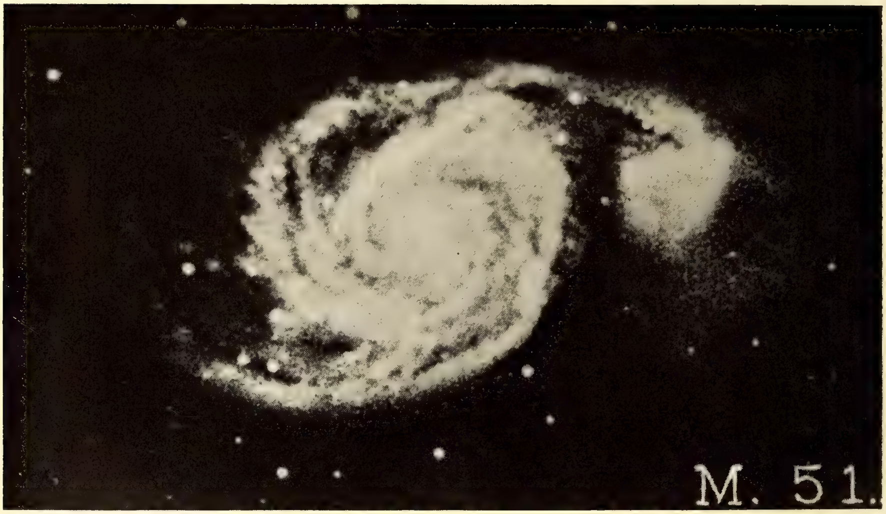
For secondary bodies of finite mass the problem presented would be one of n mutually perturbing bodies, but the principles of centre of inertia, energy, and moment of momentum would remain in force and parabolic velocities could not be attained. It would appear, therefore, that even in a system of finite masses, axially arranged, angular velocities must diminish with distance from the centre of inertia and be infinitesimal at an infinite distance. Then the axis must be distorted into a more or less irregular spiral which is asymptotic to the axis and must, therefore, have a point of inflection. These conditions would be satisfied by any curve of the form $r\varphi^{n^2}=\text{constant}$, $n^2<1$, for these spirals have a point of inflection at $\varphi_0=n\sqrt{1-n^2}$, the maximum value of which is $1/2$.

Possibly perturbations might bring about approximations to some of these curves, but since perturbations are necessarily excluded from consideration, the only curve with any standing is that in which $n^2=2/3$.

Supposing, then, that the initial configuration of the system were an axial line passing through the centre of inertia and that the orbits were circular, the configuration after the lapse of a given time would vary from the spiral $r^3\varphi^2=\text{constant}$ only as a result of perturbations and this spiral must represent, to a first approximation, the axis of such spiral nebulae as have been developed from substantially rectilinear nebulous streamers, while in some other cases it will represent displacements from an initial configuration.

The diagram shows the form of the spiral excepting that near the center where it would be almost impossible to plot the curves, a nucleus is substituted. Both branches are prolonged somewhat beyond the points of inflection. Of course, the curve in the diagram represents only the axis of the baculum. Below the diagram is shown the Whirlpool nebula[8] for comparison, and evidently a diagram could be drawn representing the mathematical spiral clothed, so to speak, with irregularly disposed nebulous material which would add to its resemblance to the photograph.





As noted above, even at the point of inflection the elongation of the original axis is in the ratio of $\sqrt{3/2}$ or 1.22, while for points nearer the origin it is more than this. Unless, therefore, the original baculum were of extraordinary uniformity, this stretching combined with local partial attractions would break up the spiral into short curved fragments. Between the point of inflection and the origin such fragments, would present their concave sides to the origin. If any such fragment were to condense to a spheroid, the centre of the spheroid would be close to the centre of inertia of the arc corresponding to the fragment; therefore the condensed mass would be nearer to the centre of attraction than in the uncondensed state and consequently the rotation of the spheroid would be positive. Beyond the point of inflection these conditions would be reversed so that condensing short arcs of the nebulous spiral would retreat and show retrograde rotation. In systems thus developed, then, positive rotation would characterize all but the outermost portions where retrograde rotation would prevail.

A bacular nebula so short or so old that its outermost portions had completed two or three revolutions round the centre of attraction would be almost or quite indistinguishable from a circular disc, marked by furrows and closely resembling Laplace's nebula after the development of rings. From this epoch onwards all the valid conclusions drawn from Laplace's great hypothesis would be applicable to the nebula under consideration, and in particular the splendid investigations of Roche, Hill, and George Darwin on unstable orbits.

So far as I can see, a baculum of very uniform composition might develop into a star without planets, though possibly attended by a disc reflecting zodiacal light; while a very heterogeneous baculum might yield a system as complex as the Pleiades. There seems to me no reason why even the most symmetrical planetary nebulae might not have been evolved from bacula closely coiled. On the other hand, a nebula much less symmetrical than a baculum as I have defined it, can be similarly discussed. Suppose the axis of figure of an elongated nebula to be any arbitrary curve of small curvature; then the equation $r^3\varphi^2 = 2r_0^3/9$ does not represent the distorted axis but does give the displacement of points on the assumed curve. Or inversely, comparison of a well-defined spiral nebula with the spiral represented by the equation should show to what simplest form the nebula could be reduced by reversing the orbital velocities. Again, if the axis of figure of the filamentous nebula were coincident with an arc of a circle, its centre of inertia might lie outside of the nebula; and in such a case, after the extremities of the nebula had revolved about the centre of inertia a few times, the figure would be indistinguishable from an empty ring; but if a portion of the mass were arrested by collisions, the ring would show a central nucleus as does the ring nebula in Lyra.

Superficial comparison of the diagram given in this paper with photographs of the Whirlpool and other nebulae is not unfavorable to the hypothesis here developed, and it seems to me that to the first order of approximation bacular nebulae must ultimately be converted into the spiral here discussed. But the geometry of this spiral is so well characterized that astronomical observers will probably be able to make a definite decision as to whether or not any of the existing spiral nebulae have developed from bacula or from less regular forms by substantially the same method. Till that is decided further discussion would be superfluous. It is possible that there are well developed spiral nebulae in which mutual interference has done little or nothing to reduce orbits to ellipses of small eccentricity. To such nebulae the analysis here indicated has no application, nor could their motions be formulated in the present state of science.[9]

---

[1] *Amer. J. Sci.*, Ser. 4, **5**, 102 (1898).

[2] Chapter VI, p. 482, of the edition of 1835.

[3] *Amer. J. Sci.*, Ser. 4, **5**, 106–107 (1898).

[4] See preface to *Worlds in the Making*, 1908.

[5] *Paris, C. R. Acad. Sci.*, **158**, 1017 (1914); and *Astrophys. J.*, **40**, 241 (1914).

[6] *Pop. Astr.*, **23**, 485 (1915).

[7] A. G. Webster, *Dynamics of Particles*, etc., 2d ed., 1912, p. 317.

[8] From a photograph taken at the Yerkes Observatory.

[9] Just in time for a reference, I have met with an interesting paper by Mr. E. J. Wilczynski, *Astrophys. J.*, **4**, 97, (1896), who pointed out that, if circular orbits are assumed, every long streak of nebulous matter must eventually be converted into a spiral as a consequence of Kepler's third law. The equation of this curve, he says, it would be easy to deduce.

A PECULIAR CLAY FROM NEAR THE CITY OF MEXICO

By E. W. Hilgard
UNIVERSITY OF CALIFORNIA

Read before the Academy, November 17, 1915. Received, November 17, 1915

In 1912 Dr. George W. Shaw, then connected with the College of Agriculture of the University of California, was requested to visit the Hacienda Santa Lucia, in the neighborhood of the City of Mexico, in order to give advice for the reclamation of certain tracts of land which were supposed to be afflicted with 'alkali,' and which had resisted the usual methods for rendering them productive. Dr. Shaw brought back samples of the soils from these alkali spots, which were as usual depressed in the middle, but found in them no excessive amounts of carbonate of soda, and that the sulphate (Glaubers salt) was mainly present in fractions of one per cent, not enough to injure vegetation.

In an attempt to leach one sample of its soluble salts he found that on pouring water on a few grams placed in a 50 cc. cylinder the substance swelled very rapidly, and over-night actually filled the cylinder to the top, making a semifluid slush. As I had never seen the like before, I undertook to investigate it.

The material, looking like an ordinary dark gray clay, adhered strongly to the tongue, and with a little water became very plastic. Unfortunately the main sample collected by Dr. Shaw came to the laboratory unlabeled and was by mistake thrown away by an assistant. A small sample of about 24 grams, however, was left; and on this the entire investigation had to be made. About half of this sample was boiled in distilled water for 24 hours for sedimentation, as there were some coarse particles, apparently of talcose schist. A reference to geological descriptions of the basin of Mexico, in which serpentines and talcose rocks are prevalent, seemed to confirm this conclusion.

The suspension did not appear to be very uniform even after protracted boiling, and under the microscope showed a multitude of dark rounded particles, very uniformly distributed through a colloidal medium of faintly yellowish tint, which when colored with a solution of 'malachite green' merely showed the fine discrete particles in larger numbers and greater fineness.

All attempts to free the colloidal ingredients from the visibly discrete particles by sedimentation proved futile. The suspension was readily coagulated and precipitated, apparently unchanged, by a solution of sodium chloride. On washing by decantation the suspension was again readily made, the microscopic character also remaining the same.

As the original material effervesced somewhat with hydrochloric acid it was conjectured that the minute grains might be earth carbonates; but the addition of drops of tenth-normal acid was slow in producing coagulation, doubtless on account of the neutralization of the acid by the earth carbonates. The filtrate gave reactions of calcium and more strongly of magnesium; yet the coagulum itself seemed to contain as many discrete particles as before, showing that the earth carbonates were very finely distributed, and not visible.

For the determination of the specific gravity some pure selected fragments weighing 1.073 grams were dried in a 25 cc. bottle. Filling the bottle, 0.481 gram of water was found to have been displaced, making the specific gravity about 2.25, nearly that of kaolinite clays. The substance was left in the gravity bottle until, with the aid of slight heating, it had absorbed the maximum amount of water, leaving a clear remainder above it. The water poured off measured 14 cc., showing that 11 cc. had been taken up by the clay, forming a coherent, gelatinous-looking mass, and giving an increase of bulk equal to about 25 times the volume of the clay. The mass was then evaporated to dryness in an air bath, with suction, at about 50°C., in order to avoid a possible molecular change; the bottle then was refilled with water. The reabsorption took place more slowly than with the crude substance, and although aided by heating to 100° for three days, did not recover the full volume of 11 cc.

A repetition of the experiment with 1.70 grams of dry clay which had been precipitated from diffusion by ammonium carbonate, gave an increase of volume to 32 times that of the clay.

A parallel experiment with some ordinary plastic clays showed an increase of only two to two and a half times the original bulk.

Chemical Composition of the Clay.—On leaching the original substance with water, it was found to contain 2.60 % of soluble salts, of which 1.74 was sodium sulphate, 0.74 sodium carbonate, and 0.12 sodium chloride. Black rings from the evaporation of the alkali solution were reported to exist in the depressed spots, as is usual in the case of 'black alkali.'

In attempting to obtain for analysis a sample free from carbonates, it was found that silica went into solution even when only cold tenth-normal acid was used. It was therefore necessary to analyze the entire mass. On ignition the loss was 19.6 %; the ignited material decrepitated when wetted, but did not swell.

Two analyses of the original material were made; one by acid digestion for five days; another, after ignition, by fusion with alkali carbonate. Unfortunately the material at command was too scanty to permit the use of more than one gram and 1.7 grams for these analyses, respectively, the results being as follows:

Partial Analyses of Clay from Mexico.

	Acid digestion.	Fusion with carbonate of soda. (Ignited substance)
Insoluble	1.83%	0.00
Soluble silica	43.00	51.43
Lime	9.06	8.97
Magnesia	17.11	27.07
Ferric oxid	1.76	10.63
Alumina	3.48	
Ignition loss (water, carbonic acid and organic matter)	19.60	0.00
Soluble salts	2.60	
Total	98.44	98.10

The above data, deficient as they are, show clearly a totally different composition from any 'clay' on record. The alumina present is far below any reasonably assumable compound with the soluble silica; the predominant base being evidently magnesia, and that greatly in excess of the lime present. There is thus an apparent relationship to the saponite or sepiolite group of minerals, but the extremely ready decomposition by acids (even by acetic acid in dilute solution) differentiates the material pointedly from that group, even apart from the exceptionally high absorptive power for water. I believe that what has been shown is sufficient to characterize a new kind of clay with a predominant magnesium instead of aluminum base, for which I suggest the name of *Lucianite*, from the locality where it was found. A better name, Auxite, referring to the exceptional increase in bulk, is too similar in sound to Augite to be desirable.

Wishing to obtain, if possible, additional material from the Mexican locality, I wrote to Don Jose G. Aguilera, the Director of the Instituto Geologico de Mexico, at the City of Mexico, transmitting also a blueprint map on which Dr. Shaw had marked the several localities from which he took samples. Dr. Aguilera very promptly and courteously responded to the request, despite the disturbed condition of the country (in 1912). Dr. Aguilera also sent statements of the analyses of clay materials from the foundations of the Legislative Palace, when building in 1904. These also show a large predominance of magnesia in the substrata of Mexico City.

The samples received, all from low 'alkali spots' in the Hacienda, resembled in general aspect the material brought by Dr. Shaw, but contained more coarse materials, among which talcose schist and black eruptive rock were readily discernable. The fine portions all showed exceptionally high expansion when wetted, one going as high as 12 times the bulk of the raw, dry mass. It was then attempted to obtain from this sample by sedimentation a clay of higher absorptive power. But the highest result so obtained was only 16 times the original bulk. Qualitative tests of this material showed again a large proportion of calcium and magnesium carbonates, with only small amounts of alumina and ferric oxide. The microscopic characters were the same as before noted in the original material, and there was the same easy decomposition of the silicate present with even weak acids, which stands in the way of obtaining a substance of definite and constant composition, free from earth carbonates.

As regards the reclamation of these alkali spots it is obvious that it cannot be accomplished by liming, as a large amount of calcium carbonate is already present and yet does not prevent the collapse of the colloidal ingredient in drying. The depressed spots probably become nearly level during the rainy season by the expansion of the colloidal ingredient. It seems as though acid treatment, instead of that with lime, were the only possible effective agent.

Circumstances have prevented a further prosecution of this investigation, which will be resumed whenever possible. Meanwhile it may be fruitful to suggest the possible practical use of these magnesian clays in cases in which a tight tamping is necessary where water has access, as in shutting off crevices in bored wells when, as in those yielding oil, undesirable flows of water interfere, and where tamping with ordinary clays fails to tighten sufficiently and permanently. Other uses for such a prodigiously swelling material readily suggest themselves.

STUDIES OF MAGNITUDES IN STAR CLUSTERS, I. ON THE ABSORPTION OF LIGHT IN SPACE

By Harlow Shapley

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, November 17, 1915

All researches on the structure of the stellar universe must take into account the probable and possible effects of the scattering or obstruction of light in its passage through interstellar space. It is recognized, for instance, that if the loss due to absorption, or scattering, by the free molecules of matter in space totals as much as a millionth of one per cent of the visual light in a journey of a hundred million miles, then, assuming the effect proportional to the distance, every star 3500 light-years away would be observed about two magnitudes fainter than its true brightness. As a consequence any uncertainty in the coefficient of scattering, especially if it is large as cited above, is very serious in studies of the distance of the faint stars and particularly in considerations of the stellar densities in various parts of the galactic system. In fact, the hypothesis that light extinction is imperceptible is prerequisite to the conclusion that the stellar universe is finite in extent.

Because of the importance of the subject several extensive investigations have been undertaken in recent years for the purpose of determining the amount of absorption. It is generally assumed that if any dimming of a star occurs it will be apparent as selective molecular scattering, which varies as the inverse fourth power of the wave length of the light.

As the effect for blue light would be about double that for yellow, an obvious method of detecting and measuring selective absorption is through the study of the colors of faint and distant stars. By means of measures of color indices various investigators have found that the fainter stars are on the average redder than the brighter ones, and as the fainter stars are also on the average, the more distant, this excess of redness might be and often is accepted as an indication of space absorption. There are, however, other possible interpretations, including the effect of intrinsic luminosity on color and the possibility of the predominance of redder spectral types among the faint stars and the more distant ones.

In all previous special studies of the problem, the chief difficulties have been the uncertainty of the distances (so few of which have been directly measured) and their relatively small values. For a given spectral type the observed color excess of the more distant stars is also small. Consequently, though positive values for the absorption coefficient have always been derived, they have in no case been considered as definitely final proof of the existence of an appreciable space absorption. The four most recent determinations (all of which depend upon studies of the colors of relatively nearby stars) give the following values for the increase of the color index for each unit of distance (32.6 light-years):

Observer	King	Kapteyn	Jones	Van Rhijn
Absorption coefficient	0.019	0.0031	0.0047	0.0015 mag.

Since the color effect is cumulative, an increase in the length of the base line increases greatly the accuracy of a measurement of absorption. If instead of using stars in our immediate stellar system, we extend the study of colors to much more distant objects, then we can readily decide whether light scattering is to play an important part in stellar investigations. The advantage of great distances is taken in the present study of globular clusters, and already it is possible to make a definite contribution to the problem of the existence of light scattering.

At the beginning of a systematic study of magnitudes and colors of stars in clusters, the great globular system in Hercules, Messier 13, has been observed photographically, both with ordinary plates sensitive to blue and violet light and with isochromatic plates which respond chiefly to the most effective visual light when the shorter wave-lengths are cut off by means of a yellow color filter. Photographic and photovisual magnitudes for about 1300 stars have been determined. The color indices thus secured are compared with those for nearby stars with identical spectra (whenever the spectra of the cluster stars can be directly observed), and the excess of redness for the cluster stars indicates the amount of space absorption. When the spectra can not be directly observed the color indices in the cluster still may serve to solve the problem. For, if the stars in the distant cluster are even approximately similar to those near the sun, the absolute values of the color indices will tell at once whether there is practically no absorption or whether there is a really appreciable amount.

The detailed discussion of the magnitudes in Messier 13, which will appear as a *Contribution from the Mount Wilson Solar Observatory*, will include a consideration of the reliability of the magnitudes and color indices and an extended analysis of the observational data. For the present communication a part of the results bearing on the absorption of light in space are shown in a condensed form in the following table. The magnitudes of the stars in the denser regions of the cluster, within 2′ of the center, are omitted, as none of the polar standards upon which the determination of the magnitude is based falls within that area, and consequently no control of the possible systematic errors of a photographic nature is assured. The color class has been defined in the September, 1915, number of these PROCEEDINGS; in brief we may say that, under the average conditions of luminosity and distance that obtain for the brighter nearby stars, color class and spectral class are practically identical, but if considerable absorption exists the two do not correspond for more distant objects, redder color being associated with bluer spectrum.

Frequency of Colors in Messier 13—Number of Stars Tabulated

DISTANCE FROM CENTER OF CLUSTER	COLOR CLASS									
	b0 to b5	b5 to a0	a0 to a5	a5 to f0	f0 to f5	f5 to g0	g0 to g5	g5 to k0	k0 to k5	k5 to m0
2′ to 3′	7	39	10	16	20	50	50	17	5	1
3′ to 5′	8	20	21	7	23	59	44	9	4	2
>5′	1	11	5	4	7	26	21	7	0	1
Totals.........	16	70	36	27	50	135	115	33	9	4

The most remarkable feature of the distribution of color class exhibited in the table is not the relative paucity of *a*'s and early *f*'s, nor the great range in color, but rather the highly significant fact that there are any negative color indices (*b*0–*b*9). Of the 495 stars more than 17 % are of color class *b*, and of these one-fifth are bluer than represented by *b*5. Moreover, we find no excessively large color indices, and none unusually small. A comparison of these colors with those derived by Parkhurst for bright stars near the north pole shows an entirely analagous distribution among the various classes. There seems to be no reason to doubt the present result, and we are left with the conviction that, so far as present data go, normal negative color indices exist in the Hercules cluster in large numbers. From the remarks of the preceding paragraphs it is at once obvious that such a condition could not co-exist with measurable selective absorption of light in space.

The distance of the Hercules cluster, we may be sure, is not less than

1000 units of stellar distance, and it may be nearly ten times this amount. If we were to adopt the value of the absorption constant derived by Kapteyn, then the smallest color indices in the cluster should be in excess of 2.5 magnitudes, and that for a star of spectrum M should be nearly five magnitudes. Actually not a single color index greater than two magnitudes has been observed. If we were to take Van Rhijn's value, the color indices in the cluster would all need to exceed a magnitude.

It seems to be necessary to conclude that the selective extinction of light in space is entirely inappreciable, at least in the direction of the Hercules cluster. If we grant, on the basis of our data, a color excess of a tenth of a magnitude, and attribute it all to space absorption, the value of the coefficient can not then exceed +0.0001 mag., an amount completely negligible in dealing with the ordinary isolated stars. In the light of this result we are probably justified in assuming that the non-selective absorption in space (obstruction) is also negligible.

The following references may be cited:

Newcomb, *The Stars*, chap. xiv, New York, 1902.
Turner, *London, Mon. Not. R. Astr. Soc.*, **69**, 61 (1908).
Kapteyn, *Astrophys. J.*, **29**, 46 (1909); **30**, 284, 316 (1909); **40**, 187 (1914).
King, *Cambridge, Mass., Ann. Obs. Harvard Coll.*, **59**, 182 (1912); **76**, 1 (1914).
Jones, *Observatory, London*, **37**, 402 (1914); *Mon. Not. R. Astr. Soc.*, **75**, 4 (1914).
Van Rhijn, *Dissertation*, Groningen, 1915.
Seares, These PROCEEDINGS, **1**, 481 (1915).

STUDIES OF MAGNITUDES IN STAR CLUSTERS, II. ON THE SEQUENCE OF SPECTRAL TYPES IN STELLAR EVOLUTION

By Harlow Shapley

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, December 9, 1915

In many investigations of globular clusters the apparent magnitudes of the individual stars, which are the quantities observed directly, may be used without error as representing the absolute magnitudes. We may consider the stars in such a system to be at the same distance from the earth, and knowledge of that distance is unnecessary in discussing the relations between changes in the absolute luminosities of the stars and their colors or spectra.

If, as is most commonly believed, the evolution of the stars progresses, chiefly through the agency of condensation and cooling, from the bluer spectral types to the redder, then, since it is definitely known that the intrinsic surface brightness of the blue stars greatly exceeds that of the red, we should normally expect to find that all the bright stars in the clusters are relatively blue and that the colors *g* and *k* do not appear until we go several magnitudes fainter than the brightest stars. This condition of increasing redness with decreasing brightness holds with but few exceptions among the stars in the open and moving clusters, such as the systems of the Pleiades, the Hyades, Ursa Major, and Messier 67. In the great globular cluster in Hercules (Messier 13), however, the interval of the four brightest magnitudes contains red and blue stars in approximately equal numbers.

In the following table the average magnitudes in Messier 13 are given for successive concentric regions and for different classes of color. The progression from blue to red is from left to right. As in the preceding paper of this series the data for the stars of the innermost regions of the cluster are not included, though the results would not be affected materially by retaining them. Each magnitude represents the mean value for about 16 stars.

It should be remembered that the system of Messier 13 is immensely remote from the sun and is enormous in its real dimensions, and also that it contains in all probability hundreds of thousands of stars. The longest exposures at Mount Wilson indicate that there are about 50,000 stars in the cluster brighter than magnitude 21, and there is no indication that the fainter limit has been approached. The same conclusions are valid for other condensed globular clusters. Hence, we must recognize that in studying stars between magnitudes 12 and 16 in Messier 13 we are dealing only with those members of the system that have the greatest intrinsic luminosity. Obviously, then, the giant red and yellow stars, which in Russell's hypothesis of the order of stellar evolution play an important part as the antecedents of the bluer stars, are present in great numbers in Messier 13.

TABLE I

AVERAGE MAGNITUDES IN MESSIER 13

LIMITS OF DISTANCE FROM CENTER	COLOR CLASS					ALL COLORS
	b	a	f	g	k	
2′.0 to 2′.5	15.18	15.12	14.88	14.36	12.11	14.76
2 .5 3 .0	15.22	14.75	14.89	14.03	12.74	14.58
3 .0 4 .0	15.16	15.14	14.81	14.18	12.19	14.67
4 .0 5 .0	15.25	15.22	14.92	14.70	13.66	14.91
5 .0 6 .0	15.22	14.96	14.84	14.15		14.62
> 6 .0	15.16	14.03	14.74	14.54	13.69	14.67
Total............	15.20	15.01	14.85	14.28	12.74	14.71

Although in the relatively near moving clusters, referred to above, there are very few of the giant red stars, their very frequent occurrence among the stars of the general galactic system is commonly recognized. From different lines of evidence we believe them to be tolerably normal stellar objects as regards mass, motions, and surface luminosities, but differing from dwarf red stars most conspicuously in density, dimensions, and total brightness. Admitting this interpretation, there is no place at all for the giant stars in the older and more conventional scheme of stellar evolution, but they harmonize almost completely with the newer evolutionary hypothesis.

From the best data available two years ago, Russell concluded that the giant stars of all spectral types have very similar absolute magnitudes. We are now in a position to make a further contribution to this subject, basing our conclusions upon results from globular clusters. Although later researches may show that our immediate galactic domain is comparable with a globular cluster to a limited extent only, there can be, nevertheless, but little doubt that each globular cluster is in itself a complete system, distinct from the galaxy, and one in which the evolution of the constituent stars probably follows analogous lines to the stellar development in our own system. In many respects it should be possible to read the observed features of the cluster directly into our interpretations of the local system, and the characteristic discussed below is probably such a case.

Table I shows the common dependence of average magnitude upon color class in the different regions of the cluster. The relation is better exhibited in Table II, which gives the average color index for successively fainter intervals of magnitude in different regions. The number of stars included in each average is in parentheses.

TABLE II

AVERAGE COLOR INDICES IN MESSIER 13

MAGNITUDE INTERVAL	DISTANCE FROM CENTER OF CLUSTER			ALL REGIONS
	2′.0 to 3′.0	3′.0 to 5′.0	> 5′.0	
11.8 to 12.6	+1.25 (8)	+1.34 (5)	(0)	+1.28 (13)
12.6 13.2	+0.81 (14)	+0.97 (6)	+0.85 (4)	+0.86 (24)
13.2 13.8	+0.95 (11)	+0.97 (6)	+0.97 (6)	+0.96 (23)
13.8 14.4	+0.71 (27)	+0.72 (36)	+0.81 (13)	+0.73 (76)
14.4 15.0	+0.67 (48)	+0.62 (37)	+0.67 (25)	+0.65 (110)
15.0 15.6	+0.31 (90)	+0.46 (97)	+0.39 (35)	+0.39 (222)

The data of the two tables show conclusively that the brightest stars in the cluster are the red and yellow giants. In fact there are 16 *g*'s and *k*'s more luminous than the very brightest *b*, which is an exceptional star; and of the 221 brightest stars in these outer regions of the cluster there are only four in color class *b* and twenty in class *a*.

Fainter than the fifteenth magnitude the bluer colors become very common and there is an indication that the reddest stars may be almost entirely absent between magnitudes 15.5 and 17.0. At any rate, between magnitudes 16.0 and 16.8 only four stars out of the hundred measured have color indices greater than half a magnitude, and none is found redder than the sun. If the result is confirmed by later study it will be very important. It would suggest that this magnitude interval represents in the cluster the special domain in the luminosity sequence of the normal first type star, and that the dwarf stars of the redder spectral types (including bodies comparable to the sun) will be found among the still fainter members of the system.

From the foregoing considerations we conclude that not only are giant second type stars present in large numbers in the globular clusters, but also on the average among the four or five brightest magnitudes the brighter the star the redder its color. This condition is the inverse of that for the dwarfs, where, without much doubt, the cooling stars are growing redder, fainter, and smaller with age. In the conventional scheme of the evolutionary transition of spectral types the present result introduces serious difficulties; for Russell's hypothesis, however, the difficulties are not so severe, but still the immediate ancestry of these giant red stars must be taken into account. More detailed considerations of the observations and their interpretation will appear in a forthcoming *Contribution from the Mount Wilson Solar Observatory*.

The following references may be cited:

Kapteyn, *Groningen Publications*, No. 14, 1904; No. 23, 1909; *Trans. Internat. Solar Union*, **3**, 225 (1911).

Eddington, *Stellar Movements and the Structure of the Universe*, chap. iv (London, 1914).

Russell, *Popular Astronomy*, **22**, 291 (1914).

Seares and Shapley, These PROCEEDINGS **1**, 487 (1915).

Shapley, *Ibid.* **1**, 459 (1915); **2**, 12 (1916).

EXPERIMENTAL EVIDENCE FOR THE ESSENTIAL IDENTITY OF THE SELECTIVE AND NORMAL PHOTO-ELECTRIC EFFECTS

By R. A. Millikan and W. H. Souder

RYERSON PHYSICAL LABORATORY, UNIVERSITY OF CHICAGO

Received by the Academy, December 3, 1915

In the fall of 1913, while studying the photo-electric properties of freshly cut surfaces of the alkali metals in extreme vacua, we observed that immediately after first cutting, the fresh surface of sodium showed very large photo-sensitiveness when tested with monochromatic light of wave length 5461 A., even when the vacuum was of the order 10^{-6} mm. as measured by a McLeod gauge. But after several weeks of experimenting and many cuttings a condition was reached in which a freshly cut surface was completely insensitive when illuminated with this wave length. The lost sensitiveness reappeared, however, in the course of not more than two minutes after cutting, and grew rapidly to a very large value in fifteen or twenty minutes. When the gas pressure was of the order of 0.01 mm. the same phenomenon occurred but the rise to a maximum value was less rapid. From these results we began to surmise that photo-electric currents must be due to the influence of some active gas, which diffused from the walls to the metal and whose action upon the surface was retarded by the presence of an inert gas.

While we were in the process of testing this hypothesis by repeating the experiment under as diverse conditions as possible the papers of K. Fredenhagen,[1] H. Küster,[2] G. Wiedmann and W. Hallwachs[3] appeared nearly simultaneously, all taking the point of view which had been suggested to us by our own experiments, and the last one of them, namely, that of Wiedmann and Hallwachs, asserting with considerable positiveness that the presence of a gas is a necessary condition for the appearance of photo-electric currents in potassium, and that the "photo-electric effect in general takes place in the mixture formed by the body and the absorbed gas."

By this time, however, we had obtained data which convinced us that the point of view taken by these authors was untenable and in April, 1914, at the meeting of the American Physical Society, we presented a paper entitled 'Effect of Residual Gases on Contact E. M. F.'s and Photo-currents,'[4] in which we described fully and exhibited to the society the curves shown in figure 1. A glance at these curves shows at once that the freshly cut surface is initially strongly photo-sensitive to lines 2535 and 2804 and that its sensitiveness for these lines diminishes continually with time; while for lines 3126, 3650, and 5461 the surface is initially relatively insensitive and then grows in sensitiveness to a maximum. Now according to Pohl and Pringsheim,[5] the selective effect in sodium shows a resonance band whose maximum is at about 340 $\mu\mu$. It is obvious then that if we wish to define the selective effect as the strong photo-sensitiveness in the neighborhood of this wave length, then these curves mean that the selective effect so defined is due to the presence of gas and grows as the gas has time to get at the surface. Since we did not wish to define the selective effect in any such arbitrary manner we interpreted our result in the following words:

> A freshly cut sodium surface is not sufficiently electro-positive to respond photo-electrically to light of wave-length 5461 A., but under the influence of an active gas, the sodium forms a new substance which is more electro-positive than the freshly cut sodium and hence is photo-sensitive to longer waves. The photo curve taken with wave-length 5461 A., represents then merely the growth and decay of this substance. For sufficiently short waves, however, the freshly cut surface is itself so photo-electrically active that its own decay curve completely masks the rise and fall curve due to the growth and decay of the more electro-positive substance resulting from the action of gas upon the sodium.

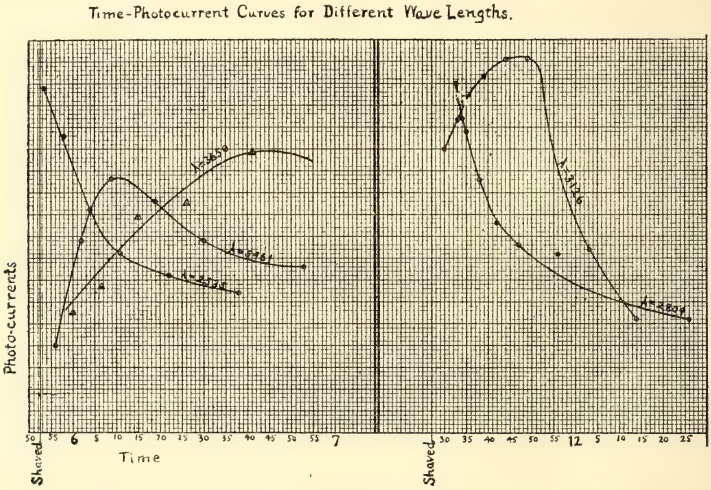


FIG. 1

In view of these results the authors raise a question as to the correctness of the conclusions of Wiedmann and Hallwachs that photo currents are obtainable only in the presence of gas. We suggest that if the Wiedmann and Hallwachs experiments are repeated with light of sufficiently short wave-lengths instead of with visible light, it is not likely that the photo-sensitiveness will be found to vanish in the way in which it did in the experiments reported by these authors.

This article formed the subject of some correspondence between Professor Hallwachs and one of us, and quite recently Wiedmann[6] working in Hallwachs' laboratory has followed out the suggestion above made and obtained by cleansing his surfaces by successive distillation instead of by cutting, as we had done previously, results altogether similar to ours, though of less pronounced character and extending over a much smaller range of wave-lengths. Wiedmann's actual observation is that potassium, whose selective resonance band, according to Pohl and Pringsheim, is at 436 $\mu\mu$, shows a relatively stronger photo-sensitiveness for line 365 $\mu\mu$ than for either 406 $\mu\mu$ or 436 $\mu\mu$, when it has been freed from gas by repeated distillation. But when distillations are less numerous and freedom from gas less complete the photo-sensitiveness of this line is some four times greater than that of the other lines. This means merely in terms of our diagrams that lines like 2535 A. and 2804 A. which have wave lengths shorter than that corresponding to the selective point are relatively strong with respect to those whose wave-lengths are at or above that of the selective point when gas has had little chance to act, but that they become relatively weak after the gas had had a chance to act.

These results from Professor Hallwachs' laboratory constitute then very interesting confirmation of both the results and the conclusions which we published one and one-half years ago, namely, that when tested with sufficiently short wave-lengths, freeing a surface from gas does not at all eliminate its photo-electric sensibility, though, as we pointed out then, it does diminish the photo-sensitiveness on the long wave-length side, i. e., at or above the region of the wave-lengths at which Pohl and Pringsheim's selective effect reaches its maximum.

Within the past year we have made experiments which show that it is very difficult if not impossible to set up any definite criterion for the selective effect since the so-called normal and selective effects merge by insensible gradations into one another. At present we prefer to regard the selective effect as nothing but the normal effect taking place in the neighborhood of an absorption band, i. e., in the neighborhood at which oscillations of one particular frequency predominate. Our reasons for this point of view are as follows:

(1) We have shown that the energy of emission is always governed by the normal equation, $\frac{1}{2}MV^2 = PDe = h\nu - P$, whether the impressed frequency coincides with the maximum of the selective effect or not.

(2) We have found that the angle of incidence or azimuth of polarization is not at all essential to the appearance of a strong maximum at the wave length at which Pohl and Pringsheim locate the frequency of the maximum selective effect, for we find this strong maximum when we work not only with unpolarized light but with normal incidence. In other words, the only condition for the copious emission of corpuscles is the coincidence of an impressed frequency with a natural frequency.

(3) We have found that with a clean surface the curve expressing the relation between the frequency and the photo-current per unit of incident energy rises steadily with increasing frequency right through the point assigned by Pohl and Pringsheim for the maximum of the selective effect and that as a surface has opportunity to be acted upon by gas a maximum slowly grows at the frequency indicated, and ultimately becomes very large. This means that if we regard the maximum as a criterion of the selective effect, then the selective effect merges by insensible gradations into the normal so that no sharp line of demarcation can be set up between them. The results stated under (1) follow from the photo-electric determination of Planck's '*h*.'[7] Those stated under (2) and (3) follow from the data plotted in figure 2. This data was taken as follows:

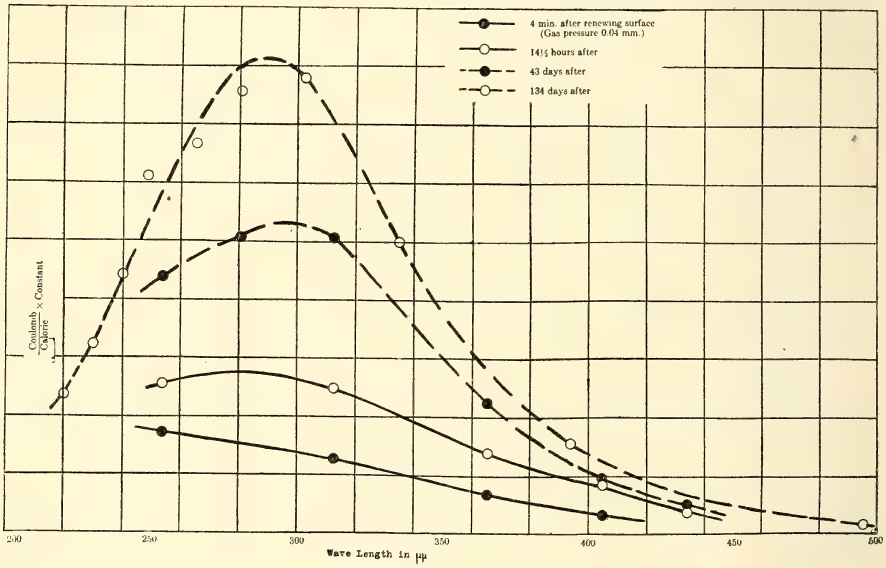


FIG. 2

Light from a 220 volt quartz-mercury lamp was passed through a Hilger monochromatic illuminator into a vacuum chamber in which the alkali metals, lithium, potassium and sodium, were arranged on the periphery of a wheel which could be rotated electro-magnetically by an electromagnet outside the chamber. Fresh surfaces of these metals were produced by means of a knife operated from outside by another electro-magnet. A Hilger thermopile used in connection with a Coblentz galvanometer sensitive to 1×10^{-11} amperes intercepted a portion of the beam and permitted energy measurements to be made simultaneously with the measurements of the photo currents.

The apparatus was the same as that used in the photo-electric determinations of Planck's '*h*' with the aid of the alkali metals.[7] As indicated above, the incidence was always normal.

The photo-currents are all saturation currents with -12 volts applied to the lithium.

The pressures at which the observation in the experiments corresponding to curves in figure 2 were made ranged from 0.04 to 0.0001 mm. While the rate at which the maximum grows depends somewhat upon the pressure, any individual curve is found at a given time to be not essentially modified, so far as the phenomenon here under consideration is concerned, by changes in pressure within the specified limits.

Changes in photo-sensitiveness bearing some similarity to these were observed by Pohl and Pringsheim[8] in the cases of barium, aluminium, magnesium and calcium but the maxima observed by them were not shown to be in the neighborhood of a wave length at which a selective effect had been found. Indeed Pohl and Pringsheim did not interpret their results as meaning that these four metals possess any selective effect at all. It will be observed, however, that the maximum in curve 2 coincides, within the limits of accuracy of Pohl and Pringsheim's determination, with the frequency 280 $\mu\mu$ at which they located the selective effect of lithium. It is not at all likely, therefore, that our normal effect can differ in any essential particular from their selective effect in lithium. It is true that we have not shown that our lithium surface shows the characteristic of yielding an energy-frequency curve having a maximum when light is polarized perpendicular to the plane of incidence combined with no maximum when it is polarized parallel to the plane of incidence. But this was not the criterion which Pohl and Pringsheim[9] used when they discovered the existence of the selective effect in lithium. Their criterion was simply the existence of a pronounced maximum at 280 $\mu\mu$ when they used oblique incidence and polarized light. It is scarcely conceivable that our maximum at 280 $\mu\mu$ would disappear if we made our tests under precisely the same conditions which they used.

Our experiments have, as yet, thrown no light on the cause of the exceptionally profuse emission shown by potassium and sodium when the electric vector of the light wave vibrates normally to the surface. But so far as we can discover the selective effect in lithium has never been shown to possess this characteristic.

Summary.—1. Photo-electric phenomena do not appear to be in general conditioned by the presence of a gas, since they appear with the use of sufficiently short wave lengths even when they do not appear with the use of long wave lengths.

2. The long wave-length limit of photo-sensitiveness is modified by the presence of a gas and this long wave-length limit is determined by the presence of a substance which gives rise to a maximum photo-sensitiveness at the so-called resonance wave-length.

3. Some of the characteristics of the selective effect have been obtained under the conditions under which only the normal effect is commonly observed.

4. The curves commonly supposed to be characteristic of the normal effect may be made to merge by insensible gradations into those of the selective effect by the action of gas on the surface of the metal.

5. All distinctions between the normal and selective effects in lithium have disappeared.

[1] K. Fredenhagen, *Physik. Zs., Leipzig,* **15,** 65 (1914).

[2] H. Küster, *Ibid.,* **15,** 68 (1914).

[3] G. Wiedmann and W. Hallwachs, *Berlin, Ber. D. physik. Ges.,* **16,** 107 (1914).

[4] R. A. Millikan and W. H. Souder, *Physic. Rev.,* Ser. 4, No. 1, **13,** July, 1914.

[5] R. Pohl and P. Pringsheim, *Die lichtelektrischen Erscheinungen,* pp. 14 and 27.

[6] G. Wiedmann, *Ber. D. Physik. Ges.,* **17,** 343 (1915).

[7] R. A. Millikan, *Physic. Rev.,* March, 1916.

[8] R. Pohl and P. Pringsheim, *Ber. D. Physik. Ges.,* **14,** 546 (1912).

[9] R. Pohl and P. Pringsheim, *Ibid.,* **46** (1912).

CONCOMITANT CHANGES IN TERRESTRIAL MAGNETISM AND SOLAR RADIATION

By L. A. Bauer

DEPARTMENT OF TERRESTRIAL MAGNETISM, CARNEGIE INSTITUTION OF WASHINGTON

Read before the Academy, November 17, 1915. Received December 3, 1915

While good progress has been made by various investigators in establishing the relationship between fluctuations of the earth's magnetism and those of the sun's activity during the sun-spot cycle, there are still outstanding a number of important questions. The magnetic quantity hitherto generally used, as for example, one of the magnetic elements (chiefly the magnetic declination), or the range of the diurnal variation (again chiefly of the magnetic declination), has not admitted always of direct physical interpretation, nor has it furnished always a convenient measure of the magnetic changes. Accordingly, the author in a preliminary examination of this relationship, made in 1909, introduced a quantity, called the 'local magnetic constant,' designated by *G*, which, under certain assumptions, is proportional to the magnetic moment of the earth, or to the intensity of magnetization.

Various recent investigations have shown that the quantity, *G*, provides an adequate measure of certain changes to which the earth's magnetism is continually subject. One interesting result of the 1909 investigation was, that increased solar activity, as measured by sun-spot frequencies, was accompanied apparently, by a decrease in the earth's magnetic constant. This is the general effect which accompanies any large magnetic disturbance. For example, during the magnetic storm of September 25, 1909, the earth's magnetic state was below normal for a period of about three months. Since magnetic storms in general increase in frequency, as well as in magnitude, with increased sun-spot activity, the general effect on the magnetic constant during the sun-spot cycle is as it was found to be.

In the present paper there are considered changes in the earth's magnetism of a considerably minor order of magnitude as compared with the magnetic perturbations just discussed; however, they are found to be not less important. The precise relationship between changes in solar radiation and possible changes in the earth's magnetism could be subjected to a definite examination only when values of the solar constant, of such accuracy as those of the Smithsonian Institution, became available. Fortunately, we now have a series of determinations at Mt. Wilson of this constant by Abbot for a period of four to five months during the years 1905 to 1914, excepting 1907. The 1913 and 1914 data were kindly supplied by him, in advance of publication, for special use in connection with the present investigation. There were likewise made available the magnetic data for the same years, recorded at the observatories of the Coast and Geodetic Survey, for which acknowledgment should be made to the Superintendent of that Bureau.

In the Balfour-Schuster theory of the diurnal variation of the earth's magnetism, it was necessary to introduce an additional hypothesis to account for the great ionization required by the theory, and solar radiation suggested itself as a possible cause. "Hence," Schuster says, "we might expect an increased conducting power in summer and in daytime as compared with that found during winter and at night." If solar radiation plays the prominent part required in the Schuster analysis of the diurnal variation of the earth's magnetism, the question naturally arises: If, at any particular moment or period, the solar radiation falling upon the earth's atmosphere, suffers, from some cause, an appreciable increase or decrease, is there a corresponding observable magnetic change? A diminution, for example, in the amount of solar radiation, could be caused by the interposition of some screening body between the sun and the earth. The interposing body might be the moon, as during a total solar eclipse, or a cooling layer above the sun's photosphere. In the first case, magnetic observations made during a total solar eclipse would shed some light, and in the second case a comparison of observed values of the solar constant with concomitant magnetic records would be of great interest. We have carried out both lines of inquiry.

It is not possible to enter here into the details of all tests applied and as to methods of computation employed. It must suffice to state the chief conclusions derived to date.

a. Changes in the earth's magnetism of appreciable amount are found associated with the changes in solar radiation as shown by values of the solar constant possessing the requisite accuracy. For the average daily change in the solar constant, which amounts to about 1.5% of its value, the magnetic constant, used as a measure of the prevailing magnetic state of the earth, suffers a change of about 0.003%, or about one digit in the fifth decimal C. G. S. units. The effect on the horizontal component of the earth's magnetic force would be about twice this.

b. Decreased solar constant appears to be accompanied by increased magnetic constant and decreased diurnal range of the earth's magnetism, in accordance with the following relations: 1% change in the solar constant is accompanied by a change of about 0.002% in the magnetic constant, and by about 1% in the magnetic diurnal range. Assuming for the present a linear relation between the solar-constant and magnetic changes, a 10% change in the solar constant, as occasionally occurs, may be accompanied by a change in the magnetic constant of about 0.02%, and by about 10% in the magnetic diurnal range. The magnetic effects observed during total solar eclipses are about equivalent to those which might be expected from about a 10% change in the intensity of solar radiation.

c. Since the changes in solar radiation are aperiodic and occur more or less spontaneously, the effect on the earth's magnetism is generally of a threefold character: (1) An alteration in the diurnal range, (2) perturbations both of the world-wide and the local kinds, (3) an outstanding residual effect such as to alter the daily mean values of the magnetic elements by an amount 10 to 100 times that caused by the regularly-progressing secular variation. The magnitude of the effects may at times exceed the average ones described in (*a*) and (*b*), dependent upon peculiar local conditions (ionizations) of the upper atmospheric layers. Changes in solar radiation may thus furnish sufficient cause for the ever-present minor perturbations and elementary waves, or pulsations, of the earth's magnetism.

d. The daily non-cyclic changes in the earth's magnetism, as found on magnetically-quiet days by previous investigators, furnish an additional check on the foregoing results, their quantities harmonizing completely, both as regards sign and magnitude with those given here. It is found that on consecutive quiet days the magnetic constant is, on the average, larger on the second day than on the first, the increase being equal to that which would be caused by an average daily change in the solar constant. Moreover, the reason why the magnetic constant, or the horizontal intensity, is larger, on the average, on the second quiet day, is because, on the average, the solar constant is slightly smaller on the second day than on the first. The relation between solar change and magnetic change during consecutive quiet days is precisely of the same sign and amount as given in (*b*).

e. If the quiet day magnetic effect were to persist throughout the year, it would cause a secular variation fully ten times that generally observed. However, the quiet days are in the minority, being exceeded three times and more by unquiet days, on which the magnetic effect is of an opposite or compensating kind to that of the quiet day. Since these acyclic effects appear to be associated with solar changes and since the latter are not periodic, but more or less sporadic, there is an outstanding effect at the end of the year which causes an irregularity in the regularly-progressing secular change. Accordingly, there should be found some correspondence between annual changes of the solar constant and annual magnetic changes. This is found to be the case. Since the solar-constant changes occur only approximately in accordance with sun-spot activity and since the magnetic changes are found to conform so closely to those in the solar constant, an explanation is obtained as to why the irregularities in the magnetic secular change do not always synchronize with changes in solar activity as measured by the sun-spot numbers nor correspond in magnitude to them.

SUB-MARINE SOLUTION OF LIMESTONE IN RELATION TO THE MURRAY-AGASSIZ THEORY OF CORAL ATOLLS

By Alfred Goldsborough Mayer

DEPARTMENT OF MARINE BIOLOGY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, December 9, 1915

In 1880 Murray[1] advanced the idea that the solvent action of sea-water for limestone was a primary factor in the deepening and widening of the lagoons of coral atolls. This idea was afterwards adopted by Alexander Agassiz[2] who became its most active advocate. In 1914 Dole[3] made a chemical study of the water of the lagoon of the atoll of Tortugas, Florida, and decided that it contained no free carbon dioxide, and Tashiro[4] pursuing a different method decided that if free carbon dioxide be present in Tortugas sea-water its amount must be slight. In view of these facts Vaughan[5] stated that lagoons of atolls could not have been dissolved out by sea-water as such, but he gave no quantitative data to support this view. The validity of the Murray-Agassiz theory depends upon the *rate* of the solution of limestone in atoll lagoons. Elschner[6] states that the solubility of calcium carbonate in sea-water is extraordinarily low but he also gives no quantitative data. I have made an attempt to determine the rate of solution of calcium carbonate in Tortugas sea-water. Carefully weighed pieces of the shell of the mollusc *Cassis* having a specific gravity of 2.88 and areas ranging from 57.6 to 85 sq. cm. were treated as follows:

Shell No. 1 was placed for 367 days in a sealed, sterilized, glass carboy containing 45 liters of doubly filtered sea-water, which after being filtered was heated within the carboy to 72.5°C. for two hours. At the end of the year this sea-water still retained 71% of its alkalinity to phenolphthalein test. No bacteria or other visible growths developed within this carboy and the shell remained clean and without slime upon its surface. This shell weighed 13.285 grams in July, 1914 and at the end of the year it had lost 0.014 gram. Thus it would have taken 948 years to dissolve the shell, and a thickness of 0.00067 mm. was removed from its surface by one year's immersion.

Shell No. 2 was placed in another similar carboy, but the sea-water was neither filtered nor sterilized, and it developed bacteria, protozoa and algæ, and at the end of the year it was acid; its free acidity over and above neutrality by phenolphthalein test being equivalent to $\frac{n}{3000}$ H_2SO_4. The shell originally weighed 15.532 grams and it lost 0.173 gram after 360 days. Hence it would have taken 90 years to dissolve the shell, and a superficial thickness of 0.007 mm. was removed in one year.

Shell No. 3 was placed for 364 days in a 15 liter glass bottle which was enclosed in a wooden dark chamber to prevent the growth of plants. The water was mechanically changed by each rise and fall of the tide, flowing in and out of small glass tubes, thus preventing the formation of strong currents within the bottle. This bottle was placed in the marine moat surrounding Fort Jefferson, Tortugas. This moat-water supports an abundant animal life and contains more CO_2 than the open ocean, but remains always alkaline to phenolphthalein. At the beginning of the experiment the shell weighed 10.5435 grams, and at the end of the year it had lost 0.0115 gram. Thus it would have taken 917 years to dissolve the entire shell and a superficial thickness of 0.00069 mm. was removed in one year.

Shell No. 4 was placed for 364 days in a similar glass bottle which remained between tides off the western wharf of Loggerhead Key, Tortugas, thus being surrounded by open sea-water. Considerable silt was drawn in through the glass tubes, and the shell was found buried beneath about 8 mm. of limestone mud which was charged with H_2S. A number of tunicates, shrimps, molluscs and worms were living within the bottle at the end of the year, but the circulation had become interrupted by the growth, within the bottle, of a tunicate across the opening of one of the tubes. The shell originally weighed 15.22 grams and it had lost 0.047 gram, amounting to a superficial thickness of 0.0019 mm. in a year. Thus it would have required 324 years to dissolve the shell.

Taking the results of this last experiment as a maximum rate of solution of calcium carbonate by sea-water as such, it appears that 19,250,000 years would be required to dissolve out a layer of calcium carbonate to a depth of 20 fathoms, this being about the average depth of atoll lagoons; and as many if not all atoll lagoons have been formed since the beginning of Tertiary times it appears that they owe their development to agencies other than that of solution by sea-water, for even if the rate of solution of reef limestone is 100 times as rapid as that of the *Cassis* shell it would require 192,500 years to dissolve out an average lagoon.

Holothurians, Echini, boring algæ, certain sponges, worms, dead organisms, and probably most important rain water washing out from forested shores are agencies which dissolve limestone but their effects have not yet been quantitatively evaluated. Thus the lagoons of barrier reefs surrounding volcanic islands may have been formed in some appreciable measure by solution, but this cannot apply in the same degree to the lagoons of atolls where the land area is insignificant in proportion to the area of the basin of the lagoon, and we know from the studies of Guppy, Wood-Jones, and Vaughan that many lagoons are filling up with sediment. In fact calcium carbonate is being precipitated from the sea-water of the Florida-West Indian region in the manner determined by Drew[7] and Kellerman and Smith;[8] the precipitate finally changing into oolite as observed by Vaughan.[9]

---

[1] Murray, J., *Edinburgh, Proc. R. Soc.*, **10**, 505 (1880).

[2] Agassiz, A., Numerous paper in the *Bulletins* and *Memoirs* of the *Museum of Comparative Zoology at Harvard* and a general statement, *London, Proc. R. Soc.*, **71**, 412 (1903).

[3] Dole, R. B., The Carnegie Institution of Washington, Publication No. 182, *Papers from the Tortugas Laboratory*, **5**, 711 (1914).

[4] Tashiro, S., Year Book of the Carnegie Institution of Washington, No. 13, p. 220 (1915).

[5] Vaughan, T. Wayland, Publication No. 182, The Carnegie Institution of Washington, p. 62, (1914); also 1914; *Bull. Geol. Soc. Amer.*, **26**, 58 (1914).

[6] Elschner, C., *The Leeward Islands of the Hawaiian Group, Honolulu*, p. 48, (1915).

[7] Drew, C. H., Carnegie Institution of Washington, *Paper from Tortugas Laboratory*, **5**, 7–45 (1914).

[8] Kellerman, K. F., and Smith, N. R., *Washington, J. Acad. Sci.*, **4**, 400 (1914).

[9] Vaughan, T. Wayland, Carnegie Institution of Washington, Publication No. 182, p. 47–54 (1914).

THE ARCHEGONIUM AND SPOROPHYTE OF *TREUBIA INSIGNIS* GOEBEL

By Douglas Houghton Campbell

DEPARTMENT OF BOTANY, STANFORD UNIVERSITY

Read before the Academy, November 16, 1915. Received November 15, 1915

Treubia insignis is a remarkable liverwort discovered by Goebel in Western Java. It is one of the largest and most striking species known and is of special interest as being in some respects the nearest to the typical leafy liverworts of any of the 'anacrogynous' Jungermanniales.

For some time after its discovery Treubia was known only from the original locality near Tjibodas on Mt. Gedeh in Western Java. It has since been discovered in several widely separated regions, e.g., New Zealand, Tasmania, Patagonia, Tahiti, and Samoa. In May 1913, I collected a single specimen on Mt. Banajao in Luzon, Philippine Islands.

The present paper is based upon material collected at the original station (Tjibodas), in 1906. The material comprised female and sterile plants with gemmae, but no male plants were found.

The archegonia are in groups, up to a dozen, protected by the characteristic dorsal scales which occur at the base of each leaf.

The development of the archegonium is much like that of other liverworts, but shows one striking difference—instead of the usual five or six rows of peripheral cells in the neck of the archegonium, there may be as many as nine, and there is not a clear line of demarcation between the neck and the ventral part of the archegonium. None of the archegonia examined by me showed more than eight neck canal cells, but in a recent paper on Treubia by Grün, it is stated that in the mature archegonium there are sixteen.

The earliest stages of the embryo were not seen. In the youngest that were found, the basal portion of the embryo was occupied by a large haustorium formed of large cells very similar to that observed in several other Anacrogynae; e.g., Pallavicinia, Podomitrium, Aneura, but not found in Fossombronia, with which Treubia has a good many points in common. The haustorium is probably formed by the first division in the embryo, and this division, as in all other Jungermanniales, is presumably transverse. The portion of the embryo above the haustorium becomes later differentiated into foot, seta, and capsule, and the young sporophyte closely resembles corresponding stages in Podomitrium or Pallavicinia. The foot is not clearly delimited from the seta.

In the upper region (capsule), the differentiation of the sporogenous tissue takes place at a rather late stage and the limits of the sporogenous tissue are much less definite than is the case, for example, in Aneura, Fossombronia or Pallavicinia. Outside the sporogenous area there are about three layers of sterile cells constituting the wall of the capsule. At the apex the number of cell layers is greater—as many as eight may be present—and a conspicuous beak is formed as in Podomitrium and Pallavicinia.

There is no elaterophore and no evident relation between the spore mother-cells and the elaters. The latter are few in number, but reach finally an extraordinary length—1.25 mm. according to Grün. There is no perianth or involucre about the archegonia, but a very massive calyptra is formed about the young sporophyte. The calyptra may finally reach a length of 1.5 cm. The ripe capsule is ovoid in outline, not globular as Andreas asserts. It opens by four broad and somewhat irregular valves.

Treubia is probably on the whole nearer the leafy liverworts (Jungermanniales Acrogynae) than is any other anacrogynous genus. Among the latter Noteroclada probably is most nearly related to Treubia, and connects it with Petalophyllum, which in turn is not very distantly related to Fossombronia. The latter shows some significant resemblances to the lowest of the liverworts—the Sphaerocarpales, and it may be necessary to separate as a special order or sub-order the series beginning with Sphaerocarpus and ending in Treubia, which in turn is almost certainly connected pretty closely with the acrogynous Jungermanniales.

BRIEF NOTES ON RECENT ANTHROPOLOGICAL EXPLORATIONS UNDER THE AUSPICES OF THE SMITHSONIAN INSTITUTION AND THE U. S. NATIONAL MUSEUM

By Aleš Hrdlička

DIVISION OF PHYSICAL ANTHROPOLOGY, U. S. NATIONAL MUSEUM

Received by the Academy, November 22, 1915

In the April and July (1915) numbers of these PROCEEDINGS I have given notes on 'Some Recent Anthropological Explorations' carried out under the auspices of the Smithsonian Institution, and on 'An Exhibit in Physical Anthropology' which was prepared on the basis of these explorations for the Panama-California Exposition; I will now complete the account by referring in brief to the scientific results of the several expeditions made in this connection.

The field work was directed towards three main objects, namely, the securing for this country of original specimens relating to earlier man in Europe and Asia and contributing thus to the advance of knowledge in this direction; the initiation of comparative study of the child among primitive peoples; and the search in Asia for possible traces of the ancient stock of humanity which gave us the Indian; while an additional aim was to complete as far as possible our collections of skeletal material bearing on prehistoric American pathology. Some of the results of these activities have already been published, at least in a preliminary form (see bibliography, April number), and need not be referred to again at this time; while the remainder can be summarized as follows:

Search for Neolithic Human Remains in Southwestern Russia.—It is well known[1] that southwestern Russia and particularly the province of Ukraina, is rich in mounds or 'kurgans,' which yield human remains dating from the early historic back into the neolithic period. It is the region which in the past has yielded bones colored red,[2] and also some crania of most interesting form, partly transitional with those of geological antiquity. The exploration was entrusted to Prof. Kazimir Stolyhwo, Chief of the Anthropological Laboratory at Warsaw, and was restricted to the district of Kiev, in the vicinity of the villages of Szulaki, Puhaczówka, Chejtowa, Zywotówka, Tackowica, Zacisze and Horodnica.

The total number of Kurgans explored was thirty-three, twenty-seven of which yielded human remains, which, however, in the majority of cases had been disturbed. The mounds averaged close to 90 feet in diameter, the range being from approximately 40 to nearly 300 feet; and nearly 5 feet in height, or from less than a foot to nearly 8 feet. Most if not all of the tumuli were originally higher, being reduced in the course of time by agriculture. The tops of several of the mounds were covered with stone, which must have been brought from a distance. The number of human burials encountered in these mounds was upwards of 70. They were located generally about or not far from the center of the mound and at varying depths, from a few inches to nearly 15 feet beneath the summit. Some of the burials were plainly intrusive. The old burials included individuals of both sexes and all ages. The bodies were as a rule in moderately contracted position. In the majority of cases the bones were colored more or less red, due to the inclusion in the graves of red ochre. The mode of burial differed. In one mound it was by incineration of several individuals; in others a fossa had been made in the surface of the ground, in which the body or bodies were placed, the grave being covered by a low shelter of wood, about and over which was piled the soil from the immediate neighborhood. In still other instances the body was simply buried in the earth. Most of the mounds contained also traces of ceremonial fire and of animal bones, some of accidental inclusion, but some probably from offerings. The archeological remains were scarce, though many may have been removed by treasure hunters previously. With two skeletons, in two separate but adjacent mounds, there was found iron; with three or possibly four, were traces (stains) of brass or copper; with one a small ornament of gold; with six there were old articles of bronze such as ear pendants and bracelets; with six there were objects of bone, such as heads, or artificially perforated teeth of carnivores; with two burials there were implements of stone, and fourteen of the mounds yielded primitive hand-made pottery. The older skeletal remains of man are of special interest. Although in poor condition, they show a uniform dolichocephalic type of people, of good stature; there is no evidence of any superposition of types or even mixture until we come to relatively recent burials. The animal bones recovered from the various mounds comprise those of three or four species now extinct in these regions, namely: *Bison bonasus*, *Equus* (prob. *gmelini*), *Ochotona pusilla*, and *Marmota bobak;* and those of some of the common living ungulates and carnivora, a few birds, with a variety of rodents. Detailed report on these finds is in preparation by Professor Stolyhwo.

Explorations in the Birusa caves and rock shelters on the Yenisei River, Siberia.—During my trip along the Yenisei River in 1912, my attention was forcibly attracted by a large number of caves showing from a distance in the cliffs of a wild region about the mouth of a small stream known as the Birusa, on the left bank of the river about 50 miles south of Krasnoyarsk. The whole locality is known as Birusa and, as I learned later, some of the caves and nearby tumuli have been previously explored in part by M. Jelenief of Krasnoyarsk, who discovered in them remains pointing in particular to the Neolithic period. As the whole region west of the Yenisei in this latitude is very rich in mounds and other remains of old populations, some of which are known to date back to the Neolithic period, it seemed most desirable to subject the Birusa caves, as far as possible, to scientific exploration, and the work was intrusted to Professor Stolyhwo, who the year before explored the mounds in Ukraina. The work was carried out during the latter part of the summer of 1913, but was greatly interfered with by adverse weather and other unfavorable conditions. Nevertheless, excavations were completed in three of the caves, four rock shelters and one mound. The results were more encouraging than conclusive. With the exception of few teeth, no skeletal remains of man were discovered; but the caves and shelters yielded numerous traces and examples of bone and stone industry. A detailed account of this exploration also is being prepared for publication by Professor Stolyhwo. Careful and extended exploration of the region is most desirable. The many remaining caves about the Birusa and the thousands of mounds over the steppes to the westward, only a few of which have as yet been touched, invite urgently the hand of the archeologist.

STUDIES ON THE DEVELOPMENT OF THE CHILD AMONG PRIMITIVE PEOPLES

The Negrito.—In 1912, after receiving due instructions, Dr. Philip Newton of Washington, D. C., was sent to the Philippine Islands to study the child among the Negrito. His stay in the Philippine Islands extended over 103 days, of which however only 65 could be spent in actual field work, the remainder being taken up by travel, storms and by illness incurred in the unhealthy environments. During the trip more than 1000 Negritos were seen, of whom 400 were measured and observed. These included 348 full bloods and a little over 50 mixed bloods. The mixed bloods represented mostly the Bataks of the Island of Palawan; the full bloods were in the main from Luzon. Among the pure bloods examined, there were 75 children. The rather small proportion of children to adults was due to the fact that many adults came for the examination for some distance, without bringing with them their families, and partly to the fact that the number of children per family among the wild Negrito is exceptionally small.

Photography and particularly cast making met with serious difficulties, due to the tropical climate and great moisture. About 300 photographs were nevertheless taken, 70 of which, however, were ruined when the boat carrying the expedition was upset during a severe storm on the Cagayan river. As to cast making, although plaster was taken along packed in supposedly air tight cans, when these were opened it was found to have absorbed so much moisture that it was quite useless; and an additional supply purchased in Manila yielded no better results. Collection of skeletal material also met with considerable difficulties. The people, and hence their burials, are widely separated, and when the latter were found the bones, in a large majority of cases, due to the excessive moisture, were in such poor condition that they could not be collected; twelve graves yielded few parts only of a single skeleton.

A great obstacle encountered in the examination of the Negrito children was the absence of reliable data as to their ages. Nevertheless, the series is an interesting and valuable one and can be confidently expected to throw light on a number of important features of the child's development among these exceedingly primitive people, who besides live in a highly specialized environment. The data are being prepared for publication.

African Negro.—Perhaps the most important large ethnic group for anthropological studies, particularly those on the child, is that of the pure African Negro, and an expedition of over 14 months' duration was carried out under my direction, among several of the more favorably situated tribes of this race, by Dr. A. Schück, of Prague. Principal attention was given to the Zulu, from South Natal to North Zululand, and the results are measurements of upwards of 1000 children and adolescents of this people. These children were reached through the various Mission schools and in nearly all cases the age could be exactly determined. The results of the measurements and observations, which are already in an advanced stage of preparation for publication, are expected to afford a solid basis of data concerning the child development of the pure blood negro of good stock, which in the future should be of considerable value in connection with studies on the development of our own colored population.

Besides the Zulu, the expedition reached a limited number of the Bushmen and was engaged on the tribes in British East Africa, including some pygmies, when the outbreak of the war in Europe brought the work forcibly to an end.

Besides measurements and observations, the results of this expedition consist of numerous photographs; a series of excellent facial casts of the Zulu, and an especially valuable series of 20 facial casts of the Bushmen. There were also made on this trip numerous collateral observations of physiological and medical nature.

The Eskimo.—About the most exceptional environment under which man lives is that of the Eskimo, and it would seem that if anywhere, it would be among these people that the effects of environment on development should be encountered. This decided me to send an expedition to one of the largest and for purposes of research the most favorably situated groups of these people, namely that of St. Lawrence Island. This island lies, as is well known, in Bering Sea, approximately 40 miles from the Siberian and a little over 100 miles from the nearest point on the Alaskan coast. The work was entrusted to Mr. Riley D. Moore, D.O., aid in my laboratory in the National Museum, and after due preparation he left for the Island in the spring of 1912, returning late in the autumn. He found the Island inhabited by 292 full-blood Eskimo, of which he measured and examined 180, including a majority of the children; with young infants the mothers objected to all handling. Fortunately, as among the Zulu, the ages of the children could in a large majority of cases be exactly determined, the Island having for years had the advantage of a Government school; in consequence the data secured can not fail to be of considerable scientific value, and are being gradually prepared for publication. Besides the measurements and anthropological observations, the expedition secured a good series of photographs of persons of all ages, a series of facial casts, a collection of skeletal material, and much general information.

Native Siberians.—In addition to the above researches, it was planned to examine into the development of the Australian child and the Chinese. Unfortunately neither of these plans could be realized, but instead it became possible to send a well qualified observer, Dr. S. Poniatowski, chief of the Ethnological Laboratory in Warsaw, to some of the tribes in the Primorskaia province of East Siberia. Unfortunately this expedition, undertaken in 1914, was broken up by the war; yet measurements were secured on 109 Golds and 25 Orochons, including a good number of children. Numerous photographs were also taken, but only one facial cast could be made on account of prevailing apprehensions regarding the purpose of the work. It was hoped that this expedition could be resumed during the fall and winter of 1915–1916, but present conditions make this impossible.

On the whole, then, we have succeeded in securing an excellent series of data on the children of the African negro; a valuable though not large series of similar data on the Eskimo child; a series of some value on the child of the Negrito; and some data among the native tribes of eastern Asia. I may add that we have already a good series of similar observations, made by the same methods, on the Indian child.[3] It may therefore be stated that, notwithstanding the many difficulties in the way of such studies among primitive peoples, there have been made some serious beginnings in this very important line of anthropological investigation. As to detailed results, nothing can be said until a careful and necessarily tedious elaboration of the data is completed.

The third main object of anthropological field work under the auspices of the Smithsonian Institution during the last three years was the search in Asia for *probable traces of the ancient stock or stocks of mankind, from which the American Indians were derived.* On this subject a number of preliminary reports have already been published[4] and it is unnecessary to do more in this place than to state that such traces undoubtedly exist in Asia to this day, that they extend over a very large territory, and that they are soon to receive further attention by the Smithsonian Institution.

[1] See bibliography in A. A. Ivanovskiji, Ob Antropologičeskom Sostavě Naselenii Rossii, Moskva, 1904.

[2] See Hrdlička's The Painting of Human Bones, etc. *Smithsonian Inst. Rep.* for 1904.

[3] Published in *Smithsonian Inst., Bull. Bur. Amer. Eth.*, No. 34.

[4] Symposium on The Problems of the Unity or Plurality and the Probable Place of Origin of the American Aborigines, *Amer. Anthrop.*, **14**, No. 1, Jan.–March, 1912; Restes, dans l'Asie orientale, de la race qui a peuplé l'Amérique, *Congrès International d'Anthropologie et d'Archéologie préhistoriques, Compte Rendu de la XIV Session*, Genève, 1912; Remains in Eastern Asia of the Race that Peopled America, *Smithsonian Inst., Misc. Coll.*, **60**, No. 16; The Derivation and Probable Place of Origin of the North American Indian, *Proc. XVIII International Congress of Americanists*, and the Genesis of the American Aborigines, *Proc. II Pan-American Sci. Congress*, Washington, 1916.

A THEORY OF NERVE-CONDUCTION

By Alfred Goldsborough Mayer

DEPARTMENT OF MARINE BIOLOGY, CARNEGIE INSTITUTION OF WASHINGTON

Presented to the Academy, November 16, 1915

This research is a continuation of the studies reported in these PROCEEDINGS, **1,** 270, May 1915, and it will therefore be unnecessary to redescribe methods of experimentation. These later experiments were made at the Marine Laboratory of the Carnegie Institution of Washington at Tortugas, Florida, in July, 1915, and the kymograph records were studied in detail in a room which the writer was privileged to occupy in Guyot Hall, Princeton University.

In the experiments of 1914 the distilled water contained between 10^{-5} and 10^{-6} H ion concentration due to carbon dioxide, thus giving an excess of free H ion. The endeavor was therefore made in July, 1915, to obtain neutral distilled water in which both the H and the OH ion concentration approached 10^{-7}. Accordingly, following the suggestion of

Prof. G. A. Hulett, the distilled water was freed in so far as possible from CO_2 by bubbling through it for 60 hours air which had passed through glass tubes containing granulated soda-lime. As the water still appeared slightly acid to phenolpthalein it was rendered alkaline by adding NaOH until the concentration of the OH ion was about $0.75\, n. \times 10^{-5}$.

Although this water was kept in a glass carboy carefully protected from contact with air containing CO_2 it nevertheless became slowly less and less alkaline, probably due to decomposition of traces of organic matter, so that in the course of 11 days the distilled water became nearly neutral, both the H and OH ion concentration approaching 10^{-7}.

The use of this nearly neutral distilled water in diluting the sea-water should enable one to determine the true effects of the sodium, calcium, and potassium cations of the sea-water without serious disturbances due to H or to OH ions, both of which in weak concentration accelerate and in strong concentration depress the rate of nerve conduction.

Thus we find that a concentration of about 6×10^{-5} OH in sea-water is a maximal stimulant for nerve conduction, but becomes a depressant if its concentration be still further increased.

Experiments were made upon 71 *Cassiopea xamachana* medusæ between July 5 and 16, 1915; the OH ion concentration in the distilled water declining during the course of the experiments from about 10^{-5} to about 10^{-7}. Thus for the first 25 medusæ, when the OH ion concentration was at the start about 10^{-5} and was approaching 10^{-6}, the curve for the decline in rate is fairly well represented by the formula

$$y = 103.37 - 0.541\,(95.2 - x)^{1.18},$$

wherein y is the rate of nerve conduction, the rate in undiluted sea-water being 100 and x the concentration of the sodium, potassium, and calcium cations, their concentration in natural undiluted sea-water being 100.

Later as the OH ion concentration continued to decline the formula became

$$y = 100 - 0.2\,(100 - x)^{1.35},$$

and finally for the last 14 medusæ when the distilled water was still alkaline but nearly neutral, the rate is given by the formula

$$y = 2.512x^{0.8}.$$

This is also equally well represented by the formula

$$y = 100 - 0.778\,(100 - x)^{1.019}.$$

Thus the curves representing the rate of nerve-conduction in sea-water diluted with alkaline distilled water gradually approach and finally become identical with Freundlich's formula for absorption $y=ax^{1/n}$ when the distilled water becomes nearly neutral. In this case $a=2.512$ and $1/n=0.8$. The data are shown in detail in Table 1 and Figure 2 at the end of this paper.

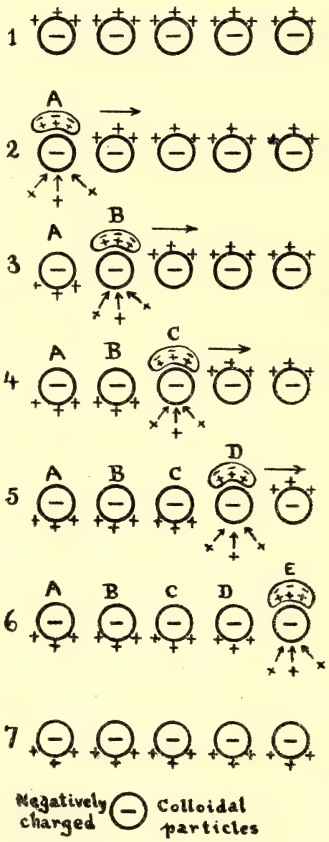


FIG. 1

Mecklenburg, in *Tables Annuelles Internationelles de Constants et données numérique*, **3,** 418 (1914), gives a list of 16 cases of adsorption in which a ranges from 0.0824 to 23.5 being usually between 2 and 3, while the exponent $1/n$ ranges from 0.167 to 0.965 being usually from 0.2 to 0.6. Hence 0.8 is high, but not beyond the range of the exponent in observed cases of adsorption.

Thus we venture to suggest that adsorption may play a fundamental rôle in nerve conduction, and that the only cations which are necessary to the reaction are the adsorbed sodium, calcium, and potassium ions, the rate of nerve conduction being proportional to the concentration of these adsorbed ions.

Hydroxyl and hydrogen ions are not adsorbed but act independently serving as accelerators of nerve conduction when in weak concentration and as depressants if concentrated to a greater degree.

A series of diagrams may serve to illustrate this hypothesis. Thus in Figure 1 the nerve is represented by a row of negative charged colloidal particles, for the colloid being normally alkaline the charge may be assumed to be negative.[1] Line No. 1 shows the nerve in its resting stage wherein the negative charge of each colloidal particle tends to be partially neutralized by the adsorbed cations of sodium, calcium, and potassium shown by + + +. The number of cations which each colloidal particle can capture and temporarily de-ionize[2] depends upon the potential of its negative charge, and also upon the concentration of the cations in the surrounding fluid. For the sake of illustration we have shown three such cations attracted to the surface of each particle, but in reality the number must be greater than this.

Line No. 2 shows the beginning of a nerve impulse wherein the adsorbed cations of particle *A* have combined with some anions to form an ion-proteid, thus neutralizing their positive charges and unmasking the negative charge of the colloidal particle. As a result other cations from the surrounding fluid (sea-water) are at once attracted and captured by the particle.

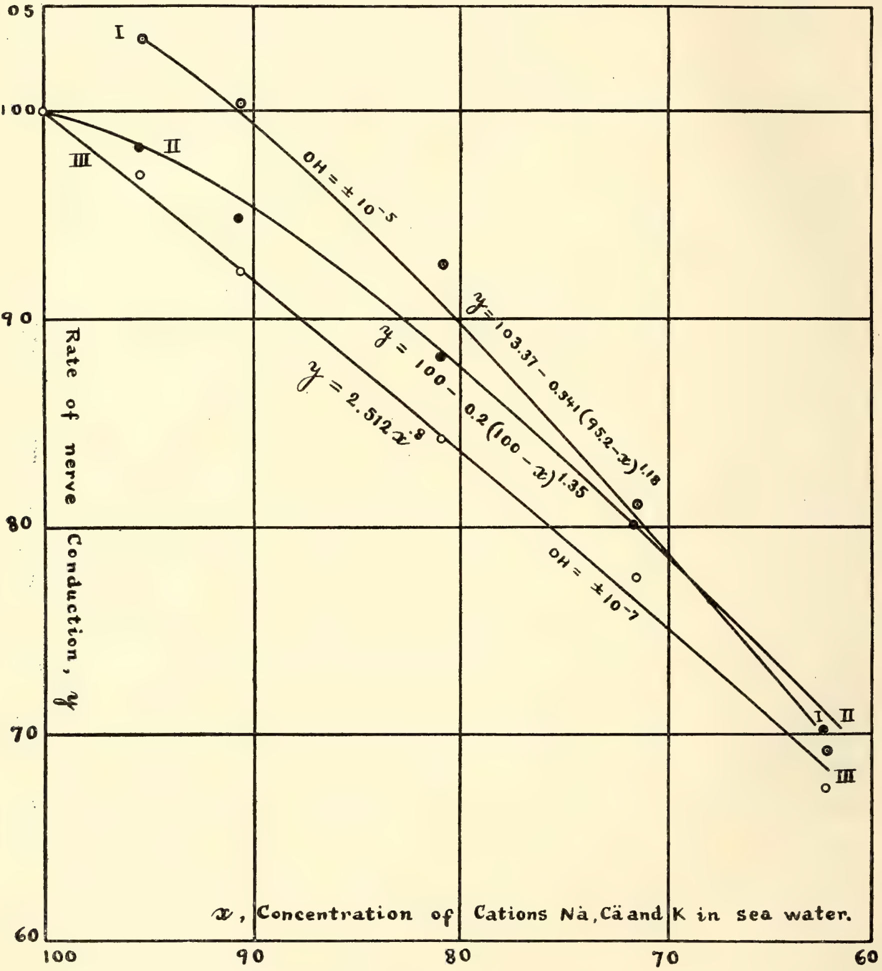


FIG. 2

In Line No. 3 the reaction has passed on to particle *B* and its negative charge is unmasked, and thus the negative charge passes through the nerve at the rate of nerve conduction until each particle has lost its original cations, and then recaptured others from the surrounding fluid as in Lines 2–7, Line 7 representing the resting nerve after the reaction has passed through it. Thus by Wilhelmy's law the rate of nerve conduction is proportional to the concentration of the adsorbed sodium, calcium and potassium cations which conduct it.

Since 1899, Loeb[3] has maintained that physiological reactions are phenomena associated with the formation of ion-proteids, but I think that while this is true for nerve conduction it is only half the truth, and that it is possibly a phenomenon of *adsorption* combined with that of an ordinary chemical reaction.

TABLE I

SHOWING THE RATES OF NERVE-CONDUCTION IN CIRCUIT-SHAPED STRIPS OF SUBUMBRELLA TISSUE OF *Cassiopea xamachana* IN SEA-WATER DILUTED WITH DISTILLED WATER. EXPERIMENTS MADE AT TORTUGAS, FLORIDA, JULY 5-16, 1915.

RATES OF NERVE-CONDUCITION

	Natural sea-water $x=100$ (Z)	(Q) 95 s. w. +5 d. w. $x=95.2$	90 s. w. +10 d. w. $x=90.5$	80 s. w. 20 d. w. $x=80.8$	70 s. w. +30 d. w. $x=71.4$	60 s. w. +40 d. w. $x=62.2$
1. Observations of Medusæ Nos. 1–25 in sea-water diluted with alkaline distilled water containing somewhat less than 10—5 OH ion concentration, July 5–9, 1915. See curve I I, Fig. 2.......	100	103.37	100.33	92.67	81.09	69.33
Rate (y) calculated from the formula: $y=103.37-0.541\ (95.2-x)^{1.18}$.		103.37	100.00	90.78	80.59	69.87
II. Observed rates of Medusæ Nos. 51–70, July 11–16 in sea-water diluted with distilled water which was less alkaline than in I....................	100	(+) 98.28 ±.96	94.86 ±1.18	88.19 ±1.83	80.12 ±1.61	70.3 ±1.96
Rate (y) calculated from the formula: $y=100-0.2\ (100-x)^{1.53}$............ See Curve II II, Fig. 2................	100	98.24	95.52	88.59	80.27	70.92
III. Observed rates of Medusæ Nos. 58–71, July 14–16, 1915 in sea-water diluted with nearly neutral distilled water.............................	100	96.86 ±.66	92.25 ±1.62	84.2 ±2.15	77.6 ±1.82	67.5 ±2.14
Rate (y) calculated from the formula: $y=2.512x^{0.3}$...................... See curve III III, Fig. 2...............	100	96.15	92.32	84.27	76.38	68.40
The ratio $\frac{x^{0.8}}{observed\ y}$	0.398	0.395	0.397	0.398	0.397	0.403
Rate (y) calculated from the formula: $y=100-0.778\ (100-x)^{1.019}$	100	96.15	92.29	84 21	76.29	68.48

Q. 95 cc. sea-water + 5 cc. distilled water.

Z. $x = 100$ means that the concentration of the cations Na, Ca, and K in natural sea-water is taken to be 100. Then the concentration of the cations in 95 per cent sea-water is 95.2, etc.

+ Probable errors are stated as ± following a determination.

Loeb indeed is not antagonistic to the view that complex changes other than those of a simple chemical reaction may accompany nerve conduction, for he says, *Comparative Physiology of the Brain and Comparative Psychology,* p. 14 (1900): "We have to remember that all life phenomena are due to motions or changes occurring in colloidal substances."

Moreover Matthews,[4] 1902, states that protoplasm consists essentially of a colloidal solution, and stimulation is accompanied by the passing of this solution to or toward a gel; and with these statements I am in accord. Matthews, however, believed the anions to be the stimulating ions, and he also thought the colloidal particles carried a positive charge. Later studies by many observers have made it apparent that the cations are the active agents in most physiological processes, and that living protoplasm is normally alkaline and thus its colloids probably carry negative charges. Moreover the phenomena of adsorption were not well understood in 1902 and Matthews makes no mention of it in respect to nerve conduction.

No one indeed had reason to support the view that adsorption plays a part in nerve conduction until the determination of the change in rate of nerve conduction of *Cassiopea* in successive dilutions of sea-water suggested this as a possibility.

My results lend no support to the theory of Sutherland[5] that the velocity of propagation of nerve impulse is that of a shear in the substance of the nerve. If this were the case its rate would vary with the viscosity of the surrounding fluid, but the decline in rate is practically the same whether the sea-water be diluted with distilled water, 0.9 molecular dextrose, or 0.4 molecular magnesium chloride.

---

[1] Hardy, W. B., *J. Physiol., Cambridge*, **24**, 296 (1899).

[2] Bayliss, W. M., *Biochem. J.*, **1**, 177 (1906), finds that electrolytes when adsorbed are non-ionized and no longer take part in the electrical conductivity of the solution. See also: *Principles of General Physiology*, p. 54–71 (1915).

[3] Loeb, J., 1899, *Festschrift für Fick*, and *Amer. J. Physiol.*, **3**, 327–338 (1900).

[4] Matthews, A. P., *Science*, **15**, 496 (1902).

[5] Sutherland, W., *Amer. J. Physiol.*, **14**, 112 (1905), and *Ibid.*, **23**, 115 (1908).

ZUÑI CULTURE SEQUENCES

By A. L. Kroeber

MUSEUM OF THE AFFILIATED COLLEGES, SAN FRANCISCO

Received by the Academy, December 8, 1915

The vicinity of the famous Indian pueblo of Zuñi in New Mexico has long been known to be rich in ruins. Many of these have been reported and described, some surveyed, and material from various sites has found its way into collections. A large body of specimens was secured through excavations by the Hemenway expedition, but this material and its data remain unpublished.

The region furnishes an unusual opportunity for an attack on the chronology, or at least the sequences of culture, in the prehistory of the

Southwest: first, because the restricted area excludes differences due to varying environments, and thus renders any observable distinctions directly interpretable in terms of time: second, because of numerous links between the historic and prehistoric periods. Several of the ruins were inhabited in Spanish times. They still bear native names that tally with those mentioned in sixteenth and seventeenth century records and some contain ruins of abandoned Catholic churches.

The tempting opportunity thus offered must of course be followed with the spade for ultimate results. I was in Zuñi during the summer of 1915. Pressure of ethnological work forbade digging; but some three thousand potsherds were gathered from the surface of about fifteen once inhabited sites within a few miles' radius of the pueblo. These were supplemented by a thousand fragments from the streets and roofs of Zuñi itself.

It was obvious that the pottery was of two well marked types, and that the surface of any one ruin yielded only such ware as plainly belonged to one or the other of the two classes. One set of sites is littered with sherds of which at least half are dull black or dark gray. The other half are as frequently red as white. Three-colored pieces—black and red on a white ground—are found. Corrugated ware is uncommon and about evenly distributed between dark and light.

On the second set of sites, black and red ware are both rare, white or whitish pieces constituting more than nine-tenths of the total. Three-color pottery has not been found. Corrugated sherds are common, but almost always of the light variety.

The first group of sites includes those which are mentioned as inhabited villages in the seventeenth century. Their sherds occur in nearly the same proportion as in modern Zuñi. These ruins therefore fall in part into the historic period. The second group of sites is wholly prehistoric. Their ware resembles that familiar as Cliff Dwellers' pottery. The two wares have been designated as type A, the later, and type B, the earlier.

The conditions of the ruins accords with this arrangement in time. Type A ruins normally include standing walls, and loose rock abounds. All type B sites are low or flat, without walls or rock, and show only pebbles in the surface soil. It seems more likely that this condition is due to the decay of age, or to the carrying away of the broken rock to serve as material in the nearby constructions of later ages, than to any habit of the period B people to build in clay instead of masonry. The latter possibility can be seriously entertained only if excavation reveals no building stone whatever in type B ruins.

Chips of obsidian are usually observable on period A sites, but have not been found on those of period B.

The proportions of different wares can be summarized thus:

	Eight Sites of Period A.	*Nine Sites of Period B.*
Wholly black	53	5
White or black on white	25	92
Containing any red	22	3
	100	100

Differences between sites of the same period can also be observed. These indicate minor periods of time. Expressed in percentages of the total number of sherds secured at each spot, the frequency of several wares is:

Period	*Site*	*Corrugated*	*Three Color*	*Black on Red*	*Any Red*	*Black*
Present	Zuñi	0[1]	12	1		
Late A	Towwayallanna	1	8	3	22	53
	Kolliwa	—	7	2		
	Shunntekya	2	7	2		
	Wimmayawa	2	4	1		
	Mattsakya	3	4	3		
	Kyakkima	4	3	2		
Early A	Pinnawa	10	1	8		
	Site W	24	—	1		
Late B	Hattsinawa	27	—	5	10	19
	Kyakkima West	12[2]	—	4	8	—[2]
Middle B	Shoptluwwayala	40	—	2	3	7
	Hawwikku B	49	—	6	12	9
Early B	Te'allatashshhanna	66	—	—	—	5
	Site X	71	—	—	3	1
	Tetlnatluwwayala	72	—	—	2	—
Uncertain	He'itli'annanna	—	—	—	—	3
	Site Y	—	—	—	—	—

[1] Present, but less than half of 1 %.

[2] Only 25 pieces altogether are available from this site.

The material as yet at hand is too slight, and too superficial in provenience, to make this classification into sub-periods more than tentative for any particular site. The statistics however do allow of three conclusions. First, the two principal periods are almost certainly subdivisible into shorter epochs. Second, these subdivisions shade into one another. Third, there is no gap or marked break between periods A and B. So far as Zuñi valley is concerned, the prehistory of Southwestern native civilization has therefore been in the main a continuous development from the earliest known time to the present.

A. V. Kidder's recent 'Pottery of the Pajarito Plateau,' in volume 2 of the *Memoirs of the American Anthropological Association*, presents analogous results, obtained by a method differing in some details, for another region of New Mexico; and at San Cristobal in still another part of the state, N. C. Nelson has excavated a stratified deposit showing four successive layers of different type. It is quite likely that some of the types at these three sites will prove to be similar, or even identical, as soon as the material can be compared. In this event a chronological frame work would be established that may prove capable of extension to accommodate a considerable part of the prehistoric data from the Southwest, and to fix distinctive and otherwise undatable local variations of ancient culture. The impression that there were at least two principal periods in the Southwest, the earlier represented by what are currently called Cliff Dweller forms, has of course long been prevalent, but the supporting evidence has been random. The three present sequential determinations promise not only definitely to establish but to elaborate the older general conviction.

The findings here discussed will be published in the Anthropological Papers of the American Museum of Natural History.

THE NUMERICAL RESULTS OF DIVERSE SYSTEMS OF BREEDING

By H. S. Jennings

ZOOLOGICAL LABORATORY, JOHNS HOPKINS UNIVERSITY

Received by the Academy, December 20, 1915

When organisms differing with respect to a pair of characters are bred for generation after generation, the relative numbers of individuals that will show any particular combination of characters in any given generation of course depend on the system of mating followed. The different classes of individuals may mate at random; or there may be assortative mating (dominants with dominants, recessives with recessives); or dominants alone may be bred; or recessives alone. Or again, self fertilization may prevail; or one of the various possible types of inbreeding may be followed. If we represent the two alternative characters by A (dominant) and a (recessive), then three types of individuals are possible, AA, Aa, and aa. The question here raised is as to the relative numbers of each of these three types of individuals after any number n of generations of mating, by any of the systems mentioned above. This depends of course on the constitution of the parents at the beginning, as well as on the system of breeding and the number of generations.

When breeding by a given system is continued for many generations, several types of results may be distinguished:

A. In some cases the proportions of the population having particular constitutions (AA, Aa, or aa) remain the same for all generations. Examples are:

(1) Random mating when the parents are at the beginning either all Aa; or half AA, half aa; or when the parental proportions are AA+2Aa +aa. In these cases after any number of generations the resulting progeny show the proportions AA+2Aa+aa.

(2) Random mating, parental proportions, rAA to $t$aa. Constant proportion in offspring: r^2AA$+2rt$Aa$+t^2$aa.

(3) Random mating, parental proportions, AA+Aa. Constant result, 9AA+6Aa+aa.

(4) Random mating, parental proportions, rAA$+s$Aa. Constant result, $(s+2r)^2$AA$+2s(s+2r)$Aa$+s^2$aa.

(5) Inbreeding, daughters bred back to father: both parents Aa. Constant result, AA+2Aa+aa.

(6) Sex-linked factors; random mating; original parents rAA$+r$A$-+$ $t$aa$+t$a$-$, in equal numbers. Constant later proportions: males, rA$-+t$a$-$; females, r^2AA$+2rt$Aa$+t^2$aa. Males and females in equal numbers.

B. In some complex cases the population changes from generation, and the best we can do is to obtain a formula which shall, when we know the proportions in a given generation n, give us the proportions in the next following generation $n+1$.

Examples:

(7) Random mating, population in nth generation: rAA$+s$Aa$+t$aa. Population in generation $n+1=(s+2r)^2$AA $+\ 2(s+2r)(s+2t)$Aa $+$ $(s+2t)^2$aa.

(8) Assortative mating; population in the generation n is rAA$+$ sAa$+t$aa. In the next generation $(n+1)$ the population is $(2r+s)^2$AA$+$ $(4rs+2s^2)$Aa$+(s^2+4rt+4st)$aa.

(9) Selection of dominants; parental population in generation n is rAA$+s$Aa$+t$aa. Population in the next generation $(n+1)$ is $(2r+s)^2$ AA$+2s(2r+s)$Aa$+s^2$aa.

C. In many cases the constitution of the population changes from generation to generation, as long as the given system of breeding lasts, and it is possible to give a simple formula in terms of n, the number of generations that the given system has been followed, from which formula the proportions in generation n can be directly computed.

Examples:

(10) Assortative mating (dominants with dominants, recessives with recessives); original parents are progeny of the cross AA × aa, so that their proportions are AA+2Aa+aa. After n assortative matings the proportions are: $(n+1)$AA$+2$Aa$+(n+1)$aa.

(11) Assortative mating; parental population is the result of the random mating of $r\mathrm{AA}+t\mathrm{aa}$. After n assortative matings the population is $r(r+nt)\mathrm{AA}+2rt\mathrm{Aa}+(n+1)t^2\mathrm{aa}$.

(12) Assortative mating; parental population all Aa. After n assortative matings the population is $n\mathrm{AA}+2\mathrm{Aa}+n\mathrm{aa}$.

(13) Assortative mating; parental population AA and Aa in equal numbers. After n matings the population is $(3n+6)\mathrm{AA}+6\mathrm{Aa}+n\mathrm{aa}$.

(14) Selection of dominants; parental population $\mathrm{AA}+2\mathrm{Aa}+\mathrm{aa}$. After n generations the population is $(n+1)^2\mathrm{AA}+2(n+1)\mathrm{Aa}+\mathrm{aa}$.

(15) Self-fertilization; parents all Aa. After n self-fertilizations the population is $(2^n-1)\mathrm{AA}+2\mathrm{Aa}+(2^n-1)\mathrm{aa}$.

(16) Self-fertilization; population at the beginning, $r\mathrm{AA}+s\mathrm{Aa}+t\mathrm{aa}$. After n generations the population is $[r(2^{n+1})+s(2^n-1)]\mathrm{AA}+2s\mathrm{Aa}+[t(2^{n+1})+s(2^n-1)]\mathrm{aa}$.

D. In most cases the constitution of the population changes from generation to generation, giving a series of values not readily expressed in terms of n (the number of generations) alone. In these cases diverse systems or diverse parents give different series of values, almost all of which are examples of certain simply derived mathematical series, or of their combinations. The results are therefore best presented by giving first these fundamental series, each with its designation. Then the results of any number of generations of any system of breeding can be given by designation of the series which it forms. The main series, with their first 10 terms, are given in the following table:

Table of the Fundamental Series in Mendelian Breeding

SERIES	HOW FORMED												
n	$x+1$	0[1]	1	2	3	4	5	6	7	8	9	10	
B	2x	1	2	4	8	16	32	64	128	256	512	1024	$(=2^n)$
C	2x+1	0	1	3	7	15	31	63	127	255	511	1023	$(=2^n-1)$
D	2x−1		2	3	5	9	17	33	65	129	257	513	$(=2^{n-1}+1)$
E	2x+1			2	5	11	23	47	95	191	383	767	$(=2^n-2^{n-2}-1)$
F	Sum of two preceding	0	1	1	2	3	5	8	13	21	34	55	
G	2x+1, then 2x−1	0	1	1	3	5	11	21	43	85	171	341	$(=B_{n-1}-G_{n-1})$
H	G−F	0	0	0	1	2	6	13	30	64	137	286	
I	B−G−F	1	0	2	3	8	16	35	72	150	307	628	$(=G_{n+1}-F_n)$
J	B_n-F_{n+1}	0	1	2	5	11	24	51	107	222	457	935	
K	B_n-F_{n+2}	0	0	1	3	8	19	43	94	201	423	880	
L	$B_n-F_{n-1}-G_{n-1}$		2	2	6	11	24	48	99	200	406	819	
M	$2x+F_{n-1}$	2	4	9	19	40	83	171	350	713	1447	2952	$(=3B_n-F_{n+2})$

The table gives in the first column a letter by which the series will be designated (as *n*, B, C, J, etc). In the next column is given the method by which any given term of the series is formed from the preceding term, this preceding term being designated *x*. Thus, under C, the expression, $2x+1$ means that to obtain any term of the series, we double the preceding term and add 1. In some of the later series the derivation here given shows that the given series is formed by some combination of two previous series. Thus, series H is formed by substracting from any term of G the corresponding term of F. Any given term of a series is designated by a subscript representing the corresponding term of the series *n*: thus, F_4 means the fourth term of the series F (it is therefore 3); J_n means the *n*th term of the series J; D_{n+1} means the $(n+1)$th term of D; etc. At the end of each series is given certain other information; showing as a rule how any term of the series may be otherwise designated; or another method of derivation.

The results of any system of breeding that falls in the present group may be given by specifying which series of the table it gives and what terms of the series correspond to a given generation. Examples will make this clear:

(17) Self-fertilization, beginning with a cross between AA and Aa. After *n* self-fertilizations the proportions of the resulting population are given by the following:

$$AA = \frac{E_{n+2}}{B_{n+2}}. \qquad Aa = \frac{1}{B_{n+1}}. \qquad aa = \frac{C_n}{B_{n+2}}.$$

This signifies that for any generation *n*, the proportion of AA will be found by dividing the term $(n+2)$ of the series E by the term $(n+2)$ of the series B. So, after one self-fertilization the proportion that are AA will be 5/8; after two, 11/16; after eight, 767/1024. After five generations the proportion that are Aa will be 1/64; of aa, 31/128; and so on.

(18) Self-fertilization in a typical Mendelian population composed of AA+2Aa+aa. After *n* generations:

$$AA = \frac{C_{n+1}}{B_{n+2}}. \qquad Aa = \frac{1}{B_{n+1}}. \qquad aa = \frac{C_{n+1}}{B_{n+2}}.$$

(19) Inbreeding, brother by sister, in the progeny derived from a cross of AA by aa. After *n* generations:

$$AA = \frac{J_n}{B_{n+1}}. \qquad Aa = \frac{F_{n+1}}{B_n}. \qquad aa = \frac{J_n}{B_{n+1}}.$$

(20) Inbreeding, brother by sister, when the original cross was AA by Aa. After *n* inbreedings:

$$AA = \frac{M_{n+1}}{B_{n+3}}. \qquad Aa = \frac{F_{n+3}}{B_{n+2}}. \qquad aa = \frac{K_{n+1}}{B_{n+3}}.$$

(21) Inbreeding, parent by offspring; parents AA by Aa; progeny mated back continually to Aa:

$$AA = \frac{D_{n+1}}{B_{n+2}}. \qquad Aa = \frac{1}{2}. \qquad aa = \frac{C_n}{B_{n+2}}.$$

(22) Sex-linked characters; a cross between AA and a−, then breeding by random mating. After *n* random matings:

$$\text{Males:}\quad A- = \frac{G_{n+1}}{B_n}\text{: thus, 1/2, 3/4, 5/8, etc.}$$

$$a- = \frac{G_n}{B_n}\text{: thus, 1/2, 1/4, 3/8, etc.}$$

$$\text{Females:}\quad AA = \frac{G_n \times G_{n+1}}{B_{2n-1}}\text{; thus 1/2, 3/8, 15/32, 55/128, etc.}$$

$$aa = \frac{G_n \times G_{n-1}}{B_{2n-1}}\text{; thus 0, 1/8, 3/32, 15/128, etc.}$$

$$Aa = 1 - AA - aa\text{; thus 1/2, 4/8, 14/32, 58/128, etc.}$$

Males and females equal in number.

(23) Sex-linked characters; a cross between AA and a−, then inbreeding, brother by sister. After *n* inbreedings:

$$\text{Males:}\quad A- = \frac{G_{n+1}}{B_n}. \qquad a- = \frac{G_n}{B_n}.$$

$$\text{Females:}\quad AA = \frac{I_{n+1}}{B_{n+1}}. \qquad Aa = \frac{F_{n+1}}{B_n}. \qquad aa = \frac{H_{n+1}}{B_{n+1}}.$$

(24) Sex-linked characters; cross between AA and a−, then inbreeding, daughters mated to fathers, sons to mothers (as could be carried out only in certain lower organisms). After *n* inbreedings:

$$\text{Males:}\quad A- = \frac{G_{2n+1}}{B_{2n}}\text{: thus, 3/4, 11/16, 43/64, etc.}$$

$$a- = \frac{G_{2n}}{B_{2n}}\text{: thus, 1/4, 5/16, 21/64, etc.}$$

$$\text{Females: AA} = \frac{G_{2n+2} - 3^{n-1}}{B_{2n+1}}; \text{ thus } 4/8, 18/32, 76/128, \text{ etc.}$$

$$\text{Aa} = \frac{3^{n-1}}{B_{2n}}; \text{ thus } 1/4, 3/16, 9/64, \text{ etc.}$$

$$\text{aa} = \frac{G_{2n+1} - 3^{n-1}}{B_{2n+1}}; \text{ thus } 2/8, 8/32, 34/128, \text{ etc.}$$

Males and females equal in number.

In all these series the proportions of any given sort gradually approach a limit; thus in the last case (24) the limit for A– is 2/3; for a–, 1/3; for AA it is 2/3; for aa, 1/3; for Aa the limit is 0.

In the full paper there is a systematic presentation of formulae for the proportions in any generation, resulting from any of the main types of breeding, and with any of the common types of parental population, giving eighty-two numbered sets of formulae. In each case the limit approached is given, together with the number of generations of breeding required to come within 1% of that limit. The complete paper appears in the first number of *Genetics*.

ON THE EFFECTS OF FEEDING PITUITARY BODY (ANTERIOR LOBE) SUBSTANCE, AND CORPUS LUTEUM SUBSTANCE TO GROWING CHICKS

By Raymond Pearl

BIOLOGICAL LABORATORY, MAINE AGRICULTURAL EXPERIMENT STATION

Received by the Academy, December 17, 1915

In connection with the studies of Pearl and Surface[1] on the effect of pituitary substance on the function of egg production in the domestic fowl it seemed desirable to see whether the *initial* activation of the ovary could be accelerated by means of this substance. A pullet of a good producing strain hatched at the proper time and well grown will begin to lay when from five to six months of age ordinarily. Can such pullets be brought to sexual activity and laying any earlier if regularly fed pituitary substance?

Forty-five pure-bred Barred Plymouth Rock pullets, all hatched the same day (April 29, 1915) were divided into three lots of 15 each. They were so chosen that the total weights of the three lots, and thus the average weight per bird, were identical at the beginning of the experiment. Further great pains were taken to get birds of the same stage of maturity and physiological development so far as could be determined. Each bird in one lot (A) received *per os* 0.082 g. per day of pituitary body (anterior lobe) substance. This was administered in No. 2 gelatine capsules with lactose as a diluent. Each bird in the second lot (B) received *per os* the same amount (0.082 g.) per day of dessicated corpus luteum substance from pregnant cows, again administered in No. 2 gelatine capsules with lactose as a diluent. Finally lot C was a control, the birds receiving no capsules. All the birds were housed in the same house, and given the same food and care except for the capsule feeding as above noted.

The experiment began August 13, 1915. The growth of the birds is shown in the table.

Table showing the weights at various intervals of birds fed pituitary body (anterior lobe) substance and of those fed corpus luteum substance.

DATE	LOT A. (PITUITARY)		LOT B. (CORPUS LUTEUM)		LOT C. (CONTROL)	
	Total weight	Mean weight per bird	Total weight	Mean weight per bird	Total weight	Mean weight per bird
	kilos	*grams*	*kilos*	*grams*	*kilos*	*grams*
Aug. 13. Initial weight...	16.78	1112.2	16.78	1112.2	16.78	1112.2
Aug. 28	22.09	1472.6	21.91	1460.6	22.14	1475.7
Sept. 4	23.81	1578.6	23.59	1571.1	24.18	1611.8
Sept. 11	24.68	1645.0	24.22	1614.8	25.40	1693.4
Sept. 18	25.72	1714.6	25.45	1696.4	26.81	1787.2
Sept. 25	27.12	1808.3	25.63	1708.5	28.26	1883.9

From this table it appears that:

1. Both pituitary body (anterior lobe) substance and corpus luteum substance retarded the growth of the birds in this experiment. Throughout the experiment all of the birds were in a perfectly healthy and active condition. It was impossible to detect any differences in these respects between the three lots. There appears to be no means of accounting for the growth differences observed except as a result of the organ substances fed.

2. The retardation of growth in the case of the birds fed corpus luteum substance (lot B) is greater than that in the lot fed pituitary substance (lot A). The differences are very small and not separately significant until the September 11 and later weighings, from which time they are relatively large.

3. After 43 days of organ substance feeding of birds starting at the same body weight those birds fed pituitary substance average 4.01% less in body weight, and those fed corpus luteum substance average 9.31% less in body weight, than normal controls.

A parallel experiment to that reported above has been run with male birds, and gave essentially the same results. Since the numbers used were too small to give reliable figures, they are not published.

Leaving now the matter of growth in body weight we may turn to a consideration of the attainment of sexual maturity in these three lots of birds. There was practically no difference between the three lots of birds in respect to time of beginning laying. The small differences exhibited cannot be regarded as significant, considering the nature of the character.

Several interesting points are brought out by this experiment. In the first place the results with pituitary body substance confirm all of our earlier work with this substance so far as concerns activation of the ovary. There is no evidence that the administration of pituitary substance hastened in any way the initial activation of the pullet ovary. The pituitary birds did not in fact begin to lay as soon as those receiving corpus luteum substance, though the two day difference between the lots cannot be regarded as significant. We have now tested the effect of pituitary (anterior lobe) substance on the ovary in three different physiological states, viz., (*a*) completely resting adult ovary during moult, (*b*) adult ovary in laying condition but with declining fecundity[2] rate, and (*c*) the inactive, immature ovary of the young pullet. In none of these physiological states has there been the slightest evidence that the pituitary has activated or accelerated the activity of the ovary in any manner or degree.

In the second place it appears that both pituitary (anterior lobe) substance and corpus luteum substance retard growth in the chick, but without affecting the attainment of sexual maturity (egg laying). The pullets in lots A and B began to lay at the same time they would have had they not received organ substance, but their body weight at the onset of laying was from 4 to 9% smaller than it would normally have been. These results are of interest in connection with the experiments of Gudernatsch[3] in feeding thyroid and thymus substance to growing tadpoles, though there is no evidence in the present experiments of differentiation being accelerated. It merely is *not* retarded, while body growth *is* retarded by pituitary and corpus luteum substance. It will be a matter of much interest to extend the period of feeding these substances, particularly corpus luteum, into earlier life. I propose to do this next year. For students of growth a means is afforded in corpus luteum feeding of notably retarding body growth without disturbing, so far as yet appears, the normal physiology or physiological development.

The complete paper describing these results and discussing them further will shortly be published elsewhere.

---

[1] Pearl and Surface, *J. Biol. Chem.*, 19, 263–278 (1914); and *Ibid.*, 21, 95–101 (1915).

[2] Pearl, R., *J. Biol. Chem.* (*in press*, 1916).

[3] Gudernatsch, J. F., *Arch. Entw.-Mech.*, 35, 457 (1912); also *Amer. J. Anat.*, 15, 431–478 (1914).

A PRELIMINARY REPORT ON FURTHER EXPERIMENTS IN INHERITANCE AND DETERMINATION OF SEX

By Richard Goldschmidt
(Of the Kaiser Wilhelm Institut für Biologie, Berlin)
OSBORN ZOOLOGICAL LABORATORY, YALE UNIVERSITY

Received by the Academy, November 29, 1915

In two former papers[1] the interesting 'gynandromorphism' produced by crossing the European and the Japanese races of the gipsy-moth (Lymantria dispar) was described and an experimental analysis of the phenomenon was attempted. From the data obtained important conclusions on the sex-problem in general were drawn. Although the main points seemed to be clear, a series of questions still remained open. One of them was, that the (apparently) same kind of crosses did not give the same results, if the material used had a different origin. It could be regarded as practically certain that the chief result, viz., the appearance of gynandromorphism in certain crosses found its right explanation in the hypothesis of a quantitatively different behavior or a different potency of the male sex-factors in the different races. Some of the experiments led me to suspect that this potency varied with the geographical distribution of the moth. Therefore it was one of my aims during a sojourn in Japan, to study the behavior of different local forms in that country in various crosses *inter se* and with different European forms.[2] These experiments are in no way to be regarded as completed, but the results so far obtained are so interesting and seem to bring the definitive solution of the problem so near, that a preliminary report on a part of them may be made.

A few words are first needed about the terminology. In previous papers I have used the term of gynandromorphism to indicate the sexual abnormalities produced in the racial crosses of these moths. It seems, however, no longer advisable to use this term, as it is applied more or less generally to quite a different phenomenon, *i.e.*, to individuals showing a mosaic of the characters of both sexes. In such a gynandromorph—see for example Boveri's late analysis of the Eugster bees[3]—a given organ or complex of cells is either male or female. But this is not the case with my moths; the entire individual represents a definite, quantitatively fixed stage intermediate between the two sexes. If we represent femaleness and maleness as the end points of a series, say one as 0 and the other as 100, a given specimen of these 'gynandromorphic' moths would be represented by the points 12 or 35, etc. These animals do not represent a mixture of the characters of the two sexes, but a definite point between the two extremes, maleness and femaleness. In the former papers some characters were thought to be a mosaic, viz., the color of the wings, which shows in certain stages definite patches of male and female design. It is now clear that this is but a consequence from the physiology of pigmentation, a fact that later will be the starting point for important theoretical discussions. So it seems advisable to introduce a new term to designate the phenomenon treated in these experiments. I shall call in the future the different sexual intermediates *intersexes;* female intersexes, if they are genetically females, but transformed to some stage towards maleness, and male intersexes if the transformation goes in the opposite direction; the whole phenomenon being called intersexualism.[4]

Before describing the new results a short account might be given of the different stages of intersexualism so far produced in the experiments. Every single step has now been bred from a normal female through the different grades of female intersexes to a normal male; also the steps starting from the normal male through male intersexes towards the female up to three-fourths of the way. Every single one of these steps can be produced now at will by crossing the right combination of races. Female intersexualism begins with animals, which show feathered antennae of medium size (feathered antennae are a male character) but which are otherwise entirely female in appearance except that they produce a smaller number of eggs, which are fertilized normally. In the next stage patches of the brown male pigment appear on the white female wings, in steadily increasing quantity. The instincts are still female, the males are attracted and copulate. But the characteristic egg sponge laid by the animal contains nothing but anal hairs, in spite of the fact that the abdomen is filled with ripe eggs. In the next stage whole sections of the wings show male coloration, with cuneiform female sectors between, the abdomen becomes smaller, contains fewer ripe eggs, the instincts are only slightly female, the males are attracted very little, and reproduction is impossible. In the next stage the male pigment covers practically the whole wing, the abdomen is almost male, but still contains ovaries with a few ripe eggs, the instincts are intermediate between males and females. Then follow very male-like animals, which still show in different organs their female origin and have rudimentary ovaries. They lead to practically male-like animals, called 'female-males' in my former paper, containing gonads which exhibit all transitional stages from an ovary to a testis. The end of the series is formed by males, which show in some minor characters, such as the shape of wings, still some traces of their female origin.

The series of the male intersexes starts with males showing a few white female spots on their wings. These become larger and larger, the amount of brown pigment correspondingly decreasing, finally (in extreme specimens) leaving only a few granules along the veins. Hand in hand with this the abdomen increases in size, reaching in the most extreme case two-thirds of the female size (without containing eggs!). The same is true for the instincts, which become more and more female. Further, the copulatory organs, which show in the female intersexes also a complete series of gradations from femaleness to maleness, and finally in the testis, which shows the first steps of transformation into an ovary.

There is another new point to be mentioned in this connection, as the fact that it was unknown is probably responsible for some errors in my former papers. It has been noticed that there are Japanese races, which contain a wing-pigmentation factor in the female sex; and the effect produced can be increased in certain crosses. In this way females with pigmented wings may arise, which may be mistaken for intersexes, and if this phenomenon occurs together with slight intersexualism, low grade intersexes may be mistaken for high grade ones. The further investigation of this will be of importance.

Going over now to the breeding results, we shall describe first only those relating to the female intersexes. It was found that there are in the European as well as in the Japanese forms, definite races possessing a special potency of the male sex-factors. If we cross forms of like or similar potency, the offspring is normal. But if we cross races with different potency of the male factors we get in F_1 female intersexes, provided the mother belongs to the race of lower potency. The degree of intersexuality depends of course upon the difference of the potencies. The European races so far known to me all show low potency; and in the Japanese races all grades are found from the lowest to the highest potency of those factors. If now any European race is crossed in any direction with one of the 'weak' Japanese races (the races K, Fu, M, H of my records) only normal females appear in the offspring. The same naturally holds true for crosses of any of these forms *inter se*. But all European females and also the females of the weak Japanese races, crossed with males of the 'strong' Japanese races (*i.e.*, G, O, A) ought to yield female intersexes, the grade of intersexuality depending upon both races involved in the cross. In detail the main results are the following; the F_1 males being left out of account:

1. All possible F_1 combinations between the weak Europeans S and F and the weak Japanese K, Fu, M, H contain only normal females.

2. Males of the moderately strong Japanese race G crossed with the very weak European females F produce nothing but high grade female intersexes ranging almost up to the above quoted 'female-males.' The same males produce, if crossed with the somewhat less weak European race S, nothing but medium female intersexes. The same males again crossed with the still less weak Japanese race H give in F_1 low grade female intersexes; and finally from the cross between these males and the highest of the weak Japanese races, K, we get only slight female intersexualism. There is to be added that the intersexuality affects every single individual that is genetically (one X-chromosome) a female.

3. Another of the Japanese races, which has a higher potency than G, produces in similar crosses with the weak European races the highest grade of female intersexuality, the so-called 'female-males.'

4. If the males used in the crosses belong to the strongest known Japanese races A and O (there is only a slight difference between these two which may be neglected here) and the females to the weak European races F and S, or the Japanese race H, all individuals genetically being females become transformed entirely into males. It may be added here that all results are based on fair numbers. For instance for this cross there are about 14 cultures with over 1000 individuals. In many cultures mortality is exactly recorded.

5. The following is to be regarded as an *experimentum crucis* for the correctness of the explanation given above: The Japanese race K proved to have a low potency of the male sex-factors, as crosses with European females produced normal offspring. Furthermore the same race K proved to be somewhat higher in potency than the European races by producing only the slightest degree of intersexuality in crosses with other males of the medium strong race G, whereas the European races gave in the corresponding cross with G-males up to high-grade intersexes. These same Europeans produce in the crosses with the strong races A and O nothing but males. Here we have an equation from which we should expect that in a cross between females from the races K and males A or O medium to high-grade intersexes will be produced. This is the actual result.

6. To these results may be added the following details:

a. The results are typical and, so far, without exception. They seem to be independent of external conditions as the same results were obtained identically (besides my former experiments carried on in Munich) in my cultures at the Bussey Institution, in Boston[6] and those of my assistant Mr. Seiler in Berlin.

b. The grade of intersexuality in one culture is subject to a regular and continuous variation around a mean. If we divide the distance between femaleness and maleness in 100 grades a given culture would show a symmetric variation of a certain range, say 20 grades, around a mean of 50 or 60, etc. What causes certain differences in the position of the modal class in sister cultures is not yet clear, but will be very important for the theoretical side of the question. It is to be expected that at the two end points of the series the individuals overlap on the normal, *i.e.*, respectively normal females and males. This was actually found in the border cases. If all individuals which are genetically females are changed into males some minus individuals may be recognizable by their somewhat female shape of the wings, proving that the variability extends into the normal, too. It is to be hoped that the statistical treatment of these facts, together with the F_2 results and the data regarding the male intersexes will open a way to an exact calculation of the relative values of potency.

c. In the crosses producing nothing but males, occasionally there appears a single normal female, hatching as the last individual of a given culture. It is very probable that we face here another case of non-disjunction (Bridges). A spermatozoon without X-chromosome ought to give a normal female with every egg (assuming naturally that the male factors are carried in the X-chromosomes). So far I have succeeded only in finding a single spermatocyte II with 30 instead of 31 chromosomes.

So far only the facts regarding the female intersexes have been recorded, and they form a strong support for the views of my earlier papers in regard to the sex-problem. But the new results about the male intersexes show that in one important point the hypotheses have to be changed. I stated there that the male intersexes appear in F_2 from the reciprocal cross, not giving female intersexes in F_1. The fact that exactly one-eighth of the males were intersexual corroborated strongly the Mendelian formula of sex-inheritance used by me, working with two pairs of sex factors. The new results give a different aspect to the facts:

1. The appearance of the male intersexes in the said crosses is a single event conditioned by the two races involved in the cross. The ratio $\frac{1}{8}$ is typical, too, only for the special combination. In other crosses any ratio between 0 and 50 percent may be typical.

2. A new fact of decisive importance is that male intersexes may be produced in F_1. So there were a few individuals in the crosses female K × male S and female O × male H. And in the cross: weak Japanese female K × weak Japanese male H, nothing but intersexual males appear in F_1.[6] These facts are of greatest importance for the whole question. Together with some other F_2 results and the insight in some former errors of interpretation they make it very probable that the female part of the formulae $F\,F\,M\,m = ♀$

F F M M = ♂ does not mendelize but is inherited plasmatically. This part too shows different potencies and the final result is the combined effect of both groups. However, it seems undesirable to insist on this point, as the decisive experiments, which may bring the complete solution, have not yet been accomplished.

---

[1] Goldschmidt, R., Erblichkeitsstudien an Schmetterlingen. I., *Zs. induct. Abstammungslehre.*, 7, (1912); and Goldschmidt, R., und Poppelbaum, H., idem. II. *Ibid.*, 11 (1914)

[2] The existence of such local forms could be shown by breeding experiments. The problem of the geographical races of this moth has been for many years the object of my principal studies. However, the results are not yet ripe for publication.

[3] Boveri, Th., Ueber die Entstehung der Eugsterschen Zwitterbienen., *Arch. Entw-Meck., Leipzig*, 41 (1915).

[4] The fact that male and female intersexes are different calls certainly for an explanation, but will not be discussed here. It may only be said that it is a question of the physiology of development.

[5] I wish to express my sincerest thanks to Professor Wheeler and all the members of the staff of the Bussey Institution, Harvard University, for their kindness in granting me the facilities for prosecuting my work.

[6] A similar cross with the same race H but another mother has been carried out already with the same result by Toyama's assistant, Dr. Machida, to whom therefore belongs the priority of this discovery. I am indebted to him, too, for the material of the race H. I do not know whether he has meanwhile published this result.

ON THE DEGREE OF INBREEDING WHICH EXISTS IN AMERICAN JERSEY CATTLE

By **Raymond Pearl** and **S. W. Patterson**

BIOLOGICAL LABORATORY, MAINE AGRICULTURAL EXPERIMENT STATION

Received by the Academy, December 17, 1915

In a series of 'Studies on Inbreeding' published during the past few years Pearl[1] has described in detail a method whereby it is possible to measure exactly the degree of inbreeding which exists in the pedigree of any particular individual animal. This is done by means of coefficients of inbreeding. These quantities may be defined as follows:

In the genetic passage from the $n + 1$'th ancestral generation to the n'th, or in other words the contribution of the matings of the $n+1$'th generation to the total amount of inbreeding involved in the production of an individual, the degree of inbreeding involved will be measured by the expression

$$Z_n = \frac{100\,(p_{n+1} - q_{n+1})}{p_{n+1}} \qquad (1)$$

where p_{n+1} denotes the maximum possible number of different individuals involved in the matings of the $n+1$ generation, and q_{n+1} the *actual* number of different individuals involved in these matings. Z_n may be called a *coefficient of inbreeding.* If the value of Z for successive generations in the ancestral series be plotted to the generation number as a base, the points so obtained will form a curve which may be designated as the *curve of inbreeding.*

It will be noted that the coefficient of inbreeding Z is the percentage of the difference between the maximum possible number of ancestors in a given generation and the actual number realized. The coefficient may have any value between 0 and 100. When there is no breeding of relatives whatever (that is, in the entire absence of inbreeding) its value for each generation is 0. As the intensity of the inbreeding increases the value of the coefficient rises.

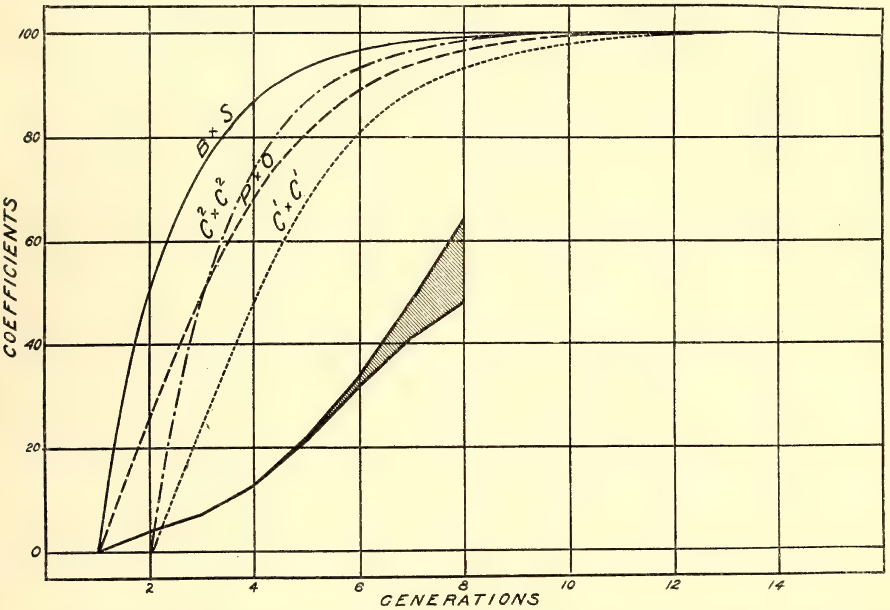


FIG. 1.—DIAGRAM SHOWING THE INBREEDING CURVES FOR A RANDOM SAMPLE OF THE GENERAL POPULATION OF AMERICAN JERSEY *COWS*. THE TWO HEAVY LINES GIVE THE UPPER AND LOWER LIMITING VALUES FOR THE SUCCESSIVE MEAN INBREEDING COEFFICIENTS. THE TRUE VALUE OF THE CURVE LIES SOMEWHERE IN THE RULED AREA BETWEEN THESE HEAVY LINES. FOR COMPARISON THE CURVES FOR CONTINUED BROTHER × SISTER (B × S), PARENT × OFFSPRING (P × O), AND FIRST COUSIN × FIRST COUSIN BREEDING ARE INCLUDED. $C^1 \times C^1$ DENOTES SINGLE COUSIN MATINGS, AND $C^2 \times C^2$ DOUBLE COUSIN MATINGS.

It is obvious that this method provides for the first time a means of examining statistically the degree of inbreeding which exists on the average in a population, or in any differentiated group of a population. It is the purpose of the present note to give in a very brief and preliminary way certain of the results of such a statistical examination of inbreeding in American Jersey cattle.

The first step in the treatment of the problem consisted in working out complete pedigrees to and including the eighth ancestral generation of 254 individual animals registered in the Herd Book of the American

Jersey Cattle Club. These animals were divided into four series as follows: (*a*) a random sample of the general population of males, (*b*) a random sample of the general population of females, (*c*) a selected sample of Register of Merit males, so chosen as to include the leading bulls in the recent history of the breed, these being bulls whose daughters were conspicuously excellent producers of milk and butter fat, (*d*) a selected sample of Register of Merit females so chosen as to include all of the cows which have recently made world's records in respect of butter fat production.

For reasons which cannot be gone into here, but will be presented in the detailed publication, it is impossible in the case of cattle to arrive at an absolutely exact value of the mean inbreeding coefficients. What we can do is to determine upper and lower limiting values, between which the true, and undeterminable value lies. Such upper and lower limiting values are presented in the table.

Table Showing Mean Coefficients of Inbreeding for American Jersey Cattle, for Both Random Samples of the General Population of Both Sexes and Samples of the Animals in the Register of Merit.

CLASS OR GROUP	NO. OF PEDIGREES INCLUDED IN SAMPLE	MEAN COEFFICIENTS OF INBREEDING							
		Z_0 Parents		Z_1 Grandparents		Z_2 Great-grandparents		Z_3 Great-great-grandparents	
		Lower limit	Upper limit	Lower limit	Upper limit	Lower limit	Upper limit	Lower limit	Upper limit
1. General population (random sample) bulls	67	0	0	1.87	1.87	5.41	5.41	11.01	11.21
2. Register of Merit bulls	63	0	0	1.59	1.59	5.36	5.62	14.88	15.91
3. General population (random sample) cows	61	0	0	4.10	4.10	6.97	6.97	12.50	12.59
4. Register of Merit cows	63	0	0	0	0	2.98	2.98	9.23	9.42

CLASS OR GROUP	NO. OF PEDIGREES INCLUDED IN SAMPLE	MEAN COEFFICIENTS OF INBREEDING							
		Z_4 Great-great-great grand-parents		Z_5 $(\text{Great})^4$ grandparents		Z_6 $(\text{Great})^5$ grandparents		Z_7 $(\text{Great})^6$ grandparents	
		Lower limit	Upper limit	Lower limit	Upper limit	Lower limit	Upper limit	Lower limit	Upper limit
1. General population (random sample) bulls	67	19.59	20.79	29.48	32.77	39.14	47.93	46.07	63.12
2. Register of Merit bulls	63	23.32	28.39	30.28	43.99	34.80	62.03	37.07	77.77
3. General population (random sample) cows	61	21.26	21.89	31.53	33.82	41.03	48.18	47.56	63.10
4. Register of Merit cows	63	15.33	16.27	24.63	28.53	33.12	44.06	39.22	61.45

From this table we get, for the first time, a definite quantitative conception of the average degree of inbreeding existing in any animal. In the complete publication which will shortly follow this note additional details will be given, including probable errors and other statistical constants.

From this analysis of Jersey inbreeding records two results, among others, stand out with particular clearness and significance. These are:

1. That American Jersey cattle, at the present time, may be said in general and on the average to be about one-half as intensely inbred, when account is taken of the eighth ancestral generation, as would be the case if continued brother × sister breeding had been followed. The *form* of the inbreeding curve is, however, very different in the two cases, the brother × sister curve being concave to the base line throughout, while the actual Jersey curves tend to have their principal curvature convex to the base.

2. That, in general and on the average, Register of Merit animals are *less* intensely inbred than the general population of Jersey cattle.

A detailed report of the work on inbreeding in Jerseys will be published shortly. The above notes are intended to be merely of a preliminary character.

---

[1] A list of references to these papers will be found in *Amer. Nat.*, **48**, 513 (1914).

UPPER LIMIT OF THE DEGREE OF TRANSITIVITY OF A SUBSTITUTION GROUP

By G. A. Miller

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF ILLINOIS

Received by the Academy, December 24, 1915

In a recent article published in the *Bulletin of the American Mathematical Society* (**22**, 68, 1915) I proved the following theorem: *A substitution group of degree $n = kp + r$, $p > k$ and $r > k$, which is neither alternating nor symmetric cannot be more than r times transitive unless $k = 1$ and $r = 2$.* From this theorem and the well known theorem that there is at least one prime number in the interval m (exclusive) and $2m - 2$ (inclusive), whenever $m \overset{=}{>} \frac{7}{2}$, it results directly that the degree of transitivity of a substitution group of degree n which does not include the alternating group of this degree must be less than $3\sqrt{n} - 2$. In the article noted above the condition $n > 12$ was added in stating this result but this condition can clearly be omitted.

The general expression $3\sqrt{n} - 2$ for an upper limit of the degree of transitivity of a substitution group of degree n which does not include the alternating group of this degree gives clearly a much smaller value for this upper limit, when n becomes large, then the expression $\frac{n}{3}+1$, which is found in the standard works of reference, for instance, in Pascal's *Repertorium der höheren Mathematik*, volume 1, (1910), page 211. It may therefore be of interest to note that it is possible to deduce from the italicized theorem noted above a general expression which gives still smaller values for the upper limit in question.

In fact, if p is a prime number in the interval $\sqrt{n}$ (exclusive) and $2\sqrt{n}-2$ (inclusive) then r can always be so selected that it does not exceed $p+n/p$. Since this expression is an increasing function of p when $p>\sqrt{n}$ and has the value $\frac{5}{2}\sqrt{n}$ when $p=2\sqrt{n}$, it results that the prime number p can be so chosen that r is always less than $\frac{5}{2}\sqrt{n}-1$. That is, the degree of transitivity of *a substitution group of degree n which does not include the alternating group of this degree is always less than* $\frac{5}{2}\sqrt{n}-1$. For all integral values of $n>43$ this theorem gives a smaller upper limit for the degree of transitivity than the formula $\frac{1}{3}n+1$, and for all values of $n>4$ it gives a smaller upper limit than $3\sqrt{n}-2$.

THE EXTENSION OF THE MONTANA PHOSPHATE DEPOSITS NORTHWARD INTO CANADA

By Frank D. Adams and W. J. Dick

COMMISSION OF CONSERVATION OF CANADA

Received by the Academy, January 10, 1916

With the development of agriculture in Western Canada there will arise in the not distant future an insistent and ever increasing demand for fertilizers and manures. Phosphoric acid is one of the most important constituents of such fertilizers. While no deposits of phosphates have hitherto been found in Western Canada, great beds of phosphate of lime have in recent years been discovered in Utah, Idaho, Wyoming, and Montana.

It was for the purpose of ascertaining whether these phosphate-bearing rocks extend northward into Canada that the present investigation was undertaken by the Commission of Conservation of Canada.

The phosphate beds from Utah to Montana occur at a certain definite horizon, usually known as the Phosphoria Formation, which is found near the summit of the Pennsylvanian subdivision of the Carboniferous system. This Pennsylvanian is usually composed of four formations, the phosphate lying at the base of the highest of these members and being usually associated with bands of chert.

The phosphate deposits in the United States which are
Canadian line are those which are found near Helena, Monta
north the great Louis overthrust comes in carrying the Can
Pre-Cambrian over the Carboniferous, which, however, reappea
Canadian boundary in the Flathead country.

The Carboniferous in Western Canada is practically confined to
Eastern slope of the Rocky Mountains in Alberta and the adjacent pa
of British Columbia.

An examination of these mountains was, therefore, made along three transverse lines of sections, viz., (1) The North Kootenay Pass; (2) The Crows Nest line of the Canadian Pacific Railway; (3) The main line of the Canadian Pacific Railway.

The North Kootenay Pass crosses the Rocky Mountains about 20 miles north of the international boundary line. A geological reconnaissance along this line was made by G. M. Dawson, when Geologist to the Commission appointed to establish the international boundary on the 49th parallel. A great development of Devono-Carboniferous rocks is shown in Dawson's section, both to the east and the west of the Flathead Valley. To the east a block of these strata is represented as being thrown down between two faults, this faulted block being capped by a series of 'red beds' regarded by Dawson as of Triassic or Permian age, having been correlated by him with a similar series of beds found by Meek in Utah. If this correlation is correct a phosphate-bearing horizon might be expected a short distance below the base of the 'red beds' in question. An examination of this district, however, showed that only one fault existed and that the section contained no rocks of Carboniferous age. A thick series of Devonian limestones directly overlie a thinner series of orange limestones and shales in which was discovered an abundant trilobite and brachiopod fauna (the Albertella fauna), which fixes the age of these strata as belonging to an horizon near the base of the middle Cambrian. The underlying quartzites 'red beds' and limestones, are, therefore, of lower Cambrian or Pre-Cambrian age. The proper horizon for the phosphate of lime does not exist, therefore, in this portion of the North Kootenay Pass.

An examination of the Crows Nest Pass section shows that here on Turtle Mountain only Devonian or Lower Carboniferous limestones are present, and these are directly overlain by a strata of Jurassic age.

In the third section, that on the main line of the Canadian Pacific Railway where this crosses the Rocky Mountains Park near Banff, the Carboniferous occurs in great volume and with a development very similar to that in Montana. There is the fourfold division as shown who has mapped this area for the Geological Survey of Canada. stratigraphical and paleontological considerations the authors were led to the conclusion that the horizon equivalent to the Phosphoria formation of the south was that at the contact between the Rocky Mountain quartzite and the Upper Banff limestone of Allan's section. These formations are repeated by faulting and appear as a set of parallel bands which cross the area four times in a general northeast and southwest direction. Forty Mile Brook crosses the contact three times, and a search was accordingly made in the shingle and loose blocks carried down by the stream, for 'float' of phosphate rock. This was found in the stream at the foot of Stony Squaw Mountain. The phosphate rock is of a hard compact variety which somewhat resembles basalt, a type found in certain of the Montana deposits. It contains 54% of tricalcic phosphate. An examination of the mountain itself showed that a heavy band of chert occurs along the contact of the Rocky Mountain quartzite and the Upper Banff limestone, and a chemical examination showed that this chert in all cases carries phosphoric acid in small amount. A thin bed of phosphate of lime was also found on the mountain in association with this chert.

The discovery of this phosphate rock was reported to the director of the Geological Survey of Canada and Mr. DeSchmidt was directed to make a detailed examination of the phosphate bearing horizon for the purpose of ascertaining whether beds of commercial value could be discovered. This gentleman had discovered two beds of which the larger was two feet in thickness, when his examination was brought to a close by the snowfall. The search, however, will be continued by the Geological Survey of Canada next spring.

CONTENTS

Page

Astronomy.—A Possible Origin for Some Spiral Nebulae. *By George F. Becker* 1

Soil Chemistry.—A Peculiar Clay from near the City of Mexico . . . . . . *By E. W. Hilgard* 8

Astronomy.—Studies of Magnitudes in Star Clusters, I. On the Absorption of Light in Space . . . . *By Harlow Shapley* 12

Astronomy.—Studies of Magnitudes in Star Clusters, II. On the Sequence of Spectral Types in Stellar Evolution . . . . . . *By Harlow Shapley* 15

Physics.—Experimental Evidence for the Essential Identity of the Selective and Normal Photo-Electric Effects . *By R. A. Millikan and W. H. Souder* 19

Geophysics.—Concomitant Changes in Terrestrial Magnetism and Solar Radiation . . . . *By L. A. Bauer* 24

Geology.—Sub-Marine Solution of Limestone in Relation to the Murray-Agassiz Theory of Coral Atolls . . . . . . *By Alfred Goldsborough Mayer* 28

Botany.—The Archegonium and Sporophyte of *Treubia Insignis* Goebel . . . *By Douglas Houghton Campbell* 30

Anthropology.—Brief Notes on Recent Anthropological Explorations Under the Auspices of the Smithsonian Institution and the U. S. National Museum . . . . *By Aleš Hrdlička* 32

Physiology.—A Theory of Nerve-Conduction . *By Alfred Goldsborough Mayer* 37

Anthropology.—Zuñi Culture Sequences . . . . *By A. L. Kroeber* 42

Genetics.—The Numerical Results of Diverse Systems of Breeding . . . . *By H. S. Jennings* 45

Zoology.—On the Effects of Feeding Pituitary Body (Anterior Lobe) Substance, and Corpus Luteum Substance to Growing Chicks . . . . *By Raymond Pearl* 50

Genetics.—A Preliminary Report on Further Experiments in Inheritance and Determination of Sex . . . . *By Richard Goldschmidt* 53

Genetics.—On the Degree of Inbreeding which Exists in American Jersey Cattle . . . . *By Raymond Pearl and S. W. Patterson* 58

Mathematics.—Upper Limit of the Degree of Transitivity of a Substitution Group . . . . *By G. A. Miller* 61

Geology.—The Extension of the Montana Phosphate Deposits Northward into Canada . . . . *By Frank D. Adams and W. J. Dick* 62

VOLUME 2 FEBRUARY, 1916 NUMBER 2

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES
OF THE
UNITED STATES OF AMERICA

Smithsonian Institution
JUN 24 1921
National Museum

EDITORIAL BOARD

ARTHUR A. NOYES, *Chairman*
EDWIN B. WILSON, *Managing Editor*
ARTHUR L. DAY, *Home Secretary*
GEORGE E. HALE, *Foreign Secretary*

J. J. ABEL
W. B. CANNON
J. McK. CATTELL
J. M. CLARKE
E. G. CONKLIN
J. M. COULTER
C. B. DAVENPORT
SIMON FLEXNER
E. B. FROST
R. G. HARRISON
W. H. HOLMES
J. P. IDDINGS
R. A. MILLIKAN
E. H. MOORE
ALEXANDER SMITH

Publication Office: Williams & Wilkins Company, Baltimore
Editorial Office: Massachusetts Institute of Technology, Boston
Home Office of the Academy: Washington, D. C.

Entered as second-class matter at the Post Office at Baltimore, Md., under the Act of August 24, 1912

INFORMATION TO CONTRIBUTORS

The Proceedings is the official organ of the Academy for the publication of brief accounts of important current researches of members of the Academy and of other American investigators, and for reports on the meetings and other activities of the Academy. Publication in the Proceedings will supplement, not replace, that in journals devoted to the special branches of science. The Proceedings will aim especially to secure prompt publication of original announcements of discoveries and wide circulation of the results of American research among investigators in other countries and in all branches of science.

Articles should be brief, not to exceed 2500 words or 6 printed pages. Technical details of the work and long tables of data should be reserved for publication in special journals. But authors should be precise in making clear the new results and should give some record of the methods and data upon which they are based. The viewpoint should be comprehensive in giving the relation of the paper to previous publications of the author or of others, and in exhibiting, where practicable, the significance of the work for other branches of science.

Manuscripts should be prepared with a current number of the Proceedings as a model in matters of form, and should be typewritten in duplicate with double spacing, the author retaining one copy. Illustrations should be confined to text-figures of simple character, though more elaborate illustrations may be allowed in special instances to authors willing to pay for their preparation and insertion. Particular attention should be given to arranging tabular matter in a simple and concise manner.

References to persons should include initials; citations of literature, numbered consecutively, should be placed at the end of the article; and short footnotes should be avoided. Indicate 'subject' for 'Contents.'

Papers by members of the Academy may be sent to Edwin Bidwell Wilson, Managing Editor, Mass. Institute of Technology, Boston, Mass. Papers by non-members should be submitted through some member.

Proof will be sent to authors only on their request, in order to avoid delay in publication. Authors are therefore requested to make final revisions on the typewritten manuscripts. The editors cannot undertake to do more than correct obvious minor errors.

Reprints should be ordered at the time of submission of manuscript. They will be furnished to authors at cost, approximately as follows:

Reprints of - -	2 pp.	4 pp.	6 pp	8 pp.	Covers extra
Charge for first 100 copies	$1.00	$1.30	$2.20	$2.20	$2.00
Charge for additional 100s	.25	.45	.85	.85	.50

INFORMATION TO SUBSCRIBERS

Subscriptions at the rate of $5.00 per annum should be made payable to the National Academy of Sciences, and sent to Williams & Wilkins Company, Baltimore, or Arthur L. Day, Home Secretary, National Academy of Sciences, Smithsonian Institution, Washington, D. C. Single numbers. $0.50.

Copyright, 1915, by the National Academy of Sciences

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES

Volume 2 FEBRUARY 15, 1916 Number 2

PERSONAL EQUATION AND STEADINESS OF JUDGMENT IN THE ESTIMATION OF THE NUMBER OF OBJECTS IN MODERATELY LARGE SAMPLES

By J. Arthur Harris

STATION FOR EXPERIMENTAL EVOLUTION, COLD SPRING HARBOR, N. Y.

Received by the Academy, December 20, 1915

Investigations in experimental evolution, involving as they necessarily do the recording of the characteristics of large numbers of individuals, afford the opportunity of obtaining quite incidentally extensive series of data upon errors of observation or estimation. The present paper states the major results of one such series of records obtained at the Station for Experimental Evolution during the course of investigation requiring the counting of large numbers of beans for mass weighings, germination tests, etc.

The chief advantages of such observations lie in the facts that they are carried out under quite natural working conditions, with none of the artificiality of the laboratory test, and that they represent far larger experiments than the average professional psychologist is able to make. Comprising as they do 28 experiments due to three observers all of whom carried on the work at considerably separated intervals over a period of two years, during which they made over 15,000 estimates with determined errors, the constants have a reliability which cannot possibly be attributed to short series.

The routine of this work was so organized that it consisted in part of a series of attempts to lay out samples of a definite number (25, 50, 100, or 200) which was constant for considerable periods. The error of each estimate was at once determined and recorded.

Two characteristics of the series of errors of estimation made by the three observers are here considered—personal equation and steadiness of judgment.

Smithson
JUN 24
National Mu

By personal equation we understand a definite bias in a given direction. In a series of estimates by an observer the errors in excess of the true value may be no more numerous and no greater in amount than those in defect. The average of the deviation of the estimates from the true number of objects will then be 0 plus or minus a small amount due to the errors of random sampling. Such an observer may be said to have no personal equation. Other individuals, however, may have a definite tendency to err on one side of verity in their evaluations. Such may be said to have a positive or a negative personal equation, as the case may be. Personal equation is measured by the mean, regarding signs, of the deviation of the samples from their ideal value.

But quite without reference to personal equation, one observer may be more erratic than another, estimating now far too high, now far too low. By steadiness of judgment we mean consistency in estimation as measured by the closeness with which the errors of estimation cluster around their mean value. Steadiness of judgment may be expressed in the absolute terms of the standard deviation of the errors of estimation about their mean (S.D.), or in the relative terms of the coefficient of variation (C.V.).

$$S.D. = \sqrt{\frac{\text{Sum of (Deviations from Mean)}^2}{\text{Total estimates}}}$$

which here is most easily calculated from the formula

$$(S.D.)^2 = \Sigma(d^2)/N - [\Sigma(d)/N]^2,$$

where Σ is the conventional summation sign, N is the number of estimates and d indicates the deviation of the estimate from the true number of objects, *i. e.*, the actual number laid out less the required number; and

$$C.V. = 100\,S.D./M,$$

where M is the constant number which the observer seeks to lay out plus or minus the observed personal equation, as the sign of the latter may indicate.

In much of the work in which personal equation is a factor the observer is not able to check his estimates against the true values, and so attempt at each successive observation to profit by his previous experience. In these experiments each observer made a persistent effort to improve. This was based on a knowledge of the immediately preceding errors, and consisted in a constant effort to lay out exactly the desired number of seeds. Thus the influence of experience upon both personal equation and steadiness of judgment may be determined from these data.

The problems here taken under consideration fall, therefore, into two groups. *First,* those having to do with the existence of personal equation in the estimation of the number of objects in samples and of differences in personal equation and steadiness of judgment from individual to individual. *Second,* the influence of previous experience upon personal equation and steadiness of judgment.

In the case of all three observers there is a slight but significant personal equation, which, notwithstanding the constant effort to improve, persisted throughout the two years during which the experiments were intermittently made. *In only three out of the twenty-eight experiments did the observer lay out samples of too small average size.* In a large number of the individual experiments the personal equation is certainly statistically significant (trustworthy) in comparison with its probable error.

From the experimental data taken as a whole one cannot conclude that there is any demonstrated difference between the personal equation of the three observers, although the figures do suggest that the bias of observer D may be slightly greater than that of either of the others. All have a bias in the direction of laying out more than the intended number of seeds, but that one is worse than another cannot be asserted.

In a high proportion of the individual experiments the differences between the personal equations of the three observers are statistically significant in relation to their probable errors. This is true in cases in which (for example) B has a greater personal equation than C, as well as in these in which she has a smaller personal equation.

The probable explanation of this result seems to be that the observers vary somewhat in their personal equation from experiment to experiment, just as they vary from time to time in general health, physiological tone, and mental vigor, alertness, or whatever one may care to call it. As a result of this variation from time to time one observer may show an abnormally high personal equation in a particular experiment in which a second observer shows an unusually low one. On another occasion the condition may be exactly reversed.

Thus *in an individual experiment* one observer may seem to be decidedly better than another. *In the long run* there is no *fully demonstrated* difference between them.

For steadiness of judgment there is no absolute standard comparable with the zero mean deviation of the personal equation. The data show a coefficient of variation of about 6.9% in the case of Observer B and C, and of 8.7% in the case of Observer *D*, who has a decidedly greater scatter in her estimates—that is a far less steady judgment—than either of the other observers. Indeed, in every individual experiment her standard deviation is higher than that of either of the two other experimenters.

Thus while there is no certain differentiation among the experimenters in personal equation, they differ distinctly in steadiness of judgment.

The influence of previous experience upon personal equation or steadiness of judgment may be most succinctly expressed in terms of the correlation between some quantitative measure of the amount of previous experience and the measures of personal equation and steadiness of judgment.

In these experiments the errors of observation were recorded in sequence. A group of fifty consecutive estimates with the accompanying determinations of the errors constituted a 'period.' In determining correlations one must deal with a number of subgroups for each period. It is most convenient to divide each half daily period of 50 estimates into five consecutive 'trials,' each of 10 estimates. For each of these 'trials' the mean personal equation and the standard deviation of the errors must be computed. Thus in obtaining the constants discussed here it was first necessary to compute 1520 means and 1520 standard deviations, which were then treated as units in computing the correlations.

The main problems involved in the question of experience are two: Is there a change in personal bias as a result of constant effort to improve and opportunity for improvement? Does the judgment become steadier, *i.e.*, does the observer make less erratic estimates, as a result of experience?

Both of these questions are really twofold. Is there an improvement from period to period? Is there an improvement within the period? In short, does the worker improve both from estimate to estimate in the same half daily period and also from period to period?

Personal equation seems to be remarkably little influenced by experience. In some experiments it increases, in others it decreases. The correlations may be either positive or negative in sign. Numerically they are generally low, and are in great part insignificant in comparison with their probable errors. Taken as a whole the results indicate a slight reduction in personal equation as a result of experience from period to period. Within the period there is no demonstrable influence of experience upon personal equation.

Steadiness of judgment is in rather conspicuous contrast with personal equation, in that it is unmistakably influenced by previous experience. The correlations between the number of previous trials within the period and steadiness of judgment and between the number of previous periods of experience and steadiness of judgment are numerically low, but almost without exception indicate that as experience becomes greater the scatter of the individual estimates about their mean value becomes less. Probably the rate of this change is not uniform, but is most rapid at first and then falls off.

The full data and discussion are appearing in two papers in the *Psychological Review.*

POLYPEPTIDE-HYDANTOINS

By Treat B. Johnson

SHEFFIELD SCIENTIFIC SCHOOL, YALE UNIVERSITY

Received by the Academy, December 24, 1915

Carbon dioxide is one of the products of decomposition when certain proteins undergo hydrolysis, under normal conditions, by digestion with aqueous solutions of acids and alkalis. Mörner[1] observed the formation of this acid anhydride during an investigation of the action of hydrochloric acid (sp. gr. 1.124) on horn at 92°, but no quantitative determination of the gas was made and no special significance attributed by him to its formation. Lippich[2] confirmed this observation several years later and showed that this anhydride is a normal product of hydrolysis of other proteins. He also made the important observation that the quantity actually formed is dependent on the nature of the hydrolytic agent employed. Quantitative determinations of the amount of the gas evolved from several proteins under specific conditions revealed the interesting fact that the maximum quantity is obtained when an alkali, as potassium or barium hydroxide, is used as the hydrolytic agent. In no case did Lippich fail to detect the presence of this substance among his products of hydrolysis. The actual percentages obtained by hydrolysis of five different proteins with potassium hydroxide solution are recorded in Table I.

For his acid hydrolyses Lippich used 33% sulphuric acid. When these same proteins were broken down by heating with this reagent entirely different analytical results were obtained. The combinations in the proteins productive of carbon dioxide were more resistant to hydrolytic changes, under these conditions, and the maximum amount of this gas was not obtained until after 25–27 hours digestion. The percentages found are recorded in Table I.

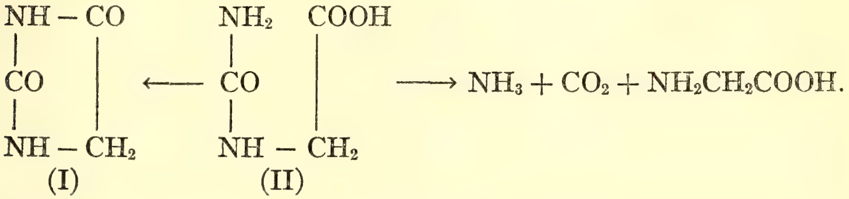
TABLE I

PROTEIN	CARBON DIOXIDE	
	By acid hydrolysis per cent	*By alkali hydrolysis per cent*
Parahämoglobin	0.352 0.332 0.370	2.09
Egg albumin	0.336	2.77
Serum globulin	0.364	2.41 2.38
Elastin	0.130	0.680
Keratin	0.372	4.63 4.53

In the course of investigations now in progress in this laboratory, dealing with the study of new hydantoin and thiohydantoin combinations of biochemical interest, I had occasion to determine whether gaseous products are evolved by hydrolysis of pure fibroin and also casein. I now find that both these proteins break down on hydrolysis with sulphuric acid with evolution of carbon dioxide and obtained from casein 0.35% of its weight in the form of this gas, while three quantitative experiments with fibroin gave the values 0.20, 0.27, and 0.25% respectively. When boiling 30% barium hydroxide solution was used for the hydrolysis of fibroin (the action of barium hydroxide on casein is being investigated), we obtained the values 1.08 and 1.05% of carbon dioxide. In other words, in the cases of the two proteins elastin and fibroin about five times as much carbon dioxide is produced by alkaline hydrolysis as is obtained by digestion with sulphuric acid. By inspection of Table I it will be seen that the greatest increase was obtained by hydrolysis of keratin. The results obtained with fibroin have special significance because the production of carbon dioxide in this case does not result from the breaking down of cystin, as this protein is free from sulphur.

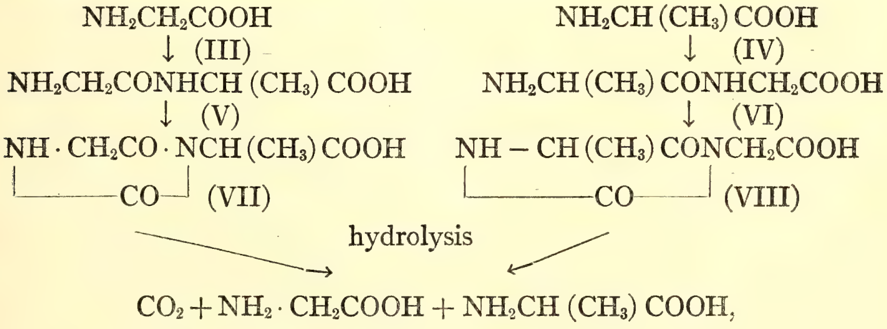
The marked difference in behavior towards acids and alkalis and the further fact that the quantity of carbon dioxide formed by hydrolysis is not proportional to the amount of arginine present in the respective proteins, suggest that urea combinations are the precursors of this gaseous hydrolytic product. Carbon dioxide would be a normal product of hydrolysis of such groupings and their destruction would be more easily accomplished by the action of alkalis than with acids. Lippich has advanced the idea that uramido acids, of which hydantoic acid II is the prototype, are the types of urea combinations which functionate in these changes. Such groupings, as is well known, are hydrolyzed by alkalis with formation of α-amino acids, carbon dioxide and ammonia.

On the other hand, when heated with acids, they easily undergo inner condensation with transformation into their corresponding anhydrides or *hydantoins* I. The latter compounds are very resistant to further hydrolysis with acids as has been shown by investigations in this laboratory.


NH – CO
CO
NH – CH₂
(I)
NH₂ COOH
CO
NH – CH₂
(II)
NH₃ + CO₂ + NH₂CH₂COOH.

While this explanation of Lippich's is in concordance with the chemical nature of hydantoic acids, on the other hand these compounds (II) do not represent the only types of uramido combinations which can undergo hydrolysis with formation of carbon dioxide and α-amino acids. One point needs to be taken into consideration here and that is the fact, that for every molecule of carbon dioxide formed by hydrolysis of a hydantoic acid II an equivalent amount of ammonia must also be obtained. Apparently this ratio $CO_2:NH_3$ has never been established quantitatively in the case of a single protein

Theoretically it is possible to link together two α-amino acids in a cyclic urea combination in such a manner that the resulting compound will undergo hydrolysis with production only of carbon dioxide and α-amino acids. If we choose glycocoll III and alanine IV as the two amino acids and combine them in the form of hydantoins as represented by formulas VII and VIII, two isomeric cyclic combinations will be obtained which will fulfil the above conditions. In other words, they are cyclic derivatives of the two isomeric dipeptides—glycylalanine V and alanylglycine VI respectively, in which the characteristic polypeptide groupings of the dipeptides have been preserved. Such combinations are the representatives of a new class of hydantoins,


NH₂CH₂COOH
(III)
NH₂CH₂CONHCH (CH₃) COOH
(V)
NH · CH₂CO · NCH (CH₃) COOH
CO
(VII)
NH₂CH (CH₃) COOH
(IV)
NH₂CH (CH₃) CONHCH₂COOH
(VI)
NH – CH (CH₃) CONCH₂COOH
CO
(VIII)
hydrolysis
CO₂ + NH₂ · CH₂COOH + NH₂CH (CH₃) COOH,

to which we have assigned the name—*polypeptide-hydantoins*. The hydantoin derivatives of the polypeptides—glycylphenylalanine IX, phenylalanylglycine X, glycyltyrosine XI and tyrosineglycine XII are now being investigated. Their formulas are:

(IX) $NH_2CH_2CO \cdot NHCH\,(CH_2C_6H_5)\,COOH$

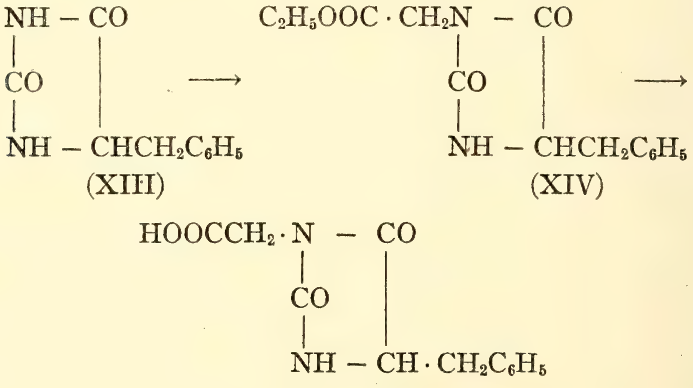
(X) $NH_2 \cdot CH\,(CH_2C_6H_5) \cdot CONH \cdot CH_2COOH$

(XI) $NH_2 \cdot CH_2CONH \cdot CH\,(CH_2C_6H_4 \cdot OH)\,COOH$

(XII) $NH_2CH\,(CH_2C_6H_4OH)\,CONHCH_2COOH$.

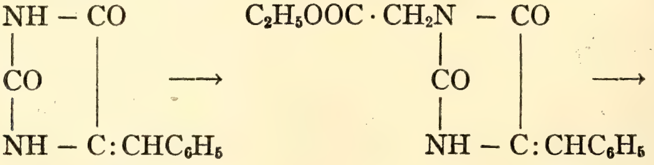
The polypeptide-hydantoin XV (m. 184–185°) has already been synthesized and two methods have been developed for its preparation which are apparently of general application:

(1) Alkylation of 4-benzylhydantoin[3] XIII with ethyl chloracetate with formation of the ester XIV (m. 157°), which is easily converted into the *polypeptide-hydantoin* XV by saponification.

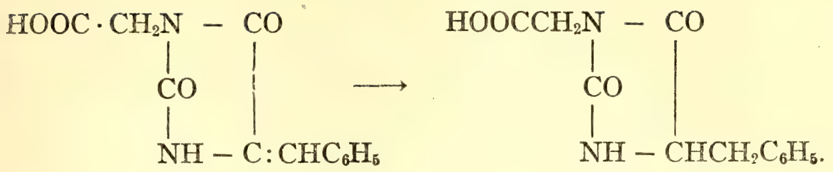

NH – CO
CO
NH – CHCH₂C₆H₅
(XIII)
C₂H₅OOC · CH₂N – CO
CO
NH – CHCH₂C₆H₅
(XIV)
HOOCCH₂ · N – CO
CO
NH – CH · CH₂C₆H₅

(XV)

(2) Alkylation of 4-benzalhydantoin[4] XVI, with ethyl chloracetate with formation of the ester XVII (m. 174°). The latter gives by saponification the unsaturated acid XVIII (m. 258°), which undergoes reduction smoothly giving the *polypeptide-hydantoin* XV.


NH – CO
CO
NH – C:CHC₆H₅
C₂H₅OOC · CH₂N – CO
CO
NH – C:CHC₆H₅

(XVI) (XVII)



(XVIII) (XV)

Other methods for synthesizing hydantoin compounds of this type (and their sulphur analogues) are being developed. This work will include not only the study of hydantoin derivatives of dipeptides but also tripeptide combinations of analogous constitution, and also an investigation of their behavior towards hydrolytic agents and enzymes.

[1] Mörner, *Zs. physiol. Chem.*, 34, 207 (1901).

[2] Lippich, *Ibid.*, 90, 441 (1914).

[3] Wheeler and Hoffman, *Amer. Chem. J.*, 45, 368 (1911).

[4] Ruhemann and Cunnington, *London, J. Chem. Soc.*, 75, 958 (1899); Ruhemann and Stapleton, *Ibid.*, 77, 246 (1900); Wheeler and Hoffman, *loc. cit.*

RECENT EXPLORATIONS IN THE CACTUS DESERTS OF SOUTH AMERICA

By J. N. Rose

DEPARTMENT OF BOTANICAL RESEARCH, CARNEGIE INSTITUTION OF WASHINGTON

Read before the Academy, November 16, 1915. Received January 6, 1916

When the cactus investigation for the Carnegie Institution of Washington was taken up a few years ago with Dr. N. L. Britton, a definite plan for field work in the deserts of North and South America was outlined. And since the Cactus family is confined to America, this meant a survey of its entire distribution.

The exploration of the deserts of South America was referred to me, and I have spent two seasons in exploring these regions; the first on the west coast, in Peru, Bolivia, and Chile, and the second on the east coast, in Brazil and Argentina. The exploration was confined chiefly to the deserts, as the cacti as a class are desert loving plants. A cactus desert, however, does not necessarily mean a desert like the Sahara or the desert of Arizona.

We found as a result of our investigation that South America contains six great cactus regions: (1) The desert of northern South America, including northern Venezuela and Colombia. This region we have not yet studied at first hand. (2) The great Peruvian and Chilean desert which extends from northern Peru to central Chile and from the Pacific Ocean to the top of the Andes, having a length of 2000 miles and a breadth of 50 to 300 miles. (3) The desert of Argentina, extending from the central part of Patagonia along the east side of the Andes well into Bolivia.

It resembles in its component parts the desert of Arizona. (4) The semi-arid region of eastern central Brazil, including most of Bahia and Pernambuco. It is very similar to the desert region of Santo Domingo and the typical genera are nearly all West Indian. (5) The desert of southern Brazil. This region we have not yet studied. (6) The states of Rio de Janeiro and São Paulo and the southern part of Minas Geraes, Brazil.

The last region is one of abundant rainfall and where all ordinary cacti would be killed. Here, however, the cacti not only grow on rocky knobs and along the beaches, but especially on the trunks of trees. Under the last named condition these plants find the same zerophytic conditions that their relatives find which grow in New Mexico and western Texas. They attach their roots to the bark of trees, their stems are reduced to long, shoe-string-like bodies, while the spines are reduced to hairs or they disappear altogether. About 40 of these epiphytic species, mostly belonging to the genus Rhipsalis, have developed in this region and they represent a most interesting group.

We have made large collections in South America in the fields visited; and we have ascertained that many species of cacti had never before been collected, and that many of those which had been collected had been poorly described and often wrongly classified.

ON THE ALBEDO OF THE PLANETS AND THEIR SATELLITES

By Henry Norris Russell

PRINCETON UNIVERSITY OBSERVATORY

Read before the Academy, November 17, 1915. Received January 10, 1916

1. The most suitable definition of albedo for astronomical purposes appears to be that proposed by Bond[1] in 1861, namely, the ratio of the whole amount of light reflected in all directions from a sphere illuminated by parallel rays to the amount of light incident on the sphere.

2. The albedo A of any planet, according to this definition, is the product of two factors, one of which depends only on the size of the planet, its distances from the earth and sun, and its brightness at the full phase, while the other depends upon the way in which the brightness varies at different phases. The first factor, which may be called p, can be calculated from known data for all the planets. Its value depends mainly upon the material of the surface, being high if this is nearly white, and low if it is dark colored. The second factor, q, can be computed only when the planet is observable over a considerable range of phase, so that the law of variation of its brightness with phase can be determined, and its values are known only for the moon and the planets

of the terrestrial group. For Jupiter and the remoter planets, however, estimates can be made which are not likely to be much more than 15% in error. This factor depends mainly upon the *texture* of the planet's surface, being high if this is smooth, and low if it is rough and covered with irregularities whose shadows darken considerable areas at phases remote from the full.

3. The relative brightness of the sun and stars, as seen from the earth, has been determined with surprising accuracy,—the results of several observers, by radically different methods, being in excellent agreement. The sun's stellar magnitude, on the Harvard scale, according to the mean of the observations of Zöllner, Ch. Fabry, Ceraski and W. H. Pickering, is -26.72 ± 0.04,—which is equivalent to saying that the sun appears to be 123 thousand million times as bright as a standard first magnitude star. The photographic magnitude of the sun, according to King and Birck, is -25.93, and its color index $+0.79$, agreeing very closely with the average for stars of similar spectrum (Class G).

4. The law of variation of the moon's brightness with phase is very well determined by the observations of J. Herschel, Bond, Zöllner, W. H. Pickering, King, Stebbins and Brown, and Wislicenus,—the results of all seven agreeing satisfactorily with a mean curve. The full moon is 8.7 times brighter than the first quarter, and 10.0 times brighter than the last quarter. The remarkable falling off in brightness between the full and half moon shows that the lunar surface must be very rough, as was first pointed out by Zöllner. The difference between the waxing and waning moon arises from the greater extent of the dark *maria* on the eastern half of the visible disk (as Stebbins has shown).

5. The results of different observers for the brightness of the mean full moon, compared with the sun or stars, are discordant. J. Herschel's observations[2] have been reduced anew, and an error which had crept into the earlier reduction corrected. The weighted mean of those determinations which are not obviously affected by systematic error makes the visual magnitude -12.55, and the ratio of sunlight to mean full moonlight 465,000, —with an uncertainty of fully 10%. The photographic magnitude, -11.37, has been well determined by King. It shows that moonlight is redder than sunlight,—in agreement with the spectro-photometric measures of Wilsing and Scheiner.

6. Müller's data for the major planets[3] have been adopted, with a correction of -0.06 mag. to reduce to the Harvard scale, except for Uranus and Neptune; Pickering's magnitudes for the asteroids,[4] and Guthnick's for the satellites of Jupiter and Saturn.[5] The color indices of Venus, Mars, Jupiter and Saturn have been derived from comparison of Müller's visual and King's photographic observations.[6]

7. Very's observations of the intensity of the earthshine[7] indicate that the mean full earth, as seen from the moon, appears 40 times as bright as the mean full moon, seen from the earth, and that the stellar magnitude of the earth, as seen from the sun, would be about −3.5, with an uncertainty of at least 25%, or 0.20 mag.

8. The intensity of sunlight from the zenith, according to H. H. Kimball, is 103,000 metre-candles. That of mean full moonlight, according to several observers, is 0.24 metre-candle. A standard candle, if of approximately the same color as the stars, would appear as bright as a star of the first magnitude if placed at a distance of 1.09 kilometres.

9. Table 1 gives the values finally derived for the albedo of the various planets and satellites, and related quantities.

TABLE I

Object	*Mag. at Mean Opp.*	*Mag. at Unit Distance*	*Semi-diameter*	*p*	*q*	*Albedo A*	*Color-Index*	*Photographic Albedo*
Moon...........	−12.55	+0.40	2″40	0.105	0.694	0.073	+1.18	0.051
Mercury........	− 2.94	−0.88	3.45	0.164	0.42	0.069	....	......
	− 2.12	−0.06		0.077	0.72	0.055	....	......
Venus...........	− 4.77	−4.06	8.55	0.492	1.20	0.59	+0.78	0.60
Mars.............	− 1.85	−1.36	4.67	0.139	1.11	0.154	+1.38	0.090
Jupiter..........	− 2.29	−8.99	95.23	0.375	1.5:	0.56:	+0.50	0.73:
Saturn...........	+ 0.89	−8.67	77.95	0.420	1.5:	0.63:	+1.12	0.47:
Uranus..........	+ 5.74	−6.98	36.0	0.42	1.5:	0.63:	....	......
Neptune.........	+ 7.65	−7.00	34.5	0.49	1.5:	0.73:	....	......
Ceres............	+ 7.15	+3.70	0.53	0.10	0.55:	0.06:	....	......
Pallas...........	+ 7.84	+4.38	0.34	0.13	0.55:	0.07:	....	......
Juno............	+ 8.95	+5.74	0.14	0.22	0.55:	0.12:	....	......
Vesta............	+ 6.04	+3.50	0.27	0.48	0.55:	0.26:	....	......
Jupiter								
Satellite I.....	+ 5.54	−1.16	2.38	0.46	1.5:	0.69:	....	......
" II....	+ 5.69	−1.01	2.08	0.51	1.5:	0.76:	....	......
" III...	+ 5.08	−1.62	3.62	0.30	1.5:	0.45:	....	......
" IV....	+ 6.26	−0.44	3.49	0.11	1.5:	0.16:	....	......
Titan	+ 8.30	−1.26	2.9	0.33	1.5:	0.50:	....	......
The Earth								
Lommel-Seeliger law......		−3.46	8.79	0.27	1.64	0.45	....	......
Lambert's law............		−3.52	"	0.29	1.50	0.43	+0.45?	0.6?
Observed law for Venus		=3.80	"	0.37	1.20	0.45	....	......
" " " Moon......		−4.40	"	0.65	0.70	0.45	....	......

Column 2 gives the stellar magnitude at mean opposition (in the case of Mercury and Venus, at full phase, unit distance from the earth, and mean distance from the sun); column 3 the magnitude for full phase and unit distance from the sun and the observer; column 4 the adopted mean angular semidiameter at unit distance; the next three columns, the values of p, q, and A, defined as above; and the last two, the color-index and the photographic albedo (A) in the cases where these are known. Two sets of values are given for Mercury, corresponding to the two empirical formulae for the variation of its brightness with phase given by Müller, and four for the earth, representing the results derived from Very's observations on the assumption that the variation of the brightness of the earth with phase follows four different laws, derived from theory or from observation of other bodies.

10. These values of the albedo of the various bodies are in entire agreement with the current views of their constitution. For Venus and the outer planets, which are generally supposed to be covered with clouds, the albedo is very near the value found by Abbot for terrestrial clouds (0.65). For Mars, Mercury and the moon the albedo is comparable with that of ordinary rocks, as it is also for three of the four asteroids. Even the high value for Vesta can be matched by some whitish terrestrial rocks; but the still higher values for the inner satellites of Jupiter are rather remarkable.

11. The value here found for the earth's albedo is intermediate between those of the cloudy and cloudless planets, and agrees very closely with Abbot's estimate[8] of 0.37, based on the known cloudiness of the earth's atmosphere. It is only half as great as that which Very has derived from the same observations, but the discrepancy is easily explicable. The discussions of the observations by Very and by the writer agree in showing that the albedo of the earth (more precisely, the value of the constant called p above) is a little more than five times as great as that of the moon (if Zöllner's value for the brightness of the latter is adopted). But the value of the moon's albedo used by Very (0.174) is what Zöllner[9] calls the "true albedo"—which is the value obtained after a large and very uncertain correction for the assumed influence of the irregularities of the surface, according to a theory which has later been found to contain a serious error.[10] Zöllner's observations themselves lead to the value 0.080 for p, and when this correction is made, the discrepancy disappears.

The full paper, with much more extensive references, will be published in the *Astrophysical Journal.*

---

[1] *Proc. Amer. Acad. Arts Sci.*, 8, 232 (1861).

[2] *Results of Astronomical Observations made at the Cape of Good Hope* (London, 1847), pp. 353–374.

[3] *Potsdam Pub. Astrophysik.*, Bd. 8, Tl. IV (1893).

[4] *Harvard Coll. Obs. Cir., No. 169* (1911).

[5] *Astr. Nachr.*, 198, 251 (1914).

[6] *Ann. Obs. Harvard Coll.*, 59, 261–264.

[7] *Astr. Nachr.*, 196, 269–290 (1912).

[8] *Smithsonian Inst., Ann. Astrophys. Obs.*, 2, 161–163.

[9] *Photometrische Untersuchungen* (Leipzig, 1865), p. 162.

[10] See Müller, *Photometrie der Gestirne* (Leipzig, 1897), p. 77.

QUANTUM RELATIONS IN PHOTO-ELECTRIC PHENOMENA

By R. A. Millikan

RYERSON PHYSICAL LABORATORY, UNIVERSITY OF CHICAGO

Received by the Academy, December 21, 1915

For the past ten years I have been engaged with experiments which were designed for the sake of subjecting Einstein's photo-electric quantum-theory equation to searching experimental tests, and although I have at times thought that I had evidence which was irreconcilable with that equation, the longer I have worked and the more completely I have eliminated sources of error the better has the equation been found to predict the observed results. I shall present herewith the barest sketch of six consequences of that equation and their experimental verification. Preliminary reports on some of these results have already been made[1] and detailed reports will be found in forthcoming numbers of the *Physical Review*.

Einstein's equation[2] grew out of a semi-corpuscular quantum theory of radiation. The assumption was that light consists of bundles or 'quanta' of electromagnetic energy which shoot out explosively from the emitting body and travel through space as localized units until they are suddenly absorbed by the atoms of matter upon which they fall. The energy in each light-unit was assumed equal to $h\nu$, in which h is Planck's 'wirkungs-quantum' and ν is the frequency of the oscillator which emits the light. Upon absorption this energy was assumed to be transformed into the kinetic energy of an escaping negative electron whose energy of escape from a metal illuminated by light of frequency ν was thus given by $\frac{1}{2}mv^2 = h\nu - p$, in which p was the work necessary to separate the electron from the surface of the metal. The maximum energy of escape is measured by $(V_0 + K)e$ in which e is the electronic charge, K the contract EMF between the emitting plate and the opposed Faraday cylinder which catches the electron, and V_0 the potential difference which must be externally applied just to stop the photo-current to this cylinder. The assertions contained in the equation $\frac{1}{2}mv^2 = h\nu - p$ are that:

1. There is a definite maximum energy of electronic emission under the stimulation of a given frequency ν. (This has recently been denied by Ramsauer.[3])

2. There is a linear relation between V_0 and ν.

3. The slope of the $V_0\nu$ line multiplied by e is exactly Planck's 'h.'

4. The intercept of this $V_0\nu$ line on the ν axis is the frequency ν_0 at which the illuminated substance first becomes photo-sensitive.

5. The contact EMF between the illuminated plate and the Faraday cylinder is given by,

$$\text{Contact EMF} = \frac{h\nu_0 - h\nu'_0}{e} - (V_0 - V'_0),$$

in which ν_0 and ν'_0 are the frequencies at which the cylinder and the

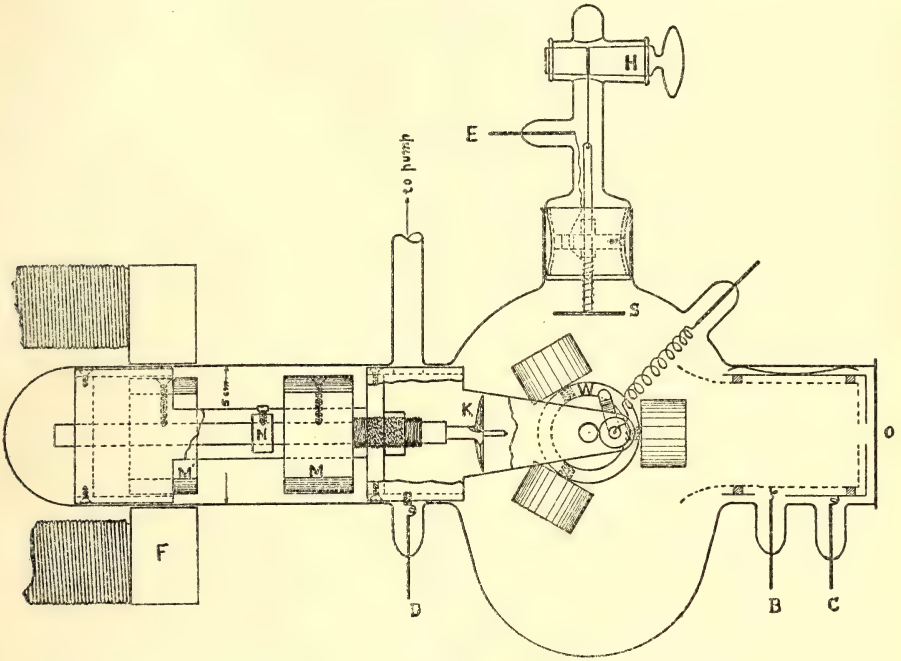
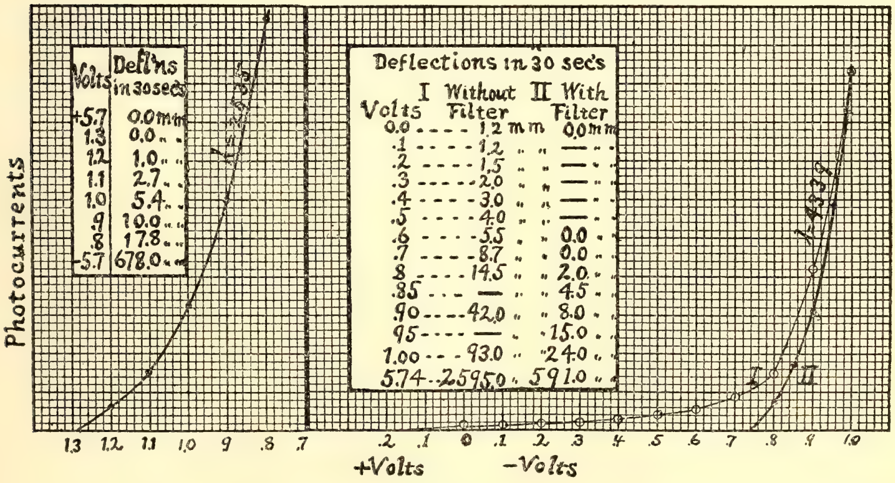


FIG. 1.



Volts	Defl'ns in 30 secs
+5.7	0.0 mm
1.3	0.0 " "
1.2	1.0 " "
1.1	2.7 " "
1.0	5.4 " "
.9	10.0 " "
.8	17.8 " "
−5.7	678.0 " "

Deflections in 30 secs

Volts	I Without Filter	II With Filter
0.0	1.2 mm	0.0 mm
.1	1.2 " "	— " "
.2	1.5 " "	— " "
.3	2.0 " "	— " "
.4	3.0 " "	— " "
.5	4.0 " "	— " "
.6	5.5 " "	0.0 " "
.7	8.7 " "	0.0 " "
.8	14.5 " "	2.0 " "
.85	— " "	4.5 " "
.90	42.0 " "	8.0 " "
.95	— " "	15.0 " "
1.00	93.0 " "	24.0 " "
5.74	2595.0 "	591.0 " "

FIG. 2.

plate respectively first become photo-sensitive, and V_0 and V'_0 are the respective maximum potentials necessary to stop discharge into the cylinder from the plate and from another plate made from the substance of the cylinder.

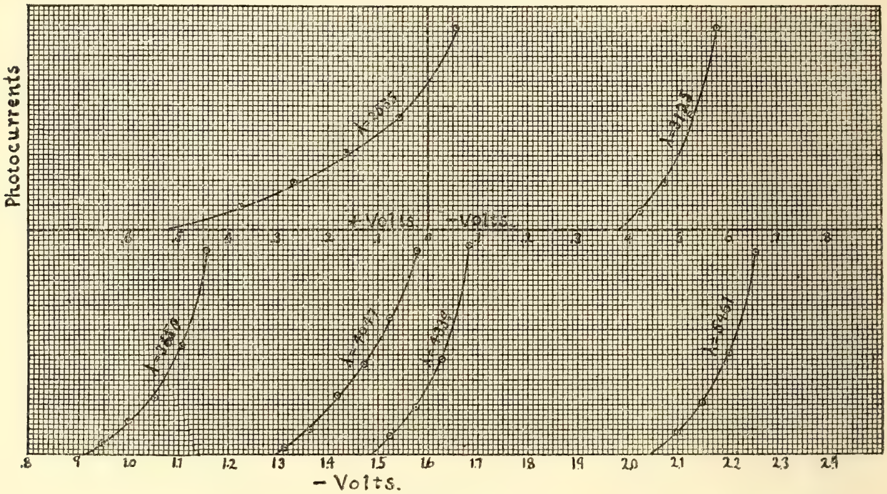


FIG. 3.

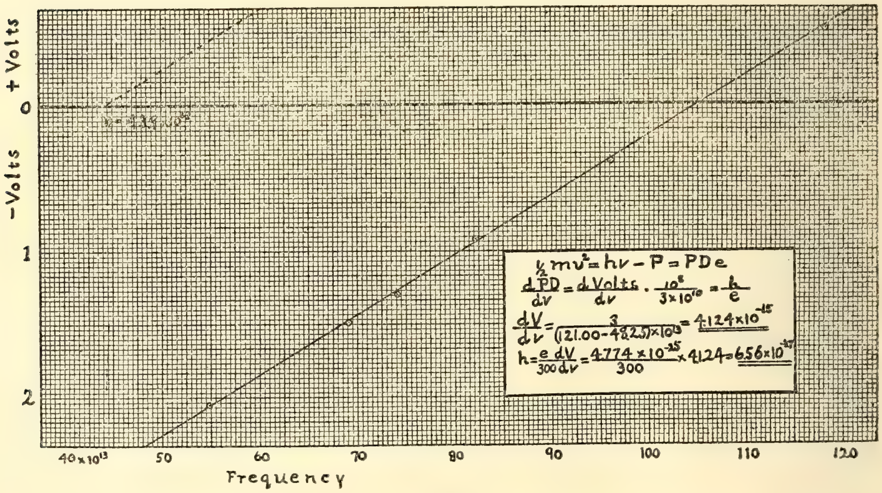


FIG. 4.

6. Contact EMF's are independent of temperature. This follows from Einstein's equation taken in connection with our now well confirmed proof[4] of the independence of photo-potentials upon temperature. Where, however, surface films cause variations with temperature of photo-potentials, there should be corresponding variations in contact EMF.

The experiments are made with a device shown in figure 1 by which clean new surfaces of potassium, sodium, and lithium can be produced by shaving in an extreme vacuum, and photo currents and contact EMF's measured immediately thereafter. The tube has a projection toward the reader not shown in the diagram with the aid of which the wheel *w* may be rotated by means of an electromagnet similar to *F* and a third armature similar to *M* and *M'*.

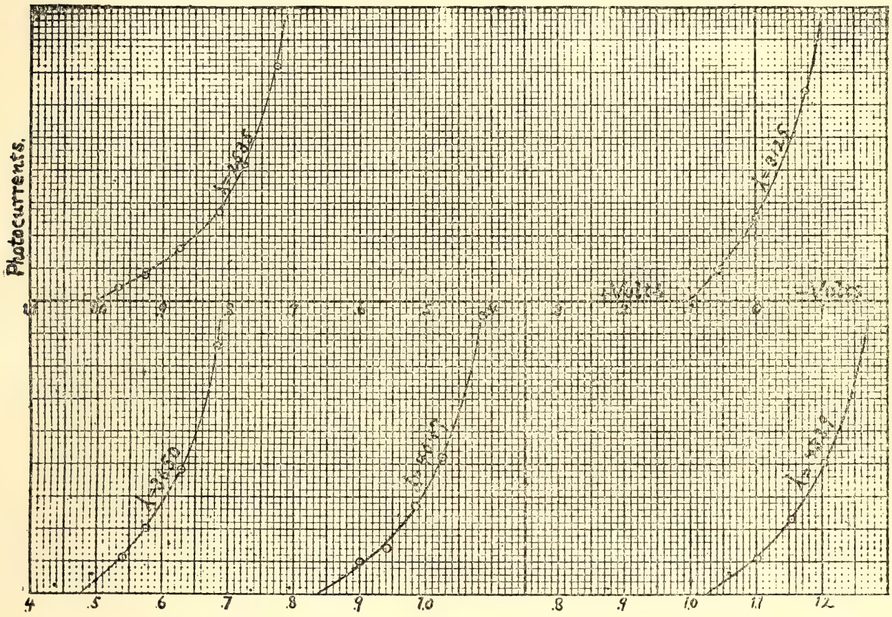


FIG. 5.

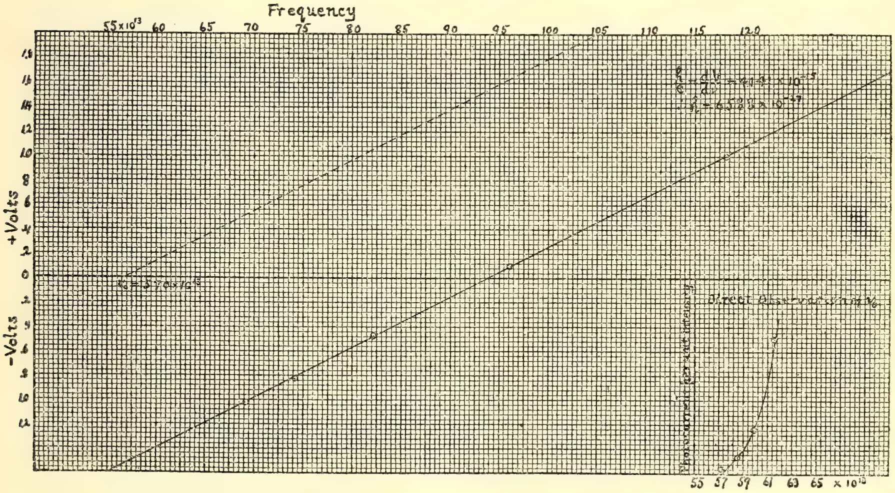


FIG. 6.

The data on lithium shown in figure 2 seems to establish assertion 1, the difference between curves I and II taken with the mercury line 4339 and a Hilgar monochromator showing how the true shape (II) of the photo-current-potential curve was entirely falsified by a little stray short wave-length light (see I) until a filter of aesculin which cut out all waves of shorter wave-length then 4339 was used.

Assertions 2 and 3 are strikingly verified in figures 3 and 4, the latter of which is the plot of the intercepts shown in figure 3, against the frequency. These figures relate to measurements on sodium, in which the saturation currents were from 75 to 500 times the largest currents in figure 3. These latter correspond to a deflection of 80 mm. in 30 seconds.

Similar measurements on lithium are shown in figures 5 and 6. The mean value of Planck's *h* thus photo-electrically determined should not be in error by more than 0.5 per cent. This value is

$$h = 6.57 \times 10^{-27}$$

(see figures 4 and 6). The value of *e* involved in this determination of *h* is the author's value[5] 4.774×10^{-10}.

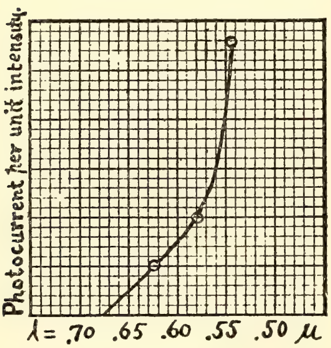


FIG. 7.

The correctness of assertion 4 is shown in figure 6 by the coincidence of the intercept $\nu_0 = 57.0 \times 10^{13}$ (see dotted line) with the direct determination of ν_0 with saturation currents shown on the lower right hand corner of figure 6. To obtain the ν_0 intercept the $V_0\nu$ line is displaced in the direction of positive potentials by the amount of the measured contact EMF. Also in the case of the sodium the ν_0 shown in figure 4 corresponds to the wave length 0.68μ. The direct determination of this long wave length limit for the sodium is shown in figure 7. The agreement is perfect.

Assertion 5 was tested for two different surfaces of lithium and one of sodium and the contact EMF computed by the equation in 5 agreed in each case with the directly observed contact EMF to within less than 2%. The details of this test will be found in the papers in the *Physical Review*.

Assertion 6 has not been tested in this work but Schottky[6] has recently measured the contact EMF's between white hot wires and cold cylinders and found results which agree, within the fairly wide limits of uncertainty, with the values which hold between the same metals at ordinary temperatures.

So far then as experiment has thus far gone, Einstein's equation seems to be an exact statement of the energies of emission of corpuscles under the influence of light waves.

Nevertheless the physical theory which gave rise to it seems to me to be wholly untenable. Be this as it may, however, the photo-electric results herewith presented constitute the best evidence thus far found for the correctness of the fundamental assumption of quantum theory, namely, the discontinuous or explosive emission of energy by electronic oscillators. They furnish the most direct and most tangible evidence which we yet have for the actual physical reality of Planck's *h*.

---

[1] R. A. Millikan, *Physic. Rev.*. Ser. 2, 4, 73 (1914); *Ibid.*, **6**, 55 (1915).

[2] Einstein, *Ann. Physik.*, Ser. 4, **17**, 132 (1905) and **20**, 199 (1906).

[3] Ramsauer, *Ann. Physik.*, **45**, 1120 (1914), also **45**, 961.

[4] Millikan and Winchester, *Physic. Rev.*, **24**, 16 (1906), and *Phil. Mag.*, Ser. **6**, **14**, 188 (1907).

[5] Millikan, *Physic. Rev.* **2**, 143 (1913).

[6] Schottky, *Ann. Physik.*, **44**, 1011 (1914).

THE CHEMICAL ACTIVITY OF THE IONS OF HYDROCHLORIC ACID DETERMINED BY ELECTROMOTIVE FORCE MEASUREMENTS

By James H. Ellis

RESEARCH LABORATORY OF PHYSICAL CHEMISTRY, MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Received by the Academy, January 22, 1916

The fact that the laws of perfect solutions which are conformed to by unionized or slightly ionized substances in dilute aqueous solutions are subject to large deviations in the case of largely ionized substances (salts, strong acids and bases) even at small concentrations makes it necessary, in the absence of any theoretical explanation of the deviations, to treat dilute solutions of these substances like concentrated solutions of other substances, namely, to determine experimentally the behavior of the separate substances, with the hope that this empirical study may then lead to generalizations. Now the most important characteristic of ionizing substances is the chemical activity which results from their ionization, or more specifically the (mass-action) effect which their ions exercise in determining chemical equilibria. This effect in the case of theoretically perfect solutes is proportional to the concentration of the ions; but in the case of deviating solutes there must be substituted for it a new quantity, which may be regarded as the 'effective ion-concentration' and which has been appropriately called by Lewis[1] the activity of the ions. This quantity has been shown by Lewis to be thermodynamically related to various other properties of solutions, thereby on the one hand increasing its practical significance and on the other affording independent means of evaluating it.

The most general of these thermodynamic relations, one indeed which may well be regarded as the best practical definition of activity, is that afforded by the equation $F_1 - F_2 = RT \log (a_1/a_2)$ in which R is the perfect-gas constant and $F_1 - F_2$ represents the decrease in the free-energy of the system attending the transfer at the absolute temperature T of one mol of any substance (thus of an ion) from a solution of any concentration in which its activity is a_1 to another solution of any concentration in which its activity is a_2; the free-energy decrease being defined in general to be equal to the maximum work W producible by the change in the state of the system under consideration diminished by the attendant increase in the product of its volume and pressure (that is, $F_1 - F_2 = W - (p_2v_2 - p_1v_1)$).

The most direct way of determining the free-energy-decrease attending the transfer of ions from one concentration to another, and thereby of determining their relative activities, is the measurement of the electromotive force of cells in which such a transfer takes place; and it is with such a study of the ions of hydrochloric acid that this investigation deals. Namely, measurements have been made of the electromotive force at 18, 25, and 35° of cells of the form H_2 (1 atm.), HCl (at various concentrations), Hg_2Cl_2 (solid) + Hg. If two such cells are considered to be placed in series in opposition to each other, the changes at the electrodes of the two cells compensate each other, and the net change in state when one faraday (F coulombs) of electricity passes through it is the transfer of 1 HCl or of 1 H^+ and 1 Cl^- from one solution to the other. Considering the ions, we have therefore the relation:

$$(E_2 - E_1)\,\mathrm{F} = F_1 - F_2 = 2\,RT \log (a_1/a_2),$$

in which $E_2 - E_1$ is the difference in the electromotive force of two cells in which the acid has the free-energies F_2 and F_1 and its ions have the activities a_2 and a_1, respectively. For a_2 and a_1 we may substitute α_2c_2 and α_1c_1 in which α_2 and α_1 are activity-coefficients (analogous to ionization-coefficients) representing the factors by which the concentrations c_2 and c_1 of the acid must be multiplied to give the activities of the ions.

The mean corrected values of the observed electromotive force in volts and the values of the free-energy-decrease in joules calculated from them by the equation $F_1 - F_2 = 96500 E \times 4.182$ are given in Table 1. These electromotive forces are probably not in error in any case by as much as 0.1 millivolt. The free-energy-decrease is that attending the cell-reaction $\frac{1}{2} H_2$ (1 atm.) + $\frac{1}{2} Hg_2Cl_2$ (solid) = Hg (liquid) + H^+Cl^- (at concentration c). The table also contains the values of the increase $(H_2 - H_1)$ in the heat-content of the cell when this reaction takes place at 25°, calculated by the fundamental thermodynamic equation:

$$\frac{H_2 - H_1}{T^2} dT = d\left(\frac{F_1 - F_2}{T}\right).$$

TABLE 1

Energy-Effects Relating to the Reaction
$\frac{1}{2} H_2$ (1 Atm.) + $\frac{1}{2} Hg_2Cl_2$ = Hg+HCl (at Various Concentrations).

MOLS HCl PER 1000 G. WATER	ELECTROMOTIVE FORCE AT			FREE-ENERGY DECREASE AT			HEAT DECREASE AT 25
	18°	25°	35°	18°	25°	35°	
4.484	0.15759	0.15506	0.15124	15208	14964	14595	25590
1.9278	0.23769	0.23589	0.23304	22937	22764	22489	30480
1.0381	0.27919	0.27802	0.27595	26942	26829	26629	32130
0.7714	0.29654	0.29571	0.29411	28616	28536	28381	32450
0.5095	0.31912	0.31865	0.31765	30795	30750	30654	33070
0.3376	0.33845	0.33836	0.33794	32661	32652	32611	33380
0.1004	0.39764	0.39884	0.40013	38373	38489	38612	34060
0.0333	0.45020	0.45258	0.45557	43444	43674	43963	34370

In Table 2 are given the corresponding values of the free energy of transfer and heat of transfer of 1 HCl from solutions of various concentrations to a solution of the concentration 0.1000 mols HCl per 1000 g. water. These are obtained from the values of Table 1 by direct subtraction (after reducing the values at the concentration 0.1004 so as to correspond to the round concentration 0.1000). In the table are included also values of the free energy of transfer at 18° for concentrations below 0.0333 molal, these having been calculated from Jahn's[2] measurements of the electromotive force of concentration-cells of the type Ag+AgCl, HCl (c_1), HCl(c_2), AgCl+Ag. In the next to last column of the table are given the corresponding values of the activity-coefficients at 18°, calculated by the equation given above on the assumption that at the smallest concentration (0.00167 molal) the activity-coefficient is equal to the ionization-coefficient (0.988) derived from the ratio of the equivalent conductance at that concentration to that extrapolated for zero concentration. The last column of the table gives the values of this conductance-ratio at 18° at the other concentrations, enabling a comparison of it to be made with the values of the activity-coefficient.

It will be seen from Table 2 that with increasing concentration the activity-coefficient first falls more rapidly than the conductance-ratio, being about 10% smaller than the latter at concentrations 0.1 to 0.5 molal. This shows that at these concentrations there is an error of this magnitude in the common practice of employing in mass-action expressions the conductance-ratio as a measure of the activity of the ions of the acid. The activity-coefficient, moreover, unlike the conductance-ratio, passes through a minimum at about 0.50 molal, and then increases rapidly with the concentration, becoming about equal to that ratio at 1 molal, and attaining at 4.5 molal a value $2\frac{1}{4}$ times as great as that at zero concentration. This large increase is in correspondence with the rapid increase of the vapor-pressure of the acid at high concentrations.

TABLE 2

ENERGY-EFFECTS ATTENDING THE TRANSFER OF HYDROCHLORIC ACID AND VALUE OF ITS ACTIVITY-COEFFICIENT

MOLS HCl PER 1000 G. WATER	FREE-ENERGY DECREASE AT			HEAT DECREASE AT 25°	ACTIVITY-COEFFICIENT AT 18°	CONDUCTANCE-RATIO AT 18°
	18°	25°	35°			
4.484	23184	23544	24037	8474	2.23	....
1.9278	15454	15744	16143	3591	1.063	....
1.0381	11449	11679	12003	1937	0.864	0.841
0.7714	9775	9971	10250	1619	0.823	0.868
0.5095	7596	7757	7978	999	0.795	0.889
0.3376	5730	5856	6021	684	0.816	0.901
0.1000	0	0	0	0	0.844	0.925
0.03332	−5053	−5167	−5331	−300	0.892	0.955
0.01668	−8224	....	....	....	0.926	0.966
0.01115	−10084	....	....	....	0.943	0.971
0.008324	−11447	....	....	....	0.953	0.976
0.006683	−12472	....	....	....	0.960	0.978
0.005569	−13325	....	....	....	0.967	0.980
0.003334	−15717	....	....	....	0.985	0.985
0.001667	−19031	....	....	....	0.988	0.988

Other exact electromotive force investigations from which ion-activities can be derived have been published by Jahn[2] on potassium chloride and sodium chloride at concentrations between 0.00167 and 0.033 molal and by MacInnes and Parker[3] on potassium chloride between 0.001 and 0.5 molal. The results of the last-named investigators give for the activity-coefficient of potassium chloride the values 0.653 at 0.5 molal, 0.738 at 0.1 molal, and 0.885 at 0.01 molal. The corresponding values for hydrochloric acid presented in this article are 0.795, 0.844, and 0.945. The value (0.738) for potassium chloride at 0.1 molal is again much smaller (namely about 14% smaller) than the conductance-ratio (0.861).

A more complete description of this research will soon appear in the *Journal of the American Chemical Society.* The preparation of the cells so as to secure constancy and reproducibility of the electromotive force values, the methods of making the measurements, the full experimental data, and thermodynamic calculations from them of other free-energy values will be there presented in detail.

This research has been carried on with the coöperation of Prof. A. A. Noyes and with the aid of a grant made to him by the Carnegie Institution of Washington. The preliminary experiments were made jointly with Dr. Louis Weisberg, and the final measurements with Mr. Frank W. Hall. For all this assistance I wish to express my great indebtedness.

---

[1] Lewis, *Proc. Amer. Acad.*, **43**, 259–293 (1907); *Zs. physik. Chem.*, **61**, 129–165 (1908).

[2] Jahn, *Zs. physik. Chem.*, **33**, 545–576 (1900).

[3] MacInnes and Parker, *J. Amer. Chem. Soc.*, **37**, 1445–1461 (1915).

EFFECTS OF CENTRIFUGAL FORCE ON THE POLARITY OF THE EGG OF CREPIDULA

By Edwin G. Conklin

DEPARTMENT OF BIOLOGY, PRINCETON UNIVERSITY

Read before the Academy, November 16, 1915. Received, January 22, 1916

If the eggs of the marine gasteropod Crepidula plana are subjected to centrifugal force of approximately two thousand times gravity the yolk is thrown to the distal or centrifugal pole, the oil and other light substances to the centripetal pole, while the nucleus and centrosphere together with most of the cytoplasm occupy the middle zone between the other two. In eggs centrifuged after fertilization and before the first cleavage the yolk zone comprises a little more than three-quarters of the volume of the whole egg, the middle zone a little less than one-quarter and the oil zone about one sixty-fourth, the relative volumes of the three being 49:14:1. In normal eggs of this stage the nucelus centrosphere and most of the cytoplasm lie near the animal pole, but in centrifuged eggs these formative substances may be displaced far from this position, the yolk, for example, being thrown to the animal pole and the protoplasm to the vegetative pole, or these displacements may take place in any other axis. Nevertheless these substances slowly come back to their normal positions provided there is sufficient time for this before the next cell division. However if cell division intervenes before the egg substances have regained their normal positions there is usually an abnormal distribution of these substances to the two daughter cells and since these substances do not pass through partition walls this abnormal distribution persists throughout later development. In each of the daughter cells, however, the protoplasm goes to the animal pole and the yolk to the vegetative one and the subsequent development is as nearly normal in every respect as is possible, although it is not possible to entirely undo the effects of the earlier dislocation. These results are in substantial agreement with the earlier work of Morgan (1907, '09, '10), Lillie (1906, '09), Conklin (1910, '12), et al., on the development of centrifuged eggs of other animals.

We have in these cases one of the simplest examples of organic regulation in which the organization concerned is merely the polarity of a single cell. The fact that polarity and pattern of organization persist in an egg after cytoplasm, nucleus, centrosphere, yolk and practically all other visible constituents of the cell have been displaced far from their normal positions is most remarkable and mysterious. Is polarity the result of some immaterial influence, some 'entelechy,' which like a divinity 'shapes our ends, rough hew them how we will?' Is it a purely physiological phenomenon, dependent as Child (1911, '12, '13, '14) believes on the rate of metabolism at different poles? Or does polarity persist in some material substance which is not moved by centrifugal force? Investigators are not inclined to close the door to scientific research by a hasty resort to 'entelechy,' and differences in the rate of metabolism or of any other physiological process at two poles, when not due to differences in the environment, must be associated with differences in the material substances at those poles. Consequently it seems necessary to conclude that there is some material substance or relation of parts in these eggs which persists with relatively little change, in spite of the dislocations caused by centrifuging, and which is capable of bringing these substances back to their normal positions when centrifuging ceases, unless they have been isolated in the meantime by the interposition of partition walls.

What is this substance or relation of parts in which polarity persists? Lillie (1906) concludes that polarity is a property of the 'ground substance' of the egg, this substance being 'a fluid which has no filar, reticular or alveolar structure' but yet is 'firmly organized' so that it is not affected by centrifuging. The substances which are dislocated by centrifugal force are mere 'inclusions' in this 'ground substance;' consequently eggs in which these inclusions are forced to occupy abnormal positions are still capable of normal development since their 'ground substance' remains unaltered. It is evident from Lillie's use of this term that he means it to include what is commonly called 'protoplasm' as contrasted with the 'inclusions' or metaplasm.

In eggs which contain relatively little yolk it may appear possible to identify this 'ground substance' with the general protoplasm, but in the eggs of Crepidula, where the yolk constitutes more than three-quarters of the whole, the protoplasm, including under this term cytoplasm, nucleus and centrosphere, is moved bodily from one pole to another when the yolk is displaced from its normal position. If there is a 'ground substance' here which is not moved by centrifuging it must constitute a relatively small part of the general protoplasm of the egg.

There is good evidence that this is indeed the case and that while the greater part of the cytoplasm is more or less free to move under the influence of centrifugal force there is a portion which is denser and more resistent which is not stratified by centrifugal force. Indeed as long as an egg remains alive this stratification is never complete and the boundaries of the zones are neither plane surfaces nor are they sharply delimited as is the case in dead eggs which have been centrifuged. In Crepidula this denser and more resistant part of the protoplasm is found in the nucleus and centrosphere, in a thin peripheral layer which retains its position as long as the egg remains unbroken, and in a framework of strands which runs through the cell and connects the nucleus and centrosphere to the peripheral layer. During non-divisional stages these strands are slender and less resistant and may be stretched or distorted but are rarely broken. They hold the centrosphere and nucleus in a definite relation to each other and to the cell axis, the centrosphere always lying on the side of the nucleus toward the animal pole and next to the free border of the cell. In whatever axis the egg may be centrifuged and however far the centrosphere and nucleus may be carried from the animal pole, the centrosphere always retains this position on the animal pole side of the nucleus and often can be seen to be connected with the peripheral layer by strands of denser protoplasm.

During cell division the strands radiating from the centrosome are stronger and more resistant than during non-divisional stages so that it is much more difficult to move the mitotic figure than it is to shift the resting nucleus and centrosphere. Indeed it is practically impossible to move this figure when once the astral radiations are well developed without destroying the power of further development. On the other hand if nucleus and centrosphere are displaced from their normal positions during non-divisional stages and are prevented from returning by prolonged centrifuging they undergo division in these new positions and give rise to cells which are more or less removed from their normal places. Thus polar bodies and ectoderm cells may be caused to form at the vegetative pole or at any other point on the surface of the egg instead of at the animal pole; nevertheless the protoplasmic portions of the yolk-containing cells return to the animal pole when centrifuging ceases unless otherwise prevented. In some instances in which centrifuging took place in the two cell stage the normal positions of protoplasm and yolk are regained in one of the cells but not in the other. All such cases indicate that it is difficult but not absolutely impossible to change the polarity of eggs and cleavage cells, and that the persistence of polarity in centrifuged eggs and the restoration of dislocated parts to their normal positions is connected with a somewhat resistent framework of protoplasmic strands which preserve the relative positions of nucleus and centrosphere in the cell axis.

Child, C. M., Studies on the Dynamics of Morphogenesis, etc. I–V, VII–VIII. *J. Exp. Zool.*, 10, 11, 13, 14, 16, 17 (1911–1914).

Conklin, E. G., Effects of Centrifugal Force on the Organization and Development of the Eggs of Fresh Water Pulmonates. *Ibid.* 9 (1910).

Lillie, F. R., Observations and Experiments Concerning the Elementary Phenomena of Embryonic Development in Chaetopterus. *Ibid.* 3 (1906). Polarity and Bilaterality of the Annelid Egg, Experiments with Centrifugal Force. *Biol. Bull.*, 16 (1909).

Morgan, T. H., Cytological Studies on Centrifuged Eggs. *J. Exp. Zool.*, 9 (1910).

THE EMISSION QUANTA OF CHARACTERISTIC X-RAYS

By David L. Webster

JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY

Received by the Academy, January 15, 1916

At the meeting of the American Physical Society last April, Duane and Hunt announced that X-rays of any given length would be excited as a part of the general radiation from a tungsten target only if the potential applied to the tube was enough to give an electron a kinetic energy as large as the Planck quantum of that wave length. This law was confirmed and extended to a potential of 100 kv by Hull. Off hand one would expect it to apply to characteristic rays also; but, since some work of Whiddington suggests an exception here, it seemed desirable to test such rays with the spectrometer.

This was done with a rhodium target in a tube which Dr. Coolidge very kindly had made with one of his hot wire cathodes. The potentials were supplied by a storage battery of 20,160 cells, and could be measured to about 1%. Neighboring potentials could be compared to about 0.1%.

The part of this work dealing with the general radiation of rhodium confirms Duane and Hunt's law exactly, and leads to a value of h of 6.52×10^{-27} erg sec, taking e as Millikan's value 4.774×10^{-20} esu. and the calcite spacing as Bragg's value 3.025 A. This lies above the values given by Duane and Hunt[1] and Planck[2] and below the values given by Millikan and Hull.[3] If the spacing of the calcite planes is recomputed on the basis of Millikan's e, it is 3.03 A, and the value of h is 6.53×10^{-27} erg sec. The wave lengths and h values given with the graphs in this paper are computed from this value of the spacing.

As a consequence of Duane and Hunt's law, if the spectrometer is set to receive one of the characteristic lines, some radiation is received as soon as the quantum potential is reached; but it is found that the intensity is at first only what one would expect from the general radiation at that wave length of no lines existed. With these rays, however, at a definite higher potential the intensity suddenly starts to increase more rapidly than before. This point therefore is the lowest potential at which any characteristic lines appear.

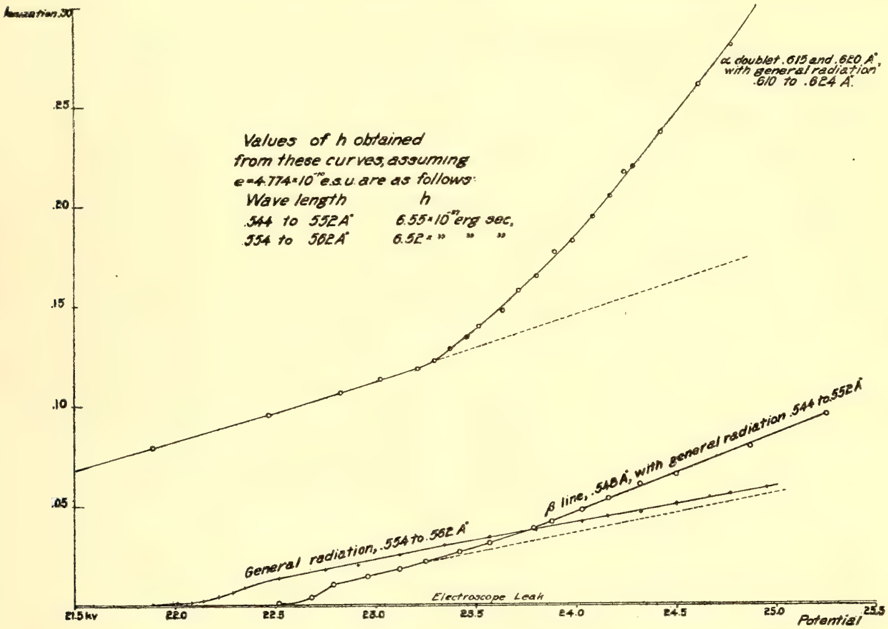


Fig. 1

This effect is shown in figure 1, the upper graph giving the intensity as a function of potential for the α doublet. The first part of the curve and its dotted continuation are due to general radiation alone, while the part to the right of the corner is due to the conbined general radiation and α doublet. The critical potential is very sharply defined, and has a value 23.3 kv, while the quantum potential, at which radiation of that wave length first begins to appear, is 20.05 kv. The same effect is shown by the β line, as one may see by comparison of its intensity-potential curve (fig. 1) with that of the general radiation of a neighboring wave length. The effect is less pronounced because the line is weaker than α. The critical potential, as nearly as one can tell, is the same in both cases.

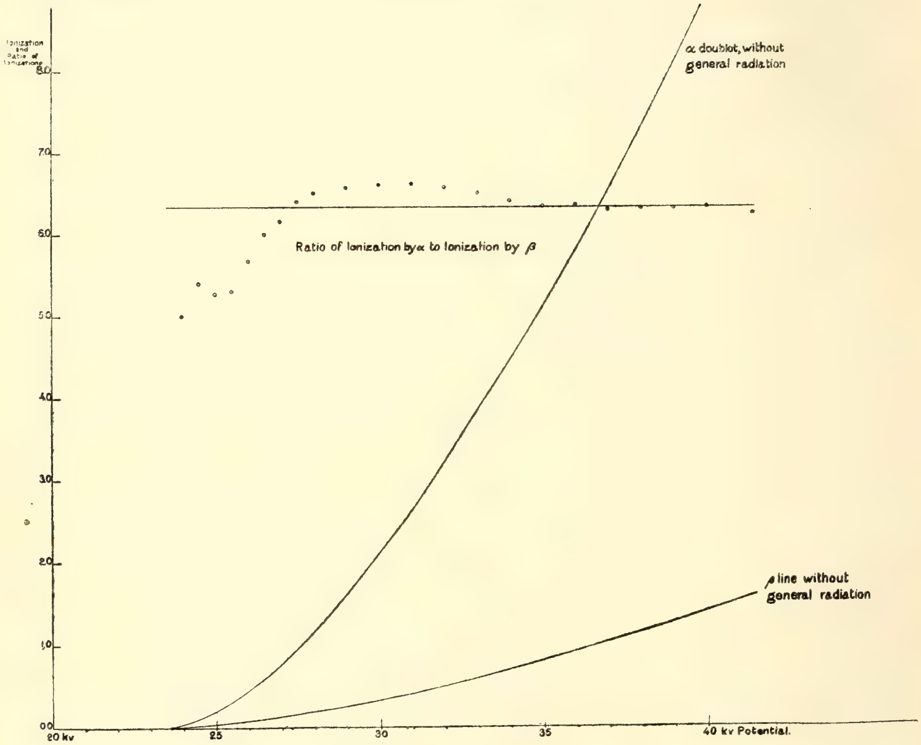


Fig. 2

To prove that the corner in the α curve really does mean the first appearance of the α lines, photographs of this part of the spectrum have been taken at different potentials above and below the critical one. These show that the ratio of the intensity of the lines to that of the background, as well as the absolute intensity of either, increases rapidly with the potential, and that, while the lines are visible against the background within 1% above the critical potential, they are absolutely invisible below it.

The next point to consider is the ratio of intensities of any two lines and its dependence on potential. From numerous photographs with different exposures it appears that the ratio of α_1 and α_2 is constant, as nearly as one can tell, from the critical potential to the limit of the battery's capacity, at about 42 kv. The ratio of the β line to the α doublet can be estimated only by allowing for the general radiation that appears in the spectrometer with each of them. This can be done, only roughly,

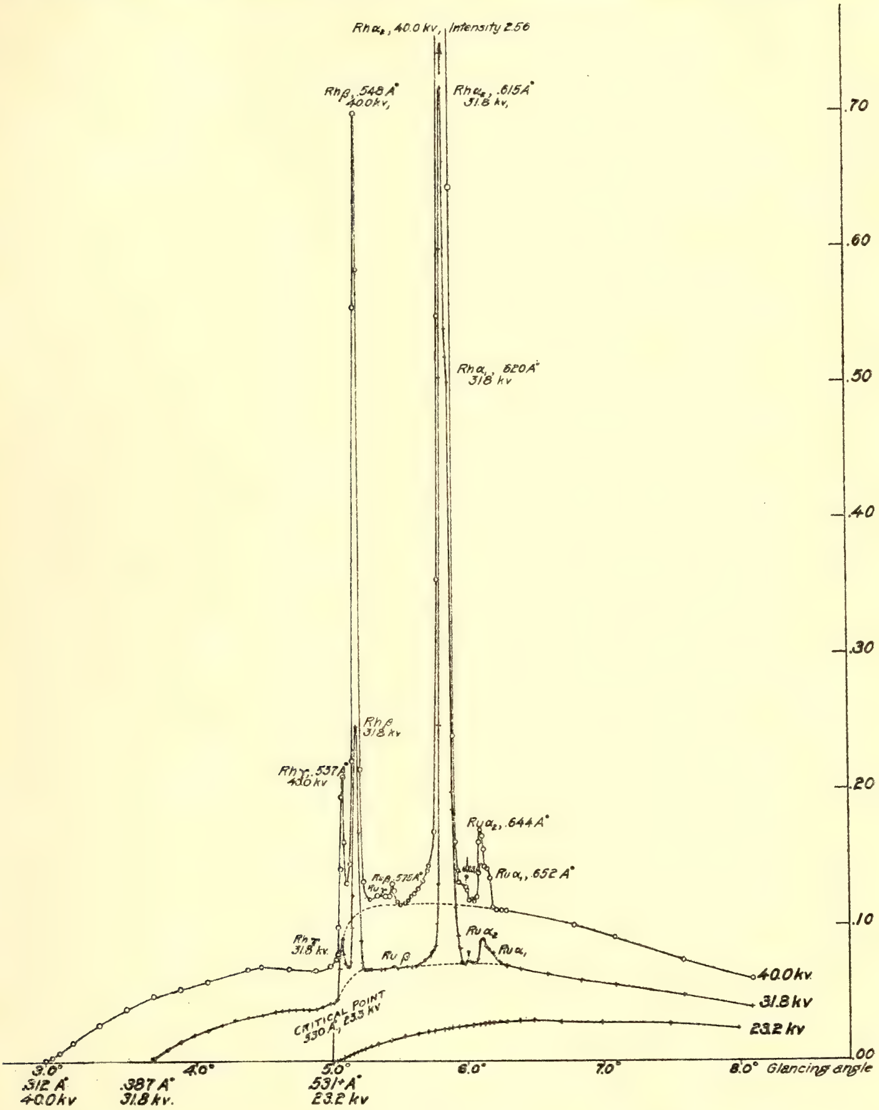


Fig. 3

by the use of the intensity-potential graphs for the combined lines and general radiation and the comparison of them with the graphs for the general radiation at neighboring points in the spectrum. The resulting graphs, for the lines alone, and the graph of their ratio against potential, are shown in figure 2. The constant ratio indicated there differs from the experimentally determined points by less than the limits of error, which are of course much larger at low potentials than at high. Since it is really a ratio of ionizations in an arbitrary amount of ethyl bromide, no importance can be attached to its absolute value, but its constancy seems very significant.

Another way of testing this point is that of plotting the spectra at different potentials as in figure 3. For various reasons this method is even less accurate than the other, but the ratios of increase of the α and β lines are again approximately the same, and here it appears that even the γ line increases similarly.

These spectra are of interest in showing also the influence of a small impurity of ruthenium in the rhodium target in producing its own characteristic lines; but the most important information they contain is the location of the wave length whose quantum potential is the critical value. Its position, 1.3% short of the γ line, is independent of any errors entering uniformly in these potential measurements. Now this part of the spectrum is known to be marked by a sudden rise of absorptive power with decreasing wave length, and indeed the drop in the general radiation at this point is undoubtedly due to the influence of this absorption on the rays leaving the target, as they do, at a very small angle to its surface. This increased absorption, moreover, is known to be accompanied by a strong characteristic fluorescence, indicating that when a higher frequency oscillator has acquired whole quantum by absorption it undergoes a drop to the characteristic frequencies for emission.

Now the results of this work show that to obtain any characteristic radiation by the impact of cathode rays, each of the latter must have have energy enough to satisfy one of these higher frequency oscillators. Hence it seems probable that in this case as well as in fluorescence the characteristic rays are produced by direct excitation of the higher frequency oscillators and their subsequent drop in frequency on emission.

To sum up, these experiments show three points. First, to excite any characteristic radiation it is necessary to use a potential above a critical value which is the value required for general radiation of a wave length 1.3% shorter than that of the γ line. Second, the lines all increase in the same ratio for any given increase of potential. Third, there is reason to believe that the characteristic rays are always a result of excitation of higher frequency oscillators, as in the case of fluorescence.

---

[1] Amer. Phys. Soc., April, 1915; *Physic. Rev.*, July, 1915. Recomputed on the basis of Millikan's e their value is 6.50.

[2] *Heat Radiation*, translation by Masius, p. 172, gives 6.415.

[3] Amer. Phys. Soc., Dec., 1915. These values are 6.57 and 6.59 respectively.

THE RESULTS OF INVESTIGATIONS OF THE ECOLOGY OF THE FLORIDIAN AND BAHAMAN SHOAL-WATER CORALS

By Thomas Wayland Vaughan

U. S. GEOLOGICAL SURVEY, WASHINGTON, D. C.

Received by the Academy, January 27, 1916

Many of the most important principles of coral ecology were long ago recognized and clearly formulated by Darwin and Dana. More recently Klunzinger, Pourtalès, Moseley, Alexander Agassiz, Verrill, Stanley, Gardiner, von Marenzeller, Duerden, Wood-Jones, and others have made important contributions.

The coral faunas which live in water less than 25 fathoms deep in coral-reef regions are separable into two subfaunas according to their ability for withstanding violently agitated water. These are (1) the strong, firmly attached, usually massive forms which can withstand breakers and the pounding of the surf; and (2) the weakly attached and branching forms which can survive only in quiet water. The forms requiring quiet water are further subdivisible according to their capacity to resist the deleterious effects of silt. A species of massive growth habit often will also live in quiet water. In some instances the same species of branching coral may be represented in both quiet and rough water, but the colonies in the rough water have shorter and stouter branches, responding to the environment by strengthening their skeletal structures. Massive, large, head-like corals, such as *Orbicella annularis*, form the strong frame-work of the reef while in the interspaces between the heads many colonies of species of smaller size grow and other organisms are present in greater or less abundance. *Acropora palmata*, a species of another growth habit, is an important reef builder in places. It forms ascending fronds which by the thickening of their basal portion become very strong. Two species which live in the quiet water on the flats behind the outer reefs or in the lagoons, are *Maeandra areolata*, which because of its small base of attachment could not remain fixed on the reef, and *Porites furcata*, which because of its fragile branches would be smashed to bits by the rough water of the outer reefs. These two species can exist where the bottom is muddy, as both possess the means necessary for ridding themselves of considerable quantities of silt. *Eusmilia fastigiata* is a species which has a fragile skeleton and requires quiet water, but as it can not endure much silt, it is restricted to areas where the bottom is cleaner. *Porites clavaria* illustrates responsive adaptation to environment, as it lives both on the reef and on the inner flats. The branches of the colonies in the former habitat are short and stumpy or the colony may be almost massive in growth form, while in quiet water the branches may be decidedly elongated.

The depth to which the more massive forms extend is between 18 and 31 meters, 18 meters is usually about the maximum for vigorous growth, but some of the branching species extend to slightly greater depths. In general the lower depth of the shoal-water coral fauna of the West Indies is about 37 meters, approximating conditions in the Pacific. The precise cause of the limit in depth has not been determined. Each of several possible factors will be discussed.

All the corals with which I have experimented possess the capacity of removing a certain amount of sediment from their surfaces. This is affected by the nonnutrient particles becoming imbedded in mucus and by cilia removing the mucus and the particles from the surface of the tissue. The capacity for cleaning their surfaces varies according to the species, it being lowest among those corals which are most important on the outer reefs—it is low in *Orbicella annularia* and high in *Maeandra areolata*. Some corals, as *Siderastrea radians*, can endure having their surfaces covered with silt for some time. This coral seems to secrete a layer of mucus which lifts the silt above the tissue surfaces and thereby protects them. The branching form of many corals prevents sediment settling on them faster than it can be removed. However, as any coral will be killed by actual burial beneath sediment, corals can not live where sedimentation is rapid; and as sediment accumulates in areas deeper than the base of strong wave action or where currents are weak, it is a factor in limiting the depth to which the littoral fauna can extend.

The mechanism of corals for catching food are as follows: (1) The ectodermal surface is beset with nematocysts, which occur on the tentacles, the oral disk, the column wall, including its downward extension called the edge-zone, and also on the margin of the mesenterial filaments. (2) The entire ectodermal surface is ciliate, the cilia in response to certain stimuli beating toward the oral apertures; in response to others, beating toward the periphery. (3) The outer surface secretes mucus in which particles may be embedded, the mucus moving under the influence of the beat of the cilia toward the oral apertures or toward the periphery, according to the nature of the response to the stimulation. (4) The tentacles are active and effective in capturing food. (5) The mesenterial filaments, which in many species of corals can be extruded through the column walls, in some instances capture food.

Many different kinds of food were offered corals, but they took only animal food; they are entirely carnivorous. The following experiment was tried many times: A piece of diatom mat was placed on one side of the oral disk and a piece of crab meat on the other. Invariably the crab meat was seized and swallowed; while the diatoms induced no reaction except ultimately to be removed from the surface. No kind of purely vegetable food was taken by any one of the numerous species investigated. However, pieces of plants coated with small animals or soaked in meat juice will be swallowed, and later the vegetal matter ejected.

As the food of corals is purely animal plankton, a decrease in the amount of this food-supply with increasing depth would limit the downward distribution of the shoal water forms, but as I do not know of any quantitative estimates of the amount of animal plankton above and below 20 fathoms in coral reef areas, there is no basis for a positive opinion.

The relation of corals to light was studied. Specimens of 17 species were put into a darkened, light-proof live-car. One of the number was dead at the end of 14 days; 3 others were dead at the end of 28 days; while 11 species survived at the end of 43 days. However, all had become pale, some even colorless, or otherwise showed abnormalities. A natural experiment, which appears conclusive, is afforded by Fort Jefferson wharf. Here corals thrive on all the outer piers where the light is strong but there are none on the central piers where there is perpetual shade. It therefore seems to me that strong light is essential for the vigorous growth of shoal water corals.

Another factor is temperature. Dr. Mayer conducted a series of experiments to ascertain the higher and lower limits of temperature which the common corals around the Tortugas can endure. These indicate that a lowering of the temperature to 13.9°C. would exterminate the principal Florida reef corals, while the most important inner flat corals would survive. He obtained similar results on the corals around Murray Island, Australia.

Dr. H. F. Moore of the U. S. Bureau of Fisheries has communicated to me temperature records made at lighthouses along the Florida reef. These show that vigorous reefs will endure a temperature as low as 18.15° C., the minimum at Carysfort light between 1879 and 1899; but at Fowey Rock, where the minimum drops to 15.6°C. although there are some corals, there is no thriving reef. The species found at the north end of the reef line are those which Dr. Mayer's experiments showed capable of withstanding the lowest temperature. The temperature records for the reef line indicate 18.15°C. as the minimum temperature which a reef will survive—this is 1.85°C. lower than the figure given by Dana. It is not probable that a reef could withstand a continuous temperature so low as this. Wherever the depth of water is great enough to lower the bottom temperature below 18.15°C., more probably about 21°C., reef corals will not live. This temperature appears to be attained around the Hawaiian Islands within a depth of 183 meters. According to Agassiz's *Three Cruises of the Blake* the bottom temperature in the Gulf of Mexico and Caribbean Sea is usually too low for the growth of reef corals at a depth of 183 meters, and in places it is too low at a depth of 87 meters. Although the possibility of control of the lower bathymetric limit of reef-building corals by decrease in temperature with increasing depth has not been adequately investigated, it appears safe to say that reef corals are usually, if not always, confined by temperature to water less than 180 meters deep.

The four possible factors which tend to limit the downward extent of reef forming corals are as follows: (1) effect of sediment, (2) decrease in supply of small animal plankton, (3) decrease in intensity of light, (4) lowering of the temperature.

The relations of corals to salinity will now be considered. The average salinity of the Tortugas water according to Dole is 36.01‰. Seventeen species of the Tortugas corals were kept in a large tank of water with a salinity of 18.28‰ for 24 hours. All were damaged or killed except *Maeandra areolata*, *Siderastrea radians*, and *Porites astreoides*; but no specimen of 16 species showed any evidence of harm after remaining 48 hours in water of a salinity of 27.87‰. Apparently corals would not be hurt if the salinity of the ocean were reduced to about 80% of its present salinity. Although I did not experiment with concentrated sea-water, the studies made by Goldforb and others on the effect of concentrated and diluted sea-water on regeneration in hydroids and in the *Cassiopea* are here pertinent. The combined results of the experiments are in accord with the deductions made by oceanographers and geologists from other data, viz., the ocean is becoming more salt, and it appears that marine organisms are now living in an environment which is considerably below the optimum condition for their existence.

In order to ascertain the amount of atmospheric exposure corals would endure, experiments were made on 16 species, any of which will endure exposure on a glass plate in the shade for half an hour without apparent damage; nearly all will stand an hour without harm, while some will stand 4 hours' exposure under the conditions stated. *Favia fragum*, *Porites clavaria*, and *Porites astreoides* have the greatest capacity for withstanding atmospheric exposure, while that of *Maeandra areolata* and *Siderastrea radians* is almost as great. A number of species withstood exposure on a glass plate in the sun for 1½ hours, the specimens being badly damaged, but not entirely killed. Although not precisely true, in general the ability to withstand atmospheric exposure is a function of the porosity of the skeleton, the species with the more porous, surviving longer than those with the denser skeletons.

The conditions necessary for vigorous coral reef development may be summarized as follows: (1) Depth of water, maximum, about 45 meters; (2) bottom firm or rocky, without silty deposits; (3) water circulating, at times strongly agitated; (4) an abundant supply of small animal plankton; (5) strong light; (6) temperature, annual minimum not below 18°C.; (7) salinity between about 27‰ and about 38‰.

In the experiments on rearing corals, the planulae were removed with a pipette from the vessel containing the parent colony to a jar on the bottom of which was a terra-cotta disk. Although the planulae will live a long time, even settle in stale water, kept at the proper salinity, it is better to change the water at least once a day. To change the water, siphons were used, a fine-mesh bolting cloth bag having been tied on the end within the jar, so as to prevent the escape of the planulae; while clean water was added through siphons from jars placed at a higher level.

Because of its bearing on the possibility of the distribution of coral species by oceanic currents, it is highly important to know the duration of the free-swimming larval stage. Observations were made on four species. The range was from 2 to 23 days. Should an ocean current have a velocity of 3 knots per hour, in 23 days planulae might be carried 1656 knots; at 2 knots per hour, 1104 knots; at 1 knot per hour, 552 knots. It is known that every species of shoal water coral in the Bermudas is found in Florida and the West Indies; while not only is the Hawaiian fauna Indo-Pacific in its facies, but several of the species (at least 4) also occur on the east coast of Africa or in the Red Sea. I seriously doubt any part of the Hawaiian fauna being peculiar to those Islands. The clue to the cause of the wide distribution of living coral species is given by the possibly long duration of the free-swimming larval stage.

The growth rate of corals was determined by planting planulae which attached themselves in the laboratory, by measuring colonies, from planulae which settled on collectors in a known season, by measuring colonies cemented to disks and fixed on the heads of stakes driven into the sea-bottom, and by measuring colonies naturally attached. The plantings around the Tortugas were made on the reef off Loggerhead Key and on the outside of Fort Jefferson moat wall, while records were made on colonies growing naturally attached at the two stations mentioned, on the piers of Fort Jefferson wharf, and in Fort Jefferson moat. The observations in the Bahamas were made on artificially planted and naturally attached colonies at the east end of South Bight, Andros Island. The Florida corals were measured annually; those in the Bahamas were measured in 1912 and again in 1914. The average growth rate for each species at each station has been computed. The size of the colonies of all species of corals seems limited, but some attain large dimensions, 2 to 3 meters or even more in diameter, and nearly as much in height, while other species are adult when a diameter of 35 to 50 mm. has been reached. *Favia fragum* and *Maeandra areolata* are instances of species which grow relatively rapidly for the first 2 to 4 years, after which they grow more slowly. *Orbicella annularis* and *Maeandra strigosa* are not so limited in size. Branching corals grow more rapidly than massive species; while of the former, the growth rate of species with perforate, loose-textured skeletons is more rapid than that of species with dense skeletons. In general the more massive and the denser the corallum, the slower the growth; while the more ramose and the more porous the skeleton, the more rapid the growth.

There is no average growth rate for corals generally speaking, as growth rate varies from species to species, and varies for the same species according to local environmental conditions. Here it may be said a colony of species of reef coral in a lagoon, if protected from sediment, may grow more rapidly than a colony of the same species does on the reef. The limitation of reef corals so largely to the outer edges of platforms is determined primarily by purity of water, i.e., freedom from silt, and by the more uniform temperature.

In order to estimate the rate at which a reef will grow, the upward growth rate of the true reef-forming species must be taken. The upward growth rate of *Orbicella annularis*, the principal builder of the Pleistocene and living reefs in Florida and the West Indies, is from 5 to 7 mm. per year, according to station. At 6 mm. per year, it would form a reef 150 feet thick in 7620 years; at 7 mm. per year it would build the same thickness of rock in 6531 years. *Acropora palmata*, which grows more rapidly, might build a similar thickness in 1800 years. The growth of corals in the Pacific appears to be more rapid and according to Stanley Gardiner they might build a reef 150 feet thick in 1000 years. The investigation of the growth rate of corals shows that any known living coral-reef might have formed since the disappearance of the last continental ice sheets.

(This summary is published by permission of the President of the Carnegie Institution of Washington and of the Director of the U. S. Geological Survey.)

CAMBRIAN TRILOBITES

By Charles D. Walcott

SMITHSONIAN INSTITUTION, WASHINGTON, D. C.

Received by the Academy, January 31, 1916

The writer has assembled data to aid in clearing up some of the problems of formations of the Appalachian region by a careful comparison of portions of their contained faunas with those of the Mississippi Valley, the Cordilleras, and other localities. No thorough study and comparison of many genera of the Cambrian faunas has been made, though collections from many outcrops have been in the writer's possession for years, awaiting the opportunity to make these studies so necessary in his work on the Cambrian Trilobites.

Two new families of trilobites are proposed, Menomonidæ and Norwoodidæ, and seven new genera: *Menomonia*, *Millardia*, *Dresbachia*, *Norwoodia*, *Saratogia*, *Vanuxemella*, and *Hanburia;* 46 new species and three new varieties are described, with 19 earlier described species and several genera. One of the marked features of the paper is the description of a number of genera of the order Proparia: *Menomonia*, *Millardia*, *Dresbachia*, and *Norwoodia*. These, taken in connection with the genus *Burlingia*, described in a previous paper (Cambrian Trilobites, *Smithsonian Misc. Coll.*, **53**, No. 2, 1908, p. 14) establish the existence of a strong group of the order in Cambrian time.

The stratigraphic position of the Weeks formation is changed from Middle to Upper Cambrian, and the problem of whether the Conasauga formation of the Coosa Valley and adjoining areas shall be restricted to the Upper Cambrian, and the Middle Cambrian beds there given a formation name, is left for further detailed study.

The discussion and comparison of the *Crepicephalus* group of trilobites is particularly interesting, including a comparison of 17 different species, 10 of them new species, 3 new varieties, and two undetermined species. The five plates of illustrations of this large trilobite also present many new and interesting features of the animal, now so long extinct.

The details of this investigation with fifteen plates continuing 280 figures will be found in *Smithsonian Misc. Coll.*, **64**, No. 3, 1916.

THE MINUTE STRUCTURE OF THE SOLAR ATMOSPHERE

By George E. Hale and Ferdinand Ellerman

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, January 10, 1916

During a total eclipse of the sun, when the light of the disk is completely cut off by the moon, the solar atmosphere is momentarily revealed. The exceedingly faint corona, extending millions of miles into space, can be seen only at such times. But the more brilliant chromosphere, the comparatively shallow atmosphere of luminous gases which completely encircles the sun, and the prominences which rise out of it, can be observed on any clear day with the aid of a spectroscope. First applied to this purpose in 1868, the spectroscope has yielded a large store of information regarding the number, distribution, and nature of the prominences and the structure of the upper chromosphere, as seen in elevation at the sun's limb. It has also permitted the observation of certain phenomena of the solar atmosphere in projection against the disk, but on account of the brilliant background, only their general outlines can be thus detected. In order to study their details we must have recourse to the spectroheliograph.

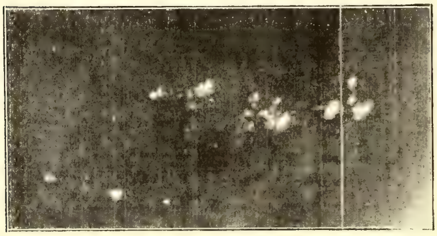


FIG. 1. DIRECT PHOTOGRAPH OF SUN-SPOT GROUP, 1915, AUG. 7, SCALE: SUN'S DIAMETER =24 CM. (Negative reproduction

With this instrument, first applied to the investigation of the solar atmosphere in 1892, a large number of new phenomena have been brought to light. The spectroheliograph may be briefly described as a moving spectroscope, driven at a uniform rate across the solar disk, and admitting to the photographic plate through a narrow slit the light of a single spectral line. Thus monochromatic images of the solar atmosphere, showing the otherwise invisible clouds of hydrogen or of calcium, iron, or other vapor (the flocculi) are recorded permanently for study. In a region on the solar image, for example, where direct observation shows nothing, or perhaps a group of sun-spots, the spectroheliograph may disclose extensive phenomena of great interest in the solar atmosphere (compare fig. 1, a direct photograph of a group of sun-spots, with fig. 5, Plate I, showing two spectroheliograms of the hydrogen flocculi above and surrounding the group). The purpose of this paper is to compare the structure thus revealed at various levels in the solar atmosphere with that of the lower-lying photosphere and sun-spots.

Langley's well-known paper[1] 'On the Minute Structure of the Solar Photosphere,' illustrated with his unrivalled drawings, is still our best source of information regarding the structural details of the photosphere and sun-spots.[2] After referring to the spots and faculae, Langley goes on to say:

> On attentive examination it is further seen that the surface of the sun everywhere . . . . is not absolutely uniform, but is made up of fleecy clouds, whose outlines are all but indistinguishable. . . . . Under high powers used in favorable moments, the surface of any one of the fleecy patches is resolved into a congeries of small, intensely bright bodies, irregularly distributed, which seem to be suspended in a comparatively dark medium. . . . .

These bright bodies, called 'rice-grains' by Stone, were found by Langley to average from 1″.4 to 2″.6 between centers, the distance decreasing with increase of telescopic power.

"In moments of rarest definition I have resolved these 'rice-grains' into minuter components, sensibly round, which are seen singly as points of light, and whose aggregation produces the 'rice-grain' structure. These minutest bodies, which I will call 'granules,' it will appear subsequently can hardly equal 0″.3 in diameter, and are probably less." In the two squares near the lower right-hand corner of the drawing (fig. 2, Plate I), the granules are the minute bodies forming the not very definite clusters or 'rice-grains.' The squares are 11″.6, corresponding nearly to 5000 miles, on a side. From a careful estimate of the size and number of the granules and 'rice-grains,' Langley concludes that "the properly luminous area is less than one-fifth of the solar surface."[3]

The sun-spots shown in the same figure combine details from several different spots, but accurately indicate the general character of the structure, which is described by Langley as follows:

"The penumbra is all but wholly made up, as it appears on a first examination, of cloud-forms whose structure makes them seem like fagots or sheaves of some elongated objects." Under the highest powers "the penumbra is resolved into 'filaments' of extreme tenuity, which by their aggregation make the 'thatch' (a term used by Dawes), just as the minute granules of the photosphere compose the 'rice-grains'." . . . . "It seems to me that there is no room for doubt, that 'filaments' and 'granules' are names for different aspects of the same thing, that filaments in reality are floating vertically all over the sun, their upper extremities appearing at the surface as granules; and that in the spots we only see the general structure of the photosphere, as if in section, owing to the filaments being here inclined."

Referring later to the extremely fine filaments, estimated at not over 0″.03 in diameter, which are sometimes seen on the umbra, Langley remarks on their resemblance to the filamentous structure depicted in spectroscopic drawings of the chromosphere. To this resemblance, which is shown by recent observations to extend to many additional phenomena, we may now direct our attention.

It has long been known that the surface of the chromosphere, as seen at the sun's limb with a spectroscope, commonly appears as a series of slender filaments like blades of grass, supposed by Secchi to correspond with the grains of the photosphere. To study the structure of these filaments in projection against the disk we may utilize the spectroheliograph, which permits their cross-sections to be photographed at several different levels. Thus, by setting the camera slit on the H_1 or K_1 line, which represents the low-lying calcium vapor, we can determine the size and form of the cross-section at a level below that seen visually in the chromosphere at the limb. The calcium lines H_2 and K_2 represent a somewhat higher level, illustrated in figure 3, Plate I. At this elevation the minute structure is similar to that of the photosphere, but the average size of the small bright flocculi is greater, if we may judge from a comparison with the grains in Langley's drawing, reproduced in figure 2 on the same scale. The smallest calcium flocculi photographed, however, are less than 1″ in diameter, and thus do not differ greatly in size from the average photospheric grains.[4]

Spectroheliograms made with light from the center of the $H\alpha$ line of hydrogen depict a still higher level, shown on the same scale in figure 4, Plate I, and on a smaller scale in the stereoscopic picture (fig. 5, Plate I). Figure 4 is enlarged to a scale of nearly a meter to the sun's diameter from an excellent photograph made under almost perfect seeing with the new 13-foot spectroheliograph, recently built in our instrument shop for the 60-foot tower telescope on Mount Wilson. The spectrum is that of the first order of a grating by Anderson, used with a plane mirror at such an angle as to give a dispersion of 3.6 mm. to the angstrom at $H\alpha$. As the camera slit transmits only the central part (about half the width) of the $H\alpha$ line, a high level in the chromosphere is represented. The diameter of the solar image at the focal plane of the 60-foot tower telescope is 17 cm., so that the scale of figure 4 corresponds to an enlargement of $5\frac{1}{2}$ diameters.

For some physiological cause it is difficult or impossible to convey a correct impression of the hydrogen flocculi from positive reproductions of spectroheliograms. For this reason the stereoscopic picture is reproduced as a negative, and in this the bright structure gives a fairly good idea of the appearance of the absorbing hydrogen (corresponding in reality to *dark* flocculi) on the original negative. Figure 4, in order to be directly comparable with figure 2 and figure 3, is reproduced as a positive, and the flocculi shown by it are therefore the dark regions, which occupy a somewhat smaller total area than the brighter spaces between them. Langley estimated that the bright 'rice-grains' cover less than one-fifth of the total area of the photosphere. The corresponding ratios for the calcium (H_2) flocculi range from 10%, when only the smallest and brightest flocculi are included, to 30% and more when the fainter flocculi are also measured. In the case of hydrogen ($H\alpha$) the ratios vary from 40 to 60%, depending upon the limit of darkness adopted for the faintest objects included. These results are derived, moreover, from only a small number of measures of minute flocculi in thinly occupied areas, and are subject to much uncertainty because of the very wide range of intensity of the flocculi, which renders it difficult to distinguish the less conspicuous ones from the background. The ratios may therefore be considerably modified as the result of a special study of this question, which is now in progress.[5]

The smallest of the dark hydrogen flocculi shown in figure 4 are about 2″ in diameter, or approximately twice the diameter of the smallest calcium flocculi shown in figure 3 at the H_2 level.

The foregoing results, in harmony with those of Langley, Secchi, and Evershed, support the view that the photosphere and the gaseous atmosphere above it are formed of columns of hot gases, rising by convection from the interior of the sun. At the photospheric level precipitation may occur of any materials refractory enough to withstand the high temperature, or if these substances do not exist, the conditions may be such as to cause the gases to emit a continuous spectrum. Higher up, where calcium and hydrogen persist, the radiation of the H and K lines in the gaseous columns still exceeds that of the intervening regions, and bright calcium flocculi are consequently recorded. At the still higher level represented by the $H\alpha$ line, the reduced temperature causes the hydrogen to show its presence mainly by absorption, so that the hydrogen flocculi are darker than the background of generally diffused gases.[6] The stereoscopic picture (fig. 5) permits this hypothesis to be further tested.

Helmholtz estimated the minimum angle between two objects just separated by the unaided eyes to be one minute of arc, but Pulfrich has shown that stereoscopic vision can be obtained when this angle is considerably smaller. Enlargement of the image by a telescope, and the effect of increased inter-ocular distance realized by combining two photographs of a rotating object like the sun, permit the angle to be reduced to surprisingly small values. Thus even on Helmholtz's basis two photographs of hydrogen flocculi on the central meridian of the sun, taken only five minutes apart at the focus of the 60-foot tower telescope, can be combined to give stereoscopic relief. At a distance of 45° from the central meridian this interval must be increased to nearly 7 minutes, but further magnification of the image can be utilized to reduce the minimum time for this region to five minutes or even less. A short time interval serves very well near the limb, in spite of the smaller displacement, because of the enhanced effect of relief due to the curvature of the sun.

The accompanying stereoscopic picture represents the hydrogen flocculi above and surrounding a southern group of sun-spots about 45° west of the central meridian on August 7, 1915. The two exposures were made at 6h. 19m. and 6h. 26m. a.m., respectively, and were thus separated by a time interval of 7 minutes. The enlargement from the original negative is 1.4 diameters, corresponding to a solar image about 24 cm. in diameter. The conditions necessary to secure stereoscopic vision are thus fully realized. It should be noted, however, that while the appearance of relief is probably genuine, so far as the chief elements of structure are concerned, certain false effects are present due to slight distortion of the images. These have the appearance of a horizontal depression running centrally across the picture, between two rounded ridges, on which lie the fringes of distended flocculi above and below the axis of the spot-group. Other false effects may arise in the case of individual details which change materially in form between the exposures. These are few in number, however, and may be readily detected.

In examining the photograph, one is struck by the general resemblance to Langley's drawings of sun-spots and the photosphere (fig. 2). While the difference in scale between the minute phenomena depicted by Langley and the coarser details of the higher atmosphere must be borne in mind, the resemblance can hardly be devoid of significance. As we have already seen, the minute hydrogen flocculi, in undisturbed regions away from spots, are granular in appearance, though larger than the minute grains of the photosphere. At some distance from the spot group (near the margin of the picture) they give place to slender filaments, extending toward the axis of the group as the much finer penum-

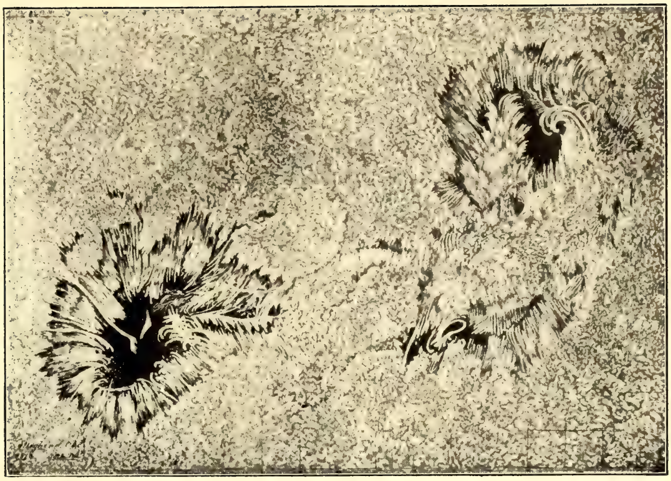


FIG. 2. MINUTE STRUCTURE OF PHOTOSPHERE AND SUN-SPOTS (LANGLEY) SCALE: SUN'S DIAMETER=94 CM. (*Amer. J. Sci.*, 1874)

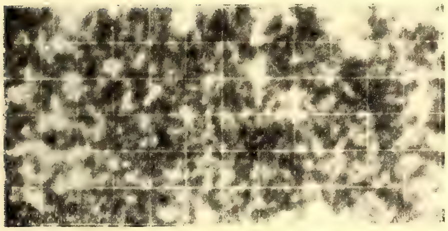


FIG. 3. MINUTE STRUCTURE OF CALCIUM (H_2) FLOCCULI. SCALE: SUN'S DIAMETER=94 CM. (From the Chicago University Press)

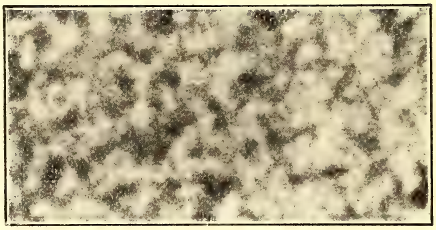


FIG. 4. MINUTE STRUCTURE OF HYDROGEN ($H\alpha$) FLOCCULI. SCALE: SUN'S DIAMETER =94 CM.

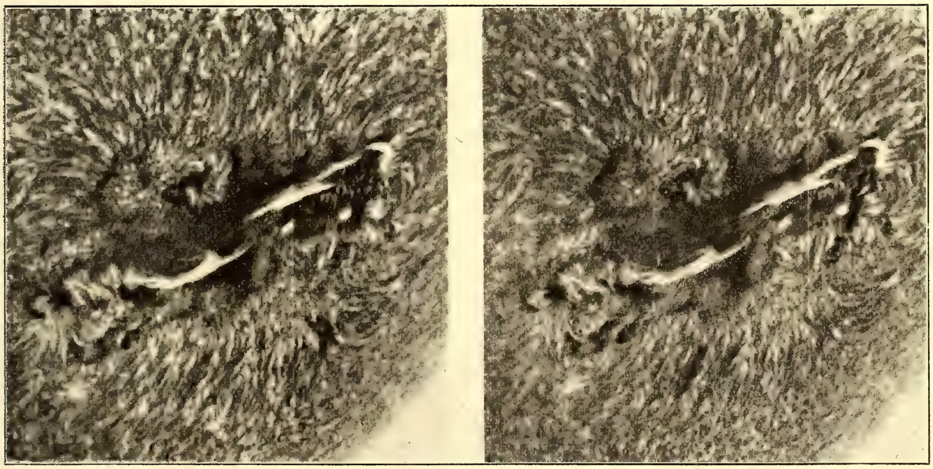


FIG. 5. STEREOSCOPIC PHOTOGRAPH OF HYDROGEN ($H\alpha$) FLOCCULI, 1915, AUG. 7, $6^h\ 26^m$ AND $6^h\ 19^m$ A.M., P.S.T. SCALE: SUN'S DIAMETER=24 CM. (Fig. 5 is a negative reproduction)

bral filaments extend toward the umbra of a sun-spot in Langley's drawing. On either side of this axis, as defined by the long dark flocculus (here bright) which appeared later as a prominence at the sun's limb, they stop abruptly at the edge of a region of honeycomb structure, out of which the long prominence rises like a high ridge. This prominence is shown by the original negatives to be composed of slender filaments, in some parts parallel for long distances, in others apparently intertwined. At its western (right-hand) extremity the prominence curves sharply in a clockwise direction toward the upper spot near the western end of the group (fig. 1), indicated at this level only by a minute white dot, much smaller than the direct image. The extreme western spot, though perhaps the largest in the group, is partially obscured in figure 5, though the hydrogen flocculi may be seen curving toward it. The neighboring spot to the east, however, is plainly visible, and its effect upon the long ridge-like prominence is shown by a second clockwise twist.

The next large spot in the group is clearly seen in the stereoscope, but its smaller companions to the east are obscured by the extensive bright hydrogen flocculi which cover much of the central part of the image, producing the apparently dark background near the middle of the negative print. It should be added that the three large spots just mentioned were shown by spectroscopic observations to be of the same magnetic polarity, indicating that their vortices were rotating in the same direction. This is in harmony with the clockwise curvature of the hydrogen flocculi shown by the stereoscope to be flowing toward each of the spots.[7]

Further to the west (above the center of fig. 1), is a bipolar spot-group, over which the hydrogen vortex is beautifully shown in figure 5 for the western (clockwise) member of the group, though the vortex above the eastern spot is less obvious. The crater-like depression at the center of the western vortex is plainly visible in the stereoscope, which also brings out the bright (dark in fig. 5) star-like boundary surrounding this member of the pair. The magnetic polarity of this spot was the same as that of the three already mentioned.

The two eastern spots near the lower left corner of figure 1 form another bipolar group, but both are partially or wholly obscured in figure 5, and the vortex structure about them is not well defined. At this end of the long ridge-like prominence some divergent filaments rise to a considerable height. These present a remarkable wave-like form in spectroheliograms taken on August 9. It is interesting to note that the highest parts of the prominence and all of the other high level phenomena are dark (bright in fig. 5), as would be expected on the hypothesis that they result from the absorptive effect of the cooler hydrogen.

It may be added that many spectroheliograms previously and subsequently obtained, especially when this region was carried to the west limb by the sun's rotation, fully bear out the above interpretation of the stereoscopic effect, which is in harmony with the conclusions already stated. Thus they confirm the usefulness of the stereoscopic method, and further substantiate the view as to the nature of these long dark flocculi (called 'filaments' by Deslandres) which we expressed when we first detected them with the $H\beta$ line at the Yerkes Observatory in 1903[8] and again when we photographed them with $H\alpha$ on Mount Wilson in 1908.[9]

We have shown in this paper that the minute structure of the quiescent solar atmosphere resembles that of the photosphere. In disturbed regions, the small granular elements (minute flocculi) are replaced by numerous slender filaments, lying side by side, and recalling the structure of the penumbra in sun-spots. While these results appear to support the hypothesis that the solar atmosphere consists of parallel columns of ascending and expanding gases, which are drawn out horizontally in spot penumbrae and in disturbed regions of the chromosphere, such questions as the dimensions of the columns and the direction of motion and velocity of the vapor in sun-spots and in the atmosphere about them are reserved for subsequent discussion.

A full account of this work, with additional observations and photographs, will be published in the *Astrophysical Journal.*

[1] *Amer. J. Sci.*, Ser. 3, **7**, 87 (1874). Figure 2, Plate I, is taken from this article.

[2] This remark applies to visual observations. The valuable photographic results of Janssen, Hansky, Chevalier and others will be discussed in another paper.

[3] Chevalier, from a series of measures of the bright grains on his excellent photographs of the photosphere, concludes that they cover approximately one-third of the solar surface; *Ann. Obs. Zô-Sè,* **8,** C 20 (1912).

[4] We have shown in a previous paper (*Pub. Yerkes Obs.*, **3**, Part I, p. 14) that the average cross-section of the calcium flocculi at the lower (H_1) level is apparently smaller than at the H_2 level. Thus the calcium vapor probably expands as it rises.

[5] Figure 3, which is taken from our paper cited above, does not give a correct impression of the area occupied by the minute calcium flocculi, as several larger aggregations are present. In determining the size of the minute flocculi, the instrumental conditions, as well as the seeing, must also be taken into account.

[6] The bright eruptive flocculi of hydrogen are not in question here.

[7] The direction of rotation of sun-spot vortices will be discussed in another paper.

[8] *Pub. Yerkes Obs.*, **3**, part I, p. 21.

[9] Solar Vortices. *Contrib. Mt. Wilson Solar Obs.*, No. 26, *Astrophys. J.*, **28**, 100 (1908).

MONOCHROMATIC PHOTOGRAPHY OF JUPITER AND SATURN

By R. W. Wood

DEPARTMENT OF PHYSICS, JOHNS HOPKINS UNIVERSITY

Received by the Academy, January 28, 1916

Photographs of the moon, which I made several years ago by means of ultra-violet light, revealed the existence of an extensive dark area surrounding the crater Aristarchus, which did not appear on plates made by means of yellow light, and could not be detected visually. Control experiments, made in the laboratory, made it appear probable that this deposit consisted of sulphur or sulphur bearing rock.

It appeared to me probable that even more interesting results would be obtained in the case of the planets, especially Mars, and preliminary experiments were carried on during the past summer at my East Hampton Laboratory with a horizontal telescope of 56 feet focal length and 16 inches aperture, figured by Mellish, and nickel plated by the method described in the *Astrophysical Journal* for October.

A deposit of nickel is necessary for the reason that silver reflects only 4% of the light in the region of the spectrum utilized, which lies between wave-lengths 3000 and 3300. This mirror was utilized in conjunction with a large Gaertner coelostat, which was placed at my disposal by the Naval Observatory. The mirror of this instrument was replaced by a 16-inch flat, also nickel plated, and photographs were made of the moon and Jupiter by means of infra-red, yellow, violet and ultra-violet light. Notwithstanding the fact that a moving plate holder, provided with eye-pieces for accurate following, was used, it was found difficult to secure sufficiently sharp definition, owing to the rather rapid periodic drift of the image which was very difficult to follow. The seeing moreover was not very good, and on the few nights when it was fair the mosquitoes were bad. Much valuable experience was gained however in the use of the monochromatic ray filters.

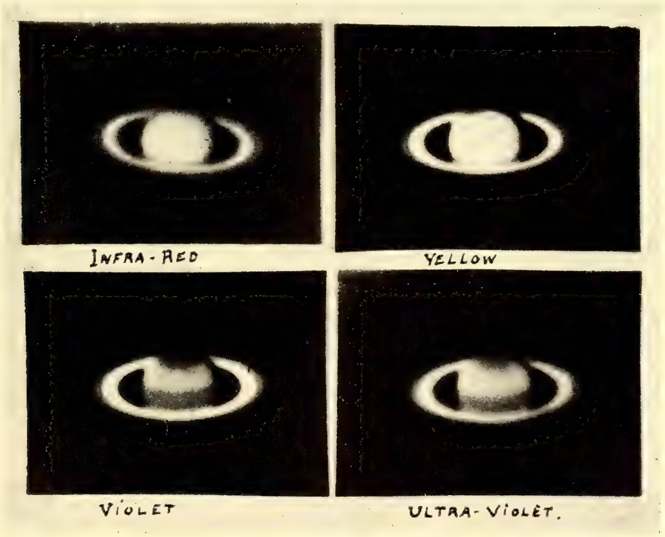
In the earlier experiments on the moon, I used, for the ultra-violet ray filter, a rather thick deposit of metallic silver on uviol glass. This was abandoned in the present work, owing to its great opacity even for the ultra-violet region transmitted, and a rectangular glass cell, closed with plates of uviol glass, and filled with dense bromine vapor, used in its place. Such a cell, when used with an ordinary (i.e., not isochromatic) plate, gives a photographic image formed by wave-lengths below 3500 exclusively. By combining it with a thin layer of a very dilute solution of chromate of potash, the transmission is practically the same as with the silver film, while the time of exposure is only about one-tenth as long.

In October, through the courtesy of Dr. G. E. Hale, Director of the Mt. Wilson Observatory, the 60 inch reflector was placed at my disposal for four nights, and by working continuously from sunset to dawn, I was able to secure a very complete set of pictures of the moon in yellow and ultra-violet light and some forty negatives of Jupiter and Saturn in infra-red, yellow, violet and ultra-violet light. The telescope was used at the eighty foot focus and as the mirrors were silvered, the region of the spectrum utilized for the ultra-violet photography was slightly less refrangible than in the earlier work. Preliminary experiments had been made on the spectrum of sunlight after three reflections from silver and transmission through the bromine cell. Full data regarding these matters will be published shortly in the *Astrophysical Journal.*

Extremely interesting results were obtained in the case of Saturn. The infra-red picture, taken by wave lengths above 7200, showed the ball of the planet almost devoid of surface markings, the merest trace of the faint narrow belts appearing. The photographs made with the yellow screen showed the belts distinctly, giving about the same impression as visual observations. On the plates made with the violet ray-filter (transmission 4000–4500), a very broad dark belt surrounded the planet's equator, occupying the region of the planet which was brightest in yellow light. In addition to this dark equatorial belt a dark polar cap of considerable size appeared in the pictures. So different were the two pictures that, were it not for the ring, it would be difficult to believe that they represented the same object. In ultra-violet light the appearance was much the same, but the dark belt was not quite so wide, the bright region between the polar cap and the belt being distinctly broader.

Photographs made with the four monochromatic filters are reproduced. Two hypotheses suggested themselves in explanation of the dark belt. We may be dealing with a fine mist or dust which forms an extension of the crepe ring down to the ball of the planet. This hypothesis appears to be favored by the circumstance that on the negatives made by ultra-violet light (and to a less degree by violet) the sky between the ball of the planet and the ring is distinctly denser than the region just outside. This would indicate that the region inside of the ring was filled with some material which reflected the short wave-lengths to a slight degree. No trace of this darkening appears on any of the plates made with the yellow screen, even on one that was many times overexposed, which appears to show that the phenomenon is real. This peculiarity was, however, not detected until the work was finished, and I should prefer to verify it, or have it verified, before setting it down as an established fact. The luminosity is much too feeble to show in the prints. Dr. Hale will have the density measured with the Hartman photometer as soon as some repairs are finished. I do not, however, believe that the dark belt is in reality due to the absorption of a dust ring, for measurements made by Mr. Ellerman showed that the belt extended higher up on the ball of the planet than the line of intersection of the plane of the rings. Moreover it seems highly probable that the belt and the dark polar cap are to be explained in the same way. The second hypothesis assumes the existence in the planet's atmosphere of some substance capable of absorbing violet and ultra-violet light. This material might be a fine mist or dust, or some gas capable of absorbing the more refrangible part of the spectrum. Such a gas would be of a pale yellow color, and sulphur vapor and chlorine naturally occur to us.

I have, however examined the absorption spectrum of both of these gases in quartz bulbs, with the result that the absorption appears to be much stronger in the ultra-violet than in the violet, which is in disagreement with the circumstance that the band appears wider in violet than in ultra-violet light. Until spectrograms are available showing the complete spectra of the different zones of the planet it is useless to speculate as to the nature of the material which causes the absorption. On the infra-red photograph the ball of the planet is much brighter in



MONOCHROMATIC PHOTOGRAPHS OF SATURN, OCTOBER 23, 1915 comparison to the brightest part of the ring than on the violet and ultra-violet pictures. This again suggests a mist or dust in the planet's atmosphere which scatters the shorter wave-lengths. My infra-red photographs of landscapes have clearly demonstrated that we can obtain clear photographs through a blue haze by means of the spectrum region above 7200. In these photographs it will be remembered that the blue sky comes out black and the grass and foliage snow white. (See my *Physical Optics*, page 626, second edition.)

Interesting results were obtained as well in the case of Jupiter. In infra-red light the belts were scarcely visible, while the violet and especially the ultra-violet pictures showed dark belts, of which no trace could be seen in pictures made with the yellow screen, or by eye observations of the planet.

In view of the interesting results obtained with these two planets I hope that similar observations will be made of Mars on the occasion of its next near approach to the earth. I made one photograph of the planet in October with ultra-violet light, but the disc was too small to show much of interest.

I have been aided in this work by a grant from the Gould Fund of the National Academy of Sciences.

CONTENTS

Page

Psychology.—Personal Equation and Steadiness of Judgment in the Estimation of the Number of Objects in Moderately Large Samples . . . *By J. Arthur Harris* 65

Chemistry.—Polypeptide-Hydantoins . . . . *By Treat B. Johnson* 69

Botany.—Recent Explorations in the Cactus Deserts of South America . *By J. N. Rose* 73

Astronomy.—On the Albedo of the Planets and Their Satellites . . . . *By Henry Norris Russell* 74

Physics.—Quantum Relations in Photo-Electric Phenomena . . . . *By R. A. Millikan* 78

Chemistry.—The Chemical Activity of the Ions of Hydrochloric Acid Determined by Electromotive Force Measurements . . *By James H. Ellis* 83

Zoology.—Effects of Centrifugal Force on the Polarity of the Eggs of Crepidula . . . . *By Edwin G. Conklin* 87

Physics.—The Emission Quanta of Characteristic X Rays . *By David L. Webster* 90

Geology.—The Results of Investigations of the Ecology of the Floridian and Bahaman Shoal-Water Corals . . . . *By Thomas Wayland Vaughan* 95

Paleontology.—Cambrian Trilobites . . . . *By Charles D. Walcott* 101

Astronomy.—The Minute Structure of the Solar Atmosphere . . . . *By George E. Hale and Ferdinand Ellerman* 102

Astronomy.—Monochromatic Photography of Jupiter and Saturn . . . . *By R. W. Wood* 109

VOLUME 2 MARCH, 1916 NUMBER 3

PROCEEDINGS

OF THE

NATIONAL ACADEMY OF SCIENCES

OF THE

UNITED STATES OF AMERICA

JUN [illegible]
National [illegible]

EDITORIAL BOARD

ARTHUR A. NOYES, *Chairman*
EDWIN B. WILSON, *Managing Editor*
ARTHUR L. DAY, *Home Secretary*
GEORGE E. HALE, *Foreign Secretary*

J. J. ABEL
W. B. CANNON
J. McK. CATTELL
J. M. CLARKE
E. G. CONKLIN
J. M. COULTER
C. B. DAVENPORT
SIMON FLEXNER
E. B. FROST
R. G. HARRISON
W. H. HOLMES
J. P. IDDINGS
R. A. MILLIKAN
E. H. MOORE
ALEXANDER SMITH

Publication Office: Williams & Wilkins Company, Baltimore
Editorial Office: Massachusetts Institute of Technology, Boston
Home Office of the Academy: Washington, D. C.

Entered as second-class matter at the Post Office at Baltimore, Md., under the Act of August 24, 1912

INFORMATION TO CONTRIBUTORS

The Proceedings is the official organ of the Academy for the publication of brief accounts of important current researches of members of the Academy and of other American investigators, and for reports on the meetings and other activities of the Academy. Publication in the Proceedings will supplement, not replace, that in journals devoted to the special branches of science. The Proceedings will aim especially to secure prompt publication of original announcements of discoveries and wide circulation of the results of American research among investigators in other countries and in all branches of science.

Articles should be brief, not to exceed 2500 words or 6 printed pages. Technical details of the work and long tables of data should be reserved for publication in special journals. But authors should be precise in making clear the new results and should give some record of the methods and data upon which they are based. The viewpoint should be comprehensive in giving the relation of the paper to previous publications of the author or of others, and in exhibiting, where practicable, the significance of the work for other branches of science.

Manuscripts should be prepared with a current number of the Proceedings as a model in matters of form, and should be typewritten in duplicate with double spacing, the author retaining one copy. Illustrations should be confined to text-figures of simple character, though more elaborate illustrations may be allowed in special instances to authors willing to pay for their preparation and insertion. Particular attention should be given to arranging tabular matter in a simple and concise manner.

References to persons should include initials; citations of literature, numbered consecutively, should be placed at the end of the article; and short footnotes should be avoided. Indicate 'subject' for 'Contents.'

Papers by members of the Academy may be sent to Edwin Bidwell Wilson, Managing Editor, Mass. Institute of Technology, Boston, Mass. Papers by non-members should be submitted through some member.

Proof will be sent to authors only on their request, in order to avoid delay in publication. Authors are therefore requested to make final revisions on the typewritten manuscripts. The editors cannot undertake to do more than correct obvious minor errors.

Reprints should be ordered at the time of submission of manuscript. They will be furnished to authors at cost, approximately as follows:

Reprints of - -	2 pp.	4 pp.	6 pp.	8 pp.	Covers extra
Charge for first 100 copies	$1.00	$1.30	$2.20	$2.20	$2.00
Charge for additional 100s	.25	.45	.85	.85	.50

INFORMATION TO SUBSCRIBERS

Subscriptions at the rate of $5.00 per annum should be made payable to the National Academy of Sciences, and sent to Williams & Wilkins Company, Baltimore, or Arthur L. Day, Home Secretary, National Academy of Sciences, Smithsonian Institution, Washington, D. C. Single numbers, $0.50.

Copyright, 1916, by the National Academy of Sciences

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES
Volume 2 MARCH 15, 1916 Number 3

THE MECHANICS OF INTRUSION OF THE BLACK HILLS (S. D.) PRE-CAMBRIAN GRANITE

By Sidney Paige
U. S. GEOLOGICAL SURVEY, WASHINGTON, D. C.

Received by the Academy, January 25, 1916

The Pre-Cambrian rocks of the Black Hills consist of a great series of slates and schists, for the most part monotonously alike, striking in a northwest direction and having steep dips generally, except in the extreme southwest, to the east. Close study shows that the persistent eastward dips represent both schistosity and bedding, the two for the most part parallel; and that the series is compressed into a number of great folds which comprise innumerable minor isoclinal folds. A sufficient number of individual beds have been traced to locate the position and nature of the greater axes of folding, and to locate two important faults.

To the south great intrusions of granite break through the strata, and around the principal mass, forming Harney Peak, a notable schistosity is developed parallel with the granite contact, and superimposed upon the normal schistosity of the region.

Field relations indicate that this granite came into its present position in the main by physical distension of the invaded rock body, under great load, and that it thus modified to an important degree the normal process of regional compressive folding, forcing the schists into closely appressed recumbent folds, parallel with the advancing surface of the magma. The rocks were deformed; they yielded to the advance of the magma, and the schistosity produced by this movement and folding lent itself to the further injection of the granite by numberless parallel dikes and by lit-par-lit intrusion. The harder rocks, the quartzites, were distended and broken apart by the upward movement, and the magma flowed in between the blocks.

JUN 24

The relation of dikes to schist layers, the fact that lit-par-lit injection tends to neutralize chemical and physical differences between the magma and the invaded rock, the probable low temperature of the granite, and the fact that positive evidence of absorption or assimilation on a large scale is lacking, all support the belief that physical displacement and not assimilation was the primary process by which the forward movement of the magma was accomplished. The possibility that large blocks of roof in the areas of harder quartzite have been engulfed in the magma, and that this process was in these areas an important one, must remain an open question. In the main it is considered that the magma actually displaced the rocks either along great rifts or by pushing the yielding softer rocks before it into recumbent folds.

ON THE FOSSIL ALGAE OF THE PETROLEUM-YIELDING SHALES OF THE GREEN RIVER FORMATION OF COLORADO AND UTAH

By Charles A. Davis

BUREAU OF MINES, WASHINGTON, D. C.

Received by the Academy, January 21, 1916

In the region extending from northwestern Colorado west into Utah and north into Wyoming, there are great areas of, generally, carbonaceous shales, which, in places, exceed 3000 feet in thickness. Scientific, as well as economic interest has been aroused in these shales, because they have recently been discovered to yield petroleum when subjected to destructive distillation in closed retorts. These shales are typically dark brown in color, very fine grained, hard, firm, tough and compact, except where weathered into thin, more or less curled and brittle laminae. Surfaces long exposed to the action of the weather bleach to whitish or yellowish gray. Some beds in the series of these shales already examined are so highly carbonaceous that they closely resemble compact lignite in appearance and burn readily when heated; other beds contain rather high percentages of finely divided mineral matter.

When freshly broken, the rock gives off a distinctly bituminous odor, but, so far as observed, it contains no free oily compounds, although nodules and particles of a substance resembling asphaltum, but insoluble in the usual solvents of that substance, are sometimes found; and, as already noted, petroleum is always found among the products of its distillation.

Careful study of the stratigraphic relationship of these shales by the U. S. Geological Survey, has shown them to belong to the Green River

Formation of Eocene time. This formation is widely known in some of its Wyoming phases for its richness in fossil fish remains.

About a year ago, samples of the shale from near De Beque, Colorado, and, later, from other localities, were submitted to me for microscopic examination. Inspection led to the conclusion that the rock was composed so largely of organic residues, that it would be practicable to apply to it methods of sectioning already in use for making thin sections for the microscopic study of peats and coals.

After some experimental work, it was found possible so to soften the material without any visible change in its original form or structure, that after it was imbedded in paraffin, sections could be made from it with razor or microtome as readily as from properly prepared fresh vegetable tissue. Sections of any desired thinness can thus be made, and in any quantity. Moreover, these sections can be so treated that when mounted they can be studied with the highest powers of the compound microscope.

Several highly interesting and important facts have been discovered by examining specimens of the shale prepared as outlined above. The ground mass of the sections was found to be of the nature of a somewhat granular, organic jelly, closely resembling in optical properties, some of the structureless, colloidal or sapropelic peats, solidified and compacted into a dense, tough, impermeable magma.

Apparently the hardening and compacting of this material from its original gelatinous condition to its present state, was a process of slow dehydration and contraction, without any violent disturbance or action of any outside metamorphosing agencies. This is shown by the undisturbed position and structure of the shale beds in the mass, as well as by the almost entire lack of visible changes in the delicate plant structures that are often abundant in the sections examined.

The ground mass, magma, or body of the shales, from its structure as seen under the microscope, seems to have been originally vegetable matter. It apparently was a mixture of finely divided and flocculent plant debris and living microscopic plants, bacteria, etc., most of which probably were more or less saprophytic, decomposing and still farther disintegrating the mass of detritus in and on which they lived.

Imbedded in this magma, as many of the sections showed, are innumerable plant cells, structures, and entire plants, almost as perfectly preserved as if they had been prepared by the most refined methods of the plant morphologist. Many of these fossils are spores, fungi or structures belonging to the higher plants, but a large percentage of them are clearly Algae of low types. Three distinct types of Algae have been discovered by the work so far done.

1. A very considerable number of cellular, filamentous and gelatinous forms, which, from their methods of reproduction and general morphology, are clearly to be placed with the Blue-Green Algae—(*Myxophyceae* or *Schizophyceae*). These include forms that show close relationship to modern types of these low Algae, not only in size and vegetative structure, but also, many of them show a series of reproductive stages so characteristic as to fully establish their taxonomic relationships. In one instance at least, a minute but highly characteristic blue-green Alga has been found that can almost certainly be referred to a living genus, namely *Spirulina*. This fossil specimen is so well preserved and the measurements and form of the plant are so closely in harmony with those given in descriptions of one of the species of the genus, that one is tempted to identify the fossil as belonging to the same species, notwithstanding the hundreds of thousands of years which must have elapsed since the Eocene time. Other fossil species that may be referable to existing genera are not uncommon.

2. The Yellow-Green Algae (*Chlorophyceae*). Somewhat less common than the low forms already mentioned are plants that can be referred with more or less certainty to the Green Algae. Among these, both the cellular and filamentous types have been discovered, and, in a few cases, the identification may be said to be positive. Among cellular forms, is one in particular that so closely resembles a species of *Pediastrum* in all essentials, that its reference to this or a closely related genus is clearly not an error.

Among the filamentous fossil forms classed as Green Algae, a type has been noted with spiral chloroplasts, so like those which characterize the well-known *Spirogyra*, that it appears at least to be a prototype, if not strictly a species of this genus. Other filamentous fossil species have been found which, because of structural peculiarities, can be safely placed among the Green Algae.

3. Unclassified Algae. A large number of plants have been found, which, from their form and structure, appear to be Algae, but which as yet, cannot be classified among known living Algae. Some of these unclassified types are the largest plants so far found in the shales, and others are very minute. Some of the more noteworthy of the large forms seem to have been thin, gelatinous films growing on the surface of the detrital material which is now the imbedding magma.

A very common and puzzling plant is one of the gelatinous film-like forms of considerable size, which seems to be without cellular structure, but which, nevertheless, has small areas on its surface which on magnification disclose definite, usually oval, spots covered with a net-like pattern formed by small crowded hexagonal depressions on the surface of the film. The meaning of these netted areas has not yet been worked out, but it seems possible that the structure is a reproductive surface from which spores arise. The whole plant after most careful examination seems to have been made up of gelatinous material forming a thallus, with no definite divisions into cells.

Besides the filmy gelatinous types, there are large numbers of seemingly gelatinous forms that were more or less filamentous. These types vary greatly in the length and width of their filaments, which are often more or less flattened, and which branch in an eccentric way, quite unusual among the modern Algae. The filaments lack visible divisions into definite cells and seem to have been wholly gelatinous or to have been sheathed by thick gelatinous envelopes, which have obliterated the evidences of cell division.

It is well known, however, that we have very slight and indefinite knowledge of the floras of present-day fresh and salt waters, especially of those which grow in and around the deeper parts of waters in which there are deposits of organic origin, and many of the plants which have been described from shallow waters are so imperfectly known that their life histories have never been worked out. These facts make the study and classification of fossil forms much more difficult than it would otherwise be, and their relationships harder to determine.

Careful study of the material under discussion indicates how perfectly plant remains, originally of the most delicate structure, have been preserved. Even very thin-walled and fragile cells and filaments show by internal evidence that they did not even collapse as they were incorporated into the mass in which they are preserved. Contraction, caused by shrinkage from the loss of water in the original substances, and the later changes which have transformed them into rock, has manifestly occurred. Dehydration has been so slow and regular, however, that it has produced no changes that have in any way interfered with the microscopic study of the properly prepared material. The perfection of the preservation of the plant remains in these rocks is such that it is quite evident that, with such specimens to work with, there need be no difficulty in learning the facts or in fully and correctly interpreting them. All doubt as to the possibility of Algae, and similar cellular plants of most delicate structure being preserved and their remains becoming a conspicuous part of the organic matter found in carbonaceous rocks is removed by the studies here recorded.

The discovery of this interesting flora of minute fossil Algae, doubtless the most extensive of its class yet reported, embracing as it does thousands of individuals and an undetermined number of species, many of which have not yet even been photographed, is in itself a noteworthy matter. The discovery seemingly has a more important bearing, however, on the broad geologic problem of the origin and development of petroleum and related carbonaceous compounds, than it has on that of the development of plant life.

Several steps looking towards the solution of this great genetic problem have been taken already and tentative advances have been made. The structure of the rock, the kind of fossils in it, and their state of preservation, make it possible to determine, to some extent at least, the sources of the abundant carbonaceous matter which it contains, and the conditions under which this matter accumulated. The state of decomposition of the original organic substance contained in the magma has, to some extent, to be assumed, since it is so largely structureless. It is practically certain from studies made on peat, that the breaking down of the original structure of this material is rather a physical than a chemical change, a comminution, not a decomposition. The true chemical condition of the organic matter of the rock is more correctly interpreted from the fossils, which probably are approximately at the same stage of carbonization as the structureless magma. The chemical condition of the finely preserved fossils, therefore, becomes highly significant, this, judging from their color, physical condition, etc., is indicative of very early stages of carbonization, which in turn, shows a very limited action by the transforming agencies originating within the earth itself, i.e., those producing metamorphism.

Finally, and of great importance in the consideration of the larger problem, is the determination of the actual substances now existing in the rock itself, from which the distillates obtained by heating, are derived. It will readily be seen that this constitutes a somewhat prolonged investigation but it is easily within the reach of modern chemical science.

It can already be stated positively that the organic matter of these shales, so far as examined, in very large part is of vegetable origin, with but very slight traces of the remains of animal organisms. The plant remains were accumulated in water of considerable depth and purity and in places apparently remote from shore lines. To a considerable extent this vegetable matter seems to have been of the nature of drift material or plankton, although it is largely mixed with the remains of Algae and similar plants that certainly grew upon and in the magmatic mass as a place of attachment, if not actually aiding in its decomposition by their growth.

The beds thus formed were built up very slowly, and when completed and solidified were so compact and fine grained as to be nearly or quite impermeable to liquids from the outside. If infiltrated by any material from without before the water originally included was drained and dried away, it is difficult to see how this could be done by liquids lighter than water, coming into it in the form of a uniform seepage from below. Exactly contemporaneous with the gradually progressive disappearance of the water and the chemical changes accompanying this, the sediments were compacted, solidified and lithified to their present texture and condition, and it seems highly probable that penetration by liquids from any outside source during the induration would be increasingly difficult. If these views are correct, the gaseous and liquid compounds obtained by distillation from the shale are not of external origin, but result from the decomposition of the carbonaceous material laid down during the formation of the rock. The absence of demonstrable liquid hydrocarbons in the shale itself would tend to strengthen this view, although not an insurmountable objection.

(Published by permission of the Director of the U. S. Geological Survey)

ARCHEOLOGICAL EXPLORATIONS AT PECOS, NEW MEXICO

By A. V. Kidder

DEPARTMENT OF ARCHAEOLOGY, PHILLIPS ANDOVER ACADEMY

Received by the Academy, February 1, 1916

During the past summer the Department of Archaeology of Phillips Andover Academy began the excavation of the ruined Indian Pueblo of Pecos in San Miguel County, New Mexico. It was the wish of the Department to undertake the thorough exploration of a site large enough and of sufficient scientific importance to justify work upon it for several years.

Pecos lies in the well-watered and fertile upper Pecos valley at an elevation of about 7000 feet. The buildings are located on the summit of a flat-topped ridge of sandstone slightly elevated above the surrounding plain, and consist of two house-clusters containing together, as nearly as can be made out in their present state of ruin, about 1100 ground floor rooms. A few hundred feet to the south stand the roofless walls of the large Spanish mission church.

The place was discovered by Coronado in 1540. At that time it was the most populous of the Pueblo villages, being able to muster 500 fighting men. The chroniclers of the expedition describe the town as a quadrangular structure, four stories in height, on the balconies of which one could make the entire circuit without putting foot to ground. The people were brave, prosperous and skilled in all the arts of the Pueblos. After Coronado, Pecos was visited by several other explorers and when New Mexico was actually settled by the Spanish at the beginning of the seventeenth century Pecos was made the seat of a mission, a large church was erected and the town became one of the most important and prosperous in the province. During the eighteenth century, however, its position on the eastern frontier exposed it to ever-increasing raids by the Comanche. Thus worn down and greatly reduced by sickness, the Pecos people managed to hold out until about 1840 when, numbering only seventeen souls, they abandoned their home and put themselves under the protection of the Jemez, a nearby tribe related to them by language and tradition.

We have, then, a historically recorded occupation from 1540 until 1840; and an examination of the potsherds scattered about the ruins shows also practically all the types of prehistoric wares known to occur in the upper Rio Grande district. The Department therefore believed that the site had been settled in very early times and that people had lived there, presumably continuously, through all the intervening centuries. So long an occupation does not seem to have occurred at any other place in New Mexico available for excavation and it was hoped that remains would there be found so stratified as to make clear the development of the various Pueblo arts and to enable students to place in their proper chronological order numerous New Mexican ruins whose culture has long been known but whose relation one to another has been entirely problematical. This hope was strengthened by the fact that Mr. N. C. Nelson of the American Museum, New York, had recently discovered very important stratified remains at San Christobal a few miles to the west and it was believed that, if similar deposits could be found at Pecos, they would serve not only to complement and, so to speak, cross-reference the finds of Mr. Nelson, but also to carry the story of Pueblo arts from 1680, the date of the abandonment of San Christobal, down through the very little known eighteenth century to the middle of the nineteenth, from which time to the present day it may still be recovered by studies among the living tribes. These were the chief reasons for the choice of Pecos.

The Regents of the State Museum of New Mexico, in whom the title to the ruins is vested, requested in making their very generous offer of the site, that the old church, greatly damaged by the weather and by vandalism, and in imminent danger of entire collapse, be so repaired and protected that no further harm should ensue. This work was cheerfully undertaken and was carried out during the summer by Mr. J. L. Nussbaum of Santa Fé; the body of the church was cleared, the walls strengthened and underpinned, and fallen portions of several arches, doorways and windows were replaced.

The archaeological work consisted entirely of excavations in the great rubbish heap on the east side of the mesa. Digging was commenced there because it was necessary to clear a place in which earth and stone, removed from the houses above, could later be dumped without danger of covering unexplored ground. This portion of the work continued longer than was expected because of the unforeseen depth and extent of the rubbish deposit. This miscalculation of the mass of the rubbish was due to the mistaken belief that the terrace surrounding the building proper formed part of the solid rock mesa (fig. 1). When, however, the trenches reached the defensive wall supposed to mark the end of the midden, it was found that the terrace, instead of being rock, was composed entirely of rubbish (fig. 2) thus approximately doubling the amount to be handled. Further complications were encountered in the form of buried kivas and old defensive walls, so that at the end of the season the main trench face was still some distance away from the

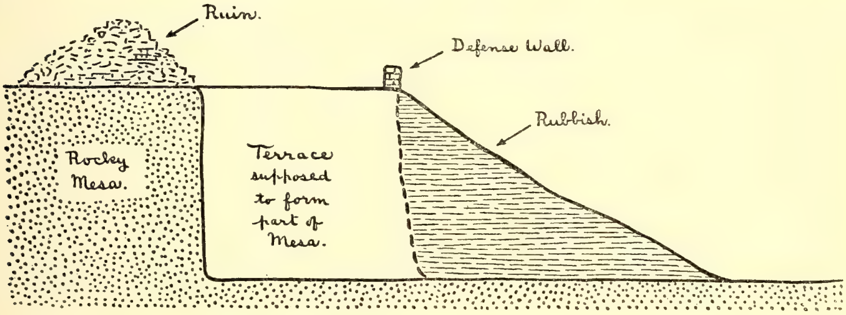


FIG. 1. SUPPOSED EXTENT OF RUBBISH BEFORE EXCAVATION

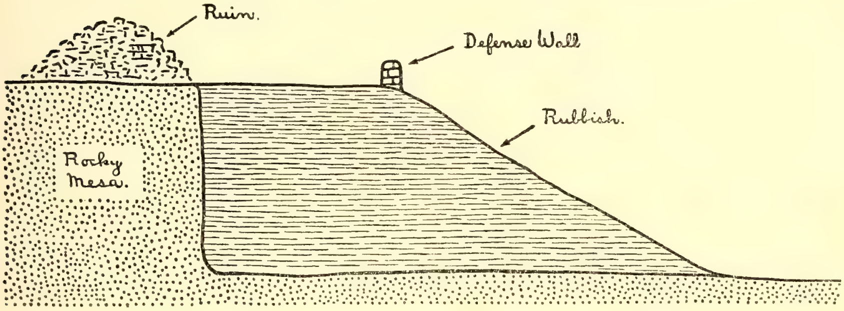


FIG. 2. ACTUAL EXTENT OF RUBBISH

mesa. Three narrow exploring trenches, however, were pushed through the terrace to the rock wall.

Against the mesa the deposit was found to vary from 15 to 20 feet in depth. It sloped away gradually toward the plain. Its composition was very uniform, soft earth darkened by charcoal and decayed organic matter, filled with potsherds, animal bones and chips of building stone. The heap was also used as a cemetery, our trench revealing over 150 burials; the skeletons lay, for the most part, in the flexed position, at depths varying according to the period of interment of from a few inches to 17 feet. Mortuary pottery occurred only in the earlier graves and not abundantly even in them. Besides skeletons and pottery there were recovered a great number of broken and many whole specimens of bone and stone tools, as well as ornaments, clay toys and figurines.

The most important results, however, were, as had been hoped, of a stratigraphical nature. It became apparent, as soon as digging had gone far enough into the refuse heap to permit observations, that the pottery at the bottom was markedly different from that at the top and that there were several distinct types between. Data on this subject were collected to some extent by examination and note taking along the face of the trench, but principally by tests made at different points as the work advanced. The tests consisted of the collection of all the sherds in a given column of débris, the fragments from each layer being placed in a separate paper bag bearing the numbers of the test and of the layer. In the early tests the face was laid off arbitrarily in 1 foot or 18 inch layers; but as it was found that this method often resulted in the splitting of strata, the later tests were not laid out beforehand and a new bag was started whenever there was encountered a zone of sand or ash or when there appeared a sherd different in type from those above. Care, of course, had to be exercised to choose for the tests places which seemed to contain no burials, and all trials were abandoned when it became clear that a grave shaft had disturbed the original deposition of the refuse.

The mass of stratigraphic material thus gathered could not be worked over in the field; final results, therefore, are not yet available. It may be said, however, that the earliest or lowest type was the black-and-white. Above this occurred a long series of wares with glazed ornamentation, running from an early type with clear cut geometric ornament in sharp glaze, up through more developed stages having heavier glaze and conventional ornament, to a late and degenerate style in which the glaze paint was so poor that the decoration lost all form and became little more than daubs and splashes. This degenerate stage extended well into historic times, as was proved by the finding, in that horizon, of European objects, but before the final abandonment of the site there occurred a sort of renaissance, when glaze painting was given up and there was produced a yellowish pottery excellently finished and tastefully decorated with dull black pigment.

The details of this long ceramic history; the growth, for example, of vessel forms; the developments in the cooking wares; the influences from without that undoubtedly produced some of the changes; all these must await closer study and more data.

The stratigraphy of the site, then, throws invaluable light on the local culture; it also may be expected to teach us much as to its external relations. In the first year's work there has been found pottery of the historic period from the Hopi villages, Acoma, Zuni, and Jemez, as well as prehistoric wares from the Little Colorado, Lower Gila (?), and San Juan. From the East we have pottery, buffalo-scapula hoes and snubnosed scrapers; from the South, clay bells and spindlewhorls strongly Mexican in type. Such finds as these give an indication of what important results may confidently be hoped for. If we can definitely recognize and chronologically arrange the successive culture stages at Pecos, we can extend that knowledge and thus fit into their proper chronological order the one-culture ruins that abound in the Rio Grande. We may also hope to learn, from trade objects found at Pecos and in the chronologically arranged one-culture ruins, the relative age of many other groups, not only in the Southwest but even well beyond its borders.

MAN AND METALS

By Walter Hough

U. S. NATIONAL MUSEUM, WASHINGTON, D. C.

Received by the Academy, January 24, 1916

In the course of a long-continued study of the uses of fire by man, I have recently completed a review of the branch referring to metallurgy, the results of which are embodied in the following brief summary.

The free metals which early man might have found and used are copper, gold, and silver. It is known that he used copper, and to some extent silver and gold, and that he worked them as other hard substances, becoming gradually acquainted with the property of ductility of these metals. Since copper was the most common metal in a free state and potentially at an early stage the most useful, it is safe to say that its employment was the beginning of the age of metals. Strictly, the age of metals begins with the working of these naturally occurring substances by means of heat. The development of metallurgy has been the development of heat, resulting in remarkable inventions and achievements, and in the increase of temperature from that of the open camp-fire to the fierce heat of the blazing arc. Fire itself, the heritage of man through all ages, though adapted mostly to the advancement of man himself, had very limited use in the early periods, but the reactions of man on fire and of fire on man must have been profound.

Metallurgy is impracticable without a concentration of heat beyond that furnished by a fire in the open supplied with limited fuel. The first and natural step was to confine fire to a limited space demarked with stones or within a shallow earth-basin. Observations would show that the heart of the fire possesses greater heat than the margin and in an especially hot focus, one day when the time was ripe, a metal, likely copper, was tried and was found to melt and run without losing its structure. From such an event as that fancied here metallurgy may have had its crudest beginnings. The objection is noted that copper which liquidifies at 1083°C. would not melt ordinarily in an open-hearth fire under natural draught and this will be commented upon later in describing the efficiency of a smothered fire massed in an earth pit. Early experiments however were far from an established practice of metallurgy, a vast time may have elapsed before metals could be reduced from the ores. The history would begin with free metals regarded as stone and continue with metals (copper) hammered into shape; smelting of free metals (copper), and casting, followed by work in combined stone and metal technique; alloys both adventitious as in tin-bearing copper and designed as in the addition of cassiterite or oxide of tin to copper producing bronze.

This imperfect résumé covers a great advance in heat production to the time when a fourth metal, tin, becomes known to man. The bronze age with its ramifications of art, of science, and of material and social progress can only be characterized as a period when the cruder tentative essays in metals were brought to a somewhat exact metallurgical formula of pre-science. The Bronze Age also illustrates the vast importance of alloys. It more especially marks for the purpose of this paper the production and regulation of heat sufficient to smelt copper and tin forming the alloy known as bronze.

It may be possible to trace the steps by which this heat was reached and it seems probable that its attainment depends on some other use of fire rather than for metallurgy. In this regard attention is called to the confinement of fire in a primitive oven for baking pottery or the cooking of vegetal substances, both very ancient arts and nearer to early needs than the working of metals. The cooking of vegetables in its primitive stage gives rise to ovens which may be ancestors of the furnace for reducing metals, namely the pit or 'gipsy' oven in which refractory fruits and vegetables were cooked. These pits were dug in the earth and heated by burning fuel in them, the coals removed and the vegetables put in and covered up, a fire often being set on top. It is observed that in modern instances the heat produced by a fire of this character is very great at the bottom of the glowing coals. There is a great concentration of heat due to the confinement of the fire in a non-conducting medium and this effect does not depend greatly upon excess oxygenation of the fuel by draught, in fact too much draught cools a fire. In such a fire bronze may easily be melted. There is used to this day a pit-smelter for melting bronze and this device would seem to be a survival. I saw one of these ovens in use in Washington a few years ago where it was employed for melting small portions of bronze for minor castings. European discoveries indicate that bronze was melted in pits in the earth and that the bronze founders had also acquired a knowledge of refractory clays which may have been contributed to some extent by the potter's art. It is evident that clay and metal working are intimately connected.

The production of higher temperatures depends upon the oxygenation of the fuel beyond that ordinarily furnished by natural draught. Natural draught in a forced fire fed with abundant fuel may give a central area of high temperature, but the question of fuel supply would militate against the common employment of this method. The simple way in the earlier periods would be to aerate the fire with a fan-like hand-blower such as are used with the braseros and small cooking fires in many parts of the world. There would follow other devices to produce draught, culminating in the modern tremendous mechanical development for forced draught. Some steps on the way may be observed in the aboriginal pit ovens for cooking previously mentioned. Most of these are without draught holes or any device for producing draught, depending for their utility upon the absorption and retention of heat in the earth wall of the pit. The pit ovens of the Pueblo tribes have a draught flue alongside the pit. A step in advance involved in building separate fire containers is the raising of the fire and the production of bottom draught. To forced draught however we must look for practical advance in metallurgy, and in comparatively recent times the use of preheated draught or blast was epoch making in the science of the production of metals. The forerunner of this was the tuyere embedded beneath the fire.

The knowledge of metals in the Bronze Age was confined to those native metals, copper, gold, and silver, and of reduced metals tin and possibly lead. The development of temperature in the Bronze Age paved the way for the reduction of iron, the most important metal known to man.

In general no extensive reduction of ores could be attained with the facilities of the Bronze Age. It is improbable that much copper ore was reduced in this age, the source of supply being nuggets of native copper. Tin in the form of the oxide resembles a native metal and would attract the attention of man. It is easily reduced at a moderate heat and requires no roasting or flux, but whether this had any bearing on the discovery of bronze is not clear. The combination of copper and tin presents no difficulties since both metals are easily accessible and of wide occurrence. Native copper containing tin and forming a natural bronze is known, but is rare and had no effect on the discovery of alloys. Copper alone is very difficult to smelt and cast so that there is no very definite copper age and not many cast artifacts of this metal have been found. The copper age would be the period when native copper was shaped by hammering as especially in North America. Bronze may have resulted from experiments with mixtures of various substances to lower the melting point and to admit of casting in a closed mould. As a rule ores consisting of oxides of metals presented little difficulty in smelting. Carbonate ores could be oxidized by roasting before smelting following the old process. The chief difficulty in the reduction of ores is due to the presence of impurities chiefly sulphur and phosphorus which remain the bane of modern metallurgists. Another difficulty is the kind and proportion of flux. The question of fuel was not pressing, charcoal being the usual and immemorial supply up to the use of coal.

The niceties of the reduction of ores remained for modern science, early metal workers confining their attention to varieties which could be reduced by the facilities and knowledge in their possession. It is possible that the bronze age even at its focus may have overlapped the iron age and it is not strange that some students should have been led to assert that iron preceded bronze. The accumulated knowledge required for the reduction of an inconspicuous ore to secure a metal not known in a free state and with properties and value unknown is greater than the production of an alloy of two metals one free and the other practically free and both known to man for untold generations. There are also the high temperature, 1200–1300°C., and the experimental data on fluxes required in the reduction of iron ore. The metallurgy of iron was a distinct advance on that of bronze and made use of the experimental knowledge and mechanical equipment acquired in the bronze age. The bellows which classical writers attribute to Daedalus and Anacharsis may have made possible the reduction of copper ores in the later bronze age and may have unlocked at another period the coming metal iron.

The Iron Age, which began before the dawn of history, ushered in the fifth metal whose effect on the material world was destined to be incalculable. Man had seen metallic iron derived from meteorites and there exist implements made from this material, but its occurrence was sporadic and limited, so that it had no effect on his arts. Heavy and lustrous hematite ores were also widely used for implements and ornaments. When and in what locality iron began to be smelted is unknown, but as has been stated, it is the logical successor to the advanced technique of the metallurgy of bronze. Primitive iron working may still be observed in Africa and has been described in India. African ore is an oxide comparatively very easy of reduction and ore beds are of general occurrence. The smelter consists of a basin-shape depression in the ground beaten down hard and leading to the center is a clay tuyere with which the rude bellows are connected. The ore is heaped with charcoal in this depression and in the midst lighted coals are placed and covered over. The bellows are started and after a time the ore is reduced to a fluxed mass in which small pellets of iron are found. The mass is pounded up and the iron sorted out to be hammered into larger pieces by a second process. This was essentially the method pursued in India. Only limited amounts of metal were secured. Casting of iron was unknown in Africa. The knowledge of smelting is quite diffused in Africa so that there are no chief centers of manufacture and distribution, although some tribes are famed as blacksmiths and a few tribes depend on alien artificers for their iron utensils. The smelting of iron ore is impossible without forced draught. The bellows therefore is a device of great interest and importance. The history of draught-producing devices embraces the hand fan, the tube for blowing, the air bag in which the air is captured and forced out by pressure through a tube, the double air bag to promote a steady stream of air, the double air bag worked by rods, in which a valve appears, the double plunger bellows, the piston bellows, the folding bellows with organ valve used by blacksmiths before the invention of the fan blower, and the fan blower which brings back the primitive hand fan much improved.

The most marked types, the complete knowledge of which would cast much light on the history of Western and Eastern foci of metallurgy, are the air bags and piston blower the former belonging to the Europe-Africa province and the latter to Malaysia.

Furnaces also have an instructive development. There may be cited the rudimentary heap-furnace with base draught and rude bellows; the pit furnace for melting bronze with fan draught; the prehistoric earth wall furnace for reduction of copper and iron ores probably with bellows draught; and the tall furnace built of masonry and called Catalan furnace with means for forced draught.

The furnace of the Iron Age has been well described by archaeologists for northern France, Switzerland, Austria and other localities in Europe. On account of convenience they were located on the vein of ore and in forests which would furnish charcoal. The furnace consisted of a pocket excavated in the side of a clay bank lined with clay and, when charged, covered with clay leaving a flue above and draught hole below. The ore and charcoal were packed in layers and the draught was probably supplied with rude bellows.[1]

There is a considerable history, which may only be indicated, in regard to the tools required in metallurgy, as those for handling—pincers, tongs, shovels; for shaping—hammer, file, rubbers, polishers; for casting—crucibles with supports, handlers and poker-stirrers, moulds of stone, metal, clay and artificial mixtures; for finishing—file, abraders, polishers, punch, drill, chisel, rivets, etc.

At some period in the Iron Age it was discovered that iron could be cemented or case hardened by reheating with organic materials away from air, the process forming a layer of steel over the softer iron or, if continued, converting the whole mass to steel. In some localities, possibly India, at remote times metallurgical processes had given rise to steel. This new alloy could be tempered, a quality foreign to any other metal known to man. Up to comparatively recent times however steel had a limited use, but at present among civilized nations steel is more common than iron.

An interesting survey is presented of the state of metallurgic art in the various world areas. It shows that the Pacific Islands, most of the Americas, Australia, and much of Asia are in the premetallic stage; Malaysia is in the beginning of the metallic age by acculturation, the first demand being weapons; native Africa is advanced in iron metallurgy, using two metals; civilized (Mediterranean) Africa advanced in the use of metals; Europe (Mediterranean) shows early development and use of five metals. There are four great ancient foci of metallurgic art; Southern Europe, Northern Africa, Western Asia, and Eastern Asia. The latter is of doubtful origin and affiliation, but the other foci were connected. American foci are Central America, Mexico, Peru, and Wisconsin.

Ancient man beginning with one important and two minor free metals acquired by heat tin and iron giving him an acquaintance with five metals. Modern man has freed 52 more, knows 85 elemental substances and predicts the eventual discovery of several others.

[1] Rollain, A., Scories de fer antéhistoriques., *Bull. Soc. Anth., Paris*, 4 s, 1899, p. 318. Discussion by M. Lionel Bonnemère.

ON THE OBSERVED ROTATIONS OF A PLANETARY NEBULA

By W. W. Campbell and J. H. Moore
LICK OBSERVATORY, UNIVERSITY OF CALIFORNIA

Received by the Academy, February 9, 1916

The geometric forms of certain classes of nebulae are such as to suggest that they are in rotation about axes passing through their effective centers. We refer especially to the spiral nebulae and to those so-called planetary nebulae which are of ring, circular, or elliptical form with relatively condensed or stellar nuclei.

In the latter part of 1915 we tested several planetary nebulae by means of observations made with the Mills 3-prism spectrograph and obtained positive evidences of rotation, as Doppler-Fizeau effects.

The planetary nebula No. 7009 in Dreyer's New General Catalogue, right ascension 20h. 58m., illustrated herewith, was submitted to the test of four spectrograms. In each case the slit of the spectrograph was placed centrally across the image of the nebula and made slightly longer than the diameter of the elliptical outline of the nebula, and the condensed nucleus of the nebula was kept central in the slit during the exposures. In two exposures the slit was placed upon the longer axis of the nebular image, parallel to the slender rectangle drawn above the nebula in the illustration to represent the slit in length and direction. In a third exposure the slit was placed east and west across the image. On these spectrograms the bright lines of nebulium (4959 and 5007A) and of hydrogen (H Beta), comprising the recorded nebular spectrum, were inclined to the 'zero' direction, as indicated by the comparison spectra of hydrogen, helium and titanium on the same plates. This inclination of the lines is illustrated (exaggerated) by the direction of the bright line on the diagram of the slit in the upper part of the figure. The section of the lines corresponding to the western parts of the nebula are displaced to the violet, and the sections corresponding to the eastern parts, to the red. Interpreted as Doppler-Fizeau effects, the western parts of the nebula are approaching us and the eastern parts are receding from us by virtue of the rotation of the nebula. The fainter ends of the lines do not maintain the inclination of the bright central section of the lines but turn backward slightly. We interpret this to mean that the corresponding faint outer strata of nebulosity do not rotate with speeds proportional to the radii of the strata—relatively to the speeds of the bright inner strata—but that they 'lag behind.'

The maximum displacements of the nebular lines correspond to points in the nebula 9 or 10 seconds of arc west and east of the central nucleus, and the observed component of rotational speed is of the order of 6 km. per second.

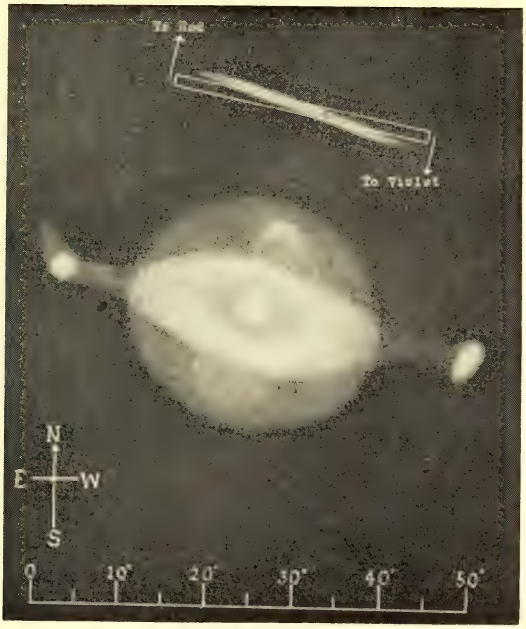
On a fourth spectrogram, with the slit upon the minor axis of the elliptical image, the nebular lines are not inclined to the 'zero' direction, and they therefore fail to indicate any components of rotational motion in the plane passing through the minor axis of the nebula and the observer—a result in accord with expectations.

The observations which we have described, considered in connection with the geometrical form of the nebular image as photographed, leave essentially no room for doubt that the nebula is rotating about an axis through the central nucleus approximately at right angles to the plane passing through the observer and the major axis of the image, the nebular materials lying nearest to us being carried from west to east by the rotation.

Measures of the rotational velocity of the nebula enable us to draw some interesting conclusions concerning its mass. If we assume that the axis of rotation is located as described above, then the orbital speed of the nebular materials lying at a distance of 9 seconds of arc from the center is about 6 km. per second. If we provisionally assume the mass of the central nucleus to equal that of the Sun, Kepler's law connecting the periodic time with the distance from the nucleus tells us definitely that the nebula is distant from us only 8.9 light years. This must be regarded as an improbably small value, in view of other evidence bearing on the question. For assumed distances of 100 and 1000 light years, which we have reason to believe are more probable orders of nebular distance, the masses of the nebula would be respectively 11.3 and 113 times that of the Sun, and the corresponding periods of rotation 1371 and 13,710 years. From these considerations it seems certain that the mass of the planetary nebula N. G. C. 7009 is several times that of the Sun. The nebula is therefore competent, from the point of view of its mass, to develop into a system more pretentious than is our solar system.

A few speculations concerning this nebula may not be without interest and value.

The faint extensions to the east and to the west of the elliptical figure suggest an encircling ring of materials whose principal plane, passing through the nucleus, passes also near our (the observers') position in space. These extensions terminate in condensed nuclei at equal distances from the nucleus and on exactly opposite sides of the nucleus. The faint extensions and condensations may be and probably are largely the effect of the edge-wise projection of such a ring, as in the case of Saturn's rings when the observer is in the plane of the rings. The forms of the two terminating condensations, and especially the wing extending up and out from the east condensation, suggest that we are not precisely in the plane of the assumed ring.



PLANETARY NEBULA N. G. C. 7009 (COMPOSITE DRAWING, FROM CURTIS'S PHOTOGRAPHS OF THE NEBULA MADE WITH THE CROSSLEY REFLECTING TELESCOPE. THE SCALE IS IN SECONDS OF ARC)

The form of the main nebula appears to be ellipsoidal and not chiefly elliptical.

The space immediately surrounding the central nucleus appears to be relatively vacuous. Aside from the nucleus, the principal mass of visible nebulosity exists in the brilliant ring, roughly elliptical as to its inner and outer boundaries, which occupies the region about midway between the nucleus and the outer edge of the nebular structure. The brilliant ring is probably in reality an ellipsoidal shell: the projection of such a shell upon a plane at right angles to the line of sight would naturally show a relatively dark central area, but the projection principle may not be the only one involved.

If this nebula is in process of development into a solar system, the indications are for a system having certain resemblances to our solar system. Our four outer planets have a combined mass 225 times as great as that of the four inner planets. Similarly in N. G. C. 7009, there is apparently a paucity of material to form planets near the nucleus and an abundance of material for planets at greater distances from the nucleus.

In the course of our determinations of nebular velocities in the past three years we have noted a relatively large number of ring forms among the planetaries, and we have observed no extremely-elongated or highly-elliptic gaseous nebulae, and no such forms of gaseous nebulae as would result from thin rings seen edge-wise. According to the probabilities of the case, if the apparent ring nebulae are really of ring form in space, we should have expected to see a number of very elongated elliptic rings and a certain number of relatively long and narrow nebulae. We are accordingly led to question whether the ring nebulae are true rings or whether they are in reality ellipsoidal shells of nebulosity, which appear to be rings wholly or in part by virtue of their projection upon the plane of sight.

Students of cometary orbits in the past five years have apparently established the Kantian view, that comets are bona fide members of our solar system. The observed fact that comets approach the Sun without marked preference for any direction of origin has seemed to be difficult to account for on the more prominent hypotheses of the evolution of the solar system. If the planetary nebula N. G. C. 7009 and other planetaries which involve the ring form have in reality three principal dimensions and are developing into solar systems, the more or less uniform distribution of cometary matter in space of three dimensions does not seem to call for surprise.

A SHORT PERIOD CEPHEID WITH VARIABLE SPECTRUM

By Harlow Shapley

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, February 14, 1916

The most suggestive indication that the stars actually develop from one physical condition represented by a definite spectral type into other closely allied conditions, if it is not in fact the only direct observational evidence of stellar evolution, is afforded by the spectra of certain short period variable stars. From one star to the next in all regions of the sky we observe continuous gradations of many constant characters, such as spectrum, intrinsic luminosity, and motion. That a star begins, passes through and ends its light-emitting career without change of spectral type is considered impossible. The universal and presumably justifiable assumption is that a given spectrum has developed by imperceptible degrees from an earlier type, and will, given time enough, develop into a later one—that every star, in the course of its life history as a luminous object, passes through many if not most of the principal types and sub-types of the spectral series. This is indeed more than an ordinary assumption; it is a scientific creed. It is now generally accepted by astronomers as an inevitable fact, so nearly axiomatic that its demonstration is no more essential than the proof of the fact of organic evolution. The essential studies concern rather the methods of stellar evolution and its numerous details.

That stellar evolution, as reflected in progressive changes of spectral type, has not been witnessed (the phenomena of novae are nebular rather than stellar) is, of course, no cause to doubt the basic assumption. Observations necessarily have been superficial, and time has been short. It is of interest, therefore, and probably of importance as a proof of stellar evolution to have observed a remarkable change in a star, not only of its light, as in the ordinary variable star, nor merely of its velocity, as in the ordinary double star, but of its very physical make-up. And it is also of importance to know that, when the physical state of a star changes and its intrinsic brightness and surface temperature are affected, the spectrum varies (from one recognizable type to another through known intermediate stages) in exactly the same way as we presume the spectra of all stars have changed or eventually will change. It should be noted, however, that differences in spectra for the most part simply represent dissimilar conditions in the relatively thin atmospheric layer that is the source of the absorption lines; indirectly the deeper structure, even to the center of the star, may be and probably is involved.

In the case under discussion in the present note the variation of spectrum is oscillatory in a period of a few hours, and does not represent, therefore, a secular, irreversible variation of spectral type. It does represent, however, an enormous change in the physical condition at the stellar surface, and exhibits the almost incredible feat of accomplishing in less than two hours time a change of spectrum that for the ordinary development of the ordinary star has been supposed to occupy unknown thousands or millions of years.

The variability of the light of the seventh magnitude star RR Lyrae, the brightest of the cluster-type sub-class of the Cepheid variables, was discovered at Harvard seventeen years ago by the unique method of setting a special automatic photographic telescope to follow the star throughout the night and make a continuous and permanent record of its light. Due to the brightness of the variable and its importance in connection with the study of stellar variation, it has received widespread attention. The light variations have been observed at Harvard, Princeton, the Lick Observatory, the University of Missouri, and at observatories in Italy, Germany, Sweden, and Ireland. Special measures of its position have been made in Finland and in Germany, and observations of the spectrum at Harvard, Lick, and Mount Wilson.

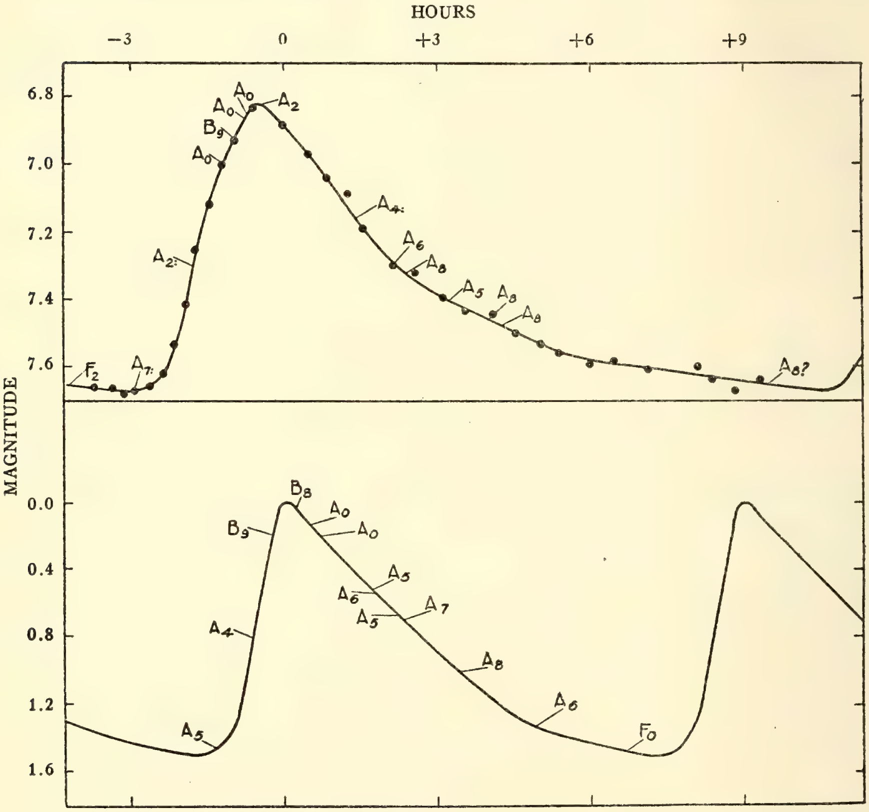


FIG. 1. TOP: MEAN VISUAL LIGHT-CURVE OF RR LYRAE (OBSERVATIONS BY WENDELL) AND VARIATION OF SPECTRUM (SHAPLEY).
BOTTOM: PHOTOGRAPHIC LIGHT-CURVE OF RS BOÖTIS (SEARES AND SHAPLEY) AND VARIATION OF SPECTRUM (PEASE).

Kiess at the Lick Observatory found the star to be a radial-velocity variable—a spectroscopic binary of the peculiar Cepheid type. This linked together the cluster-type and the similar longer-period Cepheid variables, all of which show periodic shifts of the spectral lines, permitting, on the Doppler principle, of interpretation as the result of orbital motion in abnormal but improbable double-star systems. The recent work on the spectrum at Mount Wilson with a 10-inch photographic triplet and an objective prism shows that the spectrum varies between classes A and F in the same period as the variation of light and radial velocity.

The star is, then, a periodic variable in at least three different senses. Every thirteen and a half hours, due to some cause as yet not fully understood, the light increases rapidly, more than doubling its intensity in less than two hours. During the same interval the apparent velocity of the star toward the earth increases from 47 to 91 kilometers a second, and the spectrum changes from type F to type A.

This last variation is probably the most remarkable of all, for it implies, on the basis of our present knowledge of stellar radiation, a change in the temperature at the radiating surface of some three thousand degrees centigrade. Presumably the mean color of the light of RR Lyrae changes perceptibly during these cataclysmic two hours, but the evidence on that point is not yet conclusive.

After the maximum of velocity, of light, and of spectrum have been reached, a reaction sets in, sharply at first, then more gradually, all factors decreasing for nearly half a day. The average time between outbursts is known to a fraction of a second, and the hour of a maximum a century from now could be predicted with considerable certainty. Doubtless a common cause underlies all these variations, and their proper interpretation and correlation will constitute the solution of Cepheid variation.

Though variation of spectral type is suspected in the case of a few other cluster-type variables, and in fact, on the basis of the present results, may be suspected with just cause of all short period Cepheids, the observations have definitely proved the spectrum variable for only two stars. For both of these the spectra have been studied at Mount Wilson, the data for RS Boötis being derived from spectrograms made by Pease at the principal focus of the 60-inch reflector. The results for the two variables are shown graphically in the accompanying figure, where ordinates designate brightness in magnitudes and abscissae give time in hours. The light curve of RS Boötis, along which are plotted the determinations of the spectrum class for the corresponding phases of the light variation, is derived from unpublished photographic observations by Mr. Seares and the writer. The light curve of RR Lyrae is derived from observations made at Harvard.

Although the mean period of RR Lyrae is remarkably constant, conspicuous transitory irregularities are present both in the period and in the form of the light curve, much resembling those of the cluster-type variable XX Cygni previously discussed in these PROCEEDINGS. In particular, there is a slow but definite variation in the length of the period which seems to complete its cycle in about sixteen years; and an oscillation in the time of the rise to maximum light which at times appears to have a uniform period of about forty days, and at other times to be erratic in both period and amplitude. The formula:

$$\text{Max.} = 2414856.451 + 0^{\mathrm{d}}.566831E - 0^{\mathrm{d}}.024 \sin (0^{\circ}.0340E - 104^{\circ}.5)$$
$$+ [0^{\mathrm{d}}.004 + 0^{\mathrm{d}}.013 \cos \frac{2\pi}{70} (E - 45)]$$

in which E represents the number of periods, predicts the individual maxima with high accuracy for the first seven of eight hundred periods after the initial epoch, and by dropping the short period harmonic it gives the mean maxima accurately for at least fifteen years from the initial date. The complete discussion of these features, based on all the available series of light measures and on the recent observations of the spectrum, will be published as a *Contribution from the Mount Wilson Solar Observatory*.

THE SPECTRUM OF δ CEPHEI

By Walter S. Adams and Harlow Shapley

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, February 8, 1916

About 130 years ago Goodricke discovered the periodic fluctuations in the light of the naked-eye star δ Cephei—the eighth variable known to astronomers. None of those previously recorded exhibited the type of light variation peculiar to this fourth magnitude star, but of the four or five thousand variables found since that time more than half show the same character of light fluctuations. The class has, accordingly, received the appropriate designation of Cepheid variables. A century after the discovery of the light variations, δ Cephei was also found to vary in radial velocity—the sixth spectroscopic variable to be placed on record. Again the variation differed from that of other stars. The period of the oscillation of the spectral lines is identical with that of the light variation, but the orbital elements, derived on the hypothesis that the star is a binary, permit no satisfactory explanation of the light variations. When other velocity-variables of this character were found they were classed as Cepheids, and, in fact, every light-variable of the Cepheid type, when put to the test, has proved to be a velocity-variable with the distinct Cepheid peculiarities. Any definite contribution to the explanation of the light or velocity variations of a single Cepheid, therefore, is of unusual value in that it involves the interpretation of the majority of all variable stars.

The present state of our knowledge of the causes of Cepheid variation has been discussed in former papers. The clearest outcome of recent studies of the subject is the apparent impossibility of relating the velocity variations to orbital motions. Probably the strongest evidence on this point is the synchronous variations of the magnitude and the spectral class, recently observed at Mount Wilson for two cluster-type Cepheids. Such variations of spectrum have received little consideration heretofore, notwithstanding the known change in color from maximum to minimum light that would seemingly demand the changes in spectral lines that distinguish one class of spectrum from another.

The question naturally arises whether in suitable investigations of the brightest and most typical Cepheids changes of spectrum would not be found analogous to those observed in the less typical variables RS Boötis and RR Lyrae. Might it not also be possible with a special analysis of high dispersion plates to bring out other characteristics of the spectrum that would throw light on the causes underlying the light and velocity variations? With these points in view an investigation of the spectrum of δ Cephei was made, and the results are briefly outlined in the following paragraphs. The discovery of the conspicuous change of the spectral class of the first and best known Cepheid indicates that constancy of spectrum is to be expected for none of this class of variables, and, coupled with the remarkable behavior of the spectral lines, suggests that the possibility of a completely satisfactory theory of Cepheid variation is not necessarily remote.

Two photographs of the spectrum of δ Cephei were obtained on the nights of December 23 and 24, 1915, with the Cassegrain spectrograph and 60-inch reflector. The full optical train of three prisms and a 102 cm. camera was employed. This combination gives a linear scale at $H\gamma$ of 5.3 angstroms to the millimeter. The data for the photographs are as follows:

Plate γ 4571 Dec. 23, G.M.T. 15^h4^m Exposure time 225^m
Plate γ 4578 Dec. 24, G.M.T. 14^h7^m Exposure time 120^m

The first photograph was taken under exceptionally poor conditions, and this fact is responsible for the extended exposure time. As it is, the plate, though well measurable, is somewhat under-exposed. Especial care was taken in adjusting the spectrograph to secure good definition in the region of the hydrogen line $H\gamma$. As a result the spectrum lines are very sharply defined from λ 4200 to λ 4500, but begin to show diffuseness to the red of this point. They are, however, measurable as far as λ 4600.

A comparison of the two negatives under a Hartmann spectrocomparator at once showed some important differences beween the two spectra.[1] These may be summarized as follows. The photograph of December 24 was taken near the star's maximum of light, and that of December 23 not far from minimum.

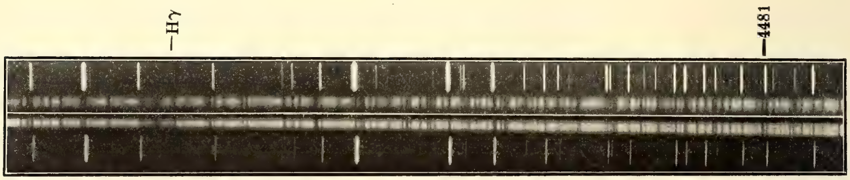


FIG. 1. SPECTRA OF δ CEPHEI
Top: Spectrum near Minimum of Light
Bottom: Spectrum near Maximum of Light

	γ 4578 (*maximum*)	γ 4571 (*minimum*)
Hydrogen line $H\gamma$	Strong	Much weakened
Enchanced lines of *Fe*, *Ti*, *Sr*, and *Cr*	Strong	Much weakened
λ 4481, enhanced *Mg*	Very strong	Much weakened
λ 4227 of calcium	Strong	Strengthened
Low temperature lines of *Ca*, *Fe*, *Ti*, and *Cr*	Weak	Strengthened
Continuous spectrum	Strong in violet	Weakened in violet

The observation on the continuous spectrum is made somewhat uncertain by the general under-exposure of the plate taken near minimum, but the result is apparently as given. The lines on this photograph appear to be broader and less sharply defined than those on the plate at maximum, and this effect we consider as probably genuine after all necessary allowance has been made for the difference in the quality of the two negatives. There is, however, no evidence of lack of symmetry in the spectrum lines either at maximum or minimum, nor of the presence of a secondary spectrum. It appears from these observations that the changes follow a definite tendency. At maximum the high temperature lines are very strong, and the low temperature lines are weak; while at minimum the reverse is the case. The conclusion appears to be justified that the temperature of the gases constituting the star's absorbing envelope is higher at maximum of light than at minimum.

A method of applying the Harvard system of classification to the determination of spectral type through the aid of numerical relationships between the intensities of the hydrogen lines and certain other selected lines will be described by one of us in another Communication. The use of this method gives in the case of δ Cephei:

At maximum F4, At minimum G2

The variation in type, accordingly, amounts to 8 divisions of the Harvard scale.

An independent determination of the spectrum and its change can be made from the comparison of the mean visual and photographic light curves. Nearly twenty visual curves have been made during the last hundred years, but only the one by Stebbins is based on measures with a photometer. The only photographic curve is due to Wirtz. It is possible to reduce both curves to the international magnitude scale; the difference between them for any phase gives the color index, which may be transformed into spectral type by known relations. Hence we derive in stellar magnitudes:

$$\text{Visual Range} = 4.25 - 3.49 = 0.76,$$
$$\text{Photographic Range} = 5.15 - 3.90 = 1.25;$$

and for the spectral class at light maximum F2, at minimum G4. The time of plate γ 4571 coincides with velocity minimum but succeeds the minima of the light and color curves by several hours. Allowing for this we derive from the color curve, for comparison with the direct classification above, the satisfactory result:

At maximum F2, At minimum G0

An important conclusion is that all genuine color variations observed in Cepheids may be directly interpreted as normal changes in spectral class.

Associated with the variation in spectral type in δ Cephei is a variation in the intensity of certain spectrum lines which have been found to fluctuate with the intrinsic luminosities of the stars in which they occur. The use of the intensities of these lines in calculations of absolute stellar magnitudes will be described elsewhere. The application of the method to the case of δ Cephei is of considerable interest because of the accurately known range of variation in apparent, and hence, of course, in absolute magnitude. Unfortunately the spectrum of the star at maximum (F4) is of a type not well suited for the use of the method and the results are necessarily approximate. The variation in absolute magnitude found in this way, however, is entitled to considerably more weight than are the magnitude values themselves. The two photographs give the following values:

Minimum M = +1.8, Maximum M = +0.7, Variation 1.1

The two conclusions to be drawn from these results are: First, that they agree with results based on other considerations such as proper motion in indicating that δ Cephei is a star of very high intrinsic brightness; second, the variation in magnitude derived from characteristics of the spectrum, 1.1, is in very fair accord with the value 0.8, the star's known range of visual magnitude.

In selecting the lines to be measured for the determination of radial velocity from the two photographs the consideration was borne in mind that different lines might give different velocities. Accordingly, an extended list was made out including: (1) a large number of enhanced lines; (2) iron lines which show a wide variety of displacements under pressure; (3) special lines such as *H*γ and λ 4227 of calcium. The photograph taken near maximum was measured by Mrs. Monk and Adams, the result given being the mean for the two observers; the other plate was measured by Mrs. Monk alone:

	No. of Lines	*Radial Velocity*
γ 4571 (minimum)	55	+ 3.8 km/sec
γ 4578 (maximum)	68	−35.2

The radial velocity for the times of the Mount Wilson photographs can be predicted by means of the elements of the velocity variation, derived by Moore[2] from Lick Observatory spectrograms, using in this computation the improved light elements by Luizet.[3] The results are:

Minimum, +3.7 km/sec, Maximum, −35.2.

The exact agreement between the observed and computed velocities not only checks the results from the two observatories and indicates that the range of velocity variation is the same now as eight years ago, but also supports Moore's conclusion that the velocity of the center of mass does not undergo the variation assigned it by Belopolsky.

A comparison of the velocities given by the individual lines on these photographs shows some interesting features. Of these the most important is the behavior of the iron lines which in the laboratory show small displacements under pressure as compared with those which show relatively large shifts. The results for the photograph taken near the star's maximum are shown in detail in Table I. The pressure shifts in angstrom units per atmosphere, given under Δ, are taken from unpublished laboratory results by Gale and Adams. The velocities, v, are in kilometers, and are the measured values, not corrected for the earth's motion.

The mean value of the velocity as derived from the lines of large pressure shift is 1.58 km greater than that from those of small shift, and it is noteworthy that not one of the velocities of the lines in the latter group reaches the mean value for the large shift lines. Similarly only two of these reach values as low as the mean for the lines of small pressure displacement.

The photograph taken near the star's minimum shows similar results, although it is entitled to considerably less weight because of its quality and the fact that but a single measurement has been made upon it.

TABLE I

Small pressure shift

LINE	Δ A	V KM	LINE	Δ A	V KM
4250.9	+0.001	−20.1	4415.2	+0.004	−20.4
4271.9	0.001	18.9	4422.7	0.001	21.2
4325.9	0.002	21.2	4466.7	0.003	19.5
4352.9	0.002	19.6	4442.5	0.005	19.6
4376.1	0.002	18.3	4482.3	0.003	20.1
4383.7	0.003	20.0	4531.3	0.003	18.3
4404.9	0.004	19.9			
Mean........				+0.0027	−19.79

Large pressure shift

LINE	Δ A	V KM	LINE	Δ A	V KM
4222.3	+0.007	−20.8	4271.3	+0.008	−22.6
4227.6	0.017	24.0	4407.8	0.012	20.6
4233.7	0.006	20.3	4430.7	0.007	19.6
4236.1	0.007	23.7	4447.8	0.006	19.4
4238.9	0.012	22.5	4494.7	0.007	20.5
4250.2	0.011	21.5	4528.7	0.006	21.7
4260.6	0.010	20.5			
Mean........				+0.0090	−21.37

Combining the results with weights of two and one we obtain the following value for the difference in velocity:

Large Δ lines (+0.009 A) − Small Δ lines (+0.003 A) = −1.22 km

The sign of this difference is opposite to that which would be found if pressure is the agent producing it and if the effective pressure at the star's surface where these lines are formed is greater than one atmosphere. It is, however, the sign which would be found if the stellar pressure is less than one atmosphere. In that case the iron comparison lines on the negatives, which of course are taken at atmospheric pressure, would be shifted toward longer wave-lengths with reference to the stellar lines by amounts corresponding to the observed pressure displacements. It seems to us probable that this is the explanation of at least the principal part of the observed difference, although differential radial currents in the star's atmosphere may account for a portion of it.

An important confirmation of this result is furnished by Mr. St. John's measurements of the displacements of solar lines relative to the arc spectrum lines of iron. He has kindly furnished us with the provisional values obtained for the lines given in Table I, and from these we find: first, that the small pressure shift lines are displaced toward the red in the sun relative to the arc lines by an average amount of 0.009 angstrom; second, that the large pressure shift lines are displaced toward the violet 0.004 angstrom. These differences are in the same direction as in the case of δ Cephei and the absolute amount is of much the same order. Thus we find:

Sun Large Δ lines − Small Δ lines = −0.013 A = −0.9 km
δ Cephei " " = −1.2 km

It is well known that the pressures in the upper portions of the sun's atmosphere where the stronger lines of the spectrum appear to find their origin are extremely low.

A comparison of the velocities given by the enhanced lines in δ Cephei with those given by such as are not enhanced indicates a distinct shift of the former toward longer wave-lengths. In the case of the photograph taken near maximum the difference amounts to +0.86 km, or about +0.012 angstrom. The other photograph gives a similar value of larger amount. A difference of the same character was found by Adams some years ago for the enhanced lines in the spectra of Sirius and Procyon.[4] The values were +0.90 km for Sirius and +0.58 for Procyon, with which the result found for δ Cephei is in very fair agreement. It is probable that the explanation of these results is to be found mainly in radial convection currents in the stellar atmospheres.

---

[1] Belopolsky and Lohmann at Pulkova have described some differences in the intensities of the lines at maximum and minimum light, but the dispersion employed makes it impossible to identify the lines with certainty.

[2] Moore, *Lick Obs. Bull.*, 7, 153 (1913).

[3] Luizet, *Ann. l'Univ. Lyon*, N. S., 33, 42 (1912).

[4] *Astrophys. J.*, 33, (1911).

INVESTIGATIONS IN STELLAR SPECTROSCOPY. I. A QUANTITATIVE METHOD OF CLASSIFYING STELLAR SPECTRA

By Walter S. Adams

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, February 8, 1916

The basis of the classification of stellar spectra is at present largely empirical. In the absence of sufficient knowledge as to the modifications of spectra produced by different physical conditions it has not been possible to establish with certainty a system of classification which will represent the actual order of stellar development. Hence the stars have been classified into types simply in accordance with the characteristics of their spectra. The appearance of new lines and the disappearance of others, systematic variations in the intensities of certain lines, the presence of bands, the intensity of the continuous spectrum, and other similar criteria have been used to separate the stars into several spectral groups.

To some extent the system of classification now in general use by astronomers, that devised by the Harvard Observatory, probably has a physical basis. Thus it is well known that the differences between the spectrum of the sun and that of a star like Arcturus are very similar to those between the spectrum of the sun and that of sun-spots. In the latter case investigations have shown that a reduction of temperature is the principal agent in producing the modifications observed. Similarly the presence of bands characteristic of certain compounds which are found in the spectra of stars like α Orionis is an indication of relatively low temperature. Accordingly it seems probable that the successive types of stellar spectra, represented by the sun, Arcturus, and α Orionis, are characterized by successively lower temperatures in the gases forming the atmospheres of these stars. This does not of necessity indicate, however, that Arcturus and α Orionis have developed from stars like our sun. Lockyer and some others consider that the curve of stellar development has both an ascending and a descending branch, and that some stars of low temperature will become hotter before beginning to cool permanently. Stars which differ greatly in size and mass must almost certainly differ in the rate, and quite possibly in the order, of their development as well.

The principal lines used in the Harvard system of classification for the separation of stars into the several types are certain lines due to calcium, the more prominent lines of such metals as iron, and, most important of all, the hydrogen lines. In accordance with this system the stars are divided into seven main types designated by the letters B, A, F, G, K, M, and N, with intermediate types indicated on a scale extending from zero to ten. Thus G5 indicates a type halfway between types G and K. The B stars are characterized by helium and hydrogen absorption lines. In the A stars the helium lines disappear, the hydrogen lines reach their maximum intensity, and faint metallic lines begin to appear. These lines grow stronger and the hydrogen lines weaker in the successive types F, G, and K, the low temperature lines in particular increasing rapidly in intensity between the G and K types. The sun is a typical G0 star. The M and N stars are distinguished by the presence of bands, in the one case of a compound of titanium, and in the other of carbon.

The Harvard system of classification in general meets the requirements of spectral observations in a most excellent way. There is, however, in published descriptions of its application a serious lack of numerical relationships between the intensities of the lines compared, and as a result a considerable uncertainty arises in the determination of spectral types. Since in many astronomical investigations a comparison is instituted between stars of very closely the same type it is important to reduce the classification of stellar spectra to as accurate a basis as possible. The following brief description of the method employed at Mount Wilson is given for two purposes: first, because it replaces to a considerable extent direct estimations of spectral type by numerical estimates of relative line intensity which may be made with much higher accuracy; and second, because the method provides the material upon which several investigations have been based. It was devised in large measure by Dr. Kohlschütter, and has been used with but slight modifications since his departure from Mount Wilson.

The material available for classification purposes consists of several thousand photographs of stellar spectra taken with a one prism slit spectrograph and the sixty-inch reflector. About two-thirds of these spectra are of types succeeding F0. On most of the photographs the region of spectrum in best definition extends from λ 4200 to λ 4900. It includes, therefore the two hydrogen lines $H\gamma$ and $H\beta$, the important calcium line at λ 4227, and some of the most prominent iron lines in the entire spectrum. Since the hydrogen lines show a rapid decrease in intensity with the successive types F, G, K and M, and form by far the most important criterion in the derivation of spectral type, accurate determinations of their intensity relative to other lines in the spectrum are essential. Accordingly several adjacent iron lines have been selected which show but a moderate change of intensity in these types, and estimates are made on an arbitrary scale, extending from zero to ten, of the differences in intensity between the hydrogen lines and this selected list. The calcium line λ 4227 is also compared with $H\gamma$ in the types F0 to G5, beyond G5 the differences becoming too great to provide satisfactory determinations. The list of pairs of lines finally adopted for classification purposes is given in Table I.

TABLE I

Lines Compared	*Range of Type*
$H\gamma$ and λ 4227, *Ca*	F0 to G5
$H\gamma$ " λ 4326, *Fe*	F3 to Ma
$H\gamma$ " λ 4352, *Fe, Mg*	F0 to Ma
$H\gamma$ " λ 4383, *Fe*	F0 to G5
$H\gamma$ " λ 4405, *Fe*	F3 to Ma
$H\beta$ " λ 4872, *Fe*	G0 to Ma
$H\beta$ " λ 4957, *Fe*	G0 to Ma

The scale of classification was adapted to the Harvard system by selecting a considerable number of stars for which Harvard determinations were available, and making estimates of the relative intensities of these pairs of lines in the stars selected. The values were then plotted against the average types of these stars, and smooth curves were drawn through the several points. These curves provide the means of converting determinations of relative line intensity into determinations of spectral type. The curves are shown in figure 1. For reasons which will appear later, they are based upon stars of large proper motion alone, and the material may, therefore, be regarded as homogeneous in character.

To illustrate the use of these curves I have selected as examples the stars Groom. 3357, Piazzi 0^h130, Groom. 145 and Lal. 19022. The estimated differences of intensity for these stars, as determined from three photographs of their spectra, are given in Table II.

TABLE II

	$\frac{4226}{H\gamma}$	$\frac{4326}{H\gamma}$	$\frac{H\gamma}{4352}$	$\frac{H\gamma}{4383}$	$\frac{H\gamma}{4405}$	$\frac{H\beta}{4872}$	$\frac{H\beta}{4957}$	*Bands*
Groom. 3357	0.0	−5.3	+7.0	+4.3	+7.0			
Pi 0^h130		+3.7	+0.7		−1.3	+3.3	+1.0	
Groom. 145		+5.3	−2.0		−4.0	−0.2	−1.8	
Lal. 19022		+6.3	−3.3		−5.0	−1.3	−3.3	1

With the aid of tables constructed from the curves we obtain the following determinations of spectral type from the separate pairs of lines:

		Mean	*Probable Error*
Groom. 3357	F7 F7 F4 F6 F5	F6	±1.0
Pi 0^h130	G7 G5 G6 G4 G5	G5	0.8
Groom. 145	K1 K1 K3 K3 K2	K2	0.6
Lal. 19022	K7 K7 Ma K6 K6	K7	1.6

The average probable error of the determination of type for these four stars is ±1.0, and this is about the value obtained for several hundred stars classified in this way. It is evident that the accuracy will be least when the lines compared differ greatly in intensity, as in the types F0 – F9 and K5 – Ma, and greatest when the lines are of nearly equal intensity.

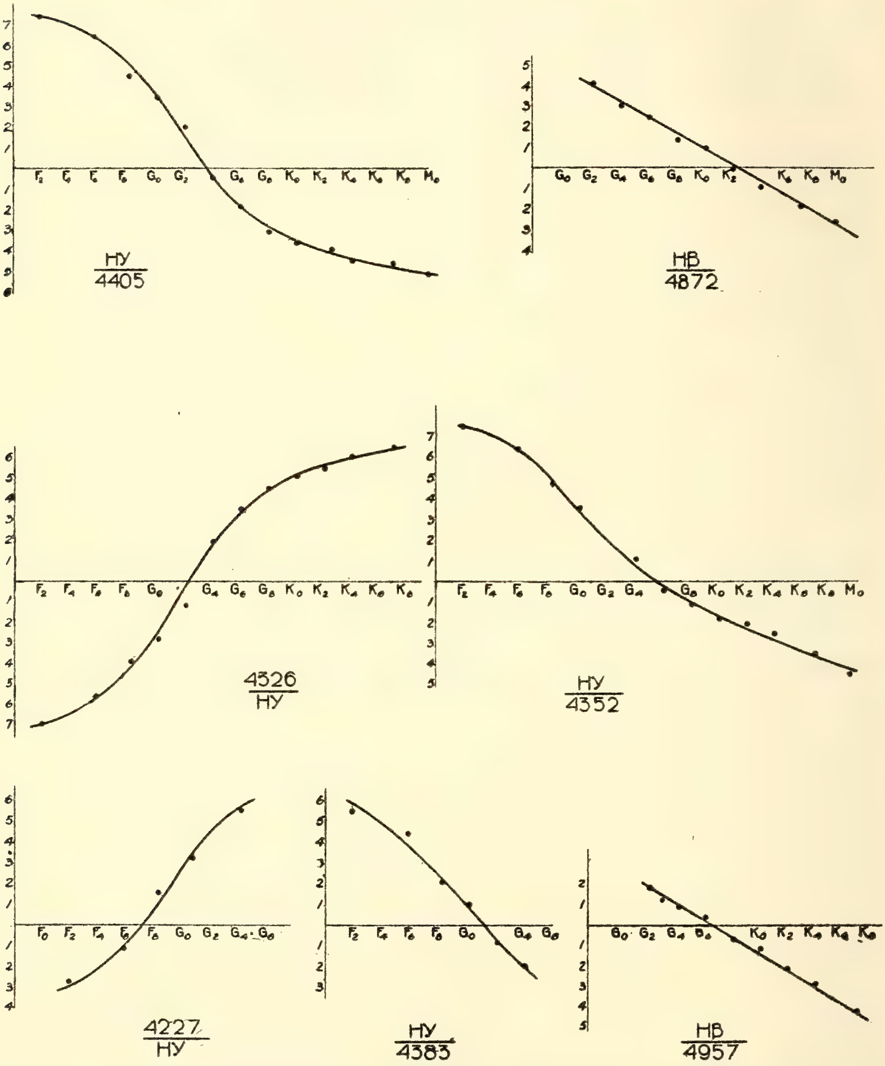


FIG. 1. CURVES USED IN STELLAR CLASSIFICATION SHOWING THE VARIATION OF THE RELATIVE INTENSITIES OF PAIRS OF LINES WITH SPECTRAL TYPE. THE ORDINATES ARE THE DIFFERENCES OF RELATIVE INTENSITY.

This simple method of classification may be recommended as being rapid of operation, and free from the difficulties connected with the comparison of separate photographs with one another. It requires the establishment of a scale of relative-intensity-estimates by the observer, but this is a very simple matter when the range employed is small. To some extent the scale will be dependent upon the dispersion of the spectrograph employed since several of the lines used are compound in character. With the single prism spectrograph at Mount Wilson the same reduction curves have been used successfully for photographs on which the linear dispersion varies from 16 to 90 angstrom units to the millimeter at the center of the spectrum.

In connection with the classification of stellar spectra a number of photographs have been made with a Koch microphotometer of the intensity curves of some of the pairs of lines employed in the comparison. There are numerous practical difficulties connected with the use of this instrument for lines as narrow and as short as those in stellar spectra, and it is doubtful whether the accuracy obtained is of so high an order as to justify the use of so laborious a method for stellar classification. It is probable, however, that it might be used to advantage in the selection of standard stars of reference in which a knowledge of the absolute intensities of certain spectrum lines would be of great value.

INVESTIGATIONS IN STELLAR SPECTROSCOPY. II. A SPECTROSCOPIC METHOD OF DETERMINING STELLAR PARALLAXES

By Walter S. Adams

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, February 8, 1916

The question whether the intrinsic brightness of a star may not have an appreciable effect upon its spectrum is one with important applications in astronomy. If two stars which have closely the same type of spectrum differ very greatly in luminosity it is probable that they also differ greatly in size, mass, and in the depth of the atmospheres surrounding them Accordingly we might hope to find in these stars certain variations in the intensity and character of such spectrum lines as are peculiarly sensitive to the physical conditions of the gases in which they find their origin, in spite of the close correspondence of the two spectra in general. If such variations exist and a relationship may be derived between the intensities of these lines and the intrinsic brightness of the stars in which they occur, we have available a means of determining the absolute magnitudes* of stars, and hence their distances.

The first attempt to detect such lines was made by Hertzsprung, who concluded that the strontium line at λ 4077 gave some indication of varying with the absolute magnitudes of the stars in whose spectra it appeared. Quite independently Dr. Kohlschütter in the course of his studies of the classification of the Mount Wilson stellar spectra found two or three lines which appeared to vary in this way, and some results of an application of these lines to the determination of absolute manitudes were published in 1914. Since that time the work has been extended greatly with the aid of the additional material available. The results of the investigation and of an attempt to utilize these criteria for the derivation of stellar distances are contained in this communication.

The first essential in beginning this research was an accurate classification of the stellar spectra into the several types. This was carried out by the method already described (These PROCEEDINGS, **1**, 481). Stars of the same type of spectrum but of very different absolute brightness were then compared with one another, and the relative intensities of the different spectral lines were examined carefully.

To illustrate the procedure we may take as an example the two stars 61^1 Cygni and α Tauri. The parallaxes of these stars are $0.''31$ and $0.''07$, respectively, and their apparent magnitudes are 5.6 and 1.1. Their absolute magnitudes may be computed from the equation

$$M = m + 5 + 5 \log \pi$$

in which M is the absolute magnitude, m the apparent magnitude, and π the parallax. The absolute magnitudes, accordingly, are 8.0 and 0.4; that is, the luminosity of α Tauri is over 1100 times as great as that of 61^1 Cygni. A comparison of the spectra of the two stars side by side on a Hartmann spectrocomparator shows several points of difference. Of these, two are most important. The calcium line at λ 4455 is very strong in 61^1 Cygni and relatively weak in α Tauri; and the strontium line at λ 4216 is weak in 61^1 Cygni and strong in α Tauri. That this difference in behavior depends upon physical conditions in the stars and is not merely accidental is made almost certain by solar investigations. The line λ 4455 of calcium is greatly strengthened in the spectrum of

---

* The absolute magnitude of a star is its apparent magnitude when reduced to unit distance. The unit commonly employed is the distance corresponding to a parallax of $0.''1$. On this scale the absolute magnitude of the sun would be 5.5, or 4.8, if more recent, and probably better, values of the sun's photometric brightness are employed.

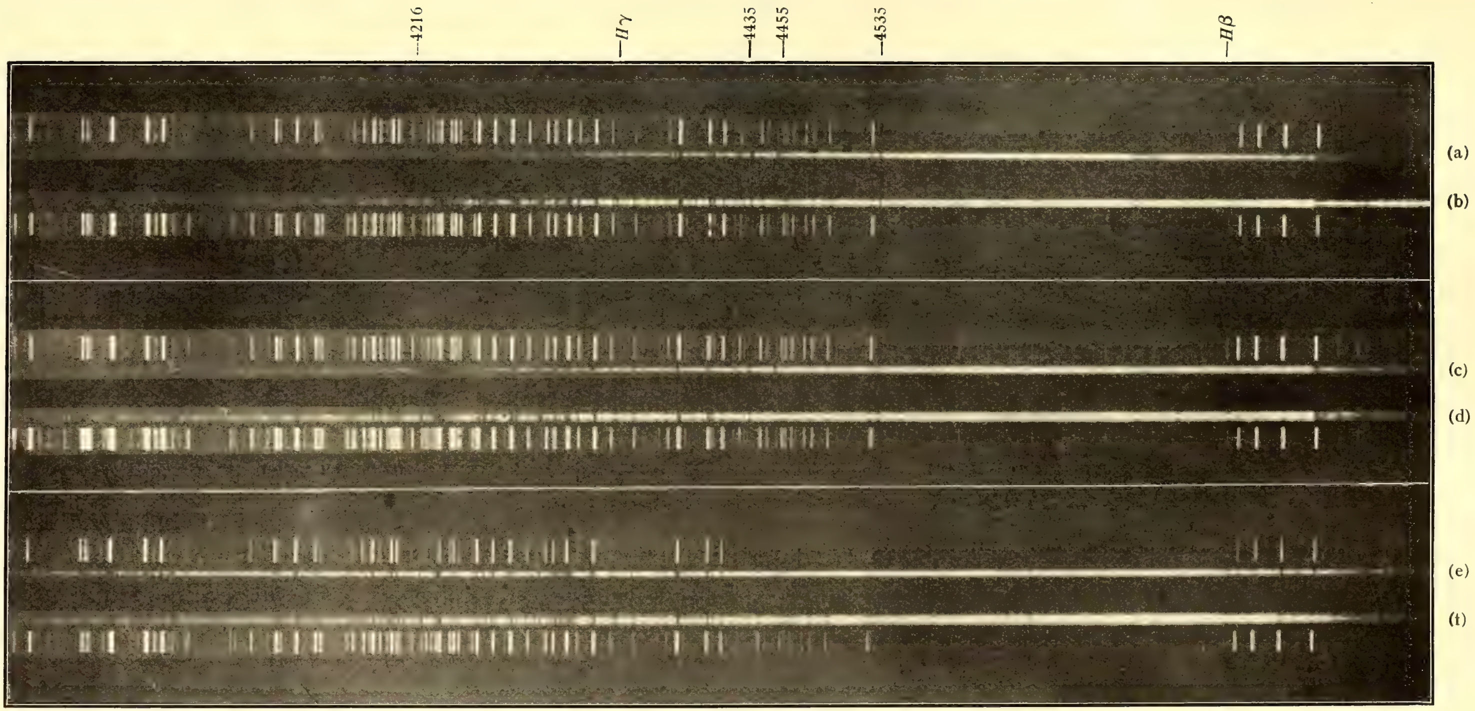

4216
Hγ
4435
4455
4535
Hβ
(a)
(b)
(c)
(d)
(e)
(t)

FIG. 1. SPECTRA OF PAIRS OF STARS OF DIFFERENT ABSOLUTE MAGNITUDE SHOWING VARIATION OF THE INTENSITY OF CERTAIN LINES WITH MAGNITUDE.

(a) Groombridge 34; (b) Boss 1560. (c) Lalande 21185; (d) Boss 2921. (e) Piazzi 5^h 146; (f) Boss 1513.

sun-spots, and increases in intensity with reduction in temperature. The line λ 4216 of strontium, on the other hand, is an enhanced line, that is stronger in the spectrum of the spark than of the arc, and is probably a high temperature line. It is very prominent in the spectrum of the sun's limb when photographed at eclipses, and also in the upper chromosphere. Numerous other smaller differences between the spectra of α Tauri and 61^1 Cygni all point in the same direction; the low temperature lines strengthened in sun-spots are stronger in 61^1 Cygni; the enhanced lines are stronger in α Tauri.

It has seemed preferable, however, for two reasons to use only these two lines in the absolute magnitude investigation. First, because they show the effect most markedly; and second, because they appear to be influenced but slightly by closely adjoining lines which blend with them. Among other lines which show the effect plainly, reference should be made to λ 4435 of calcium and λ 4535 of titanium, which are strong in intrinsically faint stars, and to two lines at λ 4395 and λ 4408 which are strong in the brighter stars. The line at λ 4395 is probably due to enhanced titanium. As will appear later, in the course of a discussion of M type stars, the hydrogen lines themselves seem to vary with absolute magnitude, at least in certain types of spectra. This should prove of fundamental importance in further investigations of stellar luminosity.

After the behavior of the two lines λ 4216 and λ 4455 had been examined in a large number of stars, and the systematic differences had been found to persist through a wide range of spectral type, the attempt was made to establish a numerical relationship between the intensities of these lines and the absolute magnitudes of the stars in which they occur. As in the case of the hydrogen lines used for classification purposes, lines were selected near λ 4216 and λ 4455, with which the intensities of these lines were compared, the differences of intensity being estimated on a scale extending from zero to ten. The pairs of lines finally adopted for all of this work are as follows:

(a)	λ 4216, *Sr* and	λ 4250, *Fe*
(b)	λ 4455, *Ca*	λ 4462, *Fe, Mn*
(c)	λ 4455, *Ca*	λ 4495, *Fe*

For convenience of reference these pairs of lines will be designated in the future as (a), (b) and (c). The value (a) = -2, for example, denotes that λ 4216 is estimated to be two units fainter than λ 4250.

As soon as the estimates had been completed a number of the stars with well-determined parallaxes were selected, their absolute magnitudes were computed, and curves were constructed in which the observed differences of intensity for each pair of lines formed the abscissae, and the absolute magnitudes the ordinates. The stars were divided into five groups according to spectral type and curves were drawn for each group. The groups are as follows:

F0–F6; F7–G7; G8–K4; K5–K9; M

The curves are so nearly straight lines in the case of the first three of these groups that straight lines have been adopted, the constants being derived by least square solutions. In the K5–K9 group the curve for (a) is a straight line but not for (b) or (c). It is probable that there are no straight lines in the M group, but this is very uncertain. The significance of a straight line is, of course, that the intensity of the spectrum line varies uniformly with the absolute magnitude.

The most serious difficulty in the construction of these curves is the scarcity of parallax determinations on stars of high luminosity. Parallax observers have confined their attention almost wholly to stars of large proper motion which promise to yield large parallaxes. With the aid, however, of the Yale observations on the very bright stars, and some most valuable determinations by Mr. van Maanen of the parallaxes of certain stars of small proper motion, a number of stars of very high luminosity were selected upon which the lower portions of the curves could be based. Particularly in the cases of the K5–K9 and the M groups these portions of the curves are still most uncertain, and must be adjusted with the aid of additional parallax observations when they become available.

The list of formulae derived for the several groups is given in Table I. The equations are from my own observations. A similar list, in which the constants differ slightly, has been obtained from the determinations of Miss Burwell, who has carried out a complete series of estimations of the line intensities in these stars. In the formulae, M is the absolute magnitude, and Δ the estimated difference of intensity for each of the pairs of lines.

TABLE I

	(a)	(b)	(c)
F0–F6	$M = -1.9\Delta + 5.2$	$M = +2.0\Delta + 3.3$	$M = +2.8\Delta - 2.6$
F7–G7	$M = -1.4\Delta + 4.9$	$M = +2.0\Delta + 3.3$	$M = +2.8\Delta - 2.6$
G8–K4	$M = -1.6\Delta + 4.7$	$M = +1.6\Delta + 5.1$	$M = +2.3\Delta - 0.3$
K5–K9	$M = -1.8\Delta + 5.0$	Curve	Curve
M (low luminosity)	$M = -1.5\Delta + 6.9$	Curve	Curve

The equation and curves in the case of the M stars are applicable only to the stars of low luminosity. In the case of the F0–F6 stars it is doubtful whether the equations given, which for (b) and (c) are the same as in the G group, are other than rough approximations. The enhanced lines in the early F stars are normally so prominent that it is not surprising that the method begins to break down at this point.

To illustrate the use of the formulae and curves we may select as illustrations a few stars of different spectral types and magnitudes. These are collected in Table II. The classification is from Mount Wilson determinations.

TABLE II

STAR	MAG.	TYPE	Δ			M			PARALLAX		
			(a)	(b)	(c)	(a)	(b)	(c)	Mean	Comp.	Obs.
Pi 10^h96....	7.6	F5	−0.7	+0.7	+3.0	+6.5	4.7	5.8	5.7	+0″.04	+0″.04
Sun........		G0	−0.5	+0.5	+3.0	+5.6	4.3	5.8	5.2		
Lal. 38287..	7.2	G5	−1.8	+1.5	+3.5	+7.4	+6.3	+6.2	7.3	+0.10	+0.09
α Arietis...	2.2	K0	+2.5	−2.4	+0.2	+1.0	1.3	+0.2	0.8	+0.05	+0.09
α Tauri....	1.1	K5	+3.0	−2.0	+0.5	−0.4	+1.9	+0.5	0.7	+0.08	+0.07
61^2 Cygni..	6.3	K8	−1.8	+5.8	+7.7	+8.2	9.3	8.9	8.8	+0.32	+0.31
Groom. 34..	8.2	Ma	−2.2	+6.8	+9.2	+10.2	10.5	10.4	10.4	+0.28	+0.28

The parallaxes are computed from the absolute magnitudes by the formula, to which reference has already been made,

$$5 \log \pi = M \dot{-} m - 5.$$

The results are given in the next to the last column of the table, and the measured parallaxes in the final column.

INVESTIGATIONS IN STELLAR SPECTROSCOPY. III. APPLICATION OF A SPECTROSCOPIC METHOD OF DETERMINING STELLAR DISTANCES TO STARS OF MEASURED PARALLAX

By Walter S. Adams

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, February 8, 1916

A definite test of the value of this method of deriving stellar parallaxes can be made only through a comparison with all available data on measured parallaxes. Since the evidence depends directly on individual values it is necessary for this purpose to present tables of a somewhat extended character.

It is evident that in the case of the stars whose absolute magnitudes, as computed from the measured parallaxes have been used in the derivation of the relationship between line intensity and absolute magnitude, the mean values of the magnitude will necessarily be identical with those derived from the formulae. The agreement of the measured and the computed parallaxes of the individual stars, however, serves as important evidence bearing on the validity of the method.

In Tables I and II are collected all of the stars with measured parallaxes equal to or exceeding +0″.05 for which we have spectral observations. Table I contains the stars used in the derivation of the curves, but in Table II the values are entirely independent, none of these stars having been used previously. This table, accordingly, serves as a most exacting test of the value of this means of computing parallaxes.

TABLE I

STAR	MAG.	TYPE	M		PARALLAX			Measured parallax	Authority
			A	B	A	B	Mean		
F8-G7									
μ Cassiop......	5.2	G3	+4.7	+5.2	+0″.08	+0″.10	+0″.09	+0″.11	K.
ι Persei........	4.2	G1	3.3	3.3	0.07	0.07	0.07	0.11	K.
Lal. 6888.......	8.4	G7	7.5	7.5	0.07	0.07	0.07	0.06	K.
α Aurigae......	0.2	G0	0.5	0.6	0.11	0.12	0.12	0.09	Y.
λ Aurigae......	4.8	G1	4.8	5.0	0.10	0.11	0.11	0.11	K.
Lal. 11196.....	6.5	G1	5.0	6.6	0.05	0.10	0.08	0.09	K.
Lal. 15565.....	6.9	G4	6.3	6.4	0.08	0.08	0.08	0.09	K.
Pi 7^{h}321.......	7.0	G2	7.0	6.1	0.10	0.07	0.08	0.05	K.
Groom. 1596...	8.2	G6	5.1	5.4	0.02	0.03	0.03	0.07	Y.
20 Leo Min.....	5.6	G4	4.7	5.6	0.07	0.10	0.08	0.07	K
Groom. 1830...	6.5	G6	5.9	5.9	0.08	0.08	0.08	0.10	K.
Groom. 1855...	7.4	G7	6.0	5.4	0.05	0.04	0.05	0.06	K.
Lal. 22908.....	7.5	G3	5.8	5.8	0.06	0.06	0.06	0.09	Y.
Lal. 30694.....	6.8	G6	5.3	5.9	0.05	0.07	0.06	0.07	K.
Lal. 31132.....	6.7	G4	6.4	7.0	0.09	0.11	0.10	0.14	Y.
72 w Herculis..	5.4	G1	6.3	5.9	0.15	0.13	0.14	0.12	K.
31 b Aquilae...	5.2	G7	5.3	4.8	0.10	0.08	0.09	0.07	K.
Lal. 37120.....	6.6	F9	4.7	5.3	0.04	0.05	0.05	0.06	Y.
Lal. 38287.....	7.2	G5	7.3	6.7	0.10	0.08	0.09	0.09	Y.
85 Pegasi......	5.9	G1	5.8	6.1	0.10	0.11	0.10	0.09	S. & M.
G8-K4									
54 Piscium	6.1	K0	+7.0	+6.4	+0″.15	+0″.11	+0.13	+0″.14	Y.
Mayer 20......	5.8	K3	7.2	7.1	0.19	0.18	0.18	0.16	K.
Pi 1^{h}159.......	5.7	K0	6.2	4.9	0.13	0.07	0.10	0.10	K.
α Arietis.......	2.2	K0	0.8	−0.1	0.05	0.03	0.04	0.05	K.
W.B. 3^{h}113....	7.8	K0	6.5	6.3	0.06	0.05	0.06	0.08	Y.
Pi 5^{h}146.......	6.4	G9	6.2	5.4	06.0	0.06	0.08	0.12	K.
Lal. 13427.....	8.2	K3	7.0	7.2	0.06	0.06	0.06	0.05	K.
β Gemin.......	1.2	G8	−0.1	1.0	0.05	0.09	0.07	0.06	Y.
Lal. 16304.....	6.0	G8	7.4	6.9	0.19	0.15	0.17	0.11	Y., F.
55 ρ^1Cancri....	6.0	K1	5.2	5.6	0.07	0.08	0.08	0.08	K.
α Boötis.......	0.2	G9	0.1	0.7	0.10	0.13	0.11	0.08	K.
W.B. 15^{h}720...	6.8	K3	5.9	5.0	0.07	0.04	0.06	0.05	R.
Lal. 30024.....	7.0	K3	7.5	7.3	0.13	0.11	0.12	0.09	Y.
Fed. 2895......	6.3	G9	6.9	5.9	0.13	0.08	0.10	0.05	Y.
Pi 20^{h}23.......	7.3	K2	7.3	7.1	0.10	0.09	0.09	0.08	Y.

TABLE I (Continued)

			M		PARALLAX				
STAR	MAG.	TYPE	A	B	A	B	Mean	Measured parallax	Authority
K5-K9									
Lal. 1799	8.0	K5	7.5	6.4	0.08	0.05	0.06	0.06	Y.
Lal. 1964	8.6	K5	8.0	8.2	0.08	0.08	0.08	0.10	Y.
Pi 2^h123	5.9	K6	6.7	5.8	0.14	0.10	0.12	0.14	K.
Groom. 745	8.3	K7	8.7	8.2	0.12	0.10	0.11	0.08	K.
α Tauri	1.1	K4	0.7	1.9	0.08	0.14	0.11	0.07	K.
Lal. 18115 Pr.	7.9	K8	9.0	8.7	0.17	0.14	0.16	0.16	K.
Lal. 18115 Fol.	7.9	K8	9.0	9.6	0.17	0.22	0.19	0.16	K.
Lal. 19022	8.2	K6	6.9	7.3	0.05	0.07	0.06	0.06	K.
A. Oe. 10603	6.8	K8	7.9	8.3	0.17	0.20	0.18	0.18	K.
O ϵ 298	7.9	K6	6.8	6.9	0.06	0.06	0.06	0.05	R.
Brad. 2179	6.7	K4	8.2	7.8	0.20	0.17	0.18	0.15	F.
W.B. 17^h322	7.8	K7	8.6	8.9	0.14	0.17	0.15	0.12	K.
70 Ophiuchi Ft.	6.0	K7	7.8	7.6	0.23	0.21	0.22	0.17	K.
Lal. 40844	9.0	K5	8.8	7.9	0.09	0.06	0.08	0.17	Y.
61^1 Cygni	5.6	K8	8.5	7.2	0.38	0.21	0.30	0.31	K.
61^2 Cygni	6.3	K8	8.8	9.0	0.32	0.35	0.34	0.31	K.
Brad. 3077	5.6	K7	6.7	6.8	0.17	0.17	0.17	0.16	K.
Ma-Mb									
Groom. 34	8.2	Ma	10.4	10.3	0.28	0.26	0.27	0.28	K.
Lal. 21185	7.6	Ma	10.6	10.6	0.40	0.40	0.40	0.40	K.
Lal. 21258	8.9	Ma	10.5	10.3	0.21	0.19	0.20	0.20	K.
Lal. 25372	8.7	Ma	10.0	9.6	0.18	0.15	0.16	0.18	K.
Krüger 60	9.2	Ma	10.3	10.0	0.17	0.15	0.16	0.26	K.
Lal. 46650	8.9	Ma	10.3	10.1	0.19	0.17	0.18	0.18	K.

TABLE II

			M		PARALLAX				
STAR	MAG.	TYPE	A	B	A	B	Mean	Measured parallax	Authority
F0-F7									
α Can. Min.	0.5	F3	+3.5	+3.0	+0″.40	+0″.32	+0″.36	+0″.32	K.
10 Urs. Maj.	4.1	F6	4.7	3.9	0.13	0.09	0.11	0.09	Y.
39 Leonis	5.8	F7	5.1	4.7	0.07	0.06	0.07	0.09	Y.
3 Cygni	6.2	F3	5.9	5.9	0.07	0.07	0.07	0.07	K.
Groom. 3042	5.7	F0	5.4	4.8	0.09	0.07	0.08	0.06	Y.
Groom. 3357	6.6	F7	5.5	5.5	0.06	0.06	0.06	0.08	J.
33 Pegasi	6.1	F5	4.7	4.2	0.05	0.04	0.05	0.06	vM.
34 Pegasi	5.8	F6	4.6	3.3	0.06	0.03	0.05	0.07	F.
F8-G7									
Brad. 3212	6.2	G7	6.7	5.9	0.13	0.09	0.11	0.15	Y.
Lal. 3022	7.8	G5	4.0	5.7	0.02	0.04	0.03	0.05	Y.
δ Trianguli	5.1	F8	5.3	5.0	0.11	0.10	0.10	0.12	K.
Lal. 5490	6.7	G2	5.6	4.5	0.06	0.04	0.05	0.06	Y.
10 Tauri	4.4	F8	3.4	4.0	0.06	0.08	0.07	0.07	Y.
Groom. 884	7.1	F9	5.8	6.7	0.06	0.08	0.07	0.09	K.
Lal. 13849	6.5	G7	3.7	3.6	0.03	0.03	0.03	0.11	Y.
Pi 6^h305	6.0	G0	5.4	4.8	0.08	0.06	0.07	0.07	K.
Groom. 1281	5.6	F9	5.4	4.8	0.09	0.07	0.08	0.07	J.
Lal. 14146	7.3	F9	6.5	5.6	0.07	0.05	0.06	0.08	Y.
Brad. 1433	5.9	F7	4.5	4.2	0.05	0.05	0.05	0.10	J., M.
Groom. 1646	6.5	G1	6.6	5.8	0.10	0.07	0.08	0.08	K.
Lal. 22585	6.4	G7	3.7	3.7	0.03	0.03	0.03	0.12	Y.

TABLE II (Continued)

STAR	MAG.	TYPE	M		PARALLAX			Measured parallax	Authority
			A	B	A	B	Mean		
Lal. 25012.....	7.5	G7	7.7	7.6	0.11	0.10	0.10	0.05	Y.
Lal. 26289.....	6.3	G2	4.8	5.4	0.05	0.07	0.06	0.11	Y., J.
χ Herculis.....	4.6	F8	4.8	3.9	0.11	0.07	0.09	0.10	Y.
Groom. 2305...	6.8	G5	6.6	7.6	0.09	0.14	0.11	0.07	Y.
Lal. 30699.....	7.8	G6	5.4	4.6	0.03	0.02	0.03	0.05	Y.
26 Draconis....	5.3	G0	5.1	5.1	0.09	0.09	0.09	0.09	Y.
16 Cygni Pr....	6.3	G1	6.0	5.0	0.09	0.05	0.07	0.05	S. & M.
16 Cygni Fol...	6.4	G3	4.2	3.7	0.04	0.03	0.04	0.04	S. & M.
Lal. 38380.....	5.7	G7	4.5	3.3	0.06	0.03	0.05	0.06	Y., F.
15 Sagittae.....	5.9	G1	5.2	4.6	0.07	0.05	0.06	0.10	K.
Groom. 3150...	6.1	G0	6.9	6.4	0.14	0.11	0.12	0.12	Y.
ρ Cygni........	4.2	G5	3.6	2.8	0.08	0.05	0.06	0.05	Y.
Fed. 4371......	7.5	G1	7.5	6.8	0.10	0.07	0.08	0.06	Y.
107 Piscium....	5.3	K2	5.8	5.9	0.13	0.13	0.13	0.13	Y.
W. B. 2^h927....	8.2	G9	5.5	5.2	0.03	0.03	0.03	0.10	Y.
ε Eridani.......	3.8	K3	6.1	5.8	0.29	0.25	0.27	0.31	Y.
δ Eridani......	3.7	G9	2.6	1.1	0.06	0.03	0.05	0.19	Y.
Lal. 10797.....	7.3	K0	5.5	5.0	0.04	0.03	0.04	0.08	Y.
Lal. 15547.....	8.6	K1	5.5	4.7	0.02	0.02	0.02	0.09	Y.
G8-K4									
83 Leonis B r...	6.5	G8	+5.8	+4.3	+0″.07	+0″.04	+0″.06	+0″.06	R.
Lal. 26196.....	7.6	K2	7.6	6.2	0.10	0.05	0.08	0.14	Y.
Lal. 27298.....	7.9	G9	6.6	7.4	0.05	0.08	0.06	0.06	K.
Lal. 27744.....	6.7	G8	7.2	5.6	0.13	0.06	0.10	0.09	F., Lee
Lal. 33439.....	6.7	K3	6.7	6.4	0.10	0.09	0.09	0.05	Y.
Lal. 38383.....	7.2	K1	7.3	7.8	0.10	0.13	0.12	0.11	Y.
η Cephei.......	3.6	K1	1.8	2.0	0.04	0.05	0.05	0.08	Y., S.
A. Oe. 25685...	7.5	K0	6.5	5.4	0.06	0.04	0.05	0.08	Y.
γ Cephei.......	3.4	K2	3.5	2.2	0.10	0.06	0.08	0.07	S. & M.
K5-K9									
Lal. 47231.....	8.6	K6	8.8	9.3	0.11	0.14	0.12	0.12	Y., S.
Lal. 1198......	8.1	K7	6.3	5.4	0.04	0.03	0.04	0.08	Y.
Lal. 6320......	7.9	K8	8.0	7.4	0.10	0.08	0.09	0.07	Y.
W.B. 4^h1189...	6.5	K7	7.1	6.4	0.13	0.10	0.12	0.11	Y.
Lal. 10299.....	8.5	K7	8.2	8.8	0.09	0.11	0.10	0.10	Y.
Lal. 13284.....	6.9	K5	6.6	8.3	0.09	0.19	0.14	0.12	Y.
83 Leonis Ft...	7.6	K7	6.3	6.5	0.05	0.06	0.06	0.06	R.
Pi 14^h212 Br...	5.8	K6	7.6	6.6	0.23	0.15	0.19	0.17	K.
Lal. 31055.....	7.9	K6	7.8	7.3	0.10	0.08	0.09	0.07	Y., F.
Lal. 34986.....	8.1	K6	7.7	7.3	0.08	0.07	0.08	0.08	Y.
B. D. +7°3967.	9.3	K5	8.7	7.6	0.08	0.05	0.06	0.04	Sch.
Lal. 39866.....	8.4	K3	8.4	6.8	0.10	0.05	0.08	0.06	Y.
Lal. 45028.....	7.8	K5	6.3	5.7	0.05	0.04	0.05	0.05	Y.
Ma									
Fed. 1384[1].....	9.2	Ma	9.9	10.4	0.14	0.17	0.15	0.10	F.
W. B. 16^h906..	8.8	Md	10.1	10.0	0.18	0.17	0.18	0.22	Y.
A. Oe. 17415...	9.1	Mb	10.6	10.8	0.20	0.22	0.21	0.27	K.
Pos. Med. 2164[1]	8.9	Mb	10.5	10.6	0.21	0.22	0.22	0.29	K.

The columns in the tables designated by A and B refer to the determinations by Adams and Miss Burwell. The final values are the means for the two observers. The measured parallaxes are taken from a variety of sources. Y. indicates Yale determinations; K., the values compiled by Kapteyn in *Groningen Publication* No. 24; Sch., the results of Schlesinger; R., those of Russell; vM., of van Maanen; S., of Slocum; M., of Mitchell; J., of Jost; and F., those of Flint. Where relative parallaxes are given the values have been reduced to absolute measure by making suitable corrections for the parallaxes of the comparison stars. The tables are arranged according to spectral type.

The comparison of the computed and the measured parallaxes shows an excellent degree of accordance for most of the stars. There are, however, occasional large discrepancies. Of these the most serious is in the case of δ *Eridani*. The spectrum observations give a much smaller parallax than is found by the Yale observers. A striking case of agreement, on the other hand, is that of ϵ *Eridani;* this parallax was computed before it was known that a measured value was available. A star which should prove of exceptional interest is Boss 6129. From spectrum observations we have obtained a parallax of $+0''.23$: no measured value has been published but the star is on the observing programme at several observatories.

The average deviation, taken without regard to sign, between the observed and the computed values of the parallaxes in Tables I and II is $0''.024$: it is $0''.026$ for the stars of Table II alone. There seems to be no marked systematic difference between the observed and the computed parallaxes; the former average somewhat larger, but this is due mainly to a few large discrepancies.

There are 25 stars with measured negative parallaxes for which we have made spectrum determinations. The largest value for any one of these stars as computed from the line intensities is $+0''.08$; the average value for all is $+0''.03$. The spectrum method, of course, gives no negative parallaxes.

It seems reasonable to conclude from these results that the method of computing absolute magnitudes and parallaxes from the variation of the intensities of lines in stellar spectra is capable of yielding results of a very considerable degree of accuracy. Especially in the K and M type stars of low luminosity, the line variations are so great that such stars may be recognized from a mere inspection of the spectrum. Stars, for example, like 61 Cygni, Groom. 34, and Krüger 60 bear very evident marks of their intrinsic faintness in the remarkable intensity of the low temperature calcium lines in their spectra. At first thought it might appear that a relationship between certain spectral characteristics and the distances of stars could hardly be credible, since it would appear like a correlation between two utterly unrelated subjects except in so far as the scattering of light in space might connect them. In fact, of course, it is not the distances but the absolute magnitudes of stars which have an influence on the character of the spectrum lines and such an effect, far from being improbable, is rather to be expected than not. The derivation of the distances is merely a by-product resulting from the combination of real, or absolute, with apparent magnitudes.

An important gain in the value of this method of determining stellar magnitudes and distances should result from an increase in the number of measured parallaxes of bright stars of small proper motion. Such stars will on the average prove to be very luminous, and, as already stated, the portion of the curves connecting line intensity with absolute magnitude is subject to much more uncertainty in the case of the high luminosity stars than in any of the others. It is probable that after such a revision has been made the method will find its most important application as a means of distinguishing these giant stars in the stellar system.

INVESTIGATIONS IN STELLAR SPECTROSCOPY. IV. SPECTROSCOPIC EVIDENCE FOR THE EXISTENCE OF TWO CLASSES OF M TYPE STARS

By Walter S. Adams

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, February 8, 1916

The principal distinguishing feature of the M type of stellar spectrum on the Harvard system of classification is the presence of absorption bands due to titanium oxide. These bands increase in intensity for the successive subdivisions Ma, Mb, and Mc. The star α Orionis, in which they are present in moderate intensity, is selected as a typical Ma star by the Harvard observers. Since these bands may be seen faintly in stars of the K5 type of spectrum it is necessarily largely a matter of judgment whether in any given spectrum they are sufficiently strong to warrant classifying the star as Ma, or whether it should still be retained within the K type.

For types of spectra previous to M the principal basis of classification is the intensity of the hydrogen lines. These reach a maximum in the A type, and grow fainter in the successive types F, G, and K. Of the hydrogen lines in α Orionis, however, Miss Cannon, in the course of her classification of the Harvard spectra, makes the statement that they are of about the same intensity as in α Tauri, a typical K5 star.[1]

The classification of the Mount Wilson stellar spectra in accordance with the Harvard system, a description of which is given in a previous communication,[2] is based upon a comparison of the intensities of the hydrogen lines with those of neighboring iron lines which are subject to relatively slight variation with type. A series of curves have been constructed giving the relationship between the relative intensities of these pairs of lines and the spectral type; and the determination of type is thus reduced to an estimation of the intensities of these lines. The stars used in the derivation of these curves are almost wholly stars of large proper motion, and in many cases have measured parallaxes of considerable size. They are, accordingly, stars of relatively low intrinsic brightness in general. This is true especially of the K5–K9 and Ma stars, nearly all of which, like 61 Cygni and Groom. 34, are of very low absolute luminosity. The curves derived in this way show a regular decrease in the intensity of the hydrogen lines throughout the range of spectrum employed, the lines in K5 stars being fainter than in K0, and in the Ma stars fainter than in K5. In fact the hydrogen lines are barely visible in most of the M stars used in the construction of the curves.

When these results are applied to the M stars of high luminosity a very anomalous condition is found. The presence of the bands places these stars definitely in the M type, but the hydrogen lines are of quite abnormal intensity. Thus α Orionis, with bands of type Ma, if classified on the basis of its hydrogen lines would become G2. This is the most remarkable case found as yet, but all of the high luminosity M stars show a strong tendency in the same direction. The results of a classification of 48 stars of types Ma to Mc on the basis of the intensities of their hydrogen lines may be summarized as follows:

TABLE I

TYPE	NO. OF STARS	TYPE	NO. OF STARS	TYPE	NO. OF STARS
G2	1	G7	9	Ma	20
G3	1	G8	10	Mb	18
G4	3	G9	4	Mc	10
G5	3	K0	4		
G6	11	K1	2		

Accordingly, the most advanced type found for any of these stars from a determination of the intensities of their hydrogen lines is K1, and the average type is G7. This is as against an average type of Mb given by the intensities of the bands. Two conclusions may be drawn at once from these results: First, that the Harvard system of classification, in which the M type stars are all included in one group on the basis of the presence of the bands, fails entirely to discriminate between the spectral peculiarities of the high and the low luminosity M stars; and second, that the intensity of the hydrogen lines in the M stars probably varies with the absolute magnitude, the brighter stars having the stronger hydrogen lines.

A method of determining the absolute magnitudes of stars from the characteristics of certain of their spectral lines has been described in a previous communication.[3] The essential feature of this method is the use of the two lines λ 4216 of strontium and λ 4455 of calcium, the intensities of which appear to be connected directly with the intrinsic brightness of the stars in whose spectra they occur. The intensities of these lines relative to other lines in the spectrum are estimated, and a numerical relationship is established between these intensity ratios and absolute magnitude by means of a selection of stars of known parallax. In this way the following formulae applicable to stars of types G8–K4 have been derived. M is the absolute magnitude, and Δ the intensity ratio for each pair of lines.

$\frac{4216}{4250}$	$\frac{4455}{4462}$	$\frac{4455}{4494}$
$M = -1.6\Delta + 4.7$	$M = +1.6\Delta + 5.1$	$M = +2.3\Delta - 0.3$

It is this set of formulae which has been used in the case of the M stars of high luminosity. The average type of these stars was found to be G7, which is sufficiently near the limits of the group to admit of the application of the corresponding equations. Summarized briefly the results for the high and the low luminosity stars are as follows:

	Average Spectrum	*No. of Stars*	*Average M*	*Range of M*
High luminosity....................	G7	48	+1.4	−1.0 to + 3.4
Low luminosity....................	Ma	10	+10.3	+9.8 to +10.7

Of the high luminosity stars only two, α Orionis and Boss 660, have negative values of the absolute magnitude, and only five stars have values exceeding 2.0. The remaining 41 stars have magnitudes ranging between 0.0 and 2.0. It is clear, accordingly, that on the basis of absolute magnitude determinations the M stars fall into two clearly defined groups, separated by an interval of about 7 magnitudes within which no intermediate values have been found.

The spectroscopic evidence, therefore, confirms the hypothesis of Hertzsprung and Russell that the M type stars are divided into two groups of 'giant' and 'dwarf' stars.[4] This hypothesis was based primarily on parallax observations. The absolute magnitudes calculated from these parallaxes showed almost a complete absence of stars of brightness intermediate between exceedingly luminous stars like α Orionis, and extremely faint stars such as Groom. 34. It has been thought probable by some astronomers that this apparent gap is due to the fact that parallax determinations have hitherto been restricted almost entirely to a few stars of great apparent brightness, and to stars of very large proper motion, while the connecting links would probably be found among stars of moderate apparent brightness and moderate proper motion. The spectroscopic evidence, however, is based upon numerous stars of just this character, and the gap still appears to persist.

These results may be summarized briefly as follows. Two groups of M stars are indicated clearly by an examination of the intensities of the hydrogen lines: in the first the hydrogen lines are very strong; in the second they are very faint. A computation of the absolute magnitudes of these stars on the basis of certain peculiarities in their spectra shows the existence of these groups distinctly. Connecting links over a range of 7 magnitudes are entirely lacking, and the conclusion seems to be unavoidable that among these stars the intensity of the hydrogen lines varies with the absolute magnitude.

The results given for the high and the low luminosity stars may be used to furnish an approximate relationship between the intensities of the hydrogen lines and absolute magnitude. Thus we have for *H*β:

	Average M	*Intensity of H*β
High luminosity stars..............................	+ 1.4	+1.9
Low luminosity stars..............................	+10.3	−3.0

Assuming a linear relationship between intensity and absolute magnitude we obtain the equation

$$M = -1.8 \, \Delta + 4.8$$

This is remarkably similar to the corresponding equation found for the line λ 4216 and given on a preceding page. It seems probable, therefore, that in the case of the M stars, at least, the hydrogen lines may be used for absolute magnitude determinations in the same way as λ 4216.

There is, however, one characteristic of the spectra of these high luminosity stars which must be taken into consideration when use is made of the relative intensities of the hydrogen lines. This is a relationship which appears to exist between the intensities of the hydrogen lines and the intensities of the bands, the hydrogen lines being stronger when the bands are stronger. There are occasional exceptions to this rule, as in the case of α Orionis, but in general the effect is well marked. Thus if we compare the intensity of the hydrogen line $H\beta$ in the stars having bands of moderate intensity with that in stars in which the bands are very strong we find the following result:

No. of Stars	*Intensity of Bands*	*Intensity of $H\beta$*
13	Moderate	+1.2
20	Strong	+1.7

It is of interest to note in this connection that the computation of the absolute magnitude shows that the Mc stars, in which the bands are exceedingly strong, are brighter on the average than those in which the bands are less intense.

Among the high luminosity stars are some with proper motions of moderate size. The absolute magnitudes of these stars should average somewhat less than those of the very small proper motion stars which constitute the remainder of the list. An analysis of the results for the 48 stars gives the following comparison. M is the absolute magnitude and m the apparent magnitude.

No. of Stars	*Average P.M.*	*Average m*	*Average M*
15	0.″155	5.06	1.54
33	0.017	5.49	1.29

After making the necessary correction for the difference in the values of the average apparent magnitude we find the large proper motion stars to be about 0.7 magnitude fainter than those of small proper motion. This furnishes a check on the accuracy of the absolute magnitude determinations.

The variations in the intensities of the hydrogen lines and of the two lines used in the computation of absolute magnitude form only a part of a more general difference in the spectral characteristics of the high and the low luminosity M stars. The results of a detailed comparison of the spectrum of α Orionis (M = −1.0) with that of Lal. 21185 (M = + 10.6) and of other intrinsically faint stars may be summarized as follows:

	α Orionis	*Lal. 21185*
Enhanced lines, especially those due to *Fe*, *Ti*, *Sr*, and *V*. . .	Strong	Weak
Hydrogen lines.	Strong	Weak
Low temperature lines of *Ca*, *Ti*, *Cr*, and *Sr*.	Weak	Strong
λ 4227 of calcium.	Weak	Very strong

Results of a character very similar to these were found in a comparison of the spectra of α Tauri (K5) and 61^1 Cygni (K8) two stars differing in brightness by nearly 8 magnitudes, and also in the case of the N and the R type stars of the Harvard classification. The differences, accordingly, appear to be fundamental in nature, and associated with the intrinsic brightness of the stars of the several types. They indicate a lower temperature in the absorbing gases constituting the atmospheres of the fainter stars, and are analogous in many respects to those observed in the spectrum of sun-spots.

The division of the M type stars into two well-defined classes of high and low luminosity stars raises the question at once whether a corresponding separation may be found among other types of spectra. From his discussion of parallax observations Russell concludes that such a separation does exist among the K stars. The spectroscopic evidence tends to support the existence of such a division at least for the K5–K9 stars. This evidence is of just the same character as that in the case of the M type stars, and is of two kinds. First, the hydrogen lines have an abnormally high intensity in the very luminous stars, and there is an absence of intermediate values of the intensity between these and the low values characteristic of the fainter K5–K9 stars. Second, computations of absolute magnitude indicate the existence of two mean magnitudes, one high and the other low, about which the values for the individual stars showed a marked tendency to gather. This effect is not so well defined as for the M stars, but still very clear. It may perhaps

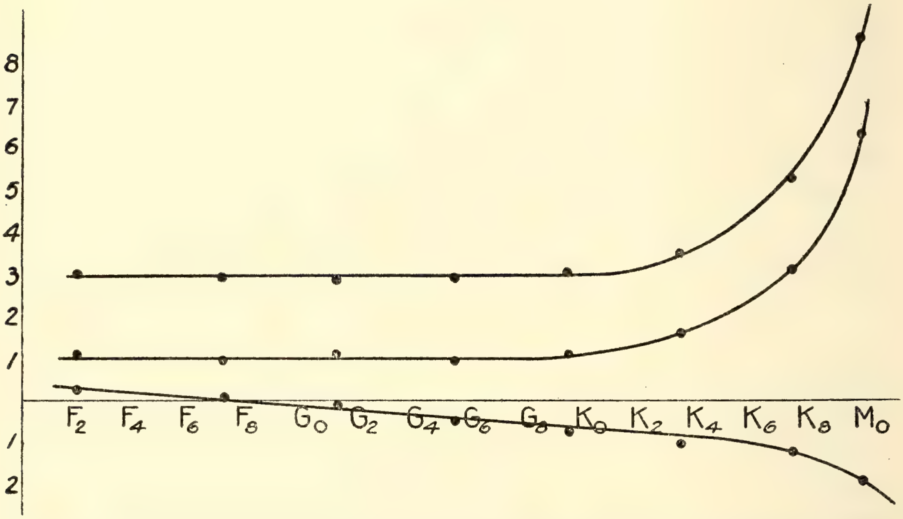


FIG. 1. CURVES SHOWING VARIATION WITH SPECTRAL TYPE OF THE RELATIVE INTENSITIES OF PAIRS OF LINES USED IN ABSOLUTE MAGNITUDE DETERMINATIONS.

Top curve, $\frac{4455}{4494}$; Middle curve, $\frac{4455}{4462}$; Bottom curve, $\frac{4216}{4250}$.

be shown to the best advantage by a reproduction of the curves representing the estimated intensity differences for the pairs of lines used in the determinations of absolute magnitude. These are given in figure 1. The curves are based upon essentially all of the stars with observed parallaxes for which we have spectral observations. Each point on the curves represents the mean for a considerable number of stars; and, as these stars differ in absolute magnitude, the corresponding intensity differences for the pairs of lines will differ. In types F and G the higher and lower luminosity values and the line differences balance one another so nearly that the successive values show but a gradual change, and the curves make but a slight angle with the horizontal axis. At about K3, however, the curves begin to bend abruptly, and the remaining types depart from the axis very rapidly. This is due to the absence of stars of even moderately high luminosity among those upon which the curves are based.

The corresponding curves for the high luminosity stars of these types run nearly parallel to the horizontal axis. We find, accordingly, both for types K5–K9 and M, a branching of the curves which points directly toward the existence of a division into two distinct groups. This evidence is based upon all of the spectroscopic material available.

In conclusion reference should be made to the necessity of adding to the symbols used in the Harvard system of classification for the M stars some character or figure which shall serve to distinguish between the spectral characteristics of the high and the low luminosity stars. The most important of these is the difference in the intensity of the hydrogen lines. Accordingly, though somewhat cumbersome in practice, I can think at present of no method which would convey the necessary information in any better way than by adding to the classification based on band intensity the corresponding classification based on hydrogen line intensity. Thus Mb (G6) would indicate a spectrum in which the bands are strong but the hydrogen lines give a type of G6. On this basis the low luminosity M stars would be of normal type and would require no suffix.

---

[1] *Ann. Obs., Harv. Coll.*, 28, 160.

[2] These *Proceedings*, 1, 481 (1915).

[3] These *Proceedings*, 2, 143 (1916).

[4] Summarized by Eddington in *Stellar Movements and the Structure of the Universe*, pp. 170–177.

THE FAILURE AND REVIVAL OF THE PROCESS OF PIGMENTATION IN THE HUMAN SKIN

By A. E. Jenks

DEPARTMENT OF SOCIOLOGY AND ANTHROPOLOGY, UNIVERSITY OF MINNESOTA

Received by the Academy, February 15, 1916

In the year 1914 I published an illustrated study entitled 'A Piebald Family of White Americans.'[1]

Further research shows that my conclusion then [one of 'the apparent positive results of this [that] research'], namely, that the spotting of the skin was due to 'progressive albinism,' was correct. To get the matter fairly before us I quote herewith my former conclusions, together with explanations which accompanied them. Attention is especially directed to number 4 below:

> Without going further into details at this time, I may summarize the apparent positive results of this research so far in hand, as follows:
>
> That in the family before us we see—
>
> 1. Hereditary spotting of the skin.
> 2. The character of spotting behaves as a simple Mendelian dominant.
> 3. The piebald persons are heterozygous for this character of spotting.
> 4. The condition of spotting is albinistic, and is progressive rather than fixed, giving progressive albinism—sometimes called dynamic leucosis.
>
> It may be well to present here definitions of albinism in its three commonly recognized phases:
>
> Complete albinism affords no visible pigment anywhere in *skin*, *hair* or *eyes.*
>
> Incomplete albinism affords visible pigment of various degrees of diffusion everywhere in *skin*, *hair*, and *eyes*.
>
> Partial or imperfect albinism affords visible pigmentation limited to areas separated by unpigmented areas. This gives 'piebald' and 'spotted' cases.
>
> Concerning the probable close interrelation between these various phases of unpigmented skin, Pearson says:
>
> When we consider the relative rareness of complete albinism, of the spotted or splashed condition and of xanthism, their relatively frequent coincidence in[2] the same stock suggests that these abnormal pigment conditions are not wholly independent, and that as a working hypothesis it is reasonable to suppose that complete albinism, partial albinism, incomplete albinism, and xanthism, all static forms of leucosis, are phases of the same process and are probably linked with leucoderma and possibly other forms of dynamic leucosis. By 'linked' we suggest that they mark the complete, incomplete, local, or progressive failure of the same metabolic process, which may never start at all, never start in certain areas, or be imperfectly started, and again being started may fail to maintain itself; further, that every variety of this failure may individually or collectively be associated with certain stocks, which may either show hereditary failure of one phase, of several, or exceptionally of all phases of pigment metabolism."[1]

Pigmentation is due to *pigment metabolism*. In 'complete albinism' pigment metabolism completely fails to start. In 'incomplete albinism' pigment metabolism occurs only incompletely. In 'partial or imperfect albinism' pigment metabolism locally fails or never starts. In 'progressive albinism,' or dynamic leucosis, pigment metabolism, though having apparently once started at some time, fails in certain areas.[3]

Before giving proofs of progressive albinism as shown by the recent research, let me add another quotation from the original study which states two of the five problems then left open for further investigation:

> 2. Whether the albinistic areas extend their borders after once having been known, or whether there is, instead, a progressive failure of pigment metabolism within a definite area.[4]
>
> 3. Whether an at-one-time albinistic area ever revives within itself the process of pigment metabolism.[5]

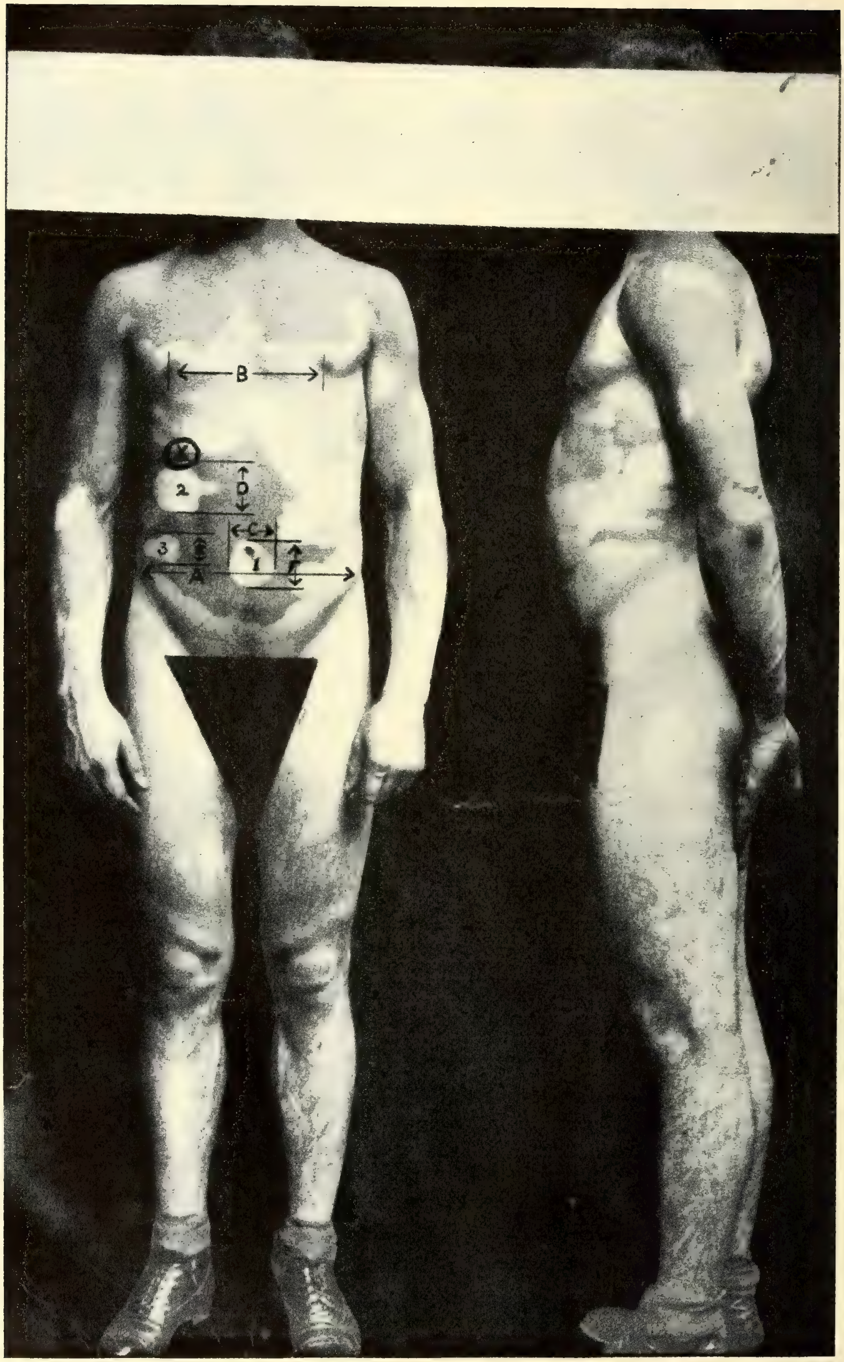
Problem number 2 has been solved. There is an extension of the albinistic areas.

Problem number 3 has also been solved, and its solution is surprising. There is a revival of the process of pigment metabolism within an at-one-time albinistic area.

The process followed in the study of the two problems (numbers 2 and 3 above) follows.

Two negatives of the same individual[6] were made as early as possible from the same points of view. The second negative was taken following a lapse of six months after the making of the first negative. The two negatives were measured by Mr. H. B. Wilcox,[7] Scholar in Astronomy at the University of Minnesota. The measurements were made on the Repsold photographic-plate measuring machine owned by the Department of Astronomy, University of Minnesota. Two lenses (one of 30 diameters, and one of 10 diameters) were used in the measurements of the distance "C" shown on Plate I. It is the horizontal measurement across the albinistic spot enclosing the navel. All subsequent measurements were made by use of the lower, or 10-diameter lens, because it resulted in better definition. However, the accuracy of the measurements by the 10-diameter lens is well confirmed by the one duplicate measure made by the 30-diameter lens, since the results obtained by the two lenses agree within the limits of error.

The error indicated is based on the theory of Least Squares—the theory of error universally employed in measurements of stellar photographs. However, attention should be called to the possibility of systematic error. This possibility of systematic error is, nevertheless, not



ABNORMALLY PIGMENTED WHITE AMERICAN SHOWING PROGRESSIVE ALBINISM

(Diagrams in black show areas measured. The black circle surrounding an "X," above area 2, has nothing to do with the present presentation.)

great, as is shown by the checking measurements made on both negatives between the points indicated by lines "A" and "B." The ratio of negative number 2 (the one made six months later than the other) to negative number 1, as shown by measurement "A" on both negatives is 1.338 ± 0.002. The ratio of negative 2 to negative 1, as shown by measurement "B" on both negatives, is 1.339 ± 0.002.

The following results were obtained by measurements of areas numbered 1, 2, and 3 on Plate I.

Results of Measurement of Albinistic Areas[8]

Area	*Measurement*	*Per cent of increase*	*Per cent of decrease*	*Per cent of error*
1	C		6.5	± .9
2	D	3.5		± .5
3	E	4.0		±1.0
1	F	1.0		±1.0

These results proclaim that the albinistic areas extended their borders, as shown by measurements "D" on area number 2, "E" on area number 3, and "F" on area number 1. The results also proclaim that the process of pigment metabolism was revived in area number 1, as shown by measurement "C," where the decrease of the size of the area was 6.5 per cent of its total height (vertical measurement of the body) with a possible error of ±0.9%.

---

[1] *Amer. Anthrop., New York,* N. S., **16,** 222–237 (1914).

[2] *Ibid.,* p. 234. [3] *Ibid.,* p. 235.

[4] *Ibid.,* p. 233. [5] *Ibid.,* p. 234.

[6] Number 32 of the Genealogical Chart, Fig. 74, p. 222 of *Op. Cit.*

[7] Mr. Wilcox is recognized as a most careful expert in the field of stellar photographic measurements.

[8] A detailed histological study of the skin of this person may be found in the article referred to in footnote number 1.

BANDED GLACIAL SLATES OF PERMO-CARBONIFEROUS AGE, SHOWING POSSIBLE SEASONAL VARIATIONS IN DEPOSITION

By Robert W. Sayles

UNIVERSITY MUSEUM, HARVARD UNIVERSITY

Received by the Academy, January 26, 1916

Near Boston, on the peninsula of Squantum, facing Quincy Bay, is a formation of tillite of Permo-Carboniferous age, described in 1914.[1] Lying conformably on the tillite is a slate formation with a known thickness of about 800 feet.

The manner of transition from the tillite to the slate, together with other evidence, makes it certain that the slate was the result of deposition in waters from the melting of the glacier which formed the tillite. The first of the transition beds are conglomerates alternating with sandstones. Then come sandstones alternating with slates. At several places it is plain that the ice readvanced over these beds, ploughing them up and often dragging upward into the mass thus formed irregular lumps of the clay. As one goes upward in the banded slate it is seen that the layers of sandstone become thinner and thinner, then disappear entirely, and finally the alternating layers are shown only in slate of dark and light bands. The microscope reveals the fact that the dark layers are composed of much finer material than the light layers. The coarse layers all, without exception so far as observed, have very fine wavy lines of bedding which are cut off and uneven in places, while the fine layers are solid in appearance without these characteristic lines. The finest part of the fine layer is usually in contact with the coarse layer upward, and the change from fine to coarse is abrupt. The change from coarse to fine is more gradual upward, as a rule, and not abrupt. These layers or bands alternate with much regularity and at any given horizon their thicknesses also show regularity.[2] The microscope also shows that the slate with the finest banding also has the finest particles of sediment, although the mineralogical composition is the same as in the rest of the slate and in the tillite. This finest slate however, has more chlorite than the rest of the slate, giving it a light green color.

That the slate was derived from the tillite there can be no doubt. A microscopical examination by Prof. J. E. Wolff has shown the same mineralogical composition for both. The principal minerals present are as follows: quartz, feldspar, sericite, epidote, melaphyre, chlorite, limonite, quartzite and calcite. The size of the grains in the slate range from about 1/300 of an inch to as fine as about one 1/6000 of an inch. The shapes of the grains are angular as in ordinary glacial clay particles. As already stated the light bands usually have coarser as well as lighter material, but next to the thinnest sandstone layers coarser dark material is usually found, and this is just as it should be according to the laws of deposition. Thus it is seen that the banding in this slate is determined by the specific gravity of the material as well as by the strength of current.

Identical mineralogical composition alone, however, would not prove that the slate was derived from the tillite, but two beds of tillite in the slate itself, together with the facts of the banding, prove this origin. The first bed of tillite is about 50 feet above the main tillite formation in well banded slate. The second tillite bed is about 150 feet from the main tillite. Each of these beds is about 5 feet thick. From the evidence presented by these tillite beds and several other beds of conglomerate in the slate, it is inferred that the final retreat of the glacier was slow and hesitating, marked by several advances after many years of retreat. It is known that the ice of the Wisconsin epoch of the Pleistocene period also began its retreat very slowly.

The cause of the disappearance of the sandstone layers and the gradual thinning of the bands can be explained by the retreat of the glacier and the consequent slackening of the currents, which would be strong enough to carry sand only in the neighborhood of the ice. The lower pebbly members of the transition beds show irregularities of deposition, especially in lenticular forms, due to the inconstant conditions of streams coming from a glacier. A regularity of alternation in deposition, however, becomes evident after the first 50 feet or so of the transition beds have been passed, where the layers indicate deeper and quieter water, and thus more uniformity in deposition. The thin individual layers now show through hundreds of feet such regularity in thickness and interval that a regularly recurring cause must be sought.

At the International Congress of Geologists at Stockholm in 1910, Gerard De Geer read a paper on the banding of the glacial clays in Sweden. He thinks, that the coarse band or layer gives a record of summer melting and deposition, and that the find band or layer gives the record of winter deposition of fine material, when the streams were slow and the fine material could easily settle on the bottom. With the co-operation of his students he was able to count the layers in the late glacial clays and then in the post-glacial deposits of the extinct Lake of Ragunda, which was drained in 1796, and gives 12,000 years as the time elapsed since the ice retreated from southern Scania to its present northerly position. The idea of measuring geological time in this manner came to De Geer in 1878. In this country B. K. Emerson advanced the same idea regarding the layers in the glacial clays of the Connecticut Valley in 1898.[3] Since that time several American geologists have published the theory. Among them may be mentioned A. P. Coleman, Frank B. Taylor, and Charles P. Berkey. As far as I can learn, however, De Geer is the first to conceive this geological time recorder, and also the first to convince the larger part of the geological world that these double bands in the glacial clays really mean years. No other explanation so far advanced, accounts for all the facts of the case. The facts observed in the slates at Squantum resemble so closely those described by De Geer and Emerson and the others, that it would seem as if there must be a very strong probability of these being similar cases, in spite of the millions of years which separate the two glacial periods.

In speaking of the banding it is not possible at this writing to say anything about the formation of slate under the main tillite. The banding in this slate appears to be somewhat different from the banding in the slate under discussion, but there has not yet been time to study it. It should show points of interest in regard to the climate responsible for the glacial advance, while the slate above the tillite should show the conditions responsible for the retreat.

Evidence of seasons during Permian times has come from New South Wales, Australia, in the discovery of well marked annual rings of growth in trees of Permian Age.[4] From Brazil similar evidence was obtained in tree trunks of the same age, and perhaps in the Triassic period which followed.[5] Such findings, together with what now appears to be good evidence of seasonal changes at the locality which is now known as Boston, would make it appear that all these localities were in a temperate zone during Permian times as now. Furthermore, the close similarity between the layers in the Pleistocene glacial clays and the layers in the slate at Squantum would make it appear that the temperatures at the close of the Pleistocene and Permian were very similar.

Not all slates which exhibit banding are of glacial origin, but a close study of the slate at Squantum should give certain definite criteria for the determination of glacial slates at other localities, and thus reveal facts hitherto unknown concerning the past climate of the earth. As glacial till is usually a land deposit and subject to rapid erosion, it is not likely to be preserved as tillite unless submerged beneath a body of water relatively soon after deposition. This is not true of a glacial slate, for slate originates in water and is thus very much more likely to be preserved than tillite. Hence, it follows that the discoveries of glacial slates should be more numerous than discoveries of tillites, and such discoveries may turn out to be as good evidences of glaciation as the tillites themselves. In this new way it may be possible to obtain very much more knowledge of past geological climates than we could ever obtain by the search for tillites alone.

[1] R. W. Sayles, The Squantum tillite, *Bull. Mus. Comp. Zool., Harvard Univ.*, **46**, No. 2, 141–175 (1914).

[2] Charles P. Berkey, Laminated Interglacial Clays of Grantsburg, Wisconsin, *The Journal of Geology*, **13**, No. 1, 36-37. January-February, 1905.

[3] B. K. Emerson, The Geology of New Hampshire County, *U. S. Geol. Surv. Monogr.*, 29, 706 (1898).

[4] Shirley and Arber, *Queensland, New South Wales, Bull. Geol. Surv.*, No. 7, 14 (1898).

[5] I. C. White, Relatorio final apresentado A S. Ex. o Sr. Dr. Lauro Severiano Müller, Comissão de estudias das minas de Carvão de Pedra de Brazil, Rio de Janeiro, 1908. (Printed in Portuguese and English.)

AN EXTENSION OF FEUERBACH'S THEOREM

By F. Morley

JOHNS HOPKINS UNIVERSITY

Received by the Academy, February 17, 1916

Feuerbach's theorem, that the four circles which touch three lines also touch a circle, may be stated thus: given four orthocentric points, forming four triangles, the 16 in-circles of these triangles touch the circle F on the diagonal points.

Now each in-circle and the omitted one of the four points is a degenerate curve of class three on the absolute points $I\,J$. There is further a rational curve of class three, on the six joins of the four points and touching the line infinity at $I\,J$, which touches F three times. Thus the theorem is suggested: *All circular line-cubics on the joins of four orthocentric points touch the Feuerbach circle.*

A proof is as follows. It is convenient to state the algebra dually. That is, we have 4 lines 1, $\pm$ 1, $\pm$ 1 and a pair of lines, ζ and $1/\zeta$, apolar to all conics on the 4 lines. Two point-cubics on the six joins of the 4 lines meet again at 3 points $x\,y\,z$, which are points of contact of a tritangent conic of either cubic. When x and y are given, z is rationally known; and when x is given and z moves on a line ζ, we know from the theory of the Geiser transformation that y moves on a rational quartic $\rho_x{}^4$ which has a triple point at x. There is then a connex of the form

$$\zeta\, x^4\, y^4$$

where z is on ζ. And if ξ be the join of x and y this connex is of the form

$$\zeta\, \xi^3\, x\, y. \tag{1}$$

If ζ be 1, 0, 0, the quartic in y is the two lines

$$\begin{vmatrix} x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \\ 0 & 1 & \pm 1 \end{vmatrix} .$$

and the conic on x and the 4 other points, that is

$$(\xi_1{}^2 - \xi_2{}^2) \begin{vmatrix} x_0{}^2 - x_1{}^2 - x_2{}^2, & x_1\, x_2 \\ y_0{}^2 - y_1{}^2 - y_2{}^2, & y_1\, y_2 \end{vmatrix} .$$

Hence the connex is explicitly

$$\begin{aligned} \sum^{3} \zeta_0\, (\xi_1{}^2 - \xi_2{}^2)\, \{\, (x_0y_1 + x_1y_0)\, \xi_1 - (x_0y_2 + x_2y_0)\, \xi_2 \\ + 2\, (x_1y_1 - x_2y_2)\, \xi_0 \,\} = 0. \end{aligned} \tag{2}$$

We now find where this curve in y meets the line $1/\zeta$. That is, we eliminate y from

$$\Sigma = 0, (\xi y) = 0, (y/\zeta) = 0.$$

We have then

$$\Sigma \zeta_0 (\xi_1^2 - \xi_2^2) \begin{vmatrix} x_1\xi_1 - x_2\xi_2, & x_0\xi_1 + 2x_1\xi_0, & -x_0\xi_2 - 2x_2\xi_0 \\ \xi_0 & \xi_1 & \xi_1 \\ 1/\zeta_0 & 1/\zeta_1 & 1/\zeta_2 \end{vmatrix} = 0,$$

or since $(x\xi) = 0$

$$\Sigma (\zeta_1/\zeta_2 - \zeta_2/\zeta_1) x_0 (\xi_0^2 - \xi_1^2)(\xi_0^2 - \xi_2^2) = 0$$

or if a be the join of ζ and $1/\zeta$,

$$\Sigma a_0 x_0 (\xi_0^2 - \xi_1^2)(\xi_0^2 - \xi_2^2) = 0. \qquad (3)$$

The values of ξ common to this equation and $(x\xi) = 0$ give the intersections y of ρ_x^4 and $1/\zeta$. Thus when $(x\xi)$ is a line of (3) then as z moves on ζ the curve ρ_x^4 touches $1/\zeta$ at the point y, and x is on the envelope sought.

Now the quartic (3) is two conics on the lines 1, ± 1, ± 1. And when

$$\Sigma \sqrt{a_i x_i} = 0, \qquad (4)$$

the two conics become one conic R whose equation is

$$\Sigma \sqrt{a_i x_i}\, \xi_i^2. \qquad (5)$$

The conic (4) occurs then twice in the envelope, the other factors being

$$[x_0^4 + 2x_1^2 x_2^2]^2,$$

$$x_0 x_1 x_2,$$

and the cubic of the system with a double point at a, namely

$$\Sigma x_0 \eta_1 \eta_2$$

where η is the join of x and a.

The conic (4) is the Feuerbach conic F, for it is on the diagonal lines of the four lines, and having the line equation

$$\Sigma a/\xi = 0,$$

it is on ζ and $1/\zeta$.

The construction of the cubics which touch both ζ and $1/\zeta$ is then as follows. Take a point x on F, and draw from x the two tangents to the conic R. The diagonals of this line-pair and the pair ζ and $1/\zeta$ give the points of contact of the two cubics. If the diagonals meet at b, then x and b are apolar to ζ and $1/\zeta$; and the line bx, being the polar of a as to R, has the equation

$$\Sigma\, ay/\sqrt{ax} = 0,$$

and is the tangent of F at x.

Dually then, ζ and $1/\zeta$ being the absolute points, the conic F the Feuerbach circle, and the conic R a rectangular hyperbola on the four given orthocentric points, and having its centre c on F, if the common diameter of F and R meet R at points dd', then these points are double foci of circular curves of class 3 on the 6 lines; the circles with centres d and d' and touching F at c are the tritangent conics; and the two cubics touch F at c.

DEFORMATIONS OF TRANSFORMATIONS OF RIBAUCOUR

By L. P. Eisenhart

DEPARTMENT OF MATHEMATICS, PRINCETON UNIVERSITY

Received by the Academy, February 5, 1916

When a system of spheres involves two parameters, their envelope consists in general of two sheets, say Σ and Σ_1, and the centers of the spheres lie upon a surface S. A correspondence between Σ and Σ_1 is established by making correspond the points of contact of the same sphere. In general the lines of curvature on Σ and Σ_1 do not correspond. When they do, we say that Σ_1 is in the relation of a transformation of Ribaucour with Σ, and vice-versa. For the sake of brevity we call it a transformation R.

It is a known property of envelopes of spheres that if the surface of centers S be deformed and the spheres be carried along in the deformation, the points of contact of the spheres with their envelope in the new position are the same as before deformation.[1] Ordinarily when S for a transformation R is deformed, the new surfaces Σ' and Σ'_1 are not in the relation of a transformation R. Bianchi[2] has shown that when S is applicable to a surface of revolution, it is possible to choose spheres so that for every deformation of S the two sheets of the envelopes of the spheres shall be in the relation of a transformation R, and this is the only case in which S can be deformed continuously with transformations R preserved. The only other possibility is that in which it is possible to deform the surface of centers of a transformation R in one way so that the sheets of the new envelope shall be in the relation of a transformation R. It is the purpose of this paper to determine this class of transformations R.

Let x, y, z denote the cartesian coordinates of Σ, expressed in terms of two parameters u and v, such that the curves u = const., v = const. are the lines of curvature on Σ and let the linear element of Σ be written $ds^2 = Edu^2 + Gdv^2$. If X, Y, Z are the direction-cosines of the normal to Σ and ρ_1, ρ_2 its principal radii of normal curvature, we have the equations of Rodriques

$$\frac{\partial x}{\partial u} + \rho_1 \frac{\partial X}{\partial u} = 0, \qquad \frac{\partial x}{\partial v} + \rho_2 \frac{\partial X}{\partial v} = 0, \tag{1}$$

and similar equations[3] in y and z.

Darboux[4] has shown that the most general transformation R of Σ is given by taking for S the surface whose coordinates x_0, y_0, z_0 are given by

$$x_0 = x - \frac{\lambda}{\mu} X, \qquad y_0 = y - \frac{\lambda}{\mu} Y, \qquad z_0 = z - \frac{\lambda}{\mu} Z,$$

where λ and μ satisfy the equations

$$\frac{\partial \lambda}{\partial u} + \rho_1 \frac{\partial \mu}{\partial u} = 0, \qquad \frac{\partial \lambda}{\partial v} + \rho_2 \frac{\partial \mu}{\partial v} = 0, \tag{2}$$

and by taking λ/μ for the radius of the corresponding sphere.

If x_1, y_1, z_1, denote the coordinates of Σ_1, and X_1, Y_1, Z_1; X_2, Y_2, Z_2, the direction-cosines of the tangents to the curves v = const., u = const. respectively on Σ, we have relations of the form

$$x_1 = x - \frac{1}{\sigma m}(\alpha X_1 + \beta X_2 + \mu X),$$

where m is a constant, and α, β, σ are functions which are in the quadratic relation

$$\alpha^2 + \beta^2 + \mu^2 = 2m\lambda\sigma,$$

and satisfy the equations

$$\begin{aligned}
&\frac{\partial \lambda}{\partial u} = \sqrt{E}\,\alpha, && \frac{\partial \lambda}{\partial v} = \sqrt{G}\,\beta, \\
&\frac{\partial \mu}{\partial u} = -\frac{\sqrt{E}}{\rho_1}\alpha, && \frac{\partial \mu}{\partial v} = -\frac{\sqrt{G}}{\rho_2}\beta, \\
&\frac{\partial \alpha}{\partial u} = -\frac{1}{\sqrt{G}}\frac{\partial \sqrt{E}}{\partial v}\beta + \mu\frac{\sqrt{E}}{\rho_1} + 2m\sqrt{\lambda\sigma}\cosh\omega,
\end{aligned} \tag{3}$$

$$\frac{\partial\alpha}{\partial v}=\frac{\beta}{\sqrt{E}}\frac{\partial\sqrt{G}}{\partial u},\qquad \frac{\partial\beta}{\partial u}=\frac{\alpha}{\sqrt{G}}\frac{\partial\sqrt{E}}{\partial v},$$

$$\frac{\partial\beta}{\partial v}=-\frac{1}{\sqrt{E}}\frac{\partial\sqrt{G}}{\partial u}\,\alpha+\mu\frac{\sqrt{G}}{\rho_2}+2m\sqrt{\lambda\sigma}\sinh\omega, \tag{3}$$

$$\frac{\partial\log\sigma}{\partial u}=\left(2\sqrt{\frac{\lambda}{\sigma}}\cosh\omega-\sqrt{E}\right)\frac{\alpha}{\lambda},\quad \frac{\partial\log\sigma}{\partial v}=\left(2\sqrt{\frac{\lambda}{\sigma}}\sinh\omega-\sqrt{G}\right)\frac{\beta}{\lambda},$$

$$\frac{\partial\omega}{\partial u}=\frac{1}{\sqrt{E}}\frac{\partial\sqrt{G}}{\partial u}-\frac{\alpha}{\sqrt{\lambda\sigma}}\sinh\omega,\quad \frac{\partial\omega}{\partial v}=\frac{1}{\sqrt{G}}\frac{\partial\sqrt{E}}{\partial v}-\frac{\beta}{\sqrt{\lambda\sigma}}\cosh\omega.$$

This system of equations is completely integrable provided that

$$\frac{\partial}{\partial u}\left(\frac{1}{\sqrt{G}}\frac{\partial\sqrt{E}}{\partial v}\right)=\frac{\partial}{\partial v}\left(\frac{1}{\sqrt{E}}\frac{\partial\sqrt{G}}{\partial u}\right).$$

This condition characterizes surfaces with the same spherical representation of their lines of curvature as isothermic surfaces. Then we establish the fundamental theorem:

In order that the surface of centers of a transformation R admits a deformation into the surface of centers of a transformation R, it is necessary that the sheets of the envelope of spheres have the same spherical representation of their lines of curvature as isothermic surfaces, and every surface of this sort admits such transformations.

From (1) and (2) it follows that μ is a solution of the differential equation satisfied by X, Y, Z, and consequently when a transformation R of a surface Σ is known, a transformation R of a surface $\bar{\Sigma}$ with the same spherical representation of its lines of curvature as Σ is determined by μ, the corresponding function λ being given by equations analogous to (2) and the other functions follow from equations of the form (3). We say that the new transformation R is obtained from the given one by a transformation of Combescure. Hence if we determine the solutions of our problem for which Σ is an isothermic surface, the others may be obtained by transformations of Combescure.

When Σ is isothermic, we may take

$$\sqrt{E}=\sqrt{G}=e^{\varphi},$$

in which case the integral of the last two of equations (3) is

$$e^{\omega}=\frac{\sqrt{\lambda\sigma}\,e^{\varphi}}{\lambda+c} \tag{4}$$

where c denotes a constant of integration.

When we take $c = 0$ in (4), and substitute this value of ω in (3), the latter reduced to the equations of transformations D_m of the isothermic surface Σ into a new isothermic surface Σ_1.[5] In this case the linear element of the sheet Σ' of the new envelope arising from the deformation of the surface of centers S of the transformation D_m is

$$ds'^2 = \left(e^{\varphi} - \frac{\lambda}{\sigma} e^{-\varphi}\right)^2 du^2 + \left(e^{\varphi} + \frac{\lambda}{\sigma} e^{-\varphi}\right)^2 dv^2,$$

and the linear element of its spherical representation is

$$d\sigma'^2 = \left(\frac{e^{\varphi}}{\rho_1} + \frac{\mu}{\sigma} e^{-\varphi}\right)^2 du^2 + \left(\frac{e^{\varphi}}{\rho_2} - \frac{\mu}{\sigma} e^{-\varphi}\right)^2 dv^2. \qquad (5)$$

The other sheet of the envelope, namely Σ'_1, is merely a point, say O. Conversely as we have shown elsewhere[6] also, a D_m is the only transformation R whose surface of centers can be deformed so that one of the sheets of the new envelope is a point.

From our fundamental theorem it follows that (5) is the spherical representation of the lines of curvature of two isothermic surfaces $\bar{\Sigma}$ and $\bar{\Sigma}'$, themselves Christoffel transforms of one another. Their respective linear elements are

$$d\bar{s}^2 = \sigma^2 e^{2\varphi} (du^2 + dv^2), \qquad d\bar{s}'^2 = \frac{e^{-2\varphi}}{\sigma^2} (du^2 + dv^2).$$

When we apply to $\bar{\Sigma}$ and $\bar{\Sigma}'$ transformations D_m determined by the function μ which is involved in the transformation from Σ' into the point O, we get two new isothermic surfaces $\bar{\Sigma}_1$ and $\bar{\Sigma}'_1$, whose linear elements are respectively

$$d\bar{s}_1^2 = \lambda^2 e^{-2\varphi} (du^2 + dv^2), \qquad d\bar{s}_1'^2 = \frac{e^{2\varphi}}{\lambda^2} (du^2 + dv^2).$$

The surface $\bar{\Sigma}_1'$ is the one which Bianchi[7] defined in a purely intrinsic manner and called the transform of $\bar{\Sigma}$ by the transformation T_m determined by the D_m of Σ into Σ_1.

Let S_0 be the surface of centers of the spheres enveloped by Σ' and O. Corresponding normals to Σ', $\bar{\Sigma}$, $\bar{\Sigma}'$ are parallel, and in like manner the normals to $\bar{\Sigma}_1$ and $\bar{\Sigma}'_1$ are parallel to the lines joining O to the points on S_0. Hence the permanent conjugate system on S_0, that is the system corresponding to the lines of curvature on Σ, project upon the unit sphere with center at O into the orthogonal curves representing the lines of curvature on $\bar{\Sigma}'_1$.

The surfaces $\bar{\Sigma}'$ and $\bar{\Sigma}'_1$ are in the relation of a transformation D_m. Let $\bar{S}$ be the surface of centers of the spheres of this transformation and $\bar{S}_0$ the deform of $\bar{S}$. Applying the above method to these surfaces, we find that the lines joining O to the points of $\bar{S}_0$ are parallel to the normals to Σ. Moreover, the surfaces $\bar{S}$ and S_0 correspond with parallelism of tangent planes; likewise $\bar{S}_0$ and S.

When $c \neq 0$ in (4), the surface Σ'_1 is an isothermic surface in the relation of a transformation T_m with Σ.

[1] Bianchi, *Geometria Differenziale*, **2,** 88.

[2] l. c., p. 117.

[3] Eisenhart, *Differential Geometry*, 122.

[4] *Théorie Générale des Surfaces*, **2,** 383.

[5] Darboux, *Ann. École Normale Supérieure*, ser. 3, **16** (1899).

[6] *Roma, Rend. Acc. Lincei,* ser. 5, **24,** 161 (1915).

[7] *Ann. Mat., Milano,* ser. 3, **12,** 19 (1906).

GEOGRAPHIC HISTORY OF THE SAN JUAN MOUNTAINS SINCE THE CLOSE OF THE MESOZOIC ERA

By Wallace W. Atwood and Kirtley F. Mather

GEOLOGICAL MUSEUM, HARVARD UNIVERSITY

Received by the Academy, February 10, 1916

The San Juan Mountains are located in the southwestern corner of Colorado. From their lofty summits one may look far off into Utah, Arizona, and New Mexico. The highest peak in the range, Uncompahgre, rises to an elevation of 14,300 feet above sea level. There are many other peaks whose summits are above 14,000 feet in elevation, and a large portion of the mountain area is more than 12,000 feet above sea level.

The core of the San Juan Mountains is composed of a complex series of pre-Cambrian gneisses, schists, quartzites, and granites, all of which have been more or less affected by intrusive bodies of igneous rocks. After this pre-Cambrian complex was so eroded that the relief in the region was low, the entire district was submerged and a great series of Paleozoic and Mesozoic strata was deposited over the once mountainous area.

At the close of the Mesozoic era and the opening of the Cenozoic era there were mountain-making movements which affected the entire Rocky Mountain province of North America, and the great dome which was then formed in the San Juan area was at once subjected to vigorous erosion. As the mountain mass rose erosion began, and as the great dome was more and more deeply dissected a mountain topography must have been produced, and those mountains may be thought of as the first generation of the San Juan range (see fig. 1). During the period of mountain growth there was some volcanism. Many porphyritic intrusions and the deposition of the great volcanic tuffs which made contributions to the Ridgway till date back to this period. The Eocene till indicates that during the dissection of these early San Juan Mountains ice formed in the range and descended to the bordering lowlands. Possibly ice formed in neighboring ranges and approached the San Juan Mountains, and possibly there were distinct glacial epochs in that period of glaciation.

After the retreat and disappearance of the early Tertiary ice, stream erosion continued, and the western portion of the San Juan Mountain area was reduced to a surface of slight relief which may be thought of as a peneplain.[1] This peneplain bordered on the west a higher area of mountainous character, which supplied the material for the Telluride conglomerate. The deposition of gravels upon this peneplain surface was probably due to some uplift and rejuvenation of the streams in the eastern portion of the range. After the deposition of the Telluride conglomerate there was further erosion in the range, and then came the three great epochs of volcanism, the San Juan, the Silverton, and the Potosi. During these epochs of volcanism a great volcanic plateau was developed. By this time the Miocene epoch had been reached and possibly passed, and with the quieting down of volcanic activity began the erosion and dissection of the volcanic plateau. During this period of dissection another generation of San Juan Mountains was carved, this time out of volcanic débris and great lava flows (see fig. 1).

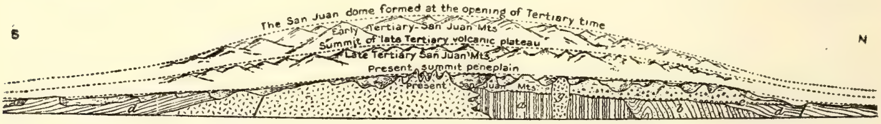

S
The San Juan dome formed at the opening of Tertiary time
Early Tertiary San Juan Mts
Summit of late Tertiary volcanic plateau
Late Tertiary San Juan Mts
Present summit peneplain
Present San Juan Mts
N
a
b
c
d
e
f
g

FIG. 1. DIAGRAMMATIC CROSS SECTION SHOWING THE PHYSIOGRAPHIC EVOLUTION OF THE SAN JUAN REGION SINCE THE BEGINNING OF TERTIARY TIME. *a*, PRE-CAMBRIAN SCHIST AND GNEISS; *b*, ALGONKIAN QUARTZITES; *c*, PRE-CAMBRIAN GRANITE INSTRUSION; *d*, PALEOZOIC AND MESOZOIC ROCKS; *e*, RIDGWAY TILL AND TELLURIDE CONGLOMERATE; *f*, MIDDLE AND LATE TERTIARY VOLCANICS; *g*, TERTIARY INTRUSIVES.

Recent physiographic studies of the range have led to the recognition of a summit peneplain in the San Juan region (see fig. 1), so it would appear that the first dissection stopped at a plain which is now represented by many of the summit areas within the region. The San Juan Mountains that were first carved out of this great volcanic plateau should then be thought of as surmounting those of today. Perhaps, if replaced, they would rise 3000 or 4000 feet above the present summits (see fig. 1).

With the redoming of the area, which involved the warping or doming of the summit peneplain, another cycle of erosion was begun. Valleys were again formed, and in these valleys snows collected which in time formed glaciers that advanced to the lowlands bordering the range. These earliest Pleistocene glaciers retreated and disappeared. The range continued to be uplifted, and the streams were so rejuvenated that they cut great canyons below the broad troughs occupied by the Cerro glaciers. Again climatic changes favored the formation of ice among the summits, and that ice (the Durango glaciers) descended through the main canyons to the foothills and later retreated and disappeared. The canyons were still more deeply cut into the mountain mass, and then climatic conditions favorable for glaciation once more returned and the Wisconsin or third series of Pleistocene glaciers formed and descended through the great canyons, nearly as far as those of the Durango stage. These glaciers have now disappeared, and there is no true glacier ice remaining in the region today, but the streams are vigorously dissecting the mountain mass to still greater depths. The vigor of that work is illustrated in many a sharp, V-shaped notch cut below the depth of ice action. The débris taken from the mountin area is being distributed along the great valleys leading away from the range.

The ice gouging of the three successive Pleistocene stages and the vigorous stream work during the interglacial intervals and since the last melting away of the ice suggest somewhat continuous mountain growth in this region during late geological time.

The physiographic study of the San Juan region has associated with it much that is of human interest. As the region has passed through successive cycles of erosion the position of the ground-water table in the great mountain mass has necessarily changed. The base of the surface zone of oxidation has therefore shifted. With the deepening of erosion and the drawing off of ground waters larger portions of the ore bodies have come to be within the surface zone of oxidation. Secondary enrichment may, therefore, have proceeded to greater and greater depths as the erosion history has progressed. This suggestion has come from the study of the physiographic history of the San Juan Mountains, but as secondary enrichment seems to have been of minor importance here, the idea must be tested in other regions that have similarly passed through several cycles of erosion, and where the secondary enrichment of ore bodies has been recognized as of greater significance.

Settlement was first made in the mountains by those seeking the mineral deposits, and the largest settlements now are mining towns. They are Ouray, Telluride, Silverton, Lake City, Creede, Rico, and Durango. About the margin of the range where agriculture has become of greater importance, and where fruit raising has been especially developed, are the towns of Montrose, Ridgway, Mancos, Ignacio, and Del Norte.

As the lowlands bordering the range have been taken up for settlement, and the broader valley bottoms within the mountains have been fenced in for ranch lands, there has come a demand for a large amount of water for irrigation purposes. Glacial lakes, in many instances, have been modified so as to form great reservoirs, and certain of the canyons have been dammed so that the surplus waters from the melting snows and heavy rains may be stored and held until late in the growing season.

The mountain slopes, where they are not too steep or too high, are clothed with magnificent forests. These are now under national control, and the ranchmen are assigned portions where they may graze their cattle, and in other portions, especially in the higher mountain areas and often in the glacial cirques or basins, thousands and thousands of sheep may be cared for during the summer. In return for this a fee is paid to the national government on a per capita basis. Where timber is cut either by the settlers or by lumbermen for commercial purposes, it is cut under the supervision of those in charge of the national forests.

At the southwest base of the range is that region of remarkable human interest now set aside as the Mesa Verde National Park. It is one of the homes of the ancient cliff dwellers. The physiographic study of the Mesa Verde region indicates that the canyons in which those people built their homes were cut during and since the last glacial period. It is evident from the study of those homes that hundreds of not thousands of people once lived on this mesa. Today there is but a single spring, and that is near one of the smaller settlements. The articles found about these homes indicate that the people raised corn and cotton, and found in the chase an abundance of deer. Today the region is without game, and all the evidence available indicates that when the region was populated there was a much more moist climate. That does not prove that the cliff dwellers occupied those homes while there were glaciers in the mountains, but it is possible that our southwest country was inhabited by men at that time. The records from anthropology and geology are coming closer and closer together, and the further study of this portion of the continent from both points of view may lead to discoveries of unusual interest.

The study of the geography of this region has necessarily a very close relationship to the geologic studies of the range, but it may lead also to a study of anthropogeography, and to a study of how the present geographic conditions are influencing the settlement and commercial development within this district.[2]

---

[1] Whitman Cross, U. S. Geol. Survey Geol. Atlas. Silverton folio (No. 120), 1905.

[2] This study is here published with the permission of the Director of the U. S. Geological Survey.

THE AGE OF THE MIDDLE ATLANTIC COAST UPPER CRETACEOUS DEPOSITS

By W. B. Clark, E. W. Berry, and J. A. Gardner

GEOLOGICAL LABORATORY, JOHNS HOPKINS UNIVERSITY

Received by the Academy, February 16, 1916

Distribution.—The Upper Cretaceous formations of the Middle Atlantic Coast area are most extensively developed in New Jersey from which state they thin rapidly to the northward through the islands off the New England coast, only a few remnants being left in southern Massachusetts; while to the south they gradually disappear, only the lowermost reaching the Potomac drainage basin. Beyond this point they have only been recognized in deep-well borings in Virginia although the deposits probably extend continuously beneath the mantle of Tertiary formations until they again appear in surface outcrops in the South Atlantic area.

Divisions.—Several formations have been recognized within the area of outcrop and have been designated under the names of Raritan, Magothy, Matawan, Monmouth, Rancocas, and Manasquan.

Numerous minor subdivisions have been described in New Jersey but have been unrecognized elsewhere. All the deposits are of shallow-water origin as shown by their contained faunas. The relatively slight differences in the faunas of these smaller units are evidently due to the varying proximity of stream mouths and sediment-bearing currents rather than to any differences in depth which may have existed.

The major divisions, on the other hand, are based on distinct differences in the faunas and floras which may be recognized when present not only in the Middle Atlantic Coast area but throughout the South Atlantic and Gulf regions as well.

The Raritan and Magothy formations are clearly separable on the basis of their contained floras while the Matawan and Monmouth, the only marine divisions with equivalents in the South Atlantic and Gulf regions, are separable by the initiation in the Monmouth of a new fauna marked by the introduction of *Belemnitella americana*, *Exogyra costata*, *Turritella vertebroides*, *Anchura pennata*, *Eutrephoceras dekayi*, and probably of *Liopistha protexta* which have been made the basis for the differentiation of the two primary zones shown on the Federal Survey maps throughout this region. While it is true that several of the Matawan forms persist far into the Monmouth this does not in the last detract from the significance of the initiation of a new element of more than local importance at the opening of the Monmouth. The time of extinction of an old fauna is properly considered as less significant than the time of initiation of a new one, and if that new one be sufficiently virile to characterize the molluscan life from New Jersey south through Georgia and west to Texas it indicates something more than a minor oscillation in a restricted area and should be given the relatively higher rank which it deserves. The Rancocas fauna is very distinct from the preceding Monmouth but it is only known in the restricted area of New Jersey and Delaware. The same is also true in the case of the Manasquan which has an even more limited development in New Jersey. These two later formations have no equivalents in the South Atlantic and Gulf areas.

Correlation.—Raritan.—The Raritan formation of the Middle Atlantic Coast contains a flora of over 200 species, on the relations of which its correlation must be based since but a few uncharacteristic invertebrates have been found in these beds. The Raritan flora has been chiefly recorded from New Jersey where the sediments were more favorable for its preservation than farther southward. It is clearly separable into older and younger florules, the latter making its appearance in the upper beds near South Amboy, New Jersey, ranging southward to the Potomac, and reappearing in the lower part of the Tuscaloosa formation of the Eastern Gulf and the Woodbine formation of the Western Gulf. The Raritan flora is conclusively correlated with the Cenomanian stage of the European section by reason of a number of forms common to such well-known Cenomanian floras as those of Niedeschoena, Saxony; Moletein, Moravia; Perutz, Bohemia; and Alcantara, Portugal. In addition to this considerable element common to the old world Cenomanian a large number of identical genera have closely related species in the two regions and many new and identical types appear at this horizon on both sides of the Atlantic.

Magothy.—The Magothy formation of the Middle Atlantic coast has furnished a rather poor fauna, principally from the New Jersey area, and an extensive flora of upwards of 300 species found from Marthas Vineyard, Massachusetts, to the Potomac. The Magothy is separated from the Raritan by an erosional unconformity. Its fauna has furnished but meager data for exact correlation but the large flora furnishes decisive evidence on this point. Compared with the underlying Raritan it may be noted that 61% of the Raritan flora becomes extinct before Magothy time and 70% of the flora of the latter, including many new and progressive types, is not found at horizons as old as the Raritan. Some of the most characteristic Magothy forms are found in the Middendorf and Black Creek beds of the Carolinas, and in the upper Tuscaloosa and lower Eutaw of the Eastern Gulf.

Table Showing the Approximate Correlation of the Middle Atlantic Coast Upper Cretaceous Formations with those of other Areas

N. J.—MD.	N. C.—S. C.	EASTERN GULF	WESTERN GULF	WESTERN INTERIOR	EUROPE
Manasquan				Laramie	Danian
Rancocas					
Monmouth	Peedee	Ripley / Selma	Navarro Taylor	Montana	Senonian
Matawan	Black Creek	Eutaw	Austin Eagle Ford	Colorado	Turonian
Magothy	Middendorf	Tuscaloosa	Woodbine	Dakota	
Raritan			Washita		Cenomanian

Compared with European floras that of the Magothy shows several types identical with and many closely related to those of the lower basin of the Rhone, Bussaco, in Portugal, and the richly plant-bearing Turonian section of Bohemia. This floral similarity as well as the stratigraphic position of the Magothy leads to the conclusion that the Magothy formation is of Turonian age.

Matawan.—The earliest fauna of any extent occurs in the Matawan from which 175 to 185 species have been described in New Jersey and Maryland. Five faunal zones have been differentiated in north-central New Jersey, where the Matawan is best developed, but it has not been possible to carry any but the lowest of these south of the Delaware-Maryland line. The Merchantville, together with the Woodbury which merges into it in southern New Jersey, has been recognized along the Chesapeake and Delaware Canal just east of the state line. Although this fauna is a small one of only about 50 species, it contains the elements of one of the most widely distributed and best characterized biotas of the entire Upper Cretaceous, a fauna represented in the *Mortoniceras subzone* of the South Atlantic and Gulf states, the Austin Chalk of Texas, the Niobrara of the Western Interior, and the Emscher beds of north-central Europe.

The late Matawan deposits were probably laid down in more shallow waters. Their contained faunas are consequently more restricted in their distribution and less cosmopolitan in their affinities. The Marshalltown of New Jersey which is best characterized by the abundance of the ponderous Ostreids, such as Exogyra and Gryphaea, is quite possibly represented in the Matawan oyster banks along the Chesapeake and Delaware Canal west of St. Georges. These beds are doubtless the time equivalent of at least a part of the *Exogyra ponderosa zone* of the South Atlantic and Gulf states, a faunal zone which has been recognized in the upper Black Creek of North and South Carolina, the lower Ripley of Georgia and eastern Alabama, the lower Selma of central and western Alabama, Mississippi, and Tennessee, and the upper Eutaw of Mississippi and Tennessee.

Monmouth.—The Monmouth formation has furnished the most prolific and diversified fauna of any of the Upper Cretaceous, over 250 species having been determined and of these about 75% are peculiar.

Three faunal zones have been differentiated in New Jersey but it is impossible to trace the New Jersey horizons southward into Maryland with any assurance. The fauna of the Sassafras River area, although very prolific, is very poorly preserved and the determinable species are none of them diagnostic of a particular facies. However, the abundance of *Belemnitella americana*, in the Eastern Shore deposits of Delaware and Maryland suggests the Navesink, while *Sphenodiscus* is peculiarly characteristic both of the Tinton of New Jersey and the prolific Prince George's County fauna in Maryland. It is exceedingly improbable, however, that there is any appreciable time interval involved in these faunal differences.

The general aspect of the Monmouth biota is more southern than that of the Matawan and it is probable that the slight depression of the seas which initiated the Monmouth broke down the barriers which had earlier prevented free communication with the inshore life of the south Atlantic.

The Monmouth is at least the partial time equivalent of the *Exogyra costata zone* which has been recognized in the Peedee sand of North and South Carolina and in the Ripley and Selma chalk of the Gulf states. A number of identical species occur in the Fox Hills of the Western Interior while the ensemble is very similar to that of the Upper Campanian of the Belgian border and the so-called Maestricht beds.

Rancocas.—Two horizons have been recognized in the Rancocas of New Jersey, the Hornerstown glauconitic marl and the Vincentown "yellow sand" but these divisions are absent south of the Delaware Bay. The diagnostic features of the fauna developed in Delaware are essentially those of the Vincentown—a prolific bryozoan fauna with *Terebratula harlani* in abundance and a very meager molluscan representation. Only about 15 species have been determined, all of them bivalve, over half of which are restricted to the Rancocas. Five have been found at older horizons, while *Ostrea vomer* and *Terebratula harlani* have been reported as surviving the break between the Mesozoic and Cenozoic although there is doubt in both cases as to the identity of the species.

The mollusca of Delaware are curiously dissimilar from those of New Jersey, none of the few characteristic species of New Jersey occurring in Delaware, while the abundant Delaware *Gryphaea* to which the characteristic Vincentown bryozoa attach themselves, is apparently not present in New Jersey.

There is no closely allied fauna in this country and its closest analogue is apparently in northern Europe. Although there is little direct evidence for its correlation with the Danian yet the general facies of the two faunas is very similar. Both are characterized by an extensive bryozoan fauna and the absence of *Belemnitella.* At the same time the fauna is not Tertiary in aspect and it seems improbable that it can belong to that horizon.

Manasquan.—The Manasquan is closely related to the Rancocas both faunally and stratigraphically. Its fauna is not characteristically Cretaceous although the general assemblage of forms suggests its association with that division rather than with the Tertiary. It is regarded as Danian. It has no equivalents as far as known among American formations elsewhere.

Conclusions.—The several Upper Cretaceous formations of the Middle Atlantic Coast represent all of the major divisions of the European series. The Raritan must be regarded from its contained flora as Cenomanian while the flora of the Magothy is considered Turonian. The fauna of the lower Matawan is regarded as lower Senonian or Emscherian because of the presence of *Mortoniceras* and associated forms. Equivalent strata to the Magothy and Matawan combined are found in the Carolinas in the Black Creek formation where beds containing the Magothy flora are interstratified throughout the upper part of the formation with beds containing a Matawan fauna. The evidence is apparently conflicting and the dividing line between the Turonian and Senonian is therefore placed midway in the Matawan although this must be regarded as a compromise position. The Monmouth fauna is Senonian in age and probably represents the middle and upper Senonian. The Rancocas and Manasquan are regarded as Danian.

UPPER CRETACEOUS FLORAS OF THE WORLD

By Edward W. Berry

GEOLOGICAL LABORATORY, JOHNS HOPKINS UNIVERSITY

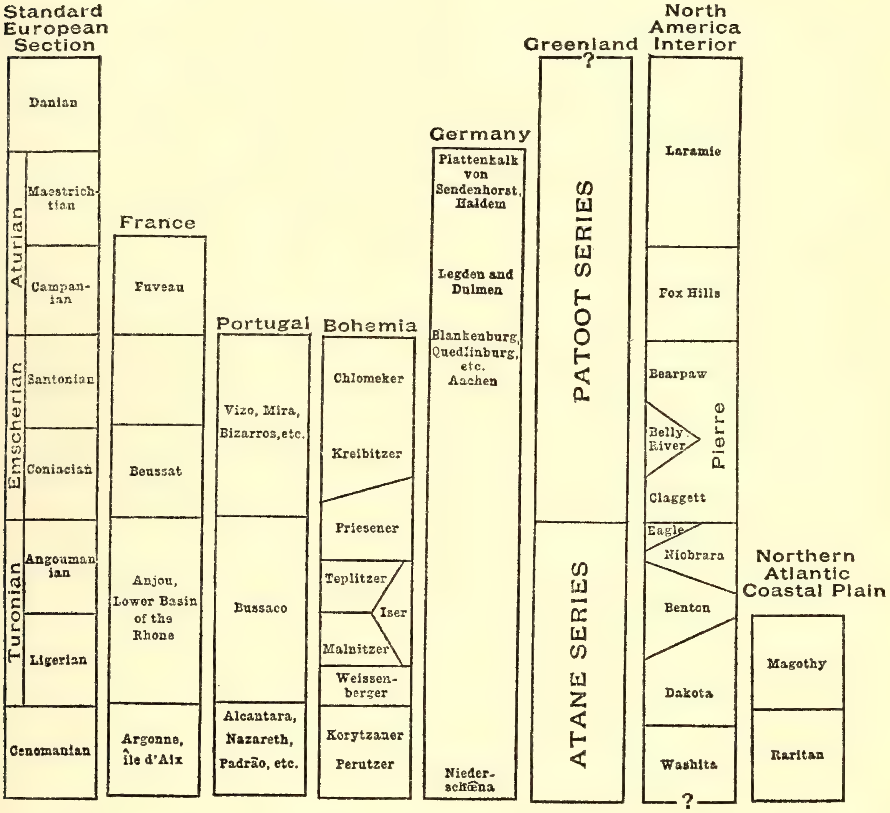
Received by the Academy, February 16, 1916

The writer has prepared a detailed account of the Upper Cretaceous floras of the world for a report on the Upper Cretaceous of Maryland which will be published later by the Geological Survey of that State.

Since the question of the age relations of the Upper Cretaceous floras of the United States has been the occasion of considerable discussion in recent years it is believed that a summary of the contemporaneous floras from other parts of the world will prove useful to paleontologists and geologists.

The stratigraphic position of the more important of these is indicated in the accompanying diagram.

The typically marine series of sediments developed in most European countries has furnished little that is of interest to the paleobotanist. The floras of the Turonian of the lower basin of the Rhone, and of the Coniacian and Campanian in the same region are well represented but largely undescribed, while the earlier Upper Cretaceous floras of France are scattered and unimportant. In Portugal extensive and important floras are intercalated in a fossiliferous marine series ranging from the Cenomanian through the Turonian and Emscherian, but these also are largely undescribed. Bohemia furnishes a splendid section from the base of the Upper Cretaceous through the Emscherian, with extensive and fully described floras in the Cenomanian, Turonian and Emscherian that are of the utmost value for purposes of correlation. Germany furnishes an extensive flora of Cenomanian age and scattered floras ranging upward to horizons as high as the Laramie of the western United States. The upper Cretaceous floras of the west coast of Greenland are very extensive but the deposits are largely continental in type and the floras offer difficulties to precise correlation in the upper part of the section since collections were not made from carefully ascertained levels.


Standard European Section
Danian
Aturian
Maestrichtian
Campanian
Emscherian
Santonian
Coniacian
Turonian
Angoumanian
Ligerian
Cenomanian
France
Fuveau
Beussat
Anjou, Lower Basin of the Rhone
Argonne, Île d'Aix
Portugal
Vizo, Mira, Bizarros, etc.
Bussaco
Alcantara, Nazareth, Padrão, etc.
Bohemia
Chlomeker
Kreibitzer
Priesener
Teplitzer
Iser
Malnitzer
Weissenberger
Korytzaner
Perutzer
Germany
Plattenkalk von Sendenhorst, Haldem
Legden and Dulmen
Blankenburg, Quedlinburg, etc. Aachen
Niederschœna
Greenland
?
PATOOT SERIES
ATANE SERIES
North America Interior
Laramie
Fox Hills
Bearpaw
Belly River
Pierre
Claggett
Eagle
Niobrara
Benton
Dakota
Washita
?
Northern Atlantic Coastal Plain
Magothy
Raritan

There are also some representation of the floras of the Upper Cretaceous in Japan and in the southern hemisphere (Patagonia, Graham Land, Australasia) but these are not included in the accompanying diagram.

OBSERVATIONS ON AMEBA FEEDING ON INFUSORIA, AND THEIR BEARING ON THE SURFACE-TENSION THEORY

By S. O. Mast and F. M. Root

ZOOLOGICAL LABORATORY, JOHNS HOPKINS UNIVERSITY

Received by the Academy, February 25, 1916

A considerable number of investigators hold that movement in Ameba is produced exclusively by changes in surface tension. Among the most prominent of these we may mention Bütschli (1892), Ryder (1894), Jensen (1905), and Verworn (1909). Many others, *e.g.*, Rhumbler (1910) and McClendon (1911), hold that while surface tension may not be the only factor involved in the process in question, it certainly is one of the most important. The results of our observations do not support these contentions.

Certain amebae at times feed almost exclusively on rotifers, at others they feed largely on paramecia. They capture the rotifers by flowing around the foot at the point of attachment to the substratum. After they have surrounded the foot they begin to flow out over the body. The rotifer responds by contracting and forcing the ameba back, after which it extends again and the ameba again begins to flow out over it, etc. In the meantime the foot begins to digest and gradually the rotifer weakens. Thus they continue sometimes for days before the rotifer is swallowed. The whole process is of such a nature that after observing it we were fully convinced that the force exerted by the amebae was far greater than could possibly be produced by changes in surface tension. The following observations prove this conviction to be correct with reference to the process of feeding on Paramecium.

When amebae are feeding on paramecia they assume a sort of mushroom shape with a serrate edge consisting of numerous short pseudopods. The paramecia tend to come to rest between and under these pseudopods by which they are usually surrounded, but sometimes the ends of the pseudopods approach each other before they are fully extended and cut the paramecium in two. This process requires approximately only ten seconds.

To cut a paramecium in two with a fine glass fiber it requires a pressure of approximately 9 mgm. Consequently, if the pseudopods have the same cutting quality as the glass fiber and if their movement is due to a change of surface tension, it would require, to perform the work involved, a reduction in surface tension of at least 1118 dynes per centimeter at the tips of the pseudopods. But if the pseudopods fuse along the edges near the ends so as to form a ring around the paramecium, and the cutting is due to constriction in this ring, and the constriction to a change in surface tension, the work involved would require a minimum reduction along the inner surface of the ring of at least 383 dynes per centimeter.

The bulk of evidence at hand seems to indicate that the paramecia are divided by the approach of two pseudopods and not by the constriction of a ring. To account for the process on the basis of the surface tension theory, therefore, the surface tension of the amebae would, in all probability, have to be considerably higher than 1118 dynes per centimeter. The surface tension of protoplasm is, however, only approximately 50 dynes per centimeter. It is, therefore, probably at best an insignificant factor in the process of feeding in Ameba.

More detailed descriptions of these observations and calculations will be published elsewhere.

The following references may be listed:

Bütschli, O., 1892, *Untersuchungen über mikroskopische Schaume und das Protoplasma*, Leipzig, 234 S.

Jensen, P., 1905, Zur Theorie der Protoplasmabewegung und über die Auffassung des Protoplasmas als chemisches System. *Anat. Hefte, Wiesbaden*, 27, Heft 83, 831–858.

McClendon, J. F., 1912, The Osmotic and Surface Tension Phenomena of Living Elements and their Physiological Significance, *Biol. Bull.*, 22, 113–204.

Rhumbler, L., 1905, Zur Theorie der Oberflächenkräfte der Amöben, *Zs. wiss. Zool.*, **83**, 1–52.

——, 1910, Die verschiedenartigen Nahrungsaufnahmen bei Amöben als Folge verschiedener Colloidalzustände ihrer Oberflächen, *Arch. Entw.-Mech.*, 30, 194–220.

Ryder, J. A., 1893, Dynamics in Evolution, *Biological Lectures*, Woods Hole, pp. 63–81.

Verworn, Max, 1909, *Allgemeine Physiologie*, Fünfte Auflage, Jena, 742 S.

THE ELECTROMOTIVE FORCE PRODUCED BY THE ACCELERATION OF METALS

By Richard C. Tolman and T. Dale Stewart

DEPARTMENT OF CHEMISTRY, UNIVERSITY OF CALIFORNIA

Received by the Academy, March 2, 1916

Modern theories of electricity have led to the belief that the passage of an electric current through a metal really consists in the progressive motion of 'free' electrons contained in the body of the metal itself. If this be true we may now expect a number of effects arising from the *mass* of these electrons which were not predictable on the basis of older theories which thought of electricity as a sort of intangible massless fluid. As examples of such effects, we should expect the rear end of an *accelerated* rod of metal to become negatively charged owing to the lagging behind of the relatively mobile electrons which the metal contains, and should expect the periphery of a *rotating* disk to become negatively charged owing to the action of centrifugal force on the electrons in the disk. Such effects would presumably be very small, however, owing to the exceedingly small mass probably associated with the electron.

In the case of electrolytes, where the mass associated with the carriers of electricity is much larger than in metals, such effects have already been detected by Colley[1] and Des Coudres[2] using respectively the accelerational and the centrifugal method, and have been subjected to a more elaborate quantitative investigation by Tolman[3] and by Tolman and Osgerby.[4]

In the case of metallic conductors, Maxwell,[5] himself, was the first, not only to discuss the nature of the phenomena which would arise if mass of the ordinary kind should be associated with electricity in metals, but also to try experiments to detect the possible effects. He had, however, no means of predicting the presumable size of such effects if they did exist, since this was before the development of the electron theory, and he merely states the negative results of his experiments without any information as to the dimensions or efficiency of his apparatus. Lodge[6] also reports a negative result for such experiments.

The first attempts to detect such effects in metals of which we have any quantitative information were made by Nichols[7] in 1906. He employed the centrifugal method, using a rotating aluminum disk and making a rubbing contact at the periphery and center with wires, which led to the electrical measuring apparatus. Such rubbing contacts, in particular the one at the rapidly moving periphery, necessarily introduce larger and variable electromotive forces; nevertheless from a series of experiments Nichols was able to conclude that the mass of the carrier in metals is less than that of the hydrogen atom.

Since in the case of metals the centrifugal method of attack almost necessarily involves some disastrous form of rubbing contact, we have been at work during the last four years developing the accelerational method of attack. In 1913[8] we were able to report that the effect in metals, if any, was so small that the mass of the carrier in metals was less than one two-hundredth part of that of the hydrogen atom. With the help of a much more sensitive galvanometer and eliminating one by one a number of accidental effects which appear when greater sensitiveness is reached, we have now apparently obtained a real effect due to the mass of the carrier in metals.

The apparatus consisted essentially of a coil of insulated copper wire, wound on the periphery of a wheel which could be rotated at a speed of about 5000 r.p.m. and brought suddenly to rest. The two ends of the coil were lead to the center of the wheel and there made connection, through wires which were allowed to twist up, with a ballistic galvanometer which measured the pulse of electric current which was produced by the tendency of the electrons to continue in motion after the wheel was stopped. For the purpose of eliminating accidental effects it was found among other things necessary to construct the apparatus of non-magnetic materials, to rotate the coil of wire in a space in which both the horizontal and vertical components of the earth's steady magnetic field has been neutralized, and to compensate for the variable part of the earth's magnetic field by connecting in series with the rotating coil a compensating coil having the same flux area but wound in the opposite direction. For details of the apparatus our complete article which has been submitted to the *Physical Review* for publication must be consulted.

The galvanometer throws were found to be in the direction predicted on the basis of a negative charge for the *mobile* carrier in metals and to agree within the limits of error with the equation

$$Q = M\frac{vl}{RF},$$

of which the derivation will be given in our complete article. Q is the pulse of electricity sent through the galvanometer on stopping, v is the velocity of the periphery of the wheel at the instant of stopping, l the length of the coil, R the total resistance in the circuit, F the value of the faraday, and M is a constant whose value ought to be fairly close to that of the mass associated with one equivalent (*i.e.*, with $F = 96{,}540$ coulombs) of electrons. For copper wire the average value of M from 131 runs was 1/1910 which may be compared with the accepted value for the mass of one equivalent of electrons in free space, namely 1/1845 (*i.e.*, the ratio of the mass of the electron to that of the hydrogen atom).

Our purpose in carrying out these measurements has not been merely to demonstrate an effect which has long been an object of search; we have also had in mind the possibility of obtaining from our experiments information as to the nature of the conducting process in metals and indeed perhaps further information as to the nature of the electron itself.The equation given above which we have tested in this work was derived on the assumption that the conducting process in metals is in the nature of a drift of 'free' electrons when acted on by an electric field, and the fact that the equation seems to fit the experimental facts is to some extent a verification of these assumptions. Such considerations are of particular interest at the present time in view of J. J. Thomson's proposal[9] of a quite different theory of metallic conduction. According to his theory, a metal contains atoms which are in the nature of electrical doublets which will orient themselves parallel to any applied electrical field. These atoms are assumed to have the power of ejecting electrons in the same direction as the axis of the doublet and hence the conducting process on the basis of this theory consists in a tendency for orientation of the doublets under the action of the applied electromotive force and a consequent ejection of electrons from one atom to another in the direction in which the current is known to flow. It seems very doubtful to us whether such a theory can be satisfactorily brought into agreement with our experimental results, since it would seem at first sight to be merely an accidental coincidence if the *mechanical* forces which we apply should produce an orientation in the right direction and of the right amount to give the pulse of electricity whose magnitude we have calculated on the basis of the other theory and actually found experimentally.

---

[1] Colley, *Ann. Physik., Leipzig,* **17,** 55 (1882).

[2] Des Coudres, *Ibid.,* **49,** 284 (1893); *Ibid.,* **57,** 232 (1896).

[3] Tolman, *Proc. Amer. Acad. Arts Sci.,* **46,** 109 (1910); *J. Amer. Chem. Soc.,* **33,** 121 (1911).

[4] Tolman, Osgerby and Stewart, *J. Amer. Chem. Soc.,* **36,** 466 (1914).

[5] Maxwell, *Treatise on Electricity and Magnetism,* 3rd edition (1892), Vol. 2, pp. 211 et seq.

[6] Lodge, *Modern Views of Electricity,* 3rd edition (1907), p. 39.

[7] Nichols, *Physik. Zs.,* **7,** 640 (1906).

[8] Tolman, Osgerby and Stewart, loc. cit.

[9] Thomson, *Phil. Mag.,* **30,** 192 (1915); Richardson, *Ibid.,* **30,** 295 (1915).

Errata: In Mr. T. W. Vaughan's article, pages 98 and 99, the familiar percentage sign (%) was printed in place of the per-thousandths sign (°/₀₀), following figures for the salinity; the salinities as printed are therefore ten times too large.

CONTENTS

Page

GEOLOGY.—THE MECHANICS OF INTRUSION OF THE BLACK HILLS (S. D.) PRE-CAMBRIAN GRANITE . . . . *By Sidney Paige* 113

ANTHROPOLOGY.—ARCHAEOLOGICAL EXPLORATIONS AT PECOS, NEW MEXICO . . . . *By A. V. Kidder* 119

ANTHROPOLOGY.—MAN AND METALS . . . . *By Walter Hough* 123

ASTRONOMY.—ON THE OBSERVED ROTATIONS OF A PLANETARY NEBULA . . . . *By W. W. Campbell and J. H. Moore* 129

ASTRONOMY.—A SHORT PERIOD CEPHEID WITH VARIABLE SPECTRUM . . . . . . *By Harlow Shapley* 132

ASTRONOMY.—THE SPECTRUM OF δ CEPHEI . *By Walter S. Adams and Harlow Shapley* 136

ASTRONOMY.—INVESTIGATIONS IN STELLAR SPECTROSCOPY. I. A QUANTITATIVE METHOD OF CLASSIFYING STELLAR SPECTRA . . . . *By Walter S. Adams* 143

ASTRONOMY.—INVESTIGATIONS IN STELLAR SPECTROSCOPY. II. A SPECTROSCOPIC METHOD OF DETERMINING STELLAR PARALLAXES . . . *By Walter S. Adams* 147

ASTRONOMY.—INVESTIGATIONS IN STELLAR SPECTROSCOPY. III. APPLICATION OF A SPECTROSCOPIC METHOD OF DETERMINING STELLAR DISTANCES TO STARS OF MEASURED PARALLAX . . . . *By Walter S. Adams* 152

ASTRONOMY.—INVESTIGATIONS IN STELLAR SPECTROSCOPY. IV. SPECTROSCOPIC EVIDENCE FOR THE EXISTENCE OF TWO CLASSES OF M TYPE STARS . . . . *By Walter S. Adams* 157

ANTHROPOLOGY.—THE FAILURE AND REVIVAL OF THE PROCESS OF PIGMENTATION IN THE HUMAN SKIN . . . . *By A. E. Jenks* 164

GEOLOGY.—BANDED GLACIAL SLATES OF PERMO-CARBONIFEROUS AGE, SHOWING POSSIBLE SEASONAL VARIATIONS IN DEPOSITION . . . . . *By Robert W. Sayles* 167

MATHEMATICS.—AN EXTENSION OF FEUERBACH'S THEOREM . . . *By F. Morley* 171

MATHEMATICS.—DEFORMATIONS OF TRANSFORMATIONS OF RIBAUCOUR . . . . . *By L. P. Eisenhart* 173

GEOLOGY.—GEOGRAPHIC HISTORY OF THE SAN JUAN MOUNTAINS SINCE THE CLOSE OF THE MESOZOIC ERA . . *By Wallace W. Atwood and Kirtley F. Mather* 177

GEOLOGY.—THE AGE OF THE MIDDLE ATLANTIC COAST UPPER CRETACEOUS DEPOSITS *By W. B. Clark, E. W. Berry, and J. A. Gardner* 181

GEOLOGY.—UPPER CRETACEOUS FLORAS OF THE WORLD . . *By Edward W. Berry* 185

ZOOLOGY.—OBSERVATIONS ON AMEBA FEEDING ON INFUSORIA, AND THEIR BEARING ON THE SURFACE-TENSION THEORY . . . . *By S. O. Mast and F. M. Root* 188

PHYSICS.—THE ELECTROMOTIVE FORCE PRODUCED BY THE ACCELERATION OF METALS *By Richard C. Tolman and T. Dale Stewart* 189

VOLUME 2 APRIL, 1916 NUMBER 4

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES
OF THE
UNITED STATES OF AMERICA

EDITORIAL BOARD

Arthur A. Noyes, *Chairman*
Edwin B. Wilson, *Managing Editor*
Arthur L. Day, *Home Secretary*
George E. Hale, *Foreign Secretary*

J. J. Abel
W. B. Cannon
J. McK. Cattell
J. M. Clarke
E. G. Conklin
J. M. Coulter
C. B. Davenport
Simon Flexner
E. B. Frost
R. G. Harrison
W. H. Holmes
J. P. Iddings
R. A. Millikan
E. H. Moore
Alexander Smith

Publication Office: Williams & Wilkins Company, Baltimore
Editorial Office: Massachusetts Institute of Technology, Boston
Home Office of the Academy: Washington, D. C.

Entered as second-class matter at the Post Office at Baltimore, Md., under the Act of August 24, 1912

INFORMATION TO CONTRIBUTORS

The Proceedings is the official organ of the Academy for the publication of brief accounts of important current researches of members of the Academy and of other American investigators, and for reports on the meetings and other activities of the Academy. Publication in the Proceedings will supplement, not replace, that in journals devoted to the special branches of science. The Proceedings will aim especially to secure prompt publication of original announcements of discoveries and wide circulation of the results of American research among investigators in other countries and in all branches of science.

Articles should be brief, not to exceed 2500 words or 6 printed pages. Technical details of the work and long tables of data should be reserved for publication in special journals. But authors should be precise in making clear the new results and should give some record of the methods and data upon which they are based. The viewpoint should be comprehensive in giving the relation of the paper to previous publications of the author or of others and in exhibiting where practicable, the significance of the work for other branches of science.

Manuscripts should be prepared with a current number of the Proceedings as a model in matters of form, and should be typewritten in duplicate with double spacing, the author retaining one copy. Illustrations should be confined to text-figures of simple character, though more elaborate illustrations may be allowed in special instances to authors willing to pay for their preparation and insertion. Particular attention should be given to arranging tabular matter in a simple and concise manner.

References to persons should include initials; citations of literature, numbered consecutively, should be placed at the end of the article; and short footnotes should be avoided. Indicate 'subject' for 'Contents.'

Papers by members of the Academy may be sent to Edwin Bidwell Wilson, Managing Editor, Mass. Institute of Technology, Boston, Mass. Papers by non-members should be submitted through some member.

Proof will be sent to authors only on their request, in order to avoid delay in publication. Authors are therefore requested to make final revisions on the typewritten manuscripts. The editors cannot undertake to do more than correct obvious minor errors.

Reprints should be ordered at the time of submission of manuscript. They will be furnished to authors at cost, approximately as follows:

Reprints of - -	2 pp.	4 pp.	6 pp.	8 pp.	Covers extra
Charge for first 100 copies	$1.00	$1.30	$2.20	$2.20	$2.00
Charge for additional 100s	.25	.45	.85	.85	.50

INFORMATION TO SUBSCRIBERS

Subscriptions at the rate of $5.00 per annum should be made payable to the National Academy of Sciences, and sent to Williams & Wilkins Company, Baltimore, or Arthur L. Day, Home Secretary, National Academy of Sciences, Smithsonian Institution, Washington, D. C. Single numbers, $0.50.

Copyright, 1916, by the National Academy of Sciences

JUN 2 [illegible]
National Mus

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES

Volume 2 APRIL 15, 1916 Number 4

PRELIMINARY REPORT UPON THE POSSIBILITY OF CONTROLLING THE LAND SLIDES ADJACENT TO THE PANAMA CANAL

By the Committee of the National Academy of Sciences Appointed at the Request of the President of the United States

Dated, February 4, 1916. Received, March 16, 1916

INTRODUCTION

The Committee of the National Academy of Sciences, appointed November 18, 1915, at the request of President Woodrow Wilson "to consider and report upon the possibility of controlling the slides which are seriously interfering with the use of the Panama Canal," submits this its preliminary report.

The Committee as originally appointed consisted of thirteen persons. For various reasons four (Messrs. A. L. Day, G. F. Becker, C. D. Walcott, and R. S. Woodward) were unable to visit the canal and participate in the deliberations of the Committee. Those who took part in the preparation of this report are as follows:

C. R. Van Hise, Whitman Cross, J. R. Freeman,
H. L. Abbott, R. C. Carpenter, J. F. Hayford,
J. C. Branner, A. P. Davis, H. F. Reid.

These members, who will be spoken of as 'the Committee' in the report, sailed from New Orleans December 11, and arrived at Panama December 19. All spent two weeks in the Canal Zone, and three of them several days longer, working upon the problem submitted to them.

The part of the Canal cut between Bas Obispo and Pedro Miguel will be called the Gaillard Cut in accordance with the official use of that term. The deepest part of the Gaillard Cut, at the continental divide, about one mile in length, will be called the Culebra District.

The hill upon which the village of Culebra stands will be called Culebra Hill.

The general direction of the Canal is nearly northwest and southeast, but nearer north and south than east and west. The various stretches vary considerably in this direction. For the sake of brevity in description, the Canal will be regarded as running north and south, and directions at right angles to the Canal will be called east and west and those parallel with it north and south.

The term 'slides' when unqualified will be applied alike to material which is now in motion and to that which once has been in motion but is now quiescent. Where it is necessary to discriminate between these two conditions of the slides, one will be called 'active' and the other 'quiescent.'

The Culebra district was visited by all members of the Committee five days, and a number of the Committee spent several additional days in this area. The attention of the Committee was primarily directed to the question of the control of the active East and West Culebra Slides, but the extensive Cucaracha Slide, now quiescent, was also examined with care. The Committee also examined the massive hills of the Culebra District and especially Gold, Culebra, Zion and Contractors Hills, all of which are adjacent to the great slides.

The work of the Committee in the field was facilitated in every way by Major General George W. Goethals, Lieut. Col. Chester H. Harding, Lieut. Col. Jay J. Morrow and Rear Admiral H. H. Rousseau; General Goethals furnished records from his office, a brief history of the slides and their movements, and much other information which the Committee desired; in short, all possible help has been given to the Committee so so that its members could carry on their work most effectively in their own way.

The Committee has profited greatly by the geological studies of Mr. Donald F. MacDonald and by conferences with him in the field and in the office.

As the uninterrupted operation of the Panama Canal is a matter of great national importance, the Committee plans a further study of the available data and expects in due time to make a fuller report; but it seems desirable to present promptly to President Wilson, a preliminary report containing the views of the Committee (so far as they can now be formulated), and such practical suggestions as the Committee is able to offer.

THE THREE GREAT SLIDES

The slides which led to the closing of the Canal on September 18, 1915, were the great East Culebra and West Culebra Slides. General Goethals has described these slides in an article prepared for the press, under date November 15, 1915, and from this article the following statements are taken:

> The East Culebra Slide began on October 14, 1914, without any warning and a section of the east bank north of Gold Hill settled vertically 20 feet. This section measured 2,000 feet (now extended to 2,700 feet) along the prism face and extended back 1,000 feet from the axis of the Canal, generally along an irregular curved line. The top of the bank was from 300 to 350 feet above sea level, and the extension of the ground eastward was relatively flat country. In the settlement, the upper portion which broke away remained practically parallel to its original position, and the benches which formed the upper part of the slide had not changed their relative positions, though they were badly broken up, while the lower strata were squeezed out across the Canal. Subsequently the broken mass moved into the cut, reducing the depth of the water from 45 feet to 9 inches at one point. Until August, 1915, the dredges were able for the most part to keep up with the movement as it came down, and probably would have been able to maintain this condition had not a movement occurred on the west bank, necessitating work on this side to the detriment of the east side.
>
> A crack was found on the slope of Zion Hill in June, 1914, but observations made upon it showed no movement and the solidity of the hill was never doubted. Subsequent to the break on the east side, a gradual but general breaking up of the west bank followed, and the crack on the slope increased in size and new ones developed farther up the hill, until finally one extended to the elevation of 480, the limit of the present break. The movement into the cut from the west bank occurred early in August, 1915, when a section of Zion Hill broke away and settled down. The edge of the break on this side is also a curve.
>
> The movements from the two sides are towards the central portion of the enclosed area, and at this central portion is the obstruction to the channel. It first appeared as an island forced up from the bottom, then as a peninsula projecting from the east bank, and finally was pushed entirely across the channel completely closing it. . . . .
>
> The length of the slides, which are directly opposite each other, is approximately 2,200 feet (the channel through which is navigable with the exception of 600 feet); the banks are 300 to 350 feet above sea level on the east and extend up to 480 feet above sea level on the west. The area of the territory affected on the east side covers 81 acres and on the west 78.5 acres.
>
> Assuming that all material lying above planes extending from the outside limits of the bottom of the prism, reference 40, up to the limits of the

breaks, will move into the cut, 7,000,000 cubic yards will have to be removed before the slides are entirely stopped. Mr. Comber, Resident Engineer of the Dredging Division, assumed a surface parallel to the surface existing on October 14, the date of the last complete survey, and 45 feet below it, on which basis 13,000,000 cubic yards would be the quantity to be handled. He thinks, however, that a mean between the two amounts may be more nearly correct, which was the method of arriving at the 10,000,000 cubic yard figure which has appeared in the press. It is at best only a guess. It must not be inferred from this that the Canal will be closed until this amount is dredged, for such is not the case; on the contrary it is the intention to pass ships as soon as the channel is secured through the remaining 600 feet, and there are reasonable grounds for assuming that a channel through the obstructed area can be maintained.

The active West Culebra Slide extends 2900 feet along the Canal, and 1350 feet at right angles to it measured from the axis of the Canal to the furthest point of the slide.

Of the slides now quiescent, the most important is the Cucaracha. General Goethals writes of it, in the article already mentioned:

> On January 20, 1913, a break occurred at Cucaracha by which the rock bluff which was holding back the upper mass of clay broke at or below the bottom level of the Canal, completely filling the prism with clay and rock, reaching to 69 feet above sea level on the opposite or west side of the cut. The length of the prism so filled was 1600 feet. Steam shovels were scarcely able to keep pace with the movement, tracks were covered and disarranged, shovels overturned, and the difficulties of transportation increased, since only tail tracks sufficient for two or three cars could be maintained. Furthermore, the soft material increased the difficulties of the dumps. As the movement continued the clay broke farther and farther up the hillside.

The Cucaracha Slide, as stated, extended 1600 feet along the Canal, and 1880 feet at right angles to it.

Dominant Importance of the Three Great Slides.—According to Mr. W. G. Comber, Resident Engineer in charge of Dredging, the acreage of the three great slides is as follows:

West Culebra Slide	60.8
East Culebra Slide	70.5
Cucaracha Slide	60.4
Total	201.7

In contrast with this, the total area of all other slides is 112 acres.

The dominance of the three great slides appears even more marked when the amounts of excavated and moving material are considered.

Mr. Comber's figures for excavation accomplished to December 30, 1915, are:

	cu. yds.
West Culebra Slide	10,931,862
East Culebra Slide	14,687,563
Cucaracha Slide	9,901,602
Total	35,521,027

In contrast with this amount, the total excavations for the other slides to the same date have been 4,852,648 cubic yards.

If the amounts of material still to be removed are compared, the dominance of the three great slides is maintained. It is estimated that, on December 30, 1915, there remained to be excavated:

	cu. yds.
West Culebra Slide	3,500,000
East Culebra Slide	5,600,000
Cucaracha Slide	500,000
Total	9,600,000

The total amount still to be removed from all other slides is estimated at the insignificant amount of 330,000 cubic yards. It thus appears that the amount of material already excavated and still to be removed from the three great slides is estimated at 45,121,027 cubic yards; whereas, the corresponding amount for all the other slides is 5,182,648 cubic yards, or about one-ninth as much.

The foregoing facts are conclusive as to the dominant importance of East Culebra, West Culebra and Cucaracha Slides.

Since the three great slides are all in the Culebra District, this is the chief area of danger. This is the natural consequence of the fact that by far the deepest part of the Canal cut is in this District; and that the weakest of the geological formations, the Cucaracha, is strongly developed here.

Slides may occur in other parts of the Canal, but they will be relatively small, and infrequent; for the banks are not high, and the unstable ground has already slid down so that its surface has approached the angle of repose. Should slides occur, they are not likely to menace the operation of the Canal. Slides, great enough seriously to obstruct traffic in the Canal, could occur only in the Culebra District, which is but one mile long. The possibility of great slides in this section has therefore claimed the most careful consideration of the Committee.

General Features of the Culebra District.—In order that the views of the Committee may be clearly conveyed, it is necessary briefly to mention and to illustrate by a map and photographs, the more important features of the Culebra District.

The Canal here traverses the highest land in its course. On the east side, rising abruptly from the Canal, is Gold Hill. This is the highest hill near the Canal; it reaches a height of 660 feet above sea level, or 620 feet above the bottom of the Canal. It is composed mainly of hard intrusive basalt and hard tuff nearly surrounded by basalt. It separates the Cucaracha slide on the south from the East Culebra slide on the north. These slides have caused great breaks in the north and south flanks of Gold Hill, leaving nearly vertical cliffs, which are 275 feet high on the south flank. On the opposite western side of the Canal there are three prominent hills, Culebra, Zion, and Contractors Hills, in order from south to north. Their elevations are:

	Above Sea Level	*Above bottom of Canal*
Contractors Hill	415	375
Zion Hill	570	530
Culebra Hill	390	350

Zion Hill is of intrusive basalt, the other two mainly of hard tuff with some basalt. Contractors Hill nearly touches the waters of the Canal opposite Cucaracha Slide. Zion and Culebra Hills stand about 1500 and 1200 feet, respectively, from the Canal, and in front of them lies the West Culebra Slide. The slide has caused breaks in both of these hills. In the eastern front of Culebra Hill a road was carried down and a number of houses had to be removed; cracks roughly parallel to the Canal have formed for a distance of about 100 feet beyond the prominent break which seems to be the western limit of the slide at the present time, and extend to within 250 feet of the summit of the hill. Zion Hill also has suffered; a large mass has fallen from its eastern face leaving a vertical cliff, whose edge is only about 50 feet from the top of the hill. Contractors Hill has not been affected by the slides.

The great slides occur in the low ground adjacent to these hills where a large amount of softer rocks had already been removed by natural erosion before the excavation of the Canal was begun.

Relations of the Great Slides to the Hills.—The Cucaracha Slide is mainly confined to the area between Gold Hill and a subordinate basalt mass to the south. It extends from the Canal for a considerable distance east of the crest of Gold Hill; and its head reaches the subordinate divide to the east. It is estimated that the Cucaracha Slide drains an area of 80 acres. The slide is sharply limited on the north by the break in Gold Hill already mentioned, which extends approximately at right angles to the Canal; its southern limit is not so well defined.

The limit of the East Culebra Slide is sharply marked on the south by the break through the north part of Gold Hill which runs approximately at right angles to the Canal. The Northern boundary is not so sharply marked. The slide extends slightly beyond the subordinate divide on the east, so that east of the slide the drainage is away from the Canal.

The West Culebra Slide is limited on the south by breaks beginning at the Canal some distance north of Contractors Hill; on the west by breaks which are sharply defined in Zion Hill; and less sharply in Culebra Hill. In the narrow valleys between Culebra and Zion Hills and between Zion and Contractors Hills the break extends beyond the divide, and the drainage is to the west.

Possible Extensions of the Great Slides.—The very important question now arises: Will the great slides extend their limits and cause further serious trouble?

In general, the Committee believes that no great extension of these slides is probable, because the soft rock constituting a very large part of the slides is quite limited in extent, except east of the East Culebra Slide, and conditions elsewhere are unfavorable for extensions.

The Cucaracha Slide cannot greatly extend its area on account of the basaltic intrusions which surround it; but its eastern and southern limiting banks are still breaking down, and the movement of the slide may be revived to a small degree. Plugs or branches of intrusive basalt standing as obstructions across the former course of the slide restrain its movement; but the strength of these obstructions cannot be determined from present exposures.

The West Culebra Slide is pretty definitely limited on the west by the hard rocks of Culebra and Zion Hills; but between the active part of the slide and Contractors Hill there is a considerable mass of the Cucaracha formation, which seems never to have taken part in the slides. The effect on this mass of the settling of the adjacent moving material cannot certainly be predicted. Indeed it is not impossible that a considerable part of it may finally be set in motion; but the mass involved will be small in comparison with the active West Culebra Slide.

East of East Culebra Slide the soft formation continues, but the slope is gently away from the Canal. Additions to the slide to the east are possible, but because of the slope and increased distance from the Canal such possible additions would be in decreasing volume. Gold Hill limits the slide to the south.

THE SOLIDITY OF THE HILLS OF CULEBRA DISTRICT

In addition to the danger of the slides, is there danger that the Canal may be blocked by the fall of the hills of Culebra District?

These hills are composed of intrusive bodies of basalt, or of masses of the hard Obispo tuff commonly associated with basalt in this district. So far as can be judged from present exposures, they do not rest on the soft Cucaracha formation, but extend far into the earth, and are self-supporting. Rock may slough off from them, but there is no evidence that they will collapse.

Culebra and Zion Hills.—The hard tuff of Culebra Hill practically limits the West Culebra Slide in front of it. Cracks have formed in the tuff and it is probable that some of the rock will break off as the slide settles. Zion Hill is a basaltic intrusion, and much rock has fallen from its face; more may follow. But the total amount that may fall will only make a relatively small addition to the upper part of the West Culebra Slide.

Gold and Contractors Hills.—Gold and Contractors Hills rise steeply from the banks of the Canal, for 410 feet and 260 feet above the bottom of the Canal, respectively; and then slope more gently to their summits. They are nearly, but not exactly, opposite each other. Gold Hill is chiefly composed of basalt, which formerly spread out near its top, and was partially supported on the softer Cucaracha formation. When the East Culebra and Cucaracha slides became active the support was removed, and a large mass of the basalt fell from the northern and southern sides of the hill. The lower part of Gold Hill on the side towards the Canal is made up of hard Obispo tuff, bounded by a basalt dike, and there is little danger that it will yield.

Contractors Hill is of hard Obispo tuff, which is separated from the Cucaracha by a fault which dips into the hill at an angle of 60 or 70 degrees with the horizontal. There is a possibility that this part of the hill depends more for its support on the Cucaracha beds than seems probable, and as a measure of precaution all reasonable means should be taken to keep the Cucaracha beds in place; and, especially, the fault fissure should be kept closed to prevent water seeping in. If the borings, suggested later, to reveal the underground structure, show that these precautions are unnecessary they can be discontinued.

The excavation of the Canal, and borings in its bottom show that a narrow belt between the two hills is composed of the soft Cucaracha beds; yet to the present time there has been no upheaval of the bottom of this part of the Canal, nor any other sign to indicate that the hills have settled. It is believed, therefore, that the great masses of Gold and Contractors Hills are self-supporting and will remain so. There is no occasion to raze them.

CAUSES OF THE SLIDES

On account of their magnitude, the land slides have received serious consideration since the early days of the Canal. But before measures for their control are taken up it is necessary briefly to discuss their causes.

The slides in the Canal Zone are essentially like many in other parts of the world; they are due to the inability of the earth or rock to support the weight of overlying material. Slow processes of natural erosion, rapid cutting by flooded streams and excavations by man frequently lead to landslides. Much of the Canal is cut through weak rocks; and in the Culebra District the prism is exceptionally deep. It is clear that the conditions there are very favorable for slides.

The weakness of the rocks is due to several causes:

Character of the Rocks of the Culebra District.—The rocks of Culebra District are of two kinds—stratified and massive. The chief material involved in the slides is the stratified Cucaracha formation. It is greenish grey in color, largely composed of clayey material with some layers of rather finely banded volcanic sandstone or tuff, only weakly consolidated. The Cucaracha beds are limited, along the line of the Canal, to the Culebra District but they have a thickness in places of over 400 feet. The soft slippery nature of its materials and their loose, unconsolidated condition, make it unusually weak and unable to sustain any considerable load.

The Cucaracha beds alone are responsible for the great slides. The Obispo tuff is a rather coarsely fragmental rock, roughly stratified; masses of the tuff and of the massive columnar basalt have broken from the hills and added some material to the slides, but they have had no part in starting them.

Structural Weakness.—The rocks, both stratified and massive, as shown by the work of Mr. MacDonald, are cut by numerous faults and this is true of the rocks throughout the Gaillard Cut. Where there is a fault, the rocks have previously been broken; and therefore, present a place of exceptional weakness. Some of the smaller slides outside the Culebra District are limited by faults.

An important source of weakness is the fracturing of the rocks by complex sets of joints. They occur in both stratified and massive rocks. The friable parts of the Cucaracha formation are broken by joints into fragments of various sizes down to a fraction of an inch. The massive rocks are fractured in every direction by joints on a much larger scale.

Earthquakes.—A consideration of the earthquakes felt in the Canal Zone and a careful examination of the instrumental records kept near the Pacific end of the Canal since the end of 1908 dispel fear of serious damage to the Canal or its accessories by earthquakes. A number of pretty sharp shocks have been felt in the Zone but they have originated at a distance of about 120 miles from the Zone. A few fairly sharp shocks had their origin about 80 miles distant; and two or three, which were not felt, and which made a very feeble record on the delicate seismographs, were much nearer. At the time of the very sharp shocks of October, 1913, a prism of earth 60 feet high and with a base of about 100 square feet broke off the steep slope near the top of the Cucaracha Slide; this is quite insignificant. There is no evidence that any of the slides have been started or increased by earthquakes.

The Heavy Rainfall.—Investigations in many parts of the world have shown that excessive water in the ground is a great promoter of landslides. The strength of the Cucaracha clays is greatly decreased by the presence of water, and the heavy tropical rains keep them nearly saturated the greater part of the year; for the broken and irregular surface of the slides and the open cracks around their borders greedily drink in the water that falls upon them.

In the Culebra District the average rainfall since 1884 has been 87.68 inches per annum, and during the time of the excavation of the Canal it has been 84.75 inches per annum. Moreover this rainfall is almost wholly concentrated in eight months of the year. The average for the rainy months during the period of excavation of the Canal by the United States, from the middle of April to the middle of December has been 80.01 inches. Where nearly seven feet of water fall upon the surface of the country within eight months of the year, it is not surprising that there is difficulty in controlling the underground seepage. Indeed during these months the ground water table is practically at the surface, except possibly on the steeper slopes; and the movement of the underground water is so slow that even in the dry season the water table is not far from the surface in the level country. The New French Company found that the water table at the two points of the East Culebra

Slide was six and sixteen feet below the surface respectively. On the summit of Gold Hill, the ground water falls 40 feet below the surface in the dry season, as indicated by the zone of weathering.

REMEDIAL MEASURES

All slide material which reaches the Canal must, of course, be removed at a certain expense and inconvenience. This work is being prosecuted by the engineers with great vigor. They estimate that about 9,000,000 cubic yards will have to be removed between the two Culebra Slides; and the dredges can remove 1,000,000 cubic yards a month. But it must not be supposed that the Canal will remain closed for nine months; as soon as the channel has been sufficiently deepened and the movement of the slides becomes so slow that the dredges, even when interrupted by the passage of ships, can more than keep pace with them the Canal may be opened for navigation. This time is probably not far off.

The Committee believes that some sliding ground will continue to enter the Canal for several years to come, though in diminishing amounts. Any relatively inexpensive measures which tend to arrest the present active slides, or which promise to reduce the charge against maintenance of the Canal in the future are fully warranted.

The composition of the rocks, their structural weakness, and earthquakes are beyond the control of man, but a partial control of ground and rain water and the relief of pressure by unloading certain areas are feasible, and the Committee will confine its suggestions to these measures.

Control of the Water.—As early as the time of the first French Company the advantage of controlling the water was recognized; and the New French Company made several attempts to keep the water out of the relatively small slides of their time by surface drains and tunnels, but with only partial success. The Committee believes that every available and practicable device should be used to turn the water falling as rain from all ground that is sliding and prevent its entering adjacent ground, and it suggests the following measures:

1. Covering Slopes with Vegetation.—Whether vegetation increases or decreases the amount of rainfall entering the ground is still a moot question. The Committee believes, however, that threatening ground bordering the slides, quiescent slides, and, so far as practicable, active slides themselves, should be sufficiently covered with vegetation to prevent surface wash.

2. Closing Peripheral Cracks.—Before extensive movements of the

ground occur, warning is frequently given by the appearance of cracks peripheral to the coming slide. In some instances cracks exist for a long time before important movements take place. They are well illustrated upon Culebra Hill, west of the great break. As soon as they are formed they should be filled up, in order that they may not intercept surface water and lead it into the slide.

3. Drainage of Undisturbed and Threatened Areas.–Undisturbed and threatened areas near the slides should be thoroughly drained both by surface and by tile drainage, to keep as much water out of them as possible, for they may become unstable and they may supply water to the slides. The drainage water should be carried from the neighborhood of the slides as directly as practicable. Experiments with tile drainage on a small scale would soon show whether it is effective enough to justify its extension. The surfaces of the ground east of the East Culebra Slide and in places west of the West Culebra Slide, slope away from the adjoining slides, and the water can be readily removed. But a considerable area above the Cucaracha Slide drains naturally into it; some of the drainage can be diverted to the east, but the rest should, so far as possible, be collected by surface and tile drains, and be carried to the Canal through a large concrete-lined surface drain.

4. Drainage of the Great Slides.—A complete system of open drains should be established on the great slides and the water carried away as directly as possible. The main drains should be made impervious on the Cucaracha and, so far as practicable, on the two Culebra Slides.

5. Drainage by Tunnels.—Drainage by means of tunnels might be adapted to a few special cases, but should be tried with caution, and extended only in cases which promise results commensurate with the cost. The tunnels should be built underneath the sliding ground in the undisturbed material and strongly timbered so as to avoid risk of collapse, which would not only destroy the tunnel but would also disturb the overlying material. From the main tunnel smaller branches may be extended into the material to be drained and frequent borings made from the surface to conduct drainage water to the tunnels from the overlying strata. Such a system would be expensive, but might be effective in draining the area tapped by it.

Relief of Pressure.—It is suggested that a cut be made by sluicing in the East Culebra Slide, starting at an appropriate point on the Canal, and diverting from it diagonally in a general southeasterly direction, in the zone of gentle slopes and in such a position as to reach the large pond which now exists on the slide. This cut would partly separate the roughly rectangular slide into two approximately triangular parts, and the adjoining ground could be sluiced along it into the Canal. It would provide a main drainage line for the ground on both sides of it, would empty the surface ponds and would lower the ground water in adjacent sliding material. The pressure of the triangle of ground in the rear of the cut against the triangle in front of it would be lessened; and it is probable that movement in the forward part of the slide would cease sooner than it would otherwise. When the cut is once established it offers a second line of defense against the slide by making it possible to work along two fronts.

A somewhat similar cut should be made in the West Culebra Slide. It should begin at the main drainage line opposite Culebra Hill and extend diagonally from the Canal in a general southerly direction, and should drain the existing pond.

STUDIES FOR THE FUTURE

In addition to the immediate remedial measures suggested above, there are certain observations and protective measures which should be undertaken with a view to the future.

Detection of Movements of Earth or Rock.—A few slides are now in motion, but many more are quiescent. A slide is not necessarily 'dead' because it has not moved notably for a few years. Its stability may hang on a delicate balance which may be disturbed by some slowly developing weakness. There is also a bare possibility that Gold and Contractors Hills are not so firmly supported as they are believed to be. Repeated surveys of properly placed signals for a number of years to come should be made in order:

(a) To indicate in good time where additional work of prevention is needed, to indicate how large a dredging fleet must be kept in readiness, and to furnish to the engineer of maintenance advance indications of emergency conditions.

(b) To test the effectiveness of remedial measures which may be taken to control or prevent slides. These tests would be more sensitive and definite than the mere occurrence or non-occurrence of slides.

(c) To furnish a reliable basis for confidence when, in the course of time, it appears that the earth and rock movements have so decreased that a condition of stability may be inferred.

Core Borings.—Definite information relative to the rock underlying Gold and Contractors Hills is wanting. Many years ago some borings were made at stations 500 feet apart along the center line of the Canal, some of them to a depth of 40 feet below sea level.

Two or three core borings should be made on each side of the Canal with a drill which would give large cores. At least one horizontal and one inclined hole should be bored on each side, their precise location to be fixed by a geologist.

Underground Water and Related Data.—As underground water is of paramount importance in promoting slides, it should be carefully studied in the Canal Zone. A satisfactory investigation of this problem would require the following determination and studies:

1. Profiles of water table for different localities of the Culebra District:
    - (a) In the wet and dry seasons;
    - (b) In areas in which remedial measures have been applied and similar areas where they have not.
2. Percentage of porosity of the several formations of the Culebra District:
    - (a) Absolute;
    - (b) With regard to size of grain.
3. The effect of tropical vegetation with the accompanying humus, its removal and its restoration, on:
    - (a) The amount of water which sinks underground in sliding and in undisturbed areas;
    - (b) Chemical action, such as oxidation, hydration, carbonation, etc.;
    - (c) The composition of the water;
    - (d) Changes in underground temperatures.
4. The nature of the changes which result in the disintegration and decomposition of the various rocks of the Culebra District when exposed to weathering agencies:
    - (a) With regard to volume;
    - (b) With regard to chemical changes, i.e., oxidation, hydration, carbonation, action of acids, etc.
5. The chemical changes which cause the so-called hot areas described by Colonel Gaillard and Mr. MacDonald:
    - (a) Direct, in production of acids, etc.;
    - (b) Indirect, i.e., the effect of produced acids upon the materials, including water and rock.
6. Observations to determine whether similar changes to those in the hot areas take place elsewhere to a less extent over large areas, and whether such changes affect the slides.

Mechanical Testing of the Rocks.—Sliding is largely dependent upon the strength of the rocks; which in turn is affected by the water content. The strength of the massive igneous rocks is well known to be great, and need not be determined. But the Obispo, Cucaracha, and other formations of the Gaillard Cut should have their strength tested when saturated with water, when moist and when dry; and under rapid and slow deformation. If the tests show that much less force is required to deform the rocks when saturated or moist than when dry, this will emphasize the importance of keeping the water from these rocks so far as possible.

The tests should be made on fresh rocks and therefore in the Canal Zone. The specimens tested should be as large as is feasible for a testing machine of 200,000 pounds capacity.

Earthquake Studies.—There are now two seismographs installed in the Administration Building at Balboa Heights. It would be an advantage if the smaller instrument should be removed to a second station, for instance Colon, in order that the origin of earthquakes, occurring in regions within two or three hundred miles of the Canal Zone, may be more definitely determined. Some of the stronger shocks felt in the Zone have thrown the needles of the delicate seismographs off the paper and left the records incomplete. A low power instrument, magnifying about four times would secure a record of the movements of the ground in these cases.

GENERAL CONCLUSIONS

It is obvious that the sliding material which enters the Canal must be removed. The important thing for the future is to prevent ground from entering the Canal. The chief remedy proposed by the Committee to retard the movement of the slides now in motion and to prevent the slides from extending their areas, is to reduce the amount of water which goes underground. Methods have been suggested by which this can be done; and they should be vigorously applied to all moving and threatening areas.

The Committee looks to the future of the Canal with confidence. It is not unmindful of the labor necessary to deal with the present slides; and it realizes that slides may be a considerable, but not an unreasonably large, maintenance charge upon the Canal for a number of years; it also realizes that trouble in the Culebra District may possibly again close the Canal. Nevertheless, the Committee firmly believes that, after the present difficulties have been overcome, navigation through the Canal is not likely again to be seriously interrupted. There is absolutely no justification for the statement that traffic will be repeatedly interrupted during long periods for years to come. The Canal will serve the great purpose for which it was constructed, and the realization of that purpose in the near future is assured.

DISCOVERY OF EIGHT VARIABLE STELLAR SPECTRA

By Harlow Shapley

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, February 23, 1916

In three earlier communications to the PROCEEDINGS various phases of the problem of Cepheid variation have been considered.[1] No further introduction to the present note is necessary beyond the statement that the periodic change in color, which appears to be typical of all Cepheids, has already been found to correspond to normal changes of spectral class for two cluster-type stars, RS Boötis and RR Lyrae, and for one variable of longer period, δ Cephei. In order to test to what extent the inconstancy of spectrum is a general phenomenon of Cepheids, some 150 spectrograms of representative variables of this class have been made with the 10-inch portrait lens and objective prism. The periods of the stars investigated range from 9 hours to 27 days; many are those for which spectroscopic orbits have been computed; some are well-known naked eye variables, others are much fainter stars; for some, changes in color have been suspected from studies of the light curves, for others the maximum intensity of the spectrum has been observed to shift toward the blue upon the approach to maximum light. For none, however, has it been suggested, so far as I know, that the spectrum changes periodically along the normal spectral series.

Following the numerical method of classification recently described by Adams,[2] the change of spectrum of Cepheids is susceptible of easy detection, even when the variation is very small. For example, in the work with low dispersion on F-type spectra the relative intensity of the *G* band and the hydrogen line $H\gamma$ is particularly capable of showing small changes of type. The results can be summarized briefly.

The table contains data relative to the spectra of eleven stars (all that are now known to have variable spectra), and some information concerning their light fluctuations. The observed range of spectrum variation is in nearly every case smaller than the probable total range, as observations at the exact time of maximum and minimum were not made. The accompanying figure contains two objective-prism spectra of δ Cephei, showing the conspicuous change, from minimum to maximum light, in the relative intensity of the spectral lines. The more detailed appearance with high dispersion of a

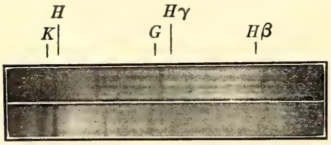


FIG 1. SPECTRA OF δ CEPHEI NEAR MAXIMUM (ABOVE) AND MINIMUM OF LIGHT (enlarged five times).

Eleven Cepheids with Variable Spectra

STAR	MAXIMUM MAGNITUDE	RANGE OF VARIATION	PERIOD	NUMBER OF SPECTROGRAMS	OBSERVED SPECTRUM VARIATION
		mag.	*days*		
TU Cassiopeiae....	7.3	1.1	2.139	9	F0 to F6
SU Cassiopeiae....	5.9	0.4	1.950	19	A8 to F5
SZ Tauri..........	7.2	0.5	3.148	11	F4 to G2
T Monocerotis...	6.0	0.8	27.012	6	F4 to F8
RT Aurigae........	5.0	0.9	3.728	12	A8 to G0
W Geminorum....	6.4	1.3	7.916	10	F3 to G0
RS Boötis.........	8.9	1.1	0.377	13	B8 to F0
X Sagittarii......	4.4	0.6	7.012	5	F2 to G
Y Ophiuchi......	6.2	0.8	17.121	4	F5 to G0
RR Lyrae.........	6.8	0.9	0.567	17	B9 to F2
δ Cephei.........	3.5	0.8	5.366	21	F2 to G3

smaller section of the spectrum has been shown in an earlier communication by Adams and Shapley.

Every variable for which the present test is sufficient was found to vary in spectrum. It appears safe to infer, therefore, that all Cepheids (including the cluster-type), besides being variable in light and in velocity, vary periodically in spectral class as well.

---

[1] Shapley and Shapley, these PROCEEDINGS, **1**, 452 (1915); Shapley, *Ibid.*, **2**, 132 (1916) Adams and Shapley, *Ibid.*, **2**, 136 (1916).

[2] These PROCEEDINGS, **2**, 143 (1916).

ON THE LINEAR DEPENDENCE OF FUNCTIONS OF SEVERAL VARIABLES, AND CERTAIN COMPLETELY INTEGRABLE SYSTEMS OF PARTIAL DIFFERENTIAL EQUATIONS

By Gabriel M. Green

DEPARTMENT OF MATHEMATICS, HARVARD UNIVERSITY

Received by the Academy, February 25, 1916

The study of an ordinary homogeneous linear differential equation of the nth order leads very naturally to the definition of the Wronskian of n solutions of the equation, and thence to the general theory of the linear dependence of n functions of a single variable. This is due to the characteristic property of the said differential equation, viz., that any solution of the equation is linearly dependent upon any fundamental set of solutions. I wish in this note to give some of the results which I have obtained in generalizing the theory of linear dependence to the case of n functions of several independent variables, and also to point out the application of these results to the study of an important class of systems of partial differential equations which are a direct generalization of the single ordinary homogenous linear differential equation of the nth order. A system of the kind referred to contains a single dependent variable and any number of independent variables, and has a fundamental system of solutions, that is a definite number of linearly independent solutions in terms of which any other solutions of the system of differential equations is expressible linearly, with constant coefficients.

The following discussion is concerned throughout with functions of p independent variables. If any of the variables be complex, we shall suppose the functions to be analytic in those variables. However, we shall state all theorems for the case in which the independent variables are real, and the functions either real or complex; the modifications which must be made if some or all of the variables be complex are easily supplied, and will need no further mention. We shall impose upon the functions no restriction other than the existence of certain partial derivatives in a certain connected p-dimensional region A of the space of the independent variables.

Let $y_1, y_2, \ldots, y_n$ be functions of the p independent variables $u_1, u_2, \ldots, u_p$. We shall denote by $y_i^{(1)}, y_i^{(2)}$, etc., partial derivatives of y_i, of any kind or order whatever. It will be unnecessary to specify just what derivative of y_i is denoted by $y_i^{(j)}$. However, in any given discussion the same superscript (j) will denote the same derivative throughout. If a derivative $y^{(j)}$ exists for each one of the set of functions y_i $(i = 1, 2, \ldots, n)$, we shall say that *the set of functions possesses that derivative.* We may now state the fundamental theorem concerning the linear dependence of functions of several variables:

THEOREM I. *Let the set of n functions $y_1, y_2, \ldots, y_n$ of the p independent variables $u_1, u_2, \ldots, u_p$ possess enough partial derivatives, of any orders whatever, to form a matrix.*

$$M \equiv \left\| \begin{matrix} y_1 & , & y_2 & , & \ldots & , & y_n \\ y_1^{(1)} & , & y_2^{(1)} & , & \ldots & , & y_n^{(1)} \\ y_1^{(2)} & , & y_2^{(2)} & , & \ldots & , & y_n^{(2)} \\ \cdot & & \cdot & & \cdot & & \cdot \\ \cdot & & \cdot & & \cdot & & \cdot \\ y_1^{(n-2)} & , & y_2^{(n-2)} & , & \ldots & , & y_n^{(n-2)} \end{matrix} \right\|$$

of n–1 rows and n columns, in which at least one of the $(n$–$1)$-rowed determinants, say

$$W_n \equiv \begin{vmatrix} y_1 & , y_2 & , \cdots, & y_{n-1} \\ y_1^{(1)} & , y_2^{(1)} & , \cdots, & y_{n-1}^{(1)} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ y_1^{(n-2)}, & y_2^{(n-2)}, & \cdots, & y_{n-1}^{(n-2)} \end{vmatrix},$$

vanishes nowhere in A. Suppose, further, that all of the first derivatives of each of the elements of the above matrix M exist, and adjoin to the matrix M such of these derivatives as do not already appear in M, to form the new matrix

$$M' \equiv \begin{Vmatrix} y_1 & , y_2 & , \cdots, & y_n \\ y_1^{(1)} & , y_2^{(1)} & , \cdots, & y_n^{(1)} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ y_1^{(n-2)}, & y_2^{(n-2)}, & \cdots, & y_n^{(n-2)} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ y_1^{(q)} & , y_2^{(q)} & , \cdots, & y_n^{(q)} \end{Vmatrix}$$

which has n columns and at least n rows, so that $q \geqq n-1$. *Then if all the n-rowed determinants of the matrix M′ in which the determinant* W_n *is a first minor vanish identically in A, the functions* $y_1, y_2, \ldots, y_n$ *are linearly dependent in A, and in fact*

$$y_n = c_1y_1 + c_2y_2 + \ldots + c_{n-1}y_{n-1},$$

the c's being constants.

The proof of this theorem is very similar to the familiar one of Frobenius for functions of a single variable [Cf. M. Bôcher, *Trans. Amer. Math. Soc.*, **2**, 139–149 (1901)]. For $p = 1$, the theorem becomes a generalization of the ordinary Wronskian theorem for functions of a single variable, and includes the latter theorem as a special case.

It should be noted that for functions of several variables it is not possible to define a single determinant which may properly be called a Wronskian; however, a Wronskian may be defined for a completely integrable system of partial differential equations, of the kind mentioned above. Before giving this definition it will be convenient to state an existence theorem for the system of partial differential equations.

Let us call a set of derivatives $y, y^{(1)}, y^{(2)}, \ldots, y^{(n-1)}$ of a function y a *normal set*, if for every element $y^{(r)}$ of the set there exists in the set at least one other element $y^{(q)}$, whose order is one less than the order of $y^{(r)}$, and from which $y^{(r)}$ may be obtained by a single differentiation. The existence theorem referred to may be stated as follows:

THEOREM II. *Suppose that in the system of partial differential equations*

$$\frac{\partial y^{(j)}}{\partial u_k} = \sum_{i=0}^{n-1} a_i^{(j,k)} y^{(i)}, \qquad (j = 0, 1, \ldots, n-1;\ k = 1, 2, \ldots, p;\ y^{(0)} \equiv y)$$

the derivatives $y, y^{(1)}, y^{(2)}, \ldots, y^{(n-1)}$ *form a normal set. Suppose further that in the closed region A the coefficients* $a_i^{(j,k)}$, *which are functions of the p independent variables,* $u_1, u_2, \ldots, u_p$, *satisfy identically the integrability conditions*

$$\frac{\partial a_\nu^{(j,k)}}{\partial u_l} + \sum_{i=0}^{n-1} a_i^{(j,k)} a_\nu^{(i,l)} = \frac{\partial a_\nu^{(j,l)}}{\partial u_k} + \sum_{i=0}^{n-1} a_i^{(j,l)} a_\nu^{(i,k)},$$

$$(\nu, j = 0, 1, \ldots, n-1;\ k, l = 1, 2, \ldots, p).$$

Let $(u_1^{(0)}, u_2^{(0)}, \ldots, u_p^{(0)})$ *be any point of A, and* $y_0, y_0^{(1)}, \ldots, y_0^{(n-1)}$ *be any set of n constants. Then there exists one and only one function y of the variables* $u_1, u_2, \ldots, u_p$ *which satisfies the system of differential equations, and whose derivatives* $y, y^{(1)}, \ldots, y^{(n-1)}$ *take on respectively the preassigned constant values* $y_0, y_0^{(1)}, \ldots, y_0^{(n-1)}$ *at the point* $(u_1^{(0)}, u_2^{(0)}, \ldots, u_p^{(0)})$.

From this theorem may be inferred at once the existence of a fundamental system of n solutions, $y_1, y_2, \ldots, y_n$, such that any other solution of the system of differential equations has the form

$$y = c_1 y_1 + c_2 y_2 + \ldots + c_n y_n.$$

Moreover, any function of this form is a solution of the completely integrable system; this system is therefore a natural generalization of the ordinary homogeneous linear differential equation of the nth order.

The derivatives $y, y^{(1)}, \ldots, y^{(n-1)}$ which appear in the right-hand members of the differential equations we shall call the *primary derivatives*. We may now define the *Wronskian* of n solutions of the completely integrable system, as the determinant formed from the primary derivatives of these solutions:

$$W \equiv \begin{vmatrix} y_1 & , y_2 & , \ldots, & y_n \\ y_1^{(1)} & , y_2^{(1)} & , \ldots, & y_n^{(1)} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ y_1^{(n-1)} & , y_2^{(n-1)} & , \ldots, & y_n^{(n-1)} \end{vmatrix}.$$

The Wronskian just defined has properties similar to those met with in the theory of an ordinary differential equation. Thus, it may be shown without difficulty that

$$\frac{\partial W}{\partial u_k} = W \sum_{j=0}^{n-1} a_j^{(j,k)}, \qquad (k = 1, 2, \ldots, p)$$

so that we may determine by a quadrature a function f such that

$$\frac{\partial f}{\partial u_k} = \sum_{j=0}^{n-1} a_j^{(j,k)}, \qquad (k = 1, 2, \ldots, p)$$

Therefore, *the Wronskian W of n solutions of the completely integrable system may be determined by a quadrature from the coefficients of the system, and is given by the expression*

$$W = \text{const. } e^f.$$

This is a generalization of the theorem of Abel for an ordinary homogeneous linear differential equation of the nth order.

We shall state one more theorem, the analogue of a familiar one concerning an ordinary differential equation. The completely integrable systems to which it applies are of somewhat less generality than those for which the existence theorem has been given.

THEOREM III. *Suppose the completely integrable system considered in Theorem II has in addition the following properties:*

1°. *The set of primary derivatives is such that, if $y^{(j)}$ be any one of the set, then all the derivatives of lower order from which $y^{(j)}$ may be obtained by differentiation also belong to the set.*

2°. *All the first derivatives of the primary derivatives exist for each of the np coefficients $a_j^{(j,k)}$ $(j = 0, 1, \ldots, n-1;\ k = 1, 2, \ldots, p)$.*

Then the system of differential equations may be transformed in but one way into a system

$$\frac{\partial \bar{y}^{(j)}}{\partial u_k} = \sum_{i=0}^{n-1} \bar{a}_i^{(j,k)} \bar{y}^{(i)} \qquad (j = 0, 1, \ldots, n-1;\ k = 1, 2, \ldots, p)$$

for which all of the quantities

$$\frac{\partial \bar{f}}{\partial u_k} \equiv \sum_{j=0}^{n-1} \bar{a}_j^{(j,k)}, \qquad (k = 1, 2, \ldots, p)$$

are zero, by the transformation of the dependent variable $y = \lambda y$, where

$$\lambda = \text{const. } e^{f/n}.$$

This last theorem is of interest in the method developed in recent years by Wilczynski for dealing with questions in projective differential geometry. In fact, the coefficients of the transformed system of differential equations are what he has generally called seminvariants of the original system; the theorem affords a means for calculating these seminvariants in a purely mechanical way. In Wilczynski's method, the geometric problem becomes the study of a completely integrable system of the kind we have been considering.

The results outlined above have been developed at length in a memoir which is to appear in the *Transactions of the American Mathematical Society.*

SYSTEMATIC MOTION AMONG STARS OF THE HELIUM TYPE

By Benjamin Boss
DUDLEY OBSERVATORY, ALBANY, N. Y.

Received by the Academy, March 1, 1916

Several investigators using different methods and different material have shown beyond a doubt that the stars evidence a preference for motion in two opposite directions in the sky. This does not mean that all stars move in one of the two directions, but that there is a stronger tendency for stars to move in the favored directions than in any other.

In such investigations the helium, or B-type, stars have presented considerable difficulties, since their motions are small, and since as a class they are situated at a great distance from the sun. It has seemed desirable, therefore, to devise a method whereby the preference of motion among the helium stars might be determined with some degree of confidence.

In the first place the zone in which all the helium stars lie was mapped off into twelve arbitrary divisions. For each division means were taken of the amount of proper motion in the two co-ordinates right ascension and declination, and these mean values were then subtracted from each proper motion. Thus the center of the velocity-figure was obtained. The rectangular co-ordinates were converted into polar co-ordinates and arranged in the order of their position-angles from the north pole. Then for thirty degree groups, 0°–30°, 10°–40°, etc., means were taken of the position-angles, and sums of the amount of proper motion. With the mean position-angles as abscissae and the sums of proper motion as ordinates, the results were plotted and smooth curves drawn to represent them. Figure 1 shows how well the observations can be fitted by mooth curves. It will also be noted that there is more than one maximum, though in the cases of the curves 5 and 10 only one maximum is real. That is, in the two cases cited, a slightly different grouping of the data would produce great changes in the curves with the exception of one maximum which is real.

Fig. 1

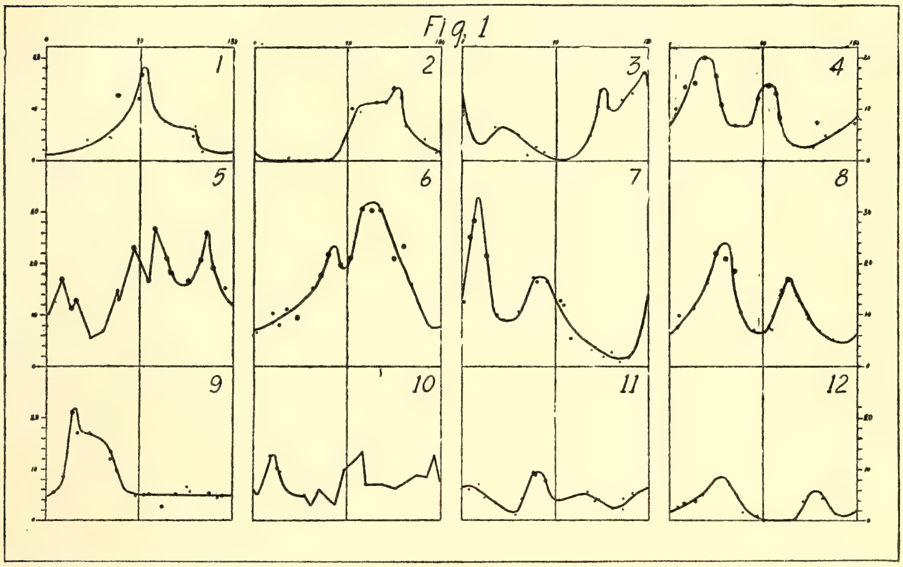


Fig. 2

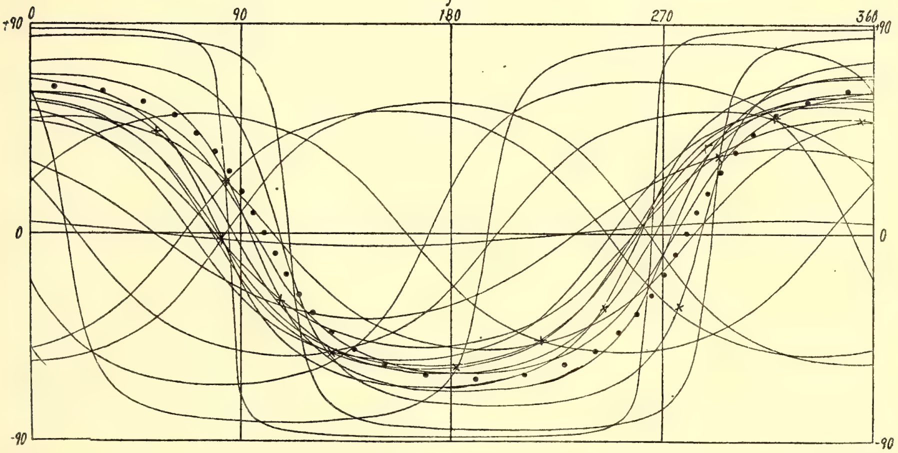


Figure 2 gives a graphic representation of the results. The crosses mark the positions of the centers of the areas treated. From these centers great circles were drawn in the direction obtained by reading off the position-angles of the maxima from the curves of figure 1. The figure then shows the paths along which there is a strong tendency for stars of the helium type to move. The black dots trace the path of the Milky Way. The paths sharply define a plane which is that of the stars used instead of the plane of the Milky Way, but in addition there are considerable tendencies of motion which carry the helium stars into other regions. In some cases this tendency seems to be the greater of the two. In the care of area 4 there is no tendency for a preference for motion in the Milky Way.

To summarize the conclusions drawn from the investigation of the systematic motions of the helium stars, there appears to be a strong tendency for these stars to move in their own plane, which should therefore be preserved, at least until the next step in the star's evolution. As a matter of fact the A-type stars, supposedly representing the next stage in evolution, exhibit a strong tendency to crowd toward this plane. But there are likewise strong tendencies for the stars of helium type to depart from the plane, so that the tendency for the stars to spread in every direction, so clearly manifest in advanced stages in the evolution of a star, has its birth in the helium stage of evolution. There is apparently nothing systematic in the motions directed away from the plane of the stars.

THE ABUNDANCE OF THE ELEMENTS IN RELATION TO THE HYDROGEN-HELIUM STRUCTURE OF THE ATOMS

By William D. Harkins

KENT CHEMICAL LABORATORY, UNIVERSITY OF CHICAGO

Received by the Academy, February 26, 1916

According to the theory already presented in a number of papers[1] the atoms of all the 91 elements of our ordinary system heavier than hydrogen are built up as intra-atomic (not chemical) compounds of hydrogen. The first of these 91 elements, helium, is the second in the system, and therefore has the atomic number 2. It has an atomic weight of 4.00, and may be considered to be composed of 4 hydrogen atoms. The element of atomic number 3, lithium, has an atomic weight of about 7. Now it has been found that in general among the elements of low atomic weight, the elements of even atomic number, beginning with helium, seem to be built up from helium atoms, and therefore may be said to have the general formula $n\mathrm{He}'$, where the prime is added to indicate that these elements are intra-atomic, not chemical, compounds. The odd numbered elements, beginning with lithium, seem in general to have the formula $n\mathrm{He}'+\mathrm{H}_3'$. Thus the elements seem to fall into two series which may be called the *even* and the *odd* series, or the helium and the lithium series, if each series is named for its first member. However, it should be noted that while the formula for the helium series is $n\mathrm{He}'$, that for the lithium series is not $n\mathrm{Li}'$.

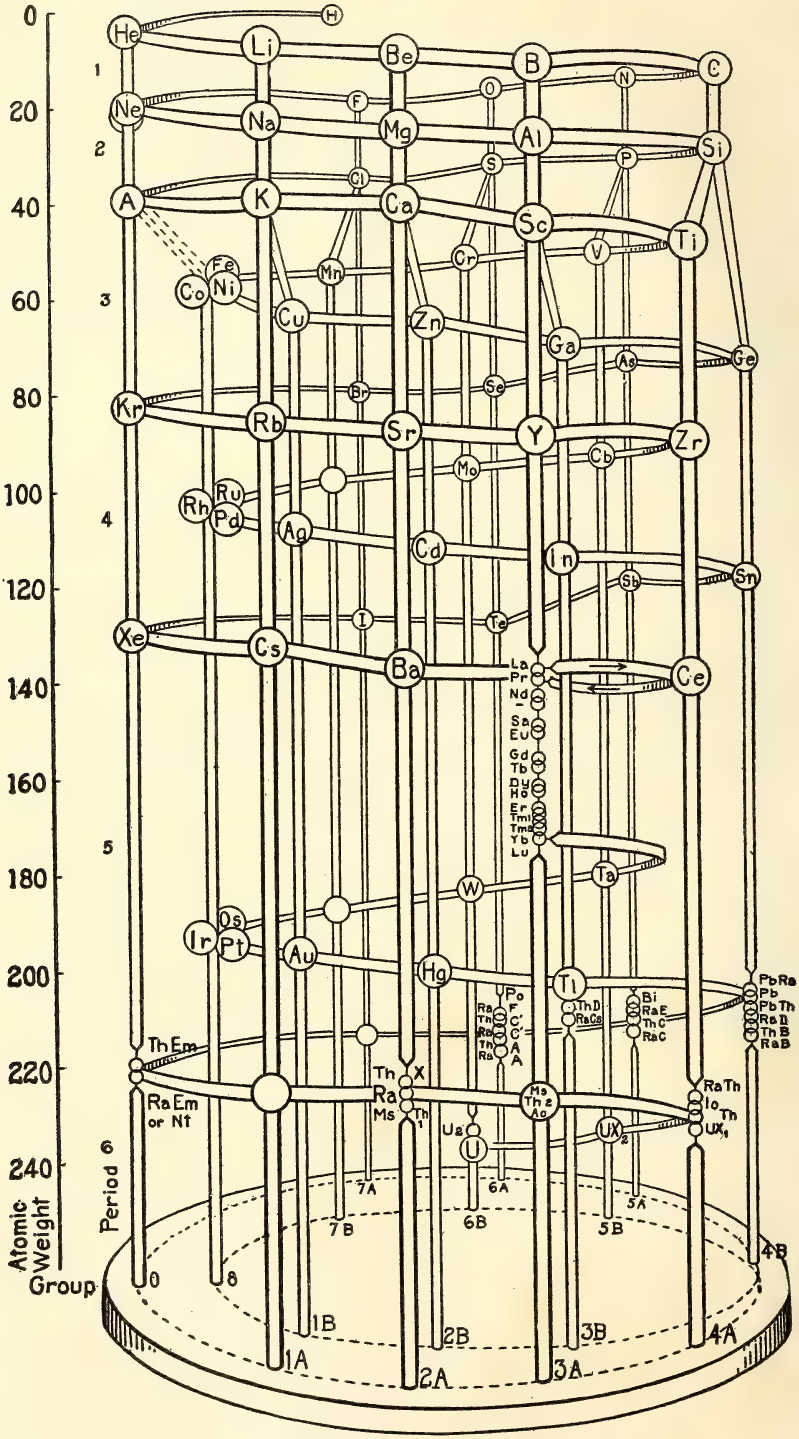
If the theory is correct it might be expected that some characteristic of the elements could be found, with respect to which there is a difference from odd to even and from even to odd, or in other words the elements should show variations in periods of 2 elements each.

In order to have a basis for the comparison of the elements in the study of this problem there has been constructed in space a periodic model of which the accompanying figure is a drawing. In this model the elements are represented by balls strung on a spiral in the order of their atomic numbers, which have recently been found to be much more characteristic of the elements than their atomic weights. The spiral is so arranged that the balls representing the elements belonging to one group and having the same maximum valence are strung on the same vertical rod. The balls are set at such heights that the vertical distance from the top down represents the atomic weight. This is essential, for otherwise the different kinds of atoms of one element, called by Soddy 'isotopes' cannot be represented. Thus in the lower right hand part of the table, on the lower part of Group 4B, the element lead is represented by 6 isotopes, with the atomic weights listed, as follows: lead from radium (uranio-Pb) 206.1; lead, 207.2; lead from thorium, 208.1; radium D, 210.1; thorium B, 212.1, and radium B, 214.1. Thus the different kinds of lead, which seem identical chemically and give the same spectrum, have atomic weights which differ by as much as 8 units, or by 4%. However, all of these isotopes have the same atomic number, 82, or according to the theory developed by various investigators, they have the same positive nuclear charge.

When arranged in this form of periodic table the elements other than hydrogen and helium are found to arrange themselves in periods as follows:

1. First short period	Li — Ne: $8 = 2 \times 2^2$ elements	Cycle $1 = 4^2$
2. Second short period	Na — A : $8 = 2 \times 2^2$ elements	
3. First long period	K — Kr: $18 = 2 \times 3^2$ elements	Cycle $2 = 6^2$
4. Second long period	Rb — Xe: $18 = 2 \times 3^2$ elements	
5. First very long period	Cs — Nt: $32 = 2 \times 4^2$ elements	Cycle $3 = 8^2$
6. Second very long period	Eka — Cs: —— (incomplete)	

It is thus seen that these periods and cycles make up a numerical system of a remarkably simple form, and it seems evident that this system


0
20
40
60
80
100
120
140
160
180
200
220
240
Atomic Weight
Group
Period
1
2
3
4
5
6
H
He
Li
Be
B
C
N
O
F
Ne
Na
Mg
Al
Si
P
S
Cl
A
K
Ca
Sc
Ti
V
Cr
Mn
Fe
Co
Ni
Cu
Zn
Ga
Ge
As
Se
Br
Kr
Rb
Sr
Y
Zr
Cb
Mo
Ru
Rh
Pd
Ag
Cd
In
Sn
Sb
Te
I
Xe
Cs
Ba
La
Pr
Nd
Sa
Eu
Gd
Tb
Dy
Ho
Er
Tm1
Tm2
Yb
Lu
Ce
Ta
W
Os
Ir
Pt
Au
Hg
Tl
Po
Ra F
Th C'
Ra C'
Th A
Ra A
Th D
Ra C2
Bi
Ra E
Th C
Ra C
Pb Ra
Pb
Pb Th
Ra D
Th B
Ra B
Th Em
Ra Em or Nt
Th X
Ra
Ms Th1
Ms Th 2 Ac
U2
U
UX2
Ra Th
Io
Th
UX1
0
8
1A
1B
2A
2B
3A
3B
4A
4B
5A
5B
6A
6B
7A
7B

Periodic Table by W. D. Harkins

must express something inherent in the structure of the atoms. However, what it is desired to emphasize here is that nearly all of the physical properties of the elements vary in periods which are either the same or nearly the same as these. The chemical properties also vary in rather long periods, which in the case of the short periods 1 and 2, are identical with those given.

From this it is seen that both the chemical and the physical properties of the elements vary in periods which are long in comparison with the change in periods of 2 elements as indicated by the division of the elements into the odd and even series. If now neither the physical nor the chemical properties vary according to these extremely short periods, what, it may be asked, is left which can so vary?

Now it might easily be shown that the hydrogen-helium system of the structure of the elements, which divides them into the odd and even series, is in reality more directly applicable to the structure of the nuclei of the atom than to the atom as a whole. If then the Rutherford theory that the nuclei of atoms are extremely minute, is used as a basis for reasoning, it would be expected that the variations in the structure of the nuclei should not cause variations in the properties of the elements except in so far as they influence the nuclear charge. This nuclear charge has been assumed to be equal to the atomic number, and therefore rises with perfect regularity from odd to even or from even to odd. It seems probable that the number of electrons external to the nucleus is equal to the nuclear charge, and that it is the change in their number and arrangement which causes the physical properties to vary according to the periods listed above. This question has been discussed in a previous paper.[2]

It might be expected, however, that the composition of the nucleus should affect its own stability, which from radioactive evidence means the stability of the atom. From this standpoint it might be reasonable to suppose that the atoms of one of the series, the even or the odd, should be more stable than those of the other. Now unfortunately there is no known method of testing the stability of the lighter atoms, but it might seem, at least at first thought, that the more stable atoms should be the more abundantly formed, and to a certain extent this is undoubtedly true. If then, at the stage of evolution represented by the solar system, or by the earth, it is found that the even numbered elements are more abundant than the odd, as seems to be the case, then it might be assumed that the even numbered elements are on the whole the more stable. However, there is at least one other factor than stability which must be considered in this connection. The formula of the even numbered elements has been shown to be $n\mathrm{He}'$, which may be written $n(4\mathrm{H}')'$. Now, since the formula for the odd numbered elements is $n\mathrm{He}'+\mathrm{H}'_3$, or $n(4\mathrm{H}')+\mathrm{H}_3'$, it is evident that, if the supply of H_3 needed by the elements was relatively small at the time of their formation, not so much material would go into this system. This would be true whether the H_3 represents three atoms of hydrogen or one atom of some other element. With regard to the latter alternative, it is at least remarkable that the H_3 occurs 11 times in the system for the first 27 elements, while H_2 and H each occur only once, and it may also be mentioned that Fabry and Buisson[3] have by interference methods determined the atomic weight of nebulium to be 2.7, and this they think indicates that its real atomic weight is 3. Also, Campbell[4] has found that in the nebula N. G. C.[4] Index 418, situated in the southern part of the constellation of Orion, the nebulium spectrum is found farther from the interior than that of helium, while the hydrogen spectrum extends out to a much greater distance still. This, he thinks, indicates that the atomic weight of nebulium lies between the values for hydrogen (1) and helium (4).

In studying the relative abundance of the elements the ideal method would be to sample one or more solar systems at the desired stage of evolution, and to make a quantitative analysis for all of the 92 elements of the ordinary system. Since this is impossible, even in case of the earth, it might be considered that sufficiently good data could be obtained from the earth's crust, or the lithosphere. However, it seems probable that the meteorites represent more accurately the average composition of material at the stage of evolution corresponding to the earth than does the very limited part of the earth's material to which we have access. At least it might seem proper to assume that the meteorites would not exhibit any special fondness for the even numbered elements in comparison with the odd, or vice versa, any more than the earth or the sun as a whole, at least not unless there is an important difference between these two systems of elements, which is just what it is desired to prove.

A preliminary study of the most recent analyses of meteorites of different classes showed that, either for any one class or for the meteorites as a whole *the even numbered or helium system elements are very much more abundant than those of the odd numbered or lithium system.* For a more detailed study use was made of the data collected by Farrington,[5] who suggests that the average composition of meteorites may represent the composition of the earth as a whole.

The results obtained by averaging the analyses of 318 iron and 125 stone meteorites, 443 in all, show that the first seven elements in order of abundance are iron, nickel, silicon, magnesium, sulphur, and calcium;

and not only do all of these elements have even atomic numbers, but in addition they make up 98.6% of the material of the meteorites. Of the remaining elements present to a great enough extent to have an appreciable effect upon the percentage values, 7 are odd and 5 are even, but in all only 1.22% are odd numbered, while 98.78% are even.

Of the iron meteorites 99.22% of the material is made up of even numbered elements, and of the stone meteorites, 97.50%. While the results for the earth's lithosphere are not so striking, they still show the same general tendency very strongly; for, of the six most abundant elements, only aluminum is odd numbered, and the elements of even atomic number make up about 86% of the material. The only odd numbered elements other than hydrogen present in the lithosphere in amounts over 0.2% are aluminum, sodium, and potassium.

Table 2 gives the average composition of iron and stone meteorites, arranged according to the periodic system. The numbers before the symbols represent the atomic numbers, and the numbers underneath give the percentage of the element. It will be noted that *the even numbered elements are in every case more abundant than the adjacent odd numbered elements.* The helium group elements form no chemical compounds, and are all gases, so they could probably not remain in large quantities in meteorites. For this reason, and also because the data are not available, the helium or zero group is omitted from the table. The only criticism which could be made of the system of averaging, which is that of including all accurate analyses, is that it places undue emphasis upon the iron as compared with the stone meteorites. However, since the two relations shown in Table 1 are true for each class of meteorites separately it is evident that they will be true whatever system of averaging may be chosen.

TABLE 1

AVERAGE COMPOSITION OF METEORITES ARRANGED ACCORDING TO THE PERIODIC SYSTEM

SERIES	GROUP ODD 1	GROUP 2 EVEN	GROUP 3 ODD	GROUP 4 EVEN	GROUP 5 ODD	GROUP 6 EVEN	GROUP 7 ODD	GROUP 8		
								Even	Odd	Even
2				6C 0.04%		8O 10.10				
3	11Na 0.17%	12Mg 3.80%	13Al 0.39%	14Si 5.20%	15P 0.14%	16S 0.49%				
4	19K 0.04%	20Ca 0.46		22Ti 0.01		24Cr 0.09	25Mn 0.03	26Fe 72.06	27Co 0.44	28Ni 6.50
	29Cu 0.01									

If attention is now turned to the heavier elements as shown in the model, it is seen that the five unknown elements eka-caesium, eka-manganese 1, eka-manganese 2 (dwi-manganese), eka-iodine, and eka-neodymium, have *odd* atomic numbers. (There is some doubt as to the discovery of thulium 2.) Not only are the unknown elements odd numbered, but among the radio-active elements, if the most stable isotope of each element is used for the comparison, the *odd* numbered elements are much less stable than the adjacent elements of even number.

If we consider the rare-earths—the elements which are most similar chemically, while at the same time their atomic numbers change in steps of one—the same result is obtained. In the following table, which includes, besides the rare-earths a number of elements adjacent to them, the letter *c* indicates common in comparison with the other elements in the table, and *r* indicates rare. *cc* represents very common, etc. The comparison is only a rough one, but it is sufficiently accurate for the purpose for it indicates that in every case the even numbered element is more abundant than the adjacent odd numbered element.

TABLE 2

ATOMIC NUMBER	ABUNDANCE	ELEMENT	ATOMIC NUMBER	ABUNDANCE	ELEMENT
55	c	Caesium	63	rr	Europium
56	ccc	Barium	64	r	Gadolimium
57	c	Lanthanum	65	rrr	Terbium
58	cc	Cerium	66	r	Dysprosium
59	r	Praseodymium	67	rrr	Holmium
60	c	Neodymium	68	r	Erbium
61	rrr	Unknown	69	rr	Thulium
62	c	Samarium			

The above results may be summarized by the statement that in the evolution of the elements much more material has gone into the even numbered elements than into those which are odd, either because the odd numbered elements are less stable, or because some constituent essential to their formation was not sufficiently abundant, or both.

It is easy to see too that most of the material has been used up in the formation of the lighter elements. Table 2 shows that in the meteorites the most abundant elements are oxygen in series 2, the elements of series 3 except neon, and the members of the first eighth group triad (iron, cobalt, nickel). Clarke[6] has found that just these same elements are the most abundant in the lithosphere, although in the lithosphere potassium and calcium in series 4 are also moderately abundant. If the lithosphere were considered alone it might be considered that the abundance of these elements is due to changes which have taken place in the lithosphere, or to the rising to the surface of the lighter elements, but these objections are not so valid when the meteorites are found to show the same relations. The density of the earth's surface rock averages between 2.70 and 2.75, the mean density of the earth is 5.516, and the density of its center has been estimated by Lunn[7] as 9.6 on the basis of Roche's law of density, and on the supposition that the chemical composition of the earth is uniform. Stone meteorites vary in density from 2.5 to 5, and iron meteorites from about 6 to more than 8, with an average density of 7.8. According to Lunn[7] the pressure at the center of the earth is 2,800,000 atmospheres, and a possible central temperature is 16,610° when both are calculated on the basis of Roche's law, $\rho = \rho_0 (1-cx^2)$. It seems probable that this law is much more in accord with the behavior of material than the simple Laplacian form usually used. Some writers have argued from the data that the center of the earth is mostly iron. However the extremely long range of extrapolation above the experimental values in both temperature and pressure, makes it seem impossible to get results in this connection which have the least value, however desirable it would be for such a problem as the one presented here if such a deduction could be properly made. Perhaps, then, the most that can be said is that in the three classes of material, the lithosphere, the stone meteorites, and the iron meteorites, in spite of variations in density from 2.5 to 8, the same two rules are found to hold, that (1) the even numbered elements, and (2) the elements of low atomic number and low atomic weight, are the elements which occur in abundance.

If an artificial line of division is made just after the first eighth group in the periodic model so as to classify the first 29 elements as of low atomic number and atomic weight, and the remaining 63 elements as of high atomic weight, then the following table, based upon data from analyses listed by Farrington and Clarke may be presented to emphasize the importance of the former class.

TABLE 3

Proportion in Various Materials of the Elements of Low Atomic Numbers

Material	Percentage of Elements with Atomic Numbers 1–29	30–92
Meteorites as a whole	99.99	0.01
Stone Meteorites	99.98	0.02
Iron Meteorites	100.00	0.0
Igneous Rocks	99.85	0.15
Shale	99.95	0.05
Sandstone	99.95	0.05
Lithosphere	99.85	0.15

It may be said that, so far as the abundance of the elements goes, the system seems to play out at the end of the first eighth group in the periodic system. It may be of interest to note here, what has been pointed out in former papers, that it is just at this point in the system that the atomic weights cease any longer to be very close to whole numbers, as they are for the lighter weight elements. Also just at this point the exact formula given for the elements ceases to hold well. These facts do not mean however, that the system fails beyond the iron group of elements; for it is just among the heaviest elements that it received its verification by the actual decomposition of the elements into helium.

The complete paper of which this is an abstract, will be published later. I wish to thank Prof. C. W. Balke of the University of Illinois for suggestions in regard to the relative abundance of the elements of the rare earth group.

[1] Harkins and Wilson, These PROCEEDINGS, **1**, 276 (1915); *J. Amer. Chem. Soc.*, **37**, 1367–1421 (1915); *Phil. Mag.*, **30**, 723 (1915).

[2] Harkins and Hall, *J. Amer. Chem. Soc.*, **38**, 186–8, 203–5, 211–14 (1916).

[3] H. Buisson, Ch. Fabry, and H. Bourget, *Astrophys. J.*, **40**, 256 (1914). See also Dempster, *Ann. Physik.*, **47**, 792 (1915).

[4] Wright, These PROCEEDINGS, **1**, 590–5 (1915).

[5] Farrington, Publications 120 and 151, Field Columbian Museum, Chicago.

[6] Clarke, *The Data of Geochemistry*, Bulletin 491, Department of the Interior (1911); see also Bulletin 616 (1915).

[7] Lunn, in 'Tidal and Other Problems,' pp. 201–18, Carnegie Institution (1909).

THE GENETIC RELATIONS OF CERTAIN FORMS IN AMERICAN ABORIGINAL ART

By Clark Wissler

AMERICAN MUSEUM OF NATURAL HISTORY, NEW YORK CITY

Received by the Academy, March 2, 1915

One of the most difficult problems in anthropology has been the working out of successive steps in the origins of particular traits of culture. The most intensive effort seems to have been made in studies on the evolution of decorative designs. By arranging designs found upon prehistoric or other pottery in order of their increasing conventionality, series have resulted, showing a clearly realistic drawing at one end and an almost entirely geometrical one at the other. Such series suggest that all these forms were arrived at by first drawing from real life and then by successive conventionalizations arriving at a pure geometric form. The weak point in this interpretation is that there are no means of dating the units of the series, their arrangement being merely a matter of selection on the part of the observer. There are still other obvious objections to the interpretation, so that the tendency of the critical is to reject these conclusions. Somewhat analogous attempts have been made in the study of industrial arts and technology, but with equally unconvincing results. Consequently, as the case stands today, we can point to scarcely a single example in which the life history of a trait can be satisfactorily demonstrated in objective data.

In the course of some technological studies in the American Museum of Natural History, the writer observed that certain structural styles of skin clothing among the Indian tribes of the Mississippi and Great Lakes areas were clearly due to the forms of the original materials. The skins of deer were used and practically always removed from the animal in the same way. Whole skins were then combined to form a garment, their natural outlines being preserved, but usually trimmed to a symmetrical form. That the cut, or style, thus resulting became a recognized feature in the native mind is shown by its survival after cloth was introduced by traders. Here the form of the material did not lend itself to the style but nevertheless was cut to conform to it. Hence, we have a case in which the evidence for the genesis of a trait is satisfactory.

Next our attention was turned to the decorations upon these costumes. Here it can be shown that these same stylistic lines, determined by the contours of the material, were followed in the embroidered decorations, resulting in peculiar curved designs. Thus on the old specimens in our collections, the design follows the cut of the skin material, but upon the modern ones it is repeated upon an even unbroken surface. So without going into details we may state that satisfactory proof can be given to show that this particular design rose from the decoration following the contour of a part of the garment.

The investigation was then extended to moccasin decoration in the same geographical area. In this case satisfactory evidence can be found for the same kind of genetic relation between three different styles of decoration and as many different types of structure. For instance, north of the Great Lakes, for a considerable distance, east and west, the moccasin is made by folding a piece of skin up over the foot and joining on the top and at the heel. The shape of the foot prevents its covering the entire instep, leaving a U-shaped space. This is closed by an insert. The decoration is placed upon this insert, for which there are good technological reasons, and so has in its entirety a U-shape. Then among some of the neighbors of these tribes, particularly the Blackfoot, we find a similar decoration upon a mocassin of an entirely different pattern and one in which it bears no necessary relation to the structure. Yet these Indians are consistent enough to make a false insert upon which the decoration is placed. In short, there are very strong reasons for concluding that the Blackfoot borrowed the decoration from their northern neighbors and that these tribes arrived at it by adjusting the decoration to the structure.

Another cut of moccasin among some tribes south of the Great Lakes requires no insert on the instep, but has a long unsightly puckered seam, extending down the middle of the foot to the toe. This is usually concealed by overlaying with a long narrow band of embroidered skin. This style extends over into the tribes of the Plains to the west where we find it upon moccasins of a pattern having no seams to hide.

Again, the Apache of New Mexico and Arizona have a moccasin with a long narrow insert reaching down the top of the foot to the toe. This gives two converging seams which are concealed by fringes and very narrow embroidered bands. Then among their northern neighbors we find moccasins without any insert whatever bearing exactly the same decoration.

Thus, in moccasin decoration we find three different examples of decorative designs developed from the structure.

We may summarize this investigation as revealing several good examples of the genesis of specific decorative designs. With one possible exception, they differ from the previous genetic studies of design in that the origin was not strictly in attempts at realistic art but merely grew out of attempts to embellish surfaces of fixed contour and to conceal unsightly lines.

The data in full will appear in the *Anthropological Papers of the American Museum of Natural History.*

THE SITUATION IN REGARD TO ROWLAND'S PRELIMINARY TABLE OF SOLAR SPECTRUM WAVE-LENGTHS

By Charles E. St. John

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, March 6, 1916

The appearance of the *Preliminary Table of Solar Wave-lengths* marked an epoch in the history of spectroscopy and for twenty years it has been an instrument of precision in the hands of the solar and stellar investigator, to such a degree that it has become an integral part of the literature of spectroscopy. In the transition to the international system, all that is of permanent value in this work of great magnitude and importance should be kept and it becomes necessary to discriminate between that which is possible to retain and that which must pass.

Several attempts have been made to determine in some general way the systematic differences between the Rowland wave-lengths and later determinations based upon the international system. Fabry and Perot[1] plotted the ratio $(\lambda R)/(\lambda$ international). To reduce the Rowland wave-lengths to the new standard, they proposed to divide the Rowland values by the corresponding ratios read from the curve. Hartmann[2] suggested the reverse procedure, namely, that wave-lengths expressed in the new system be multiplied by these ratios, reducing them to the approximate Rowland values and thus minimizing the break with the historical standard. Instead of the ratios suggested by Fabry and Perot, Albrecht in a recent paper[3] uses as ordinates the quantities, Rowland minus international, and from the mean curve derives directly the systematic difference for any line.

In many present-day discussions of solar problems, the third decimal place, when wave-lengths are expressed in angstroms, is drawn upon for the requisite data. One purpose of the present communication is to direct attention to some observations which show that a general transformation from one system to the other is a matter of the greatest difficulty if the required degree of precision is to be reached, even though the relative wave-lengths in each system were free from error, and that statistical comparison between different systems is a procedure fraught with the possibilities of introducing residuals that may be quite misleading.

For several years direct comparison between the lines of the iron arc and the solar spectrum have been in progress at this Observatory, and recently more precise determinations of the wave-lengths of iron lines dependent upon arc conditions have been made. The sun-arc comparisons show that for the lines of the same pressure group the displacements between sun and arc are, among other relations, a function of the line intensity.[4] The differences, then, between the Rowland and the international wave-lengths must be unequal even for lines in the same group and spectral region. In Table I are shown the data for two strong lines situated between weaker lines of the same group. The larger sun-arc displacements paralleling the larger Rowland-International differences for the strong lines are typical and the sun-arc data accord with the mean from a long series of determinations.

Similar systematic differences appear when lines of different groups, but in the same region, are considered, as shown in Tables II and III.[5]

TABLE I

Iron Lines of the Same Group and Spectral Region but Differing in Solar Intensity

Group	*λ Rowland*	*Intensity*	*λ International*	*R−I*	*Sun−Arc*
b	4337.216	5	.052	0.164	+0.004
b	4352.908	4	.740	0.168	+0.005
b	4383.720	15	.548	0.172	+0.012
b	4404.927	10	.753	0.174	+0.010
b	4427.482	5	.314	0.168	+0.003
b	4443.365	3	.198	0.167	+0.003
Mean		12		0.173	+0.011
Mean		4		0.167	+0.004
Intensity 12—Intensity 4				+0.006	+0.007

TABLE II

Iron Lines of Group a and d in the Same Region

Group	*λ Rowland*	*International*	*R−I*	*Sun−Arc*
d	5340.121	.945	0.176	−0.003
a	5341.213	.029	0.184	+0.009
d	5393.375	.178	0.197	−0.006
a	5397.344	.135	0.209	+0.008
Mean Group a			0.196	+0.008
Mean Group d			0.186	0.004
Mean Group a—Group d			+0.010	+0.012

TABLE III

Iron Lines of Groups a and e in Same Region

Group	*λ Rowland*	*International*	*R−I*	*Sun−Arc*
e	5407.357	.136	0.221	+0.015
a	5405.989	.780	0.209	+0.006
e	5411.124	.904	0.220	+0.016
e	5415.416	.195	0.221	+0.026
e	5424.290	.064	0.226	+0.026
a	5429.911	.702	0.209	+0.007
a	5434.740	.529	0.211	+0.009
Mean Group a			0.210	+0.007
Mean Group e			0.222	+0.021
Mean Group e—Group a			+0.012	+0.014

It is evident from the data given that no factor of transformation nor any curve can yield true differences between the Rowland and international wave-lengths for all lines even for a limited spectral region.

A comparison has recently been made between the separations of close pairs as given in the Rowland tables and as measured upon plates of higher dispersion than that used by Rowland. For pairs consisting of lines of intensities 3 and 4, whose separations are 0.0 to 0.1 A, 0.1 to 0.2 A, and 0.2 to 0.35 A, the Rowland values exceed those found at

Mount Wilson by 0.011 A, 0.007 A, and 0.004 A, respectively. These systematic differences furnish new ground for not adopting any form of operator for transforming the wave-lengths in Rowland's Preliminary Table to the international standard, and indicate some possible pitfalls incident to statistical methods of comparison.

Though the Rowland wave-lengths cannot be transformed so as to reproduce rigorously the wave-lengths expressed in the international system, it does not follow that the *Preliminary Table of Solar Wave-lengths* has outlived its usefulness. It will be some years before complete tables of solar wave-lengths, based upon the new standards, will be available, and even then the Rowland table will serve indefinitely as a reference for other things than wave-lengths. It has been assumed that the accidental errors in it are considerably greater than ±0.01 A, but recent comparisons between certain lines in Rowland's table and their wave-lengths as measured here upon plates of a very high dispersion, 1 mm. = 0.3 angstrom, using as standards neighboring free-standing lines, show that this is an over-estimate. For 54 lines in pairs with separations between 0.25 and 0.50 angstrom, the mean variation from Rowland's values is ±0.003 angstrom. As more than 200 lines were used as standards, it appears that, for the types of lines involved, the accidental errors in the Rowland wave-lengths are much less than 0.01 angstrom. It is probable, as Frost and Adams remark,[6] that errors of this magnitude occur but rarely and mainly then for lines whose measurement is inherently difficult, such as the very weak and the strong shaded lines. When it was first noticed that the differences between the Rowland wave-lengths in the Preliminary Table and those found by the newer arc determinations were not constant over even a short spectral region, it appears to have been assumed that the large variations were due to errors in the Rowland values, but data similar to those reported in Tables I, II, and III show that the apparent discrepancies represent real differences in the behavior of lines and tend to increase rather than destroy confidence in the accuracy of the relative wave-lengths of lines not presenting special difficulties of measurement and sufficiently separated from others to be within the power of spectrographs in common use.

[1] Ch. Fabry and A. Perot, *Astroph. J.*, **15**, 272 (1902).

[2] J. Hartmann, *Astroph. J.*, **18**, 167 (1908).

[3] Sebastian Albrecht, *Astroph. J.*, **41**, 333 (1915).

[4] Charles E. St. John, *Mt. Wilson Contr.*, No. 93, p. 35; *Astroph. J.*, **41**, 63 (1915).

[5] Charles E. St. John and L. W. Ware, *Mt. Wilson Contr.*, No. 61, pp. 31–32; *Astroph. J.*, **36**, 45–46 (1912).

[6] E. B. Frost and W. S. Adams, *Publications of Yerkes Observatory*, vol. 2, p. 155.

CHANGES IN THE FORM OF THE NEBULA N. G. C. 2261

By Edwin P. Hubble

YERKES OBSERVATORY, UNIVERSITY OF CHICAGO

Received by the Academy, March 9, 1916

A comparison of photographs has established changes in structural detail in the nebula N.G.C. 2261 (h 399) R.A. 6^h 35^m, Dec. $+8°50'$. This nebula is the finest example of a cometary nebula in the northern skies. The nucleus is known as the variable star R Monocerotis, said to range from magnitudes 9 to 13.5, with an irregular period. Lassell states that the nucleus is not a star, and Professor Barnard confirms this opinion from observations with the forty-inch refractor.

Plates obtained by me with the twenty-four inch reflector during the last six months were compared in the Blink-Mikroskop with an excellent plate taken with the same telescope in March, 1908, by Mr. F. C. Jordan. In the interval, the following edge of the nebula has bulged out to a greater convexity; a bright portion of the nebulosity just north of the nucleus has shifted about $5''$ toward the east; the north preceding part has moved toward the south following. In the center, however, is a sharply defined brighter wedge-shaped portion pointing to the east, which shows no motion.

In compliance with the request of the Director of this Observatory, Mme. Dorothea Roberts had the great kindness to prepare and send us both positive and negative copies of the plate of this nebula taken by the late Dr. Isaac Roberts at Starfield on January 27, 1900, and shown in *Knowledge*, vol. 24, p. 181. These amply confirm the reality of the phenomena and further establish that the motion is progressive both in direction and amount. The first impression is that the nebula is turning about its own axis after the manner of a top, and there is some indication of a helical motion toward the nucleus. The observed shifts seem to be rather of mass than of illumination and are independent of the variability of the nucleus.

Such changes are so novel that a question at once arose as to whether they might not be due to differences in exposure times or other photographic conditions rather than in the nebula itself. Fifteen plates obtained during the last six months under widely varying conditions of steadiness, transparency, and exposure time, agree perfectly in detail when compared among themselves. Of these, three were taken on the same night, with half, full, and double the normal exposure time, and show no differences other than the symmetrical building up of the image. Further, Director Frost and Professor Barnard have examined the plates and give me authority to say that in their opinion the changes are in the nebula itself.

The position of the nebula is in a dark lane in the sky, connected with the nebulosity around 15 Monocerotis. This seems significant in that the two nebulae in which variability has certainly been established both lie in dark regions. These are N.G.C. 1555, known as Hind's variable nebula (close to T Tauri), and N.G.C. 6729 in Corona Australis. In one other case, that of the planetary N.G.C. 7662, Professor Barnard has found that the nucleus varies through several magnitudes.

Preceding the nucleus (R Monocerotis) by $4\overset{s}{.}4$ and north 97″ is a star somewhat fainter than the 15th magnitude with a proper motion of the order of 30″ per century. North following 9′ and 10′, respectively, are two variable stars whose maxima are at about 15.5 mag.; and north preceding 17′ is still another variable with a range of at least from the 11th to the 17th magnitude; all of which add to the interest of this remarkable region of the sky.

The plates are being measured and a more detailed investigation of the data, with reproductions of the photographs, will be published in the *Astrophysical Journal.*

ON THE EFFECT OF REMOVAL OF THE PRONEPHROS OF THE AMPHIBIAN EMBRYO

By Ruth B. Howland

SWEET BRIAR COLLEGE, SWEET BRIAR, VA.

Received by the Academy, March 9, 1916

The following note embodies the results of a series of experiments, performed at Yale University at the suggestion of Dr. R. G. Harrison, on embryos of the spotted salamander, Amblystoma punctatum. The particular problems in mind were first, to determine whether the head kidney or pronephros is a functioning organ necessary to the life of the embryo, and second, to investigate the correlation of the development of this organ with that of other components of the excretory system.

The embryos used for the experiments were nearly of a uniform age, varying slightly in size and degree of development from the condition in which the first loop of the pronephric tubules appears as a slight, ventrally directed curve of the duct, to the stage in which the two funnels, together with the first loop, appear as a broadened Y. (Fig 1.) The tail-bud was clearly defined, and the pronephric swelling distinctly visible. In all cases, however, embryos were used before contraction of the body muscles began, for contraction not only hindered the operation, but often tore open the wound after successful removal of the kidney.

Two methods were employed in removing the pronephroi. In the first, three straight cuts were made, one beneath and one along each side of the pronephric prominence. The flat of skin thus defined was lifted up and the organ removed from below. In the second and more satisfactory method, a single incision was made dorsal to or immediately over the thickening, the needle inserted, and the tubule raised upward from the ventral side and excised.

Conditions ensuing upon the removal of the head kidney of both sides in Amblystoma larvae show clearly that these organs are necessary to the life of the embryo, although the presence of one pronephros suffices to keep the organism alive and in a healthy condition. All embryos from which both head kidneys had been excised died within eight to twelve days, evidencing during that interval symptoms of weakened heart-action—oedema and effusion into the pericardial and abdominal cavities—presumably brought about by uremic poisoning. Pricking the body wall to relieve the dropsical condition was resorted to in many cases from five to seven days after operating, but it proved ineffective.

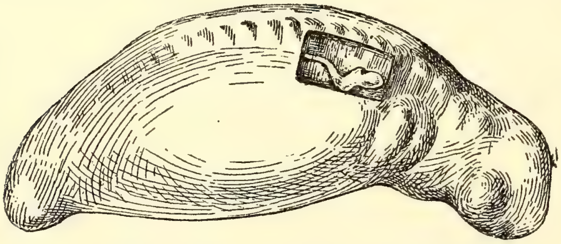


FIG. 1.—AMBLYSTOMA EMBRYO IN THE OPERATING STAGE, SHOWING THE PRONEPHROS EXPOSED AFTER REMOVAL OF ECTODERM. × 13.

The pronephros remaining after the removal of one head kidney evidently takes over the function of excretion usually performed by the two organs, and, concomitant with the increased physiological activity, presents marked morphological changes. The size of the organ which functions alone is greatly increased, indicating the occurrence of compensatory hypertrophy. In the normal pronephros, the walls of the tubules are thick, and consist of cuboidal cells, the central ends of which often bulge out into the narrow lumen. The hypertrophied tubules are thin walled, the cells flattened as is the case in normal larvae of greater age, and the lumen accordingly is nearly twice that of the unoperated specimen. To determine the nature and extent of the change brought about in the functioning pronephros through the removal of the organ on one side, wax models of this organ (× 200) of an operated specimen and of a normal embryo were constructed. The control was chosen from a large number of normal embryos which, on sectioning, showed the several organs (retina, lens of eye, digestive tract, etc.) to be in a stage of development identical with that in the operated individual. On comparison of the models, the evidence of hypertrophy shown in microscopic examination was strengthened. The model of the operated individual not only showed a considerable increase in length of the coiled tubule as contrasted with the control, but by its size indicated a great increase in volume. The length of the tubules making up the models was determined by taking the average of five measurements. The larger model measures 188 cm. (× 200), the smaller model 155 cm. (× 200), showing an increase of 21 per cent over the normal conditions. Difference in weight also indicates hypertrophy, the normal model weighing 62.9 gr. as contrasted with the larger, which has a weight of 115.9 gr.

The formation of the pronephric glomeruli under normal conditions as outgrowths from the aorta toward the newly forming kidneys suggested the possibility that in the operated specimens the glomeruli of one side might be lacking. The removal of one pronephros has, however, no effect on this development, for in all of the operated embryos both glomeruli were present. The glomerulus on the operated side is not as uniform in size and shape as the normally functioning one, its form evidently being largely determined by the width of the coelomic cavity in this region. The outer layer of the glomerular capillaries and the epithelial lining of the body wall often coalesce. Absence of the pronephros enlarges the cavity on that side, and the glomerulus is then found bridging this cavity as a long strand instead of appearing as a more compact tuft of capillaries.

In the removal of the embryonic pronephros, as much of the tissue was scraped out as seemed possible without disturbing the Anlage of the aorta. In spite of this fact, a large number of the operated embryos possessed well developed anterior and posterior nephrostomes and portions of the funnels, indicating that these structures are probably regenerated from the adjacent coelomic epithelium. Of the sixteen embryos examined, twelve showed well formed anterior funnels and nine had posterior funnels. In one instance the anterior funnel was of a peculiar double form, suggesting the normal condition in those amphibia with three pronephric openings.

The effect of excision of one pronephros on the formation of the segmental duct on the operated side varied widely. The process of development of the non-functioning ducts is carried on only to a limited extent. In the embryos examined, every gradation of development was found from a condition in which the lumen, though small and flattened dorsoventrally, appears throughout the entire length, to a condition where only the occasional presence of a few degenerating cells indicates the location of the atrophied duct.

Increased activity of a single kidney also has a definite effect on the segmental duct of that side. Cross sections of the duct of an individual with unilateral operation, when compared with either of the ducts of a normal larva of the same stage, show a marked increase in diameter.

The mesonephroi of both sides develop normally, at least in the early stages, even after excision of one pronephros. Non-development of the segmental duct in one instance left the mesonephric tubules disconnected on that side, and with no outlet for excretory products. No specimens have yet been kept a sufficient length of time to determine the ultimate outcome of this abnormal condition.

In brief then, the following conclusions may be drawn.

1. Removal of both pronephroi in Amblystoma larvae induces conditions leading to oedema and subsequent death, though the presence of one head kidney is sufficient to keep the embryo in a condition of health.
2. Excision of one head kidney brings about an increase in size in the remaining organ, and also in the diameter of the segmental duct on that side.
3. Removal of one pronephros has no essential effect on the development of the pronephric glomerulus of that side, but the segmental duct appears in varying stages of atrophy.
4. Anterior and posterior nephrostomes may regenerate from the coelomic epithelium.
5. Early developmental stages of the mesonephros are normal, even after excision of one head kidney.

ON THE PRESENCE OF A MEDIAN EYE IN TRILOBITES

By Rudolph Ruedemann

NEW YORK STATE MUSEUM, ALBANY

Received by the Academy, March 7, 1916

The entirely extinct sub-class Trilobita of the Crustaceans, comprising about 1840 species divided into some 185 genera, has always held a central place in the phylogenetic history of all classes of Crustacea. It is becoming *not only* more probable that the Ostracoda and Cirripedia were developed from the trilobites, but that also the insect subphylum has taken its origin from this ancient class of arthropods. Some authors derive the Xiphosura and Eurypterida and through them the scorpions and all later arachnidians from the trilobites, and others again, as **Patten,** trace the vertebrates back to the Xiphosura and Eurypterida. The phylogenetic importance of the trilobites, which were the dominant animal class in the Cambrian era, is therefore assured, and it is for this reason that observations of new structures in the trilobites are of greater interest than they might otherwise be.

An organ not heretofore recognized in the trilobites is the median or parietal eye on the glabella, and yet the question of its presence or absence in the trilobites is of considerable phylogenetic importance. Beecher, through whose observations we have learned so much of the ventral anatomy of the trilobites, would ally them closely with the Phyllopods (Branchiopoda) and Walcott, who has made us acquainted with an unexpected wealth of Cambrian trilobites and other crustaceans, would directly derive them from primitive Branchiopods correlated for convenience with Apus-like forms. On the other hand we find authors such as Kingsley, Bernard and Jaekel, who advance arguments to prove that the trilobites can be traced back directly to arthropods more primitive than any crustaceans, as for example, the annelids. Bernard would even not place them in the class Crustacea.

In these discussions the absence of median or parietal eyes in the trilobites has been emphasized as a feature distinguishing them from most primitive crustaceans, notably the Phyllopoda, as well as from the Merostomes. It is therefore of interest that the presence of such median eyes can be demonstrated in this group.

The median eye appears, in the majority of cases, as a single tubercle upon the glabella. This tubercle has so far been recognized in upward of thirty genera, among them all genera of the Asaphidae and representatives of most other families of the trilobites. In studying the structure of the tubercle it was found that the median eye presents all stages of development seen in other crustaceans, from mere transparent thinner spots of the test to a lenticular body covered by a thin cornea. The lenticular body is frequently recognized by the pit in the interior cast of the tubercle; it was seen in sections of *Cryptolithus* (*Trinucleus*) *tesselatus* from the Trenton limestone of New York. Here it appears as a glossy, pearly body in the interior cast of the eye tubercle when the thin cornea is removed. In sections it is seen to be composed of the same substance as the matrix, but bounded on the under side by a carbonaceous layer; and it is also observed that the test above the lenticular body is thinned to one-half its normal thickness, thus forming a thin cornea. From the absence of a separate crystalline structure in the lens, and the presence of the carbonaceous division line, we infer that there was no hard chitinous lens that would become a separate center of crystallization in fossilization, as in the lateral eyes of the trilobites, but one corresponding to that of the parietal eye of other crustaceans, and especially of the phyllopods, which is a lens- or pear-shaped sac, usually filled with water. The parietal eye of the trilobites is hence not at all comparable to the larval ocelli of insects or the parietal eye of *Limulus*, the eurypterids and arachnids in general, where the chitinous integument thickens into an exterior lens, but it agrees well in its structure with the median eye of the crustaceans; the thin black layer at the base of the lenticular body being derived from the pigment of the retina. Whether the lenticular cavity was filled with sea-water or a body fluid is not known, but there are indications that some trilobites may have possessed a pore on the apex of the tubercle, giving the sea-water access to the interior.

Indirect evidence for the visual function of the tubercle is seen in the following facts:

The eye tubercle is the sole prominence on the otherwise smooth glabella in the *Asaphidae*, *Trinucleidae*, etc. It must therefore have a function requiring prominence for its performance. It is always situated at the highest spot of the carapace, either on the apex of the bulging frontal lobe of the glabella as in *Cryptolithus* (*Trinucleus*), or where the glabella abruptly bends downward on the prominent posterior portion between the last glabellar lobes (*Isotelus* and *Asaphus*). It is, further, generally situated between the posterior portions of the lateral or compound eyes. This position is explained by the fact that the median or parietal eye according to its origin is always nearest to the brain and this is, in the phyllopods and most other crustaceans, situated in the dorsal region of the head, beneath or between the lateral eyes. As in the eurypterids, so also in the trilobites, the median eye is often found at the posterior extermity of a distinct crest, extending backward a short distance upon the glabella. This crest is probably an analogue of the 'eye-line' of the lateral eyes of the primitive trilobites, and at least in part, marks the path of the nerve leading to the median eye. As in the crustaceans and eurypterids, the median eye tubercle is relatively largest and most prominent in the earliest growth stages, and in the later stages it may entirely disappear, as in *Isotelus gigas*. It is likewise better developed in the more primitive orders of the trilobites; and the phylogenetically late families of the highest order, the Proparia, viz., the *Calymenidae* and *Phacopidae*, seem to have practically lost the median eye. The Ordovician and Silurian trilobites show well developed median eye tubercles; the Devonian forms lack them; the median eye, as in the higher crustaceans, having been either reduced again to a transparent spot in the test and wandered inward or become altogether effaced. In the Cambrian trilobites, the tubercle, for the most part, is also absent; the median eye from the present evidence, being still in its most primitive form of one or two transparent spots of the test.

It is a distinct fact pointing to a visual function of the median tubercle that the genera usually considered as blind because of reduced or absent lateral eyes, are apt to show these median eye tubercles most distinctly, as notably *Cryptolithus*, *Trinucleus*, *Dionide*, *Dindymene*, and also *Agnostus*, *Microdiscus* and *Ampyx*, while on the other hand the genera, *Phacops* and *Dalmanites* with their highly developed lateral eyes, show the least trace of the median eye. Its constant presence in a great number of genera is further evidence of its important function; and finally the fact that all lower crustaceans typically possess the median or parietal eye and that for that reason zoologists of standing have already simply assumed the presence of this organ in the trilobites, makes it a reasonable inference that these primitive early crustaceans should have also possessed the median eye, in at least some stages of their evolution, and that is what the writer hopes to have demonstrated.

A fuller account of this investigation is being printed in a New York State Museum Bulletin.

THE NATURE OF MECHANICAL STIMULATION

W. J. V. Osterhout

LABORATORY OF PLANT PHYSIOLOGY, HARVARD UNIVERSITY

Received by the Academy, March 13, 1916

The effects of certain kinds of stimuli can be referred directly to chemical changes which they produce in the protoplasm, but there are other kinds which appear to operate by physical means only. In the latter category are such stimuli as contact, mechanical shock and gravitation. While their action appears at first sight to be purely mechanical, they are able to produce effects so much like those of chemical stimuli that it appears probable that in every case their action must involve chemical changes.

The chief difficulty which confronts a theory of mechanical stimulation appears to be this, How can purely physical alterations in the protoplasm give rise to chemical changes? It would seem that a satisfactory solution of this problem might serve to bring all kinds of stimulation under a common point of view, by showing that a stimulus acts in every case by the production of chemical reactions.

An answer to this question is suggested by some observations of the writer. These were originally made on the cells of the marine alga *Griffithsia Bornetiana*. A cell of this alga is shown in figure 1. Within the cell wall (*a*) is a thin layer of protoplasm (*b*) which encloses the large central vacuole (*e*). The protoplasmic layer includes numerous chromatophores (*c*). The latter contain chlorophyll and a red pigment (phycoerythrin) which is soluble in water. Under normal conditions the surface of the chromatophore is impermeable to the red pigment, which is thus confined to the chromatophore and prevented from escaping into the surrounding protoplasm or into the vacuole.

The writer has observed when one of the larger cells is placed under the microscope (without a cover glass) and touched near one end (with a needle or a glass rod or a splinter of wood) a change occurs in the chromatophores directly beneath the spot which is touched. The surfaces of the chromatophores in this region become permeable to the red pigment, which begins to diffuse out into the surrounding protoplasm. This change begins soon after the cell is touched. As the red pigment diffuses through the protoplasm it soon reaches neighboring chromatophores and it may then be seen that their surfaces also become permeable and their pigment begins to diffuse out. In this way a wave—which may be compared to a wave of stimulation—progresses along the cell until the opposite end is reached.

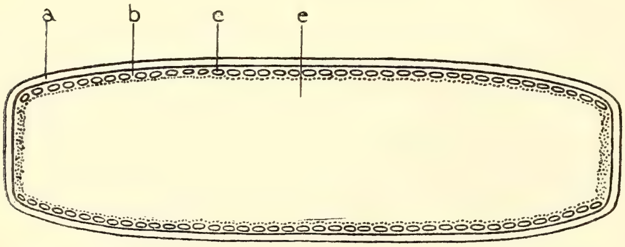


FIG. 1.—A CELL OF *GRIFFITHSIA BORNETIANA* (IN OPTICAL SECTION). *a*, CELL WALL; *b*, PROTOPLASM; *c*, CHROMATOPHORE CONTAINING CHLOROPHYLL AND A RED PIGMENT (PHYCOERYTHRIN) WHICH IS SOLUBLE IN WATER; *e*, VACUOLE FILLED WITH CELL SAP. (DIAGRAMMATIC.)

The rate of propagation of this wave corresponds to that of the diffusion of the pigment. It would seem that at the point where the cell is touched, pigment, and probably other substances, are set free, diffuse out and set up secondary changes as they progress. These changes are doubtless chemical in nature.

The important question then arises, How does the contact initiate the outward diffusion of the pigment or other substances?

It seems to the writer that this may be due to a mechanical rupture of the surface layer of the chromatophore which is either not repaired at all or only very slowly. Many cases are now known in which the surface layers of protoplasmic structures behave in this way.* If therefore, such structures exist within the cell, it is evident that any deformation of the protoplasm which is sufficient to rupture their surface layers will permit their contents to diffuse out into the surrounding protoplasm. A great variety of cellular structures (plastids, vacuoles, 'microsomes,' inclusions, etc.), possess surface layers of great delicacy and it is easy to see how some of these may be ruptured by even the slightest mechanical disturbance.

It is therefore evident that deformation of the protoplasm may rupture the surface layers of certain protoplasmic structures and cause their contents to diffuse out. If the substances which thus diffuse out meet other substances from which they were separated by the semipermeable surface layer before it was ruptured it is easy to see how reactions may be set up which in certain cells may bring about the responses characteristic of mechanical stimulation. The occurrence of such reactions seems probable, since many cases are known where substances in close juxtaposition are prevented from reacting by the presence of such semipermeable layers; but when these layers are destroyed (by crushing the cells) the reaction at once takes place.

If these processes occur it is evident that purely physical alterations in the protoplasm can give rise to chemical changes. Responses to contact and mechanical stimuli may thus be explained; and since gravitational stimuli involve deformation of the protoplasm we may extend this conception to geotropism.

In this conception of mechanical stimulation the essential things are (1) substances which are more or less completely prevented from reacting by semipermeable surfaces, (2) a deformation of the protoplasm sufficient to produce in some of these surfaces a rupture which is not at once repaired, (3) a resulting reaction which produces the characteristic response to the stimulus.

---

* In many cases rupture of the plasma membrane causes the protoplasm to disintegrate and mix with the surrounding medium. In other cases the surface layer is at once reconstituted.

HEREDITARY REACTION-SYSTEM RELATIONS—AN EXTENSION OF MENDELIAN CONCEPTS

By R. E. Clausen and T. H. Goodspeed

DIVISION OF GENETICS, COLLEGE OF AGRICULTURE AND DEPARTMENT OF BOTANY, UNIVERSITY OF CALIFORNIA

Received by the Academy, March 11, 1916

The most important as well as the most consistent and intelligible series of Mendelian conceptions are those which Morgan and his associates have formulated on the basis of their extensive studies of heredity in the common fruit fly, *Drosophila ampelophila.* During the progress of their investigations they have observed the origin of over a hundred factor-mutations in this species, and they have determined the hereditary interrelations of a large number of them. They have established, for the fruit fly, the validity of the fundamental conception of Mendelism that the units contributed by two parents separate in the germ cells of the offspring without having had any effect on one another, that long and intimate association in the same chromosomal mechanism does not modify the fundamental constitution or relations in the hereditary mechanism of the units of which it is made up. They have also demonstrated that the known behavior of the chromosomes furnishes a most satisfactory basis for an explanation of the distribution of hereditary units to the germ cells. Furthermore, from the linkage-relations displayed among the factors, Morgan has succeeded not only in demonstrating that the number of groups of factors corresponds to the number of pairs of chromosomes, but he has also succeeded in preparing a map of the relative linear positions of the factors within the chromosomes. It, therefore, follows that, so far as heredity is concerned, the chromosomes are made up of a linear series of loci which bear at least some specific relation to one another as is indicated by this aggregation into chromosomes. Hereditary modifications of characters in the individual depend upon changes in the loci, a particular type of change in some particular locus corresponding to each different character-modification. Now, since a changed locus maintains the same formal relations with the other loci in the system as it does in its normal unchanged condition, it is clear that the chromosome conception of heredity furnishes a consistent explanation of the fundamental nature of allelomorphism and of the mechanistic basis of Mendelian segregation. Further the evidence of somatogenesis seems to indicate that the hereditary units form a physico-chemical reaction-system of which the elements, the loci of the hereditary system, bear more or less specific relations to one another. In *Drosophila*, for example, the development of the normal abdomen under certain environmental conditions in spite of the presence of the factor for abnormal abdomen in the reaction system indicates the existence of compensatory relations among the factors of the system. Such compensatory relations are even more strikingly evidenced in the case of maize seedlings of the yellow-green chlorophyll reduction type. Normally these die, but under favorable conditions the system is able to overcome the disturbance incident upon the presence of the chlorophyll reduction factor and to go on and develop the normal chlorophyll coloration in the plant. Similarly, the lethal effect of changes in certain loci, the similarity in effect of different changes in the same locus displayed in multiple allelomorphism, the apparently universal significance of the multiple-factor conception of character development, and a variety of other considerations indicate that important physiological relations exist among the loci of the system, and that character expressions depend upon the reaction-system relations of Mendelian factors. The product of somatogenesis, the individual, represents the reaction end-product of such a physico-chemical system working under particular conditions; the specific hereditary differences between individuals of the same species indicate particular differences in some one or more elements in such a reaction-system. Normal Mendelian behavior, then, would follow as a result of hybridization phenomena involving a contrast between a relatively few particular differences within a reaction system which is fundamentally identical in the races under consideration. If in contrast to this type of behavior it should be possible to secure contrasts of fundamentally different reaction systems, then conceivably the elements, although playing definite parts in their own systems, might fail to establish the harmonious inter-relations which are necessary for normal development and reproduction. Such incompatibility of elements would give rise to a peculiar type of behavior in inheritance which could not well be accounted for by the customary formal treatment based on the Mendelian viewpoint. The experimental data which we have collected seem to indicate that such a situation actually does obtain in certain cases of hybridization between distinct species.

For ten years a number of species and varieties of *Nicotiana* have been grown in the University of California Botanical Garden. Among many others this collection has included *N. sylvestris* and a considerable array of varieties of *N. Tabacum.* The varieties of *Tabacum* display notable morphological differences throughout—differences so marked that to regard them as distinct species would be entirely justifiable, even though they do show evident group relationships. On the other hand *sylvestris* apparently is monotypic and is distinctly different from the *Tabacum* group. Now, the study of a large number of varietal crosses within the *Tabacum* group has demonstrated that most characters are expressed in intermediate degrees in the F_1 hybrids and subsequent segregation in further generations indicates that these phenomena, although complex, are in accord with normal Mendelian expectation. The differences within the *Tabacum* group, therefore, apparently depend upon certain factor differences within a common reaction system. When, however, any one of this array of *Tabacum* varieties is crossed with *sylvestris*, the F_1 hybrid very nearly or completely reproduces on a larger scale the characters of the particular *Tabacum* variety concerned in the hybrid. This has been found true for a number of *Tabacum* varieties; viz. *angustifolia*, *calycina*, *macrophylla*, *macrophylla purpurea*, 'Cavala,' 'Cuba,' and 'Maryland;' descriptions and plates of which based on material grown in the University of California Botanical Garden have been given elsewhere by Setchell. The completeness of the domination of the *Tabacum* parent in the somatogenesis of these F_1 *Tabacum-sylvestris* hybrids is shown particularly in the crosses involving characters which are normally recessive in *Tabacum* variety hybrids. When *calycina*, which produces abnormal, split, 'hose-in-hose' flowers, is crossed with *Tabacum* varieties producing normal flowers, the F_1 hybrids produce the normal type of flowers with few exceptions. On the other hand in marked contrast to the type of behavior in varietal crosses, *calycina* when crossed with *sylvestris* gives an F_1 hybrid which produces only calycine flowers. Similarly, the partially parthenocarpic tendency of 'Cuba,' which is manifested in the retention and normal development of many fruits without pollination, although recessive in varietal crosses, is so impressed on the 'Cuba'-*sylvestris* hybrid that all the fruits mature normally in spite of the fact that no functional pollen is produced. Somatogensis in F_1 hybrids of *Tabacum* with *sylvestris* seems, therefore, to be dominated by the *Tabacum* system as a unit, so that any particular modification of the *Tabacum* reaction system displays its full possibilities in the development of such hybrids.

Now, if these F_1 hybrids of *Tabacum* with *sylvestris* represent the reaction end-product of two fundamentally dissimilar reaction systems, then the relations of these two systems, as manifested by the domination of the *Tabacum* system to nearly or quite the exclusion of the *sylvestris* system, indicate a rather extensive mutual incompatibility of the elements of the two systems. This deduction is borne out by the fact that the F_1 *Tabacum-sylvestris* hybrids produce only a very few functional ovules, the number of which is apparently constant within rather narrow limits. Assuming that segregation and recombination take place normally and in accordance with the chromosome view of heredity, these functional ovules represent the *Tabacum* and *sylvestris* extremes of a recombination series, the vast majority of the members of which fail to function because they are built up from incompatible elements derived from both systems. The evidence for such a constitution of the functional ovules is furnished by the results of back-crosses of the hybrid with the parents. When the F_1 hybrid is crossed back with *sylvestris*, the progeny consists of abnormal, sterile individuals and a few typical *sylvestris* individuals which are completely fertile and breed true. On the other hand when the *Tabacum* parent is crossed back onto the F_1 hybrid the progeny consists of *Tabacum* forms some of which are completely fertile and others of which are sterile like the F_1 hybrids. The hereditary phenomena, therefore, displayed by these F_1 species hybrids confirm the conception that they represent a contrast between reaction systems, the elements of which display a considerable degree of mutual incompatibility. It follows, then, that the type of behavior displayed by species hybrids may be considered as dependent upon the degree of incompatibility of the elements of the reaction systems therein involved. Sterility in such cases is merely a logical consequence of this same incompatibility, and the degree of sterility may be regarded as an expression of its extent.

The adoption and application of such a reaction system conception to hereditary phenomena has far reaching consequences. When, for example, this conception is applied to the *Oenothera* phenomena, it at once follows that the widespread occurrence of partial sterility, the significance of which has never been definitely determined, must be of primary importance in the formulation of any consistent explanation of the hereditary phenomena displayed in *Oenothera*. Until it is possible to define clearly the exact significance of this partial sterility, it is obviously useless to attempt to apply any rigid Mendelian analysis. Moreover, the *Oenothera* phenomena belong to several different categories, three of which, at least, may be clearly distinguished. In the first place, there are some strict factor mutations in *Oenothera*, such as *rubricalyx*. These mutations depend upon a particular change in some locus in the hereditary mechanism, and they display normal Mendelian behavior when tested with the forms from which they were derived. The extensive observations of Morgan and his associates on mutation in *Drosophila*, to say nothing of other well authenticated cases in both plants and animals, seem to establish the validity and nature of this type of mutation beyond cavil or doubt. In the second place, there is a considerable series of forms which depend upon duplication of one or more or even all of the chromosomes to the extent of tetraploidy in some forms. The particular type of behavior displayed by such forms appears to depend upon changes in the proportions of the elements within the reaction systems, rather than upon actual changes in germinal substance. In the third place, there is a complicated group of phenomena which appear to be best considered as due to complex segregation of a type analogous to that displayed in wide crosses. In contrast to the simple and definite behavior of factor mutants, the forms resulting from this segregation are often distinctly different throughout from the forms from which they arose, and when tested with them, they exhibit a complicated but orderly type of hereditary behavior. There are two facts which stand out prominently with respect to this behavior—first the mutations affect the total ontogenetic development of the individual and second they tend to recur in relatively constant ratios in certain races. The definite ratio relations in the production of 'mutant' forms, the peculiar but orderly behavior of the hybridization phenomena, and the universal occurrence of partial sterility make together a series of facts which seem at least as consistently explainable on the basis of substratum hybridity as on assumptions of general germinal change. If the conceptions applied above to the behavior of species hybrids be extended in a somewhat modified form to the *Oenothera* phenomena, the occurrence of the 'mutants' and their subsequent behavior in hybridization admit of logical arrangement and interpretation without any necessity for assumptions of extensive germinal changes.

The experimental data cited above were obtained from cultures made possible by a portion of the Adams' Fund allotted to the Department of Botany by the Department of Agriculture of the University of California. A more detailed statement of the general position here outlined has been prepared and will appear in the near future.

POINT SETS AND ALLIED CREMONA GROUPS (PART II)

By Arthur B. Coble

DEPARTMENT OF MATHEMATICS, JOHNS HOPKINS UNIVERSITY

Received by the Academy, March 13, 1916

In Part I of this account[1] the ordered set P_n^k of n discrete points in a projective space S_k was studied with particular reference to its invariants, its association with a set Q_n^{n-k-2}, and its mapping upon a space $\Sigma_{k(n-k-2)}$. In this space Σ there was induced by permutation of the points of P_n^k a Cremona group $G_{n!}$ which for $n = 5, 6$ led to new versions of the solutions of the quintic and sextic equations.

In Part II the novel idea of the *congruence* of point sets is introduced. This notion can be defined at once for a space S_2. Two mutually ordered point sets P_n^2 and $P_n'^2$ are congruent under a ternary Cremona transformation C with $\rho \leqq n$ fundamental points (F = *points*) if ρ of the pairs comprise the F-points of C and C^{-1} and if the remaining $n-\rho$ pairs of the sets are pairs of ordinary corresponding points of C. Thus the number of types of congruence depends upon the number of types of C and the number of ways of ordering the two sets. There is a natural pairing of the ρ F-points of C with the ρ F-points of C^{-1} and this is utilized to state in quite compact form the conditions for congruence up to $n = 9$. These conditions imply when $\rho = n$ a construction for the two sets of F-points and when $\rho < n$ a construction for the transformation C. The importance of the notion of congruence is due to the fact that two sets congruent in some order to a third are congruent in some order to each other. Thus if all the sets $P_n'^2$ congruent in some order to a given set P_n^2 be mapped upon points P' of $\Sigma_{2(n-4)}$ they form a conjugate set of points under the operations of a Cremona group $G_{n,2}$ in $\Sigma_{2(n-4)}$ which contains the $G_{n!}$ of Part I as a subgroup.

This definition of congruence cannot be extended immediately to sets in S_k. Let us first define a *regular* Cremona transformation C in S_k to be one which can be generated as a product of projectivities and of inversions of the variables. The regular transformations constitute a *regular Cremona group in* S_k. They are determined by their F-points precisely as in S_2. It is now possible to define as above congruence of sets P_n and $P_n'^k$ under regular Cremona transformation in S_k and the group $G_{n,k}$ in $\Sigma_{k(n-k-2)}$.

The effect of a regular transformation on spreads in S_k of order x_0 with multiple points of orders $x_1, \ldots x_n$ at the points of P_n^k is represented by a linear transformation on these variables with integral coefficients. In this way a group $g_{n,k}$ is derived which is isomorphic with $G_{n,k}$. This group reveals the striking analogy between the general transformation in S_2 and the regular transformation in S_k.

All sets $P_n'^k$ congruent to a set P_n^k upon an elliptic norm-curve in S_k can be projected upon the same curve. The transition from P_n^k to $P_n'^k$ is then effected by a linear transformation on the elliptic parameters $u_1, \ldots, u_n$ with rational coefficients. In this way a group $e_{n,k}$ is derived which also is isomorphic with $G_{n,k}$. The group is useful in determining the cases in which congruence implies projectivity.

The close relation of associated sets P_n and Q_n^{n-k-2} is again apparent in that the groups $G_{n,k}$ and $G_{n,\,n-k-2}$ are identical while the groups $g_{n,\,k}$ and $g_{n,\,n-k-2}$, as well as the groups $e_{n,\,k}$ and $e_{n,\,n-k-2}$, are linear transforms of each other.

The group $G_{n,\,k}$ is in general infinite and discontinuous. The only finite types are the $G_{6,\,2}$, the $G_{7,\,2} = G_{7,\,3}$ and the $G_{8,\,2} = G_{8,\,4}$. These are identified with the well known groups of respectively the lines on a cubic surface, the bitangents of a quartic, and the tritangent planes of a sextic of genus 4 on a quadric cone. For these cases $G_{n,\,k}$ is in immediate algebraic relation to the corresponding geometric configuration. This advantage is used in the case of P_7^2 to determine the simplest system of irrational invariants of the point set which are invariants of the allied quartic as well. A similar method will be employed in Part III to handle the P_6^2 and the allied cubic surface. The first cases of infinite order, the $G_{9,\,2} = G_{9,\,5}$ and the $G_{8,\,3}$ can be adequately discussed by means of the isomorphic group $e_{n,k}$ and their structure has been determined.

Some interesting by-products are obtained. By means of $g_{n,\,k}$ a determination of all types of regular Cremona transformations with a single symmetrical set of F-points is made. The new types thus found are a transformation in S_4 of order 49 with 8 F-points of order 30 and a transformation in $S_{2\,k-1}$ of order $(2k^2-1)$ with $2\ (k+1)$ F-points of order $2k\ (k-1)$. Also the discussion of $e_{9,\,2}$ leads to a determination of the infinite number of types of ternary Cremona transformations with 9 or fewer F-points in terms of 8 independent integers. Theorems such as the following:—*A pencil of plane cubic curves can be transformed by ternary Cremona transformation into only* 960 *projectively distinct pencils of cubics*—are proved for special sets P_9^2. Similar facts can be derived for P_8^3. Furthermore the general methods developed can be employed in the problem of determining the finite groups of regular transformations in S_k. For if a point P in $\Sigma_{k(n-k-2)}$ is fixed under a certain subgroup of $G_{n,k}$ the corresponding set P_n^k in S_k defines a regular Cremona group in S_k isomorphic with the given subgroup.

---

[1] These PROCEEDINGS, **1,** 245 (1915) and *Trans. Amer. Math. Soc.*, **16,** 155 (1915). This investigation has been carried out under the auspices of the Carnegie Institution of Washington, D. C.

ON A THEOREM OF LUCAS

By M. B. Porter

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF TEXAS

Received by the Academy, March 10, 1916

In a paper on the *Geometry of Polynomials*[1] Lucas has an interesting generalization of Rolle's theorem, to wit: *That the zeros of any polynomial $F'(z)$ lie inside any closed convex contour inside of which the zeros of $F(z)$ lie.*

Many proofs[2] of this theorem have been given, but no one seems to have pointed out that the theorem is applicable to integral transcendental functions of the type $I_0(z) = \Pi_1^\infty (1 - z/\alpha_1)$ where $\Sigma_1^\infty |1/\alpha_i|$ is convergent, i.e., functions of zeroth order (*genre* zero).

We shall show that this theorem can be generalized so as to give information concerning the distribution of the zeros of the derivative of certain rational functions and certain transcendental functions of the type $I_0(z)/\bar{I}_0(z)$.

We begin by giving a very elementary proof (perhaps new) of Lucas' theorem.

Proof. Since, in the finite part of the complex plane $F'(z)/F(z) = \Sigma_1^n (z-\alpha_i)^{-1}$, where $\alpha_1, \ . \ . \ . \ , \alpha_n$ are the zeros of $F(z)$, can vanish only when $F'(z)$ vanishes, we have only to show that $\Sigma_1^n (z-\alpha_i)^{-1}$ can vanish only *inside* the convex contour mentioned. Now since the contour is convex, all the vectors $z-\alpha_i$ drawn from a point z *outside* the contour lie *inside* the arms of an angle *less* than 180°; the same thing will be true of the vectors $(z-\alpha_i)^{-1}$ (obtained by inverting and reflecting in the axis of reals through the point z). But such a set of vectors cannot form a closed[3] polygon, and hence the theorem is proved. It is now at once evident that the theorem is true for functions of the type $I_0(z)$ and we have, for example, a theorem of Laguerre's that: *If the zeros of $I_0(z)$ are all real, so are those of $I'_0(z)$.*

If $F(z) = P_n(z) / P_m(z)$, where P_n and P_m are polynomials or functions of the type $I_0(z)$ whose zeros α_i and β_i respectively lie inside of closed convex contours C_n and C_m which are *external* to each other, the proof given above shows that, if $\Phi(z) = P_n(z)/P_m(z)$, then

$$\frac{\Phi'(z)}{\Phi(z)} = \left(\log \frac{P_n(z)}{P_m(z)}\right)' \equiv \sum_1^n \frac{1}{z-\alpha_i} - \sum_1^m \frac{1}{z-\beta_i}$$

can have no zeros in the region swept out by such tangent straight lines to C_m as can be moved parallel to themselves into tangency with

C_n without cutting either C_m or C_n or passing through the position of the line infinity. This is a region which can be easily marked out on the complex plane and will have inside of it neither of the contours C_m and C_n.

Thus our theorem asserts that if the zeros of P_n are inside of one branch of an hyperbola and the zeros of P_m are inside the other branch, all the zeros of Φ' are *inside* of the hyperbola, or again, if all the zeros of P_n are real and lie in the interval (1) $x > a$, while all the zeros of P_m are real and lie in the interval (2) $x < b \leqq a$, then $\Phi'(z)$ has no complex zeros and all of its zeros lie in the intervals (1) and (2).

---

[1] *J. Ec. Polytech., Paris,* 28.

[2] All these proofs save one by Hayashi (*Annals of Mathematics,* March, 1914) are based on dynamical considerations. Fejér, Ueber die Wurzel vom kleinstein absoluten Betrage, etc., *Leipzig, Math. Ann.,* 65, 417, attributes the theorem to Gauss and gives a bibliography for it.

[3] If this is not at once intuitionally evident it can be shown by resolving the vectors in question into components parallel to the arms of the angle above mentioned.

INTERPRETATION OF THE SIMPLEST INTEGRAL INVARIANT OF PROJECTIVE GEOMETRY

By E. J. Wilczynski

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF CHICAGO

Received by the Academy, March 13, 1916

If $y = f(x)$ is the cartesian equation of a plane curve, the integral

$$s = \int_{x0}^{x_1} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx, \tag{1}$$

which represents the length of the arc of this curve between the points $P_0(x_0, y_0)$ and $P_1(x_1, y_1)$, obviously remains unchanged when the curve is subjected to a plane motion. Therefore we may speak of s as an integral invariant of the group of motions, or as a *metric integral invariant.*

In the present paper we shall show how to find integrals connected with a given plane curve, whose values are not changed when the points of the plane are subjected to an arbitrary projective transformation. We shall speak of these integrals as *projective integral invariants.*

Let y_1, y_2, y_3, be the homogeneous coördinates of a point P_y, and let y_1, y_2, y_3 be given as linearly independent analytic functions of a parameter x. As x changes P_y will describe a non-rectilinear analytic curve C_y. There exists a uniquely determined linear homogeneous differential equation of the third order

$$y''' + 3p_1y'' + 3p_2y' + p_3y = 0 \tag{2}$$

of which y_1, y_2, y_3 form a fundamental system of solutions, and of which C_y shall be said to be an integral curve. All other integral curves of (2), associated with different fundamental systems of solutions, are projective transforms of C_y.

Since the coordinates y_1, y_2, y_3 are homogeneous, only those combinations of the coefficients p_1, p_2, p_3 can be of interest for the geometry of the curve C_y which depend only upon the ratios $y_1 : y_2 : y_3$. These combinations, the so-called *seminvariants* of (2), are all expressible as functions of

$$P_2 = p_2 - p_1^2 - p_1', \quad P_3 = p_3 - 3p_1p_2 + 2p_1^3 - p_1'' \tag{3}$$

and of their derivatives. The seminvariants of (2) are not altered if (2) is transformed by putting $\bar{y} = \lambda(x)y$ where $\lambda(x)$ is an arbitrary function of x.[1]

Although the seminvariants depend only upon the ratios $y_1 : y_2 : y_3$, they are still not adequate to represent the purely geometric properties of the curve C_y. The values of P_2 and P_3 depend also upon the special parametric representation which has been chosen for C_y. We may change this parametric representation in the most general way by putting $\bar{x} = \xi(x)$, where $\xi(x)$ is an arbitrary function of x. Those combinations of the seminvariants, called *absolute projective differential invariants*, which are left unaltered by all possible transformations of this sort, express intrinsic properties of the curve C_y. Moreover these properties are projective properties, since any projective transform of C_y may be regarded as an integral curve of (2).

Every absolute projective differential invariant can be expressed as a quotient of two relative invariants. The simplest of these relative invariants is[2]

$$\theta_3 = P_3 - \frac{3}{2}P_2'. \tag{3}$$

The property of θ_3 which justifies us in speaking of it as a relative invariant, is the following. Let us transform (2) by putting

$$\bar{x} = \xi(x), \; \bar{y} = \lambda(x)y, \tag{4}$$

where $\xi(x)$ and $\lambda(x)$ are arbitrary functions of x. From the coefficients of the resulting differential equation between $\bar{x}$ and $\bar{y}$ let us form the quantity $\theta_3(x)$ according to the same rule which was used in forming θ_3 from the coefficients of (2). We shall find[3]

$$\bar{\theta}_3(\bar{x}) = \frac{\theta_3(x)}{(\xi')^3}. \tag{4}$$

This equation may be written

$$\bar{\theta}_3(\bar{x})\ (d\bar{x})^3 = \theta_3(x)dx^3.$$

Consequently the integral

$$p = \int_a^b \sqrt[3]{\theta_3\ (x)}\, dx \tag{5}$$

will not change its value under the transformations (4). Thus we see that the integral p is intrinsically connected with some geometric property of that arc of the curve C_y which corresponds to the interval $a \leqq x \leqq b$. It is also clear that this integral and its geometric significance will remain unaltered by any projective transformation of the plane, since it is expressed entirely in terms of the coefficients p_1, p_2, p_3 of (2) which are invariants of the projective group.

Therefore *the integral p, defined by (5), is a projective integral invariant.* If I is any absolute differential invariant of the curve C_y, the integral $\int I dp$ is again an integral invariant, and all integral invariants are expressible in terms of those obtained in this way.

We wish to explain the geometrical significance of the invariant integral p. For this purpose we need one further preliminary notion, namely that of the eight-pointic nodal cubic of a given point of a given curve.

A cubic curve is in general determined by nine of its points. If eight points only are given, there exist infinitely many cubics, forming a pencil, which pass through these points. In particular there exists a pencil of cubics, such that each cubic of the pencil has eight-pointic or seventh-order contact with the given curve C_y at a specified non-singular point P_y. One and only one of the cubics of this pencil has P_y, the point of contact, as double point. We call this cubic the *eight-pointic nodal cubic* of the point P_y, or the *penosculating nodal cubic* of P_y.[4]

The significance of the integral p is contained in the following theorem which we shall state without proof, but all of the terms of which have now been explained.

Consider an arc of an analytic curve corresponding to the interval $a \leqq x \leqq b$ of the independent variable. Divide this interval into n parts by means of the values $x_0 = a, x_1, x_2, \ldots x_{n-1}, x_n = b$, such that $\lim \delta x_k = \lim\ (x_{k+1} - x_k) = 0$ as n grows beyond bound. Let $A, P_1, P_2, \ldots P_{n-1}, B$ be the points on the curve which correspond to these $n + 1$ values of x. Let t_k be the tangent and C_k the eight-pointic nodal cubic of P_k. The three points of inflection of the cubic C_k are on a line i_k which interesects t_k in a point I_k. Denote by t_k one of the three inflectional tangents of C_k and let T_k be its intersection with

t_k. The line P_kP_{k+1} will intersect i_k and t_k in two points, I_k' and T_k', and the cross-ratio $(I'_k, T_k', P_k, P_{k+1})$ turns out to be equal to

$$1 - \frac{3}{\sqrt[3]{10}} \sqrt[3]{\theta_3 (x_k)\, \delta x_k} \tag{6}$$

except for terms of higher than the first order in δx_k.

By a perspective correspondence the three points $I'_{n-1}, T'_{n-1}, P_{n-1}$ of $P_{n-1} B$ may be projected into the points $I'_{n-2}\, T'_{n-2}, P_{n-1}$ of $P_{n-2}\, P_{n-1}$. Let B_{n-1} be the point of $P_{n-2}\, P_{n-1}$ which, in this perspective, corresponds to B. Then project similarly $I'_{n-2}, T'_{n-2}, P_{n-2}, B_{n-1}$ into the four points $I'_{n-3}, T'_{n-3}, P_{n-2}, B_{n-2}$ of $P_{n-3}\, P_{n-2}$, and continue in this way. We shall finally obtain upon the line AP_1 a point B_1 determined from B by this sequence of perspectives. As n grows beyond bound, B_1 will approach a limiting position Q on the initial tangent t_0 of the arc AB. The cross-ratio

$$k = (I_0, T_0, A, Q) \tag{7}$$

will be the limit which the product

$$\prod_{k=1}^{n} \left(1 - \frac{3}{\sqrt[3]{10}} \sqrt[3]{\theta_3 (x_k)\, \delta x_k}\right) \tag{8}$$

approaches when n grows beyond bound. Consequently we find

$$\log k = \frac{3}{\sqrt[3]{10}} \int_a^b \sqrt[3]{\theta_3\ (x)}\, dx. \tag{9}$$

This equation contains the desired interpretation of the integral p.

From a theoretical point of view the expression (5) for the integral p is the simplest and most general. We shall however give, in conclusion, three other expressions for p in terms of more familiar variables.

If the curve is given by means of its cartesian equation in the form $y = f\ (x)$, we may write

$$p = -\frac{1}{3\sqrt[3]{2}} \int \frac{\sqrt[3]{9\,(y'')^2 y^{(5)} - 45\, y''y'''y^{(4)} + 40\, y^{(3)\,3}}}{y''\sqrt{1 + y'^2}}\, dx, \tag{10}$$

where $y' = dy/dx$, $y'' = d^2y/dx,^2$ and so on.

If the curve is given by means of parametric equations of the form $x = \varphi(s)$, $y = \psi(s)$, where s denotes the length of arc, and if r is the radius of curvature at the point which corresponds to the value s of the parameter, we find

$$p = \frac{1}{\sqrt[3]{6}} \int \sqrt[3]{\frac{r'''}{r} - \frac{r'r''}{r^2} + \frac{4}{9}\frac{r'^3}{r^3} + 4\frac{r'}{r^3}}\, ds, \tag{11}$$

where $r' = dr/ds$, $r'' = d^2r/ds^2$, etc..

Finally we may write

$$p = \int \frac{1}{\rho_0} \sqrt[3]{\frac{r_0}{2r}}\, ds, \qquad (12)$$

where r has the same meaning as in (11), where ρ_0 is the distance from the point P of the curve to the center M of the corresponding osculating conic, and where r_0 is the radius of curvature at M of the locus which M describes when P moves along the given curve.

[1] E. J. Wilczynski, *Projective Differential Geometry of Curves and Ruled Surfaces*, p. 58.
[2] Loc. cit., p. 59. [3] Loc. cit., p. 60. [4] Loc. cit., pp. 67-68.

SIZE INHERITANCE IN GUINEA-PIG CROSSES

By W. E. Castle

BUSSEY INSTITUTION, HARVARD UNIVERSITY

Received by the Academy, March 9, 1916

For several years my pupils and I have been engaged in studying the inheritance of size and weight differences among animals, these being characteristics of much economic importance and of peculiar theoretical interest. Preliminary studies published in 1909 showed that size and weight in rabbits do not follow the Mendelian rules of dominance and segregation as unit-characters. But Lang subsequently suggested that multiple Mendelian factors may be concerned in such cases, extending to animals a principle already recognized by Nilsson-Ehle in dealing with certain categories of characters in plants. Punnett and Bailey (1914) accept this principle in explaining weight inheritance in crosses of bantam fowls with those of ordinary size. They believe that four differential factors are concerned in a particular cross studied, three dominant factors which tend to increase size being found in the larger race, one such factor being found in the bantam race. By recombination in F_2 some individuals are obtained smaller than the bantam race, and others in F_3 larger than the larger race. But there are some reasons for questioning the validity of this analysis which assigns very definite quantitative values to the several hypothetical factors, without however making any allowance for physiological changes of size due to non-genetic causes, or for possible quantitative variation in the factors themselves. Moreover, let it be granted for the sake of argument that these four Mendelizing factors exist and that each is an independent agency for increasing size. On the Mendelian hypothesis there should be obtained from the cross in question individuals which lack *all four* of these factors. What, it may be asked, will their size be? Will they be without size or devoid of genetic variation in size? No race of animals has yet been shown to exist which is devoid of genetic variation in size,—Jennings' supposed demonstration of such a condition in asexually reproducing Paramecium having been disproved by the work of Calkins and Gregory and his own subsequent work on Difflugia. If when all discoverable Mendelian factors are wanting size still varies genetically, a Mendelian explanation of size inheritance proves itself inadequate. There also exists a possibiity that *bantamness* in fowls involves discontinuous variation such as does not occur in ordinary size differences. For Phillip's results with ducks do not show such segregation of the extremes of size as Punnett and Bailey record, though his crosses were made between races of ducks both very pure and very different from each other in average size. What Phillips observed was a blend with a slight increase of variability in F_2 but without evidence of complete segregation or recombination of size factors.

Some excellent material for the study of size inheritance has become available to the writer in certain very pure races of guinea-pigs differing widely as to size and it is the purpose of this note to describe briefly the more important results obtained and their possible significance. Attention is invited to the nature of the growth curves observed for the races crossed and to non-genetic as well as to genetic factors affecting size.

Three distinct and unrelated races were used in the crosses to be described. First, a wild race obtained at Arequipa Peru in December 1911, and identified as Cavia Cutleri Bennet. This has been bred for three generations in captivity at the Bussey Institution and has shown itself very uniform in size and other characters. Second, a race of guinea-pigs which we may call race B, bred distinct for many years at the Bussey Institution, very uniform in size, and varying as regards Mendelian factors only in respect to the color factor. Some animals of this race are black, others are albino. Both sorts are alike as regards other characters. A race C, also long bred distinct at the Bussey Institution, is of about the same average size as race B. It as well as race B has been used in crosses with C. Cutleri, but the hybrids from these two crosses have not been interbred, though the data concerning them have been combined for statistical treatment.

Figure 1 shows separately growth curves for the two sexes of race B and Cavia Cutleri. These curves have been obtained by combining the individual growth curves of several different animals reared in captivity and weighed at intervals of one or two weeks. They represent *averages* and the curves have been smoothed somewhat. It will be observed that the young of Cutleri grow rapidly for the first 40 or 50 days of their lives after which increase in weight slows up gradually and finally ceases altogether at the age of about one year. Both sexes are of about the same size at birth but females grow at first faster than males, attaining sexual maturity some weeks earlier. At about fifty days of age the sexes are again of the same average size, but from this time on the males are heavier. The general form of the growth curve in both sexes is, as in animals generally, at first concave upward but later becomes convex upward as the growth rate declines. Growth is completed at about one year, after which time no change in weight occurs except as caused by health or food conditions, or pregnancy of the females. The growth curve of C. Cutleri may be characterized as a sharply rising, flat curve. Compared with this the growth curve of the domesticated race B rises less rapidly at first but continues its rise longer. The same difference in the growth rate of the sexes is observed in race B as in the wild C. Cutleri. Females grow faster up to 40 or 50 days of age, after which time males permanently take the lead. The adult weight of race B animals is approxmately double that of Cutleri individuals.

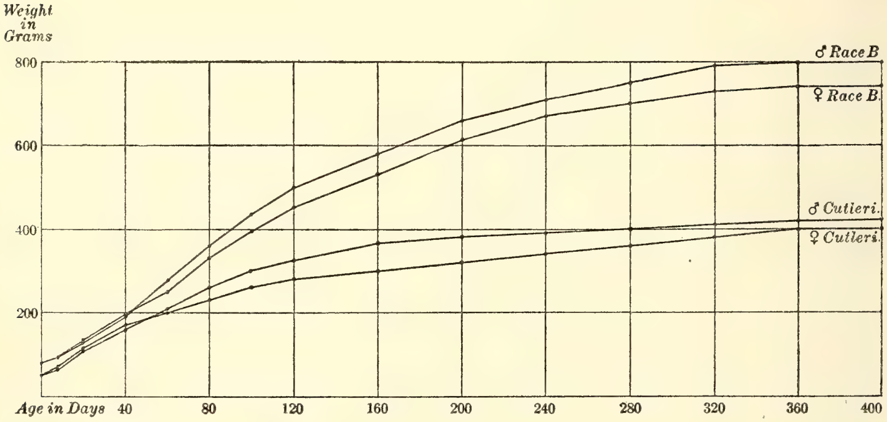


FIG. 1. GROWTH CURVES OF RACE B GUINEA-PIGS AND OF CAVIA CUTLERI.

In figure 2 are reproduced for comparison the growth curves of the females of both race B and Cutleri and along with these are plotted the growth curves of F_1 and of F_2 females produced by crossing Cutleri males with females of race B and race C. These curves show that F_1 females are larger at birth than females of either parent race and that they remain continuously larger having at maturity a size slightly larger than that of either parent race. That this increased size is not due to heredity, but to a growth stimulus produced by the crossing of unrelated parent races, a 'law of hybridization' formulated by Focke (1881), is shown by an examination of the F_2 growth curve, which is everywhere lower than the F_1 growth curve. The F_2 females are indeed, like F_1 females, at first larger than young of either pure race and continue this lead for three or four months, but from that time on their growth rate slows up notably and falls below that of race B so that at maturity they are nearly intermediate in size between the parent races. The F_2 females, as a group, have a rapid early growth like that of the Cutleri parent but lack the staying qualities of race B as regards growth so that from an age of about 150 days on they fall below race B in size. Accordingly, though the growth curves of both parent races and of the F_1 females are so far parallel in general course that they do not cross each other, the growth curve of the F_2 females crosses and falls below that of race B attaining at length an intermediate position between Cutleri and race B. This, so far as heredity is concerned, we may assume to be its true position, the added size of F_1 females being due not to heredity but to a growth impetus arising from hybridization, in accordance with Focke's law.

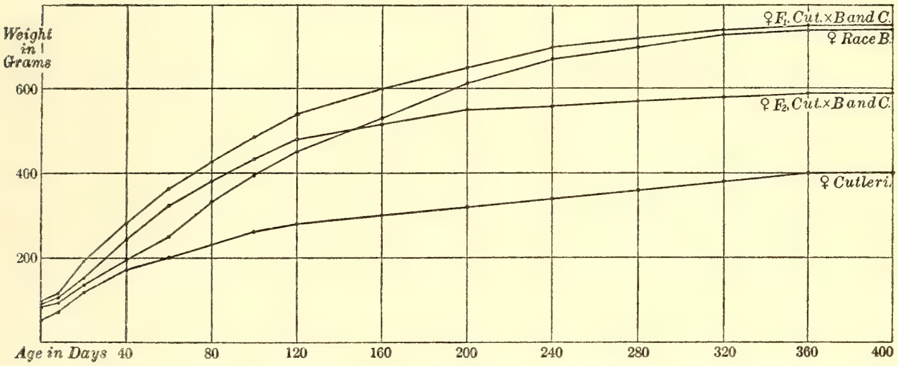


FIG. 2. GROWTH CURVES OF RACE B AND CUTLERI FEMALES AND OF THEIR FEMALE HYBRIDS BOTH F_1 AND F_2.

Figure 3 shows that the statements made concerning female hybrids are true also for male hybrids in crosses between Cutleri males and females of race B or race C. In this case the superior size of F_1 hybrids, as compared with the parent races, is even more strongly in evidence, while F_2 hybrids, though larger than either parent race for about four months, from that time on are of intermediate size. But for some reason, possibly not due to heredity, their ultimate size is nearer to that of race B than to that of the Cutleri parent.

It might be supposed that the size inheritance in crosses of a wild species with a domesticated race may be different from that in crosses between different domesticated races. To test this matter crosses were made between females of race B and a male guinea-pig obtained in the cabin of a native in Arequipa, Peru, this Arequipa guinea-pig being of a size about one half greater than that of race B males. The growth curve of race B males is shown in figure 4 where it can be compared with the growth curves of F_1 and F_2 male animals obtained by crosses of the Arequipa male with race B females. It will be observed that the growth curve of the F_1 males runs a course in general parallel with that of the growth curve for race B males but at a much higher level. The growth curve of the F_2 males starts in at a still higher level, perhaps in part because the vigorous F_1 mothers supply an abundance of nourishment to the young. But from weaning time on (age about 20–30 days) the F_2 males grow less rapidly. Their curve crosses the F_1 curve at 40 days and continues thence below it so that its ultimate position is

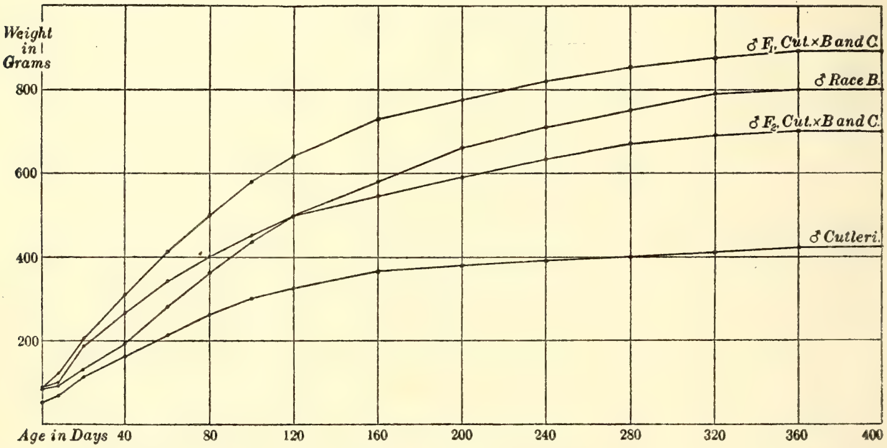


FIG. 3. GROWTH CURVES OF RACE B AND CUTLERI MALES AND OF THEIR MALE HYBRIDS BOTH F_1 AND F_2.

intermediate between the F_1 and the race B growth curves. It is evident that here, as in the Cutleri crosses, the F_1 animals are larger than heredity alone would make them, that they have an added size due to a growth stimulus produced by hybridization, but that this added stimulus is largely dissipated in F_2, which accordingly shows blended or intermediate heredity. It is evident that Mendelian dominance is wanting in these size crosses. It remains to inquire whether segregation also is wanting. The individual growth curves of F_1 and F_2 animals give no very clear evidence on this point. Too many environmental factors enter into the problem, such as time of year, food conditions, accumulation of fat in old age, and the like, which, while they do not affect the *average* growth curves already discussed, do obscure the question of segregation in individual cases. Bone measurements are believed to be less affected by extraneous influences than the growth curves of individuals. Consequently our inquiry as to evidences of segregation will be confined to them. Three different bone measurements have been studied for each full grown individual (age 13 months or over, in most cases). These are maximum skull-length, maximum skull-width, and femur length. The facts arrived at are contained in Tables 1–3.

In the first part of Table 1 is shown the variation in skull-length of 10 full grown Cutleri females and of 28 full grown race B females. Measurements were made with a caliper rule furnished with a vernier, and are accurate within 0.1 mm., the possible observational error shown by repeated measurement. The measurements are classified in classes of 0.5 mm. range numbered in order of increasing size. The lowest class, that numbered 1, would include measurements 48.5–48.9 mm.; class 2 would include measurements 49.0–49.4, etc. The absolute measurements included within each class of the table may thus be readily calculated, if desired. They are omitted for simplicity. The Cutleri females measured range in skull-length from class 1 to class 10. The race B females range from class 11 to class 25. Thus the two races do not overlap in range. Race B begins where Cutleri leaves off. Their means are 6.6 mm. apart.

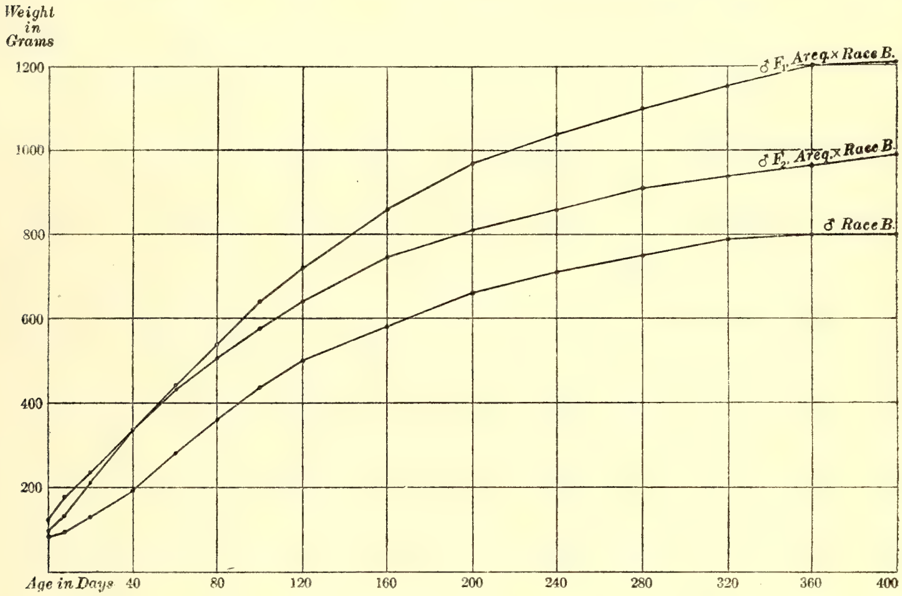


FIG. 4. GROWTH CURVES OF RACE B MALES AND OF THE MALE HYBRIDS BOTH F_1 AND F_2, PRODUCED BY A CROSS WITH THE AREQUIPA RACE.

TABLE 1

VARIATION IN SKULL-LENGTH OF CAVIA CUTLERI AND OF CERTAIN RACES OF GUINEA-PIGS AND OF THEIR HYBRIDS

CLASS 1 INCLUDES MEASUREMENTS 48.5–48.9 MM.

	0.5 MM. CLASSES																																						TOTALS	MEANS	σ
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38			
♀ C. Cutleri.........	1			2			2	2	2	1																													10	51.55	13.50
♀ Race B...........											1		1	2	2	1		2	1	3	3	4	3	1	4														28	58.14	19.75
♀ F_1 Cutleri × Race B												1		4		2		1	3	4	2	4	2	1															24	57.70	16.85
♀ F_2 Cutleri × Race B	1						1	3	2	1	1	6	7	4	1	2	3	1																					33	54.35	17.20
♂ C. Cutleri.........							2		2		2	1																											7	52.91	9.45
♂ Race B...........																		2		3	6	7	9	9	7	4	7	4	1	2	1	1							63	60.35	15.05
♂ F_1 Cutleri × Race B																				1		1	3	2	7	5	2	2	1	1	1								26	61.20	12.15
♂ F_2 Cutleri × Race B										1		1	1	1	2	2	3		3	4	4	1								1									24	57.26	20.00
♀ F_1 Areq. × Race B.																							1		3	4	3		2	2	3								18	61.92	12.10
♀ F_2 Areq. × Race B.															1		1		2	4	3	4	4	2	1	3	5	2	4	3	1	1							41	60.44	20.65
♂ F_1 Areq. × Race B.																										1	1		3	2	3	5	1	4	4		1	2	27	64.40	15.15
♂ F_2 Areq. × Race B.																		1				2	7	5	3	3	9	5	5	5	4	2	3	1	1				56	61.86	17.75

TABLE 2

Variation in Skull Width of Cavia Cutleri and of Certain Races of Guinea-Pigs and of Their Hybrids

Class 1 Includes Measurements 29.5–29.9 mm.

	0.5 mm. classes																											TOTALS	MEANS	σ
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27			
♀ C. Cutleri	2	2	2	1	2		1																					10	30.84	9.35
♀ Race B							1	3	4	4	3	8	1	2	2													28	34.68	10.56
♀ F_1 Cutleri × Race B							1		3	1	5	3	6	1	2	1	1											24	35.24	11.60
♀ F_2 Cutleri × Race B			1	1	1	1	8	9	5	2	3			2														33	33.26	11.45
♂ C. Cutleri			2		3	1	1																					7	31.63	6.80
♂ Race B									1	2	4	6	11	13	11	2	8	1	2	2								63	36.33	11.90
♂ F_1 Cutleri × Race B													1	4	2	2	5	5	2	3	1	1						26	37.79	11.70
♂ F_2 Cutleri × Race B							1		2	1	9	1	5	1	1	1	2											24	35.24	12.05
♀ F_1 Areq. × Race B															2	2	1	3	6	1	3							18	38.37	9.40
♀ F_2 Areq. × Race B												2	2	10	7	5	1	5	1	3	4		1					41	37.35	14.05
♂ F_1 Areq. × Race B																	5	1	5	2	5	3	3	2	1			27	39.40	11.95
♂ F_2 Areq. × Race B																3	10	7	17	5	4	4	3	2			1	56	38.87	11.40

TABLE 3

Variation in Femur-Length of Cavia Cutleri and of Certain Races of Guinea-Pigs and of Their Hybrids
Class 1 Includes Measurements 35.5–35.9 mm.

	0.5 mm. classes																								totals	means	σ
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24			
♀ C. Cutleri		1	1	1	1	1	1	3	3																12	38.45	12.05
♀ Race B							3		2	3	2	6	5	5	1	1	1								29	41.16	12.50
♀ F_1 Cutleri × Race B											2	2	2	4	2	4	6	1							23	42.63	10.35
♀ F_2 Cutleri × Race B	1					2	1	5	3	6	2	6	3		3		1								33	40.38	15.60
♂ C. Cutleri					2		2	2		1															7	38.77	8.20
♂ Race B									1		2	6	9	10	12	4	3	3	1	1					52	42.39	10.70
♂ F_1 Cutleri × Race B											1				4	2	5	3	4						19	43.57	9.80
♂ F_2 Cutleri × Race B								2	1	2	5	4	6		2	1						1			24	41.32	14.20
♀ F_1 Areq. × Race B														3	3	3	1	4	4	1	2		2		23	44.07	13.35
♀ F_2 Areq. × Race B										2	1	5	3	6	4	5	3	5	3	1	2	1			41	42.95	15.30
♂ F_1 Areq. × Race B																2	1	5		5	8	2	1	1	25	45.16	10.05
♂ F_2 Areq. × Race B										1	1	1	4	9	8	7	10	6	5	1			1		54	43.15	11.80

F_1 female hybrids between Cutleri and race B are practically as large in skull-length as race B females. Their means differ by less than half a millimeter. This harmonizes with the observations on general size as indicated by the weights given in the growth curves. The empirical range of the F_1 hybrids is from class 12 to class 24, just within the limits of variation of race B. The F_2 hybrids are considerably lower in mean skull-length than their F_1 parents: indeed they are strictly intermediate between pure Cutleri and race B, in complete agreement with the growth curves (fig. 2). Their mean skull-length is 54.35 mm.; the middle point betwen the skull-length of Cutleri and race B is just 0.5 mm. greater. Their range (if we leave out of consideration one aberrant individual) extends from the middle of the range of Cutleri to the middle of the range of race B, and would show no increase of variability over F_1 or race B. But the one aberrant individual makes F_2 more variable than F_1; it is as small as the smallest Cutleri female in skull-length and also in femur length (Table 3). It was however not so small in skull-width (Table 2). Nevertheless it might pass for a very good size segregate closely resembling the Cutleri ancestor. There can be no doubt about its hybrid origin or that it was a genuine F_2, not an accidental back-cross of F_1 with Cutleri, for it was of a color variety, cinnamon, which could not be obtained from a back-cross or from either uncrossed parental race. We shall consider its significance further.

In the second section of Table 1 are classified the skull-length measurements of Cutleri and race B males and of male hybrids between these races, both F_1 and F_2. These show that the skull-length of males regularly exceeds that of females of like ancestry but that the relations of race to race, as regards skull-length, are the same for males as for females. Cutleri and race B do not overlap in range of skull-length in either sex. Their F_1 hybrids equal or exceed race B in mean skull-length, but show no greater variability. F_2 is intermediate in skull-length between the parental races, lacking the vigor of F_1 due to crossing but not a matter of heredity. There is however, among the male F_2 hybrids, one aberrant individual with a long skull comparable with that of the largest race B animals and Table 3 shows that one of the F_2 males (the same one in fact) even surpassed race B males in femur length. But in Table 2 we look in vain for an aberrant F_2 male of unusual skull-width. The animal in question accordingly had an unusually long but not an unusually wide skull. It also had an unusually long femur. Obviously some agency is at work which produces variation in *length* of skull and femur without greatly affecting width of skull. The aberrant F_2 female as well as the aberrant F_2 male indicate this. Whether the hypothetical agency is genetic or purely physiological (like that which produces the large size of F_1) is at present uncertain. If it should prove to be genetic, it may conceivably be a Mendelizing factor of greater or less stability, like those which affect the shape of fruits in tomatoes, peppers, and squashes (Gross, Emerson). But this will remain a matter of uncertainty until further evidence is forthcoming. Aside from these two individuals with skeletons of unusual *form*, everything so far indicates that the inheritance of skeletal dimensions is completely blending with physiological increase of size in F_1, this being however not a matter of heredity.

We have already anticipated the result of the Cutleri × race B cross as regards skull-width. See Table 2. Here also the parent races scarcely overlap in range of variation. The mean of F_1 slightly exceeds that of race B. F_2 is intermediate between the parent races and scarcely more variable than F_1, with no very aberrant individuals.

In the lower portion of Tables 1–3 will be found the bone-measurements for the F_1 and F_2 generations produced by a cross between race B females and the Arequipa male, 1002, still living and so not available for bone measurements.

But bone measurements of a pair of animals of the Arequipa race are slightly less than the corresponding average measurements of the F_1 animals. Hence it is probable that a considerable physiological increase of size occurs in F_1 in this cross, as in the Cutleri cross, causing the production of an F_1 larger than either parent race. Comparison of the growth curves of F_1 and F_2 (figs. 3 and 4) and of their respective bone measurements confirms this idea. The mean of F_2 is in all measurements less than that of F_1, the difference amounting to one or two millimeters. This indicates that the increase of size due to crossing, seen in F_1, does not persist in F_2.

But the important question theoretically is whether there is evidence of the segregation or recombination in F_2 of distinct genetic size factors. If such segregation occurs, it might be expected to show itself either (1) in increased amplitude of the variation, provided that multiple factors occur which lack dominance but segregate and recombine independently of each other, or (2) in the formation of multi-modal F_2 variation curves with the production of isolated aberrant individuals, provided that the genetic factors concerned are few in number or show dominance.

As regards the first possibility, the F_2 animals do show somewhat greater variability than the F_1s, though the difference as measured by the standard deviation (σ) is not great. In the case of Cutleri × race

B females, the standard deviation of F_2 in skull-length (Table 1) is less than that of uncrossed race B females and is not half a millimeter greater than that of the F_1s. The difference can scarcely be regarded as significant. In skull-width the F_2 females have a smaller standard deviation than the F_1s, while in femur-length alone is F_2 larger than F_1 and either parent race.

Among the male hybrids produced by the Cutleri × race B cross, the evidence for increased variability in F_2 is rather better. In skull-length (Table 1) and femur-length (Table 3) the standard deviation of F_2 exceeds that of F_1 or either uncrossed race. In skull-width the difference is not significant.

It should be noted in passing that the very low standard deviations of pure Cutleri animals do not necessarily indicate lack of variability, but are to be explained in part as due to the small numbers of animals available for measurement. For Pearson has shown that with numbers less than 25, the empirical standard deviation is as a rule too small. It will be observed in Tables 1–3 that the highest standard deviations among the pure Cutleri animals are found where the numbers studied are largest. As between the F_1 and F_2 animals studied, the numbers are not sufficiently different to make any allowance of this sort necessary. In the Arequipa × race B crosses, F_2 has a higher standard deviation than F_1 in five out of six cases, the F_1 animals being more variable only in the skull-width measurements of males (Table 2).

We may conclude that on the whole F_2 shows consistently a higher variability than F_1 in the croses studied, which is in agreement with the observations made in numerous other hybridization experiments with both animals and plants. But it does not follow that the difference observed is due wholly to multiple genetic factors affecting size. For increased variability in F_2 would occur if the physiological increase of size observed in F_1 persisted to some extent in F_2 but persisted unequally (i.e., in different degrees) among the different F_2 zygotes. Now there is some reason to think that the non-genetic or excess vigor of F_1 does persist slightly into F_2, for in 5 out of 6 cases in the Cutleri × race B cross (Tables 1–3), the F_2 mean is *greater* than the intermediate point between the means of the uncrossed races. But it is clear that if this non-genetic vigor is found to a greater extent in some F_2 zygotes than in others, *it will increase the variability of the F_2 zygotes as a group.*

As against the multiple factor hypothesis it may be urged further that an increased variability of F_2 may be satisfactorily accounted for in still other ways without involving multiple factors, as for example by quantitative variation in a single factor affecting total growth energy of the zygote.

We come now to the alternative question whether evidence is forthcoming of a *few* genetic factors or of single genetic factors affecting size, showing themselves in multimodal variation or in aberrant individuals. There is in Tables 1–3 no evidence of multimodal variation. As regards aberrant individuals we have two and only two notable cases, which are found in F_2 of the Cutleri × race B cross, as already stated. These are (1) a female with very short skull and femur closely resembling pure Cutleri females in these measurements, but not in skull-width, and (2) a male with very long skull and femur, closely resembling race B animals in these respects but not in skull-width. It seems quite possible that some special factor affecting duration of growth is concerned in the production of these extreme individuals. Thus castration of cattle is known to permit abnomally prolonged growth of the skeleton *in length.* Early sexual maturity is probably correlated with an opposite change. Whether physiological factors affecting the sexual system are concerned in the production of these unusual individuals, or whether genetic agencies are concerned is at present uncertain. It is regrettable that the exceptional character of these two individuals was not recognized while they were still alive and so capable of genetic tests. Possibly further investigations now in progress may throw light on these questions.

If the foregoing reasoning is sound, we have from these crosses no evidence showing either the existence of *numerous* multiple Mendelian factors affecting size, or the existence of a *few* Mendelian factors affecting size, or the existence of a *single* Mendelian factor affecting size. We have however clear evidence of a physiological factor causing a marked increase of size in F_1 and probably persisting to a small extent in F_2 and so increasing slightly the variability of F_2. This factor, though its existence has long been known, has been largely neglected in those recent studies of size inheritance which seek to give a Mendelian interpretation of the phenomena observed in size crosses, and it is quite possible that this neglect has caused erroneous conclusions to be reached.

Castle, W. E., and H. E. Walter, R. C. Mullenix, and S. Cobb, Studies of inheritance in rabbits, Carnegie Inst. of Wash., *Publ.* No. 114 (1909).

Lang, A., Die Erblichkeitsverhältnisse der Ohrenlänge der Kaninchen nach Castle and das Problem der intermediaren Vererbung and Bildung konstanter Bastardrassen. *Zs. indukt. Abst. Vererbungs-lehre,* **4**, 1–23. (1910.)

Nilsson-Ehle, H., Kreuzungsuntersuchungen an Hafer und Weizen, I. *Lunds Universitats Årsskrift,* N. F., 7 (1909).

Phillips, J. C., Inheritance of size in ducks. *J. Exp. Zool.*, **12**, 193 (1911); A further study of size inheritance in ducks with observations on the sex ratio of hybrid birds. *Ibid.*, 16, 131–148 (1914).

Punnett, R. C., and P. G. Bailey, On inheritance of weight in poultry. *J. Genetics*, **4**, 23–39, (1914) Pl. IV.

CONTENTS

Page

PRELIMINARY REPORT UPON THE POSSIBILITY OF CONTROLLING THE LAND SLIDES ADJACENT TO THE PANAMA CANAL 193

ASTRONOMY.—DISCOVERY OF EIGHT VARIABLE STELLAR SPECTRA . . . . . *By Harlow Shapley* 208

MATHEMATICS.—ON THE LINEAR DEPENDENCE OF FUNCTIONS OF SEVERAL VARIABLES, AND CERTAIN COMPLETELY INTEGRABLE SYSTEMS OF PARTIAL DIFFERENTIAL EQUATIONS . . . . *By Gabriel M. Green* 209

ASTRONOMY.—SYSTEMATIC MOTION AMONG STARS OF THE HELIUM TYPE . . . . . *By Benjamin Boss* 214

CHEMISTRY.—THE ABUNDANCE OF THE ELEMENTS IN RELATION TO THE HYDROGEN-HELIUM STRUCTURE OF THE ATOMS . . . . . *By William D. Harkins* 216

ANTHROPOLOGY.—THE GENETIC RELATIONS OF CERTAIN FORMS IN AMERICAN ABORIGINAL ART . . . . *By Clark Wissler* 224

ASTRONOMY.—THE SITUATION IN REGARD TO ROWLAND'S PRELIMINARY TABLE OF SOLAR SPECTRUM WAVE-LENGTHS . . . . *By Charles E. St. John* 226

ASTRONOMY.—CHANGES IN THE FORM OF THE NEBULA N. G. C. 2261 . . . . . . *By Edwin P. Hubble* 230

ZOOLOGY.—ON THE EFFECT OF REMOVAL OF THE PRONEPHROS OF THE AMPHIBIAN EMBRYO . . . . *By Ruth B. Howland* 231

PALEONTOLOGY.—ON THE PRESENCE OF A MEDIAN EYE IN TRILOBITES . . . . . *By Rudolph Ruedemann* 234

BOTANY.—THE NATURE OF MECHANICAL STIMULATION . . . *W. J. V. Osterhout* 237

GENETICS.—HEREDITARY REACTION-SYSTEM RELATIONS—AN EXTENSION OF MENDELIAN CONCEPTS . . . . *By R. E. Clausen and T. H. Goodspeed* 240

MATHEMATICS.—POINT SETS AND ALLIED CREMONA GROUPS (PART II) . . . . *By Arthur B. Coble* 244

MATHEMATICS.—ON A THEOREM OF LUCAS . . . . *By M. B. Porter* 247

MATHEMATICS.—INTERPRETATION OF THE SIMPLEST INTEGRAL INVARIANT OF PROJECTIVE GEOMETRY . . . . *By E. J. Wilczynski* 248

GENETICS.—SIZE INHERITANCE IN GUINEA-PIG CROSSES . . . *By W. E. Castle* 252

VOLUME 2 MAY, 1916 NUMBER 5

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES
OF THE
UNITED STATES OF AMERICA

JUN

EDITORIAL BOARD

Arthur A. Noyes, *Chairman* Edwin B. Wilson, *Managing Editor*
Arthur L. Day, *Home Secretary* George E. Hale, *Foreign Secretary*

J. J. Abel
W. B. Cannon
J. McK. Cattell
J. M. Clarke
E. G. Conklin
J. M. Coulter
C. B. Davenport
Simon Flexner
E. B. Frost
R. G. Harrison
W. H. Holmes
J. P. Iddings
R. A. Millikan
E. H. Moore
Alexander Smith

Publication Office: Williams & Wilkins Company, Baltimore
Editorial Office: Massachusetts Institute of Technology, Boston
Home Office of the Academy: Washington, D. C.

Entered as second-class matter at the Post Office at Baltimore, Md., under the Act of August 24, 1912

INFORMATION TO CONTRIBUTORS

The Proceedings is the official organ of the Academy for the publication of brief accounts of important current researches of members of the Academy and of other American investigators, and for reports on the meetings and other activities of the Academy. Publication in the Proceedings will supplement, not replace, that in journals devoted to the special branches of science. The Proceedings will aim especially to secure prompt publication of original announcements of discoveries and wide circulation of the results of American research among investigators in other countries and in all branches of science.

Articles should be brief, not to exceed 2500 words or 6 printed pages. Technical details of the work and long tables of data should be reserved for publication in special journals. But authors should be precise in making clear the new results and should give some record of the methods and data upon which they are based. The viewpoint should be comprehensive in giving the relation of the paper to previous publications of the author or of others and in exhibiting where practicable, the significance of the work for other branches of science.

Manuscripts should be prepared with a current number of the Proceedings as a model in matters of form, and should be typewritten in duplicate with double spacing, the author retaining one copy. Illustrations should be confined to text-figures of simple character, though more elaborate illustrations may be allowed in special instances to authors willing to pay for their preparation and insertion. Particular attention should be given to arranging tabular matter in a simple and concise manner.

References to persons should include initials; citations of literature, numbered consecutively, should be placed at the end of the article; and short footnotes should be avoided. Indicate 'subject' for 'Contents.'

Papers by members of the Academy may be sent to Edwin Bidwell Wilson, Managing Editor, Mass. Institute of Technology, Boston, Mass. Papers by non-members should be submitted through some member.

Proof will be sent to authors only on their request, in order to avoid delay in publication. Authors are therefore requested to make final revisions on the typewritten manuscripts. The editors cannot undertake to do more than correct obvious minor errors.

Reprints should be ordered at the time of submission of manuscript. They will be furnished to authors at cost, approximately as follows:

Reprints of - -	2 pp.	4 pp.	6 pp.	8 pp.	Covers extra
Charge for first 100 copies	$1.00	$1.30	$2.20	$2.20	$2.00
Charge for additional 100s	.25	.45	.85	.85	.50

INFORMATION TO SUBSCRIBERS

Subscriptions at the rate of $5.00 per annum should be made payable to the National Academy of Sciences, and sent to Williams & Wilkins Company, Baltimore, or Arthur L. Day, Home Secretary, National Academy of Sciences, Smithsonian Institution, Washington, D. C. Single numbers, $0.50.

Copyright, 1916, by the National Academy of Sciences

Smithsonian Institution
JUN 24 1921
National Museum

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES

Volume 2 MAY 15, 1916 Number 5

THE HIGH FREQUENCY SPECTRUM OF TUNGSTEN

By Albert W. Hull and Marion Rice
RESEARCH LABORATORY, GENERAL ELECTRIC COMPANY

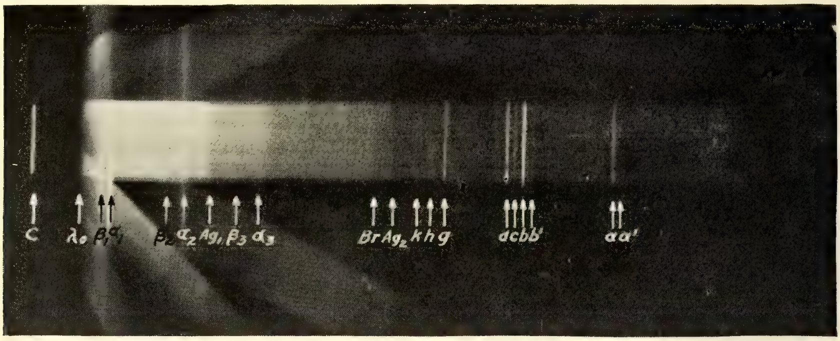
Received by the Academy, March 20, 1916

Mosely[1] has shown that the high frequency spectrum given out by the target of an X-ray tube consists of two series of lines superimposed upon a continuous spectrum. The lines, which are known as the *K* and *L* series, respectively, are characteristic of the material of the target. Mosely measured the wave lengths of most of the *K* lines for elements having atomic weights between aluminum and silver, and of the *L* lines for elements from calcium to gold, and showed that for all the lines measured the square roots of the frequencies of corresponding lines are proportional to the atomic numbers of the elements emitting them. Malmer[2] has added to this list the *K* lines of six more elements between silver and lanthanum, and W. H. Bragg[3] and others have studied in great detail the lines of a few of these elements, especially rhodium and platinum.

The continuous or band spectrum was observed qualitatively by Mosely (l.c.), and its short wave-length limit at different voltages measured by Duane and Hunt,[4] who found this limiting frequency, ν_{max}, to be exactly proportional to the voltage on the tube, and given accurately by the quantum relation $h\nu_{\text{max}} = eV$, where V is the voltage on the tube, e the charge of an electron, and h Planck's constant.

The spectrum of tungsten is of special interest on account of its use as target material in X-ray tubes, and it has been the subject of several recent investigations. Barnes[5] measured the *L* lines, but was unable to find any *K* lines, although his voltage, 96,000, was sufficiently high for their excitation. Gorton[8] also measured the *L* lines. Rutherford, Barnes and Richardson,[6] using the coefficient of absorption method, measured the effective wave-length of the 'end radiation,' i.e., the short wave-length limit of the continuous spectrum, for different voltages up to 180,000, and found that this minimum wave-length did not decrease continuously with increase of voltage, but approached asymptotically a limiting value of 0.172 angstrom units. As will be shown below, the wave-lengths found by the spectrometer are much shorter, and do not appear to approach any limiting value.

The spectrum shown in figure 1 was taken in the usual manner with a rock salt crystal in continuous slow rotation, photographic plate stationary at 19.13 cm. distance from the crystal, collimating slits 0.2 mm. wide and 20 cm. apart, with a Coolidge tube running at 1 milliampere and 100,000 volts constant potential. The horizontal band gives the reflection from the cubic (100) planes of the crystal, those at 45° from the dodecahedral (110) planes, and those between from the tetrahexahedral (210) planes, etc.



WAVE-LENGTHS OF LINES SHOWN IN FIG. 1

L = Designation of line, *D* = Distance from *C* in cm., λ = Wave-length in angstroms

L	λ_0	β_1	α_1	β_2	α_2		Ag_1	β_3	α_3		Br
D	0.964	1.332	1.496	2.614	2.932	2.994	3.332	3 948	4.386	4.494	6.524
λ	0.142	0.196	0.219	0.192	0.215	0.220	0.488	0.192	0.212	0 217	0.463

L	Ag_2	k	h		g	d	c	b	b′	a	a′
D	6.886	7.368	7.604	7.662	7.884	9.074	9.224	9.382	9.558	11.10	11.19
λ	0.487	1.033	1.065	1.073	1.100	1.242	1.260	1.280	1.300	1.468	1.480

The wave-lengths of the lines and bands in the horizontal strip are given in the accompanying table. *C*, in the photograph, is the undeviated central beam, diminished in intensity by passage through a lead strip 2 mm. thick and 5 mm. wide, placed in front of the plate. The shadow of this strip extends only one fourth of the distance from *C* to λ_0 and is not visible on the plate.

The short wave-length limit of the spectrum, marked λ_0, is the shortest wave-length that is produced by electrons of velocity corresponding to the operating voltage (100,000) and its value, 0.142 angstrom units, agrees very closely with the value given by Duane and Hunt's formula $eV = h\nu_{\text{max}} = hc/\lambda_{\text{min}}$. I have already shown[7] that the proportionality between frequency and voltage holds accurately up to 100,000 volts, and these measurements have since been extended, with less accuracy, up to 150,000 volts. The shortest wave-length so far observed is 8×10^{-10} cm., or 0.08 A.U.

The lines marked $\alpha_1\beta_1$, $\alpha_2\beta_2$, and α_3 β_3 are the first, second, and third order reflections respectively of the K_α and K_β lines of tungsten, the α line being a doublet. They are more clearly shown in figure 2, which was taken with narrower slits and greater distance of photographic plate from crystal, so that the doublet is clearly resolved. In this photograph the second order was given an extra exposure, which accounts for the apparent band in the middle. In the photograph of figure 1, all parts were given the same exposure. The wave-lengths of the lines are about 6% less than would be required by Mosely's formula. Slight deviations from the formula have already been noted for the K lines measured by Mosely and Malmer. All these lines, including tungsten, can, however, be correctly represented by the empirical formulae

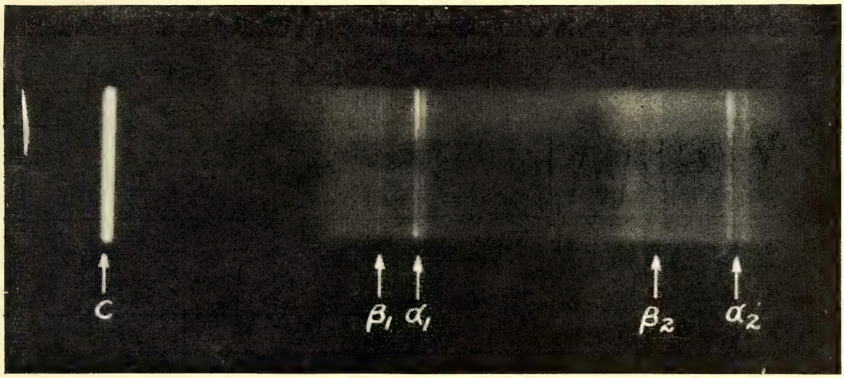


FIG. 2

$$\nu_\alpha = 1.64 \times 10^{15} N^{2.10} \text{ for the } \alpha \text{ lines,}$$

and

$$\nu_\beta = 1.56 \times 10^{15} N^{2.15} \text{ for the } \beta \text{ lines,}$$

where ν is the frequency and N the atomic number. These formulae, while they have no theoretical significance, may be useful for interpolation.

The bands which terminate on the long wave-length side at Ag_1 and Ag_2 are due to the silver in the photographic plate and are produced by the reflection, in the first and second order respectively, of those wave-lengths which are capable of stimulating the characteristic radiation of silver, that is, those which are shorter than the gamma line of silver. The edge of the band therefore marks the position of the γ line of silver, 0.488 A.U. In the same way the band terminated at Br is

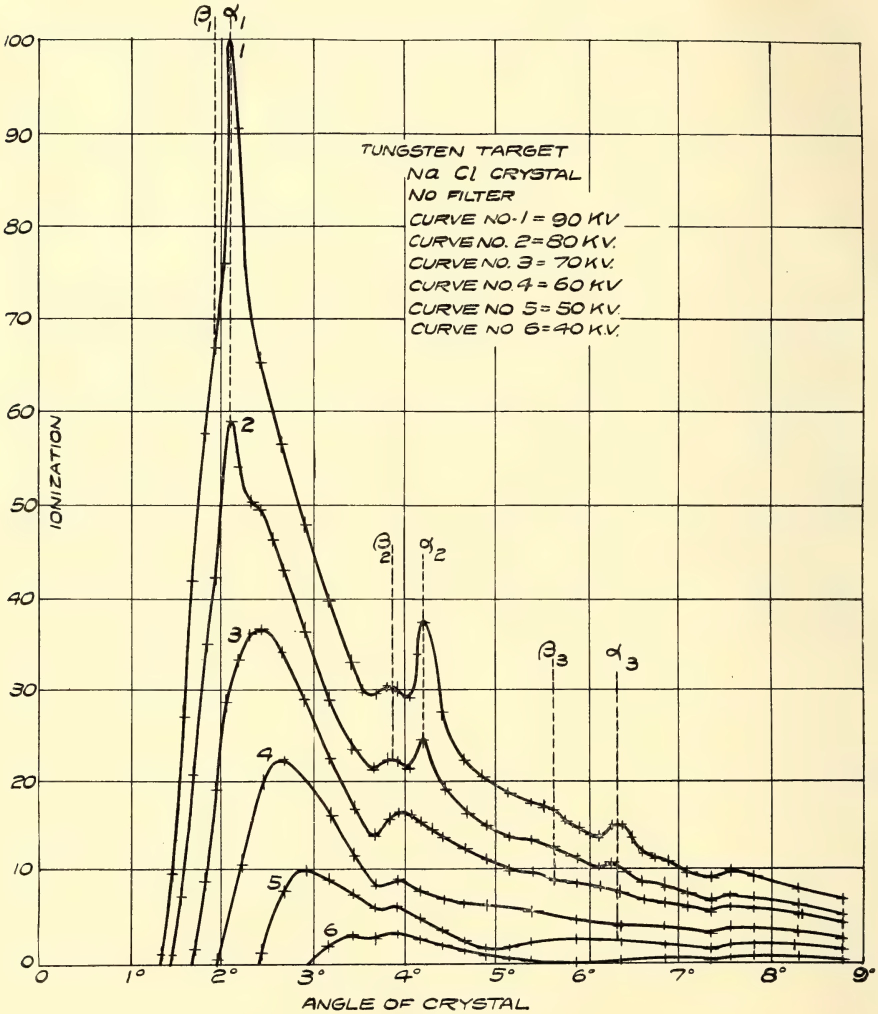


FIG. 3

due to the bromine in the photographic plate, and marks the position of the γ line of bromine.

The remaining nine lines, *a* . . . .*k*, belong to the *L* spectrum of tungsten, and agree with those found by Barnes,[5] with the inclusion of two faint lines not observed by Barnes. The line marked *h* appears to be a doublet. Gorton's[8] values are all about 2% smaller.

The position of the K lines, and their relation to the general radiation at different voltages, has been studied by means of the ionization chamber also. Figure 3 shows the ionization current as a function of the angle of incidence of the rays on the crystal, for five different voltages, and figure 4 a part of the same in the second order. The position of the K lines in the first, second and third order respectively, is shown by the dotted lines marked α_1, β_1, α_2, β_2, etc. There is no trace of the lines at 70,000 volts, but at 80,000 they are clearly visible, and increase in intensity as the voltage increases. It is probable that the lowest voltage at which the lines appear is the 'quantum' voltage, i.e., that given by Duane and Hunt's equation, for the γ line, about 70,000 volts for tungsten. This would be in harmony with the mechanism of radiation suggested by Kossel[9] and has already been found by Webster[10] to hold for rhodium. It may be mentioned that the quantum relations

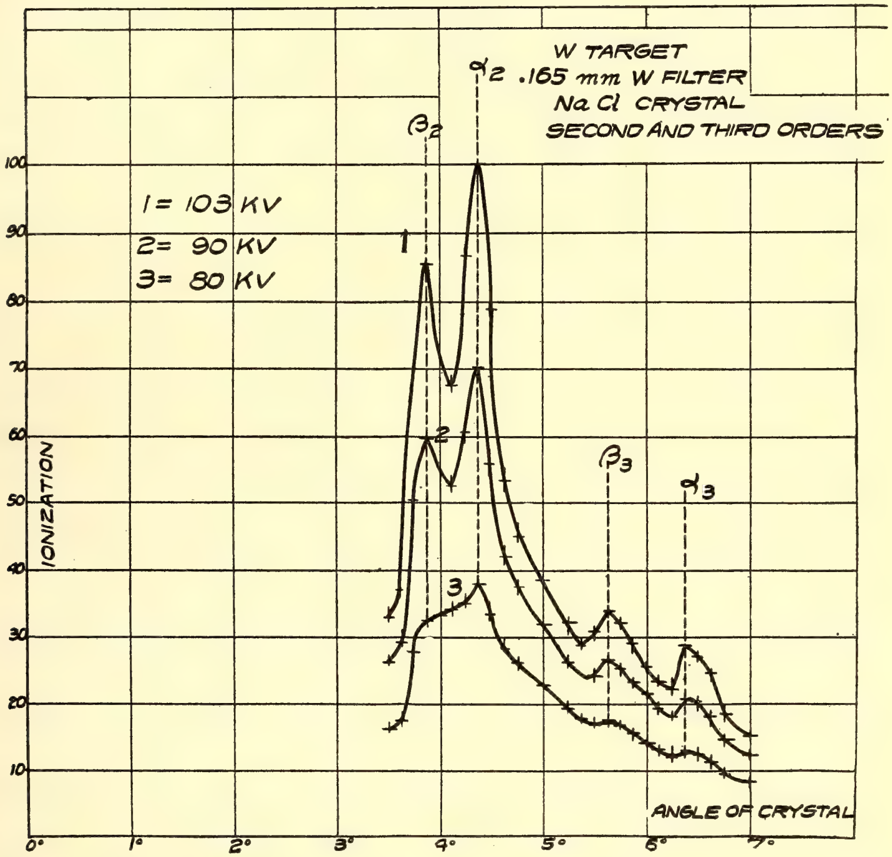


FIG. 4

established by Kossel between the frequencies of the K and L lines hold true for the tungsten lines.

A more detailed account of these and other experiments on the tungsten spectrum, including the distribution of energy in the continuous spectrum, will be published shortly in the *Physical Review*.

---

[1] Mosely, *Phil. Mag.*, **26**, 210 and 1024 (1913); **27**, 710 (1914).

[2] I. Malmer, *Phil. Mag.*, **28**, 787 (1914).

[3] W. H. Bragg, *Phil. Mag.*, **29**, 407 (1915).

[4] Duane & Hunt, *Physic. Rev.*, **6**, 166 (1915).

[5] Barnes, *Phil. Mag.*, **30**, 368 (1915).

[6] Rutherford, Barnes and Richardson, *Phil. Mag.*, **30**, 339 (1915).

[7] Hull, *Physic. Rev.*, **7**, 156 (1916).

[8] Gorton, *Physic. Rev.*, **7**, 203 (1916).

[9] Kossel, *Ber. D. Physik. Ges.* **16**, 953 (1914).

[10] Webster, these Proceedings, **2**, 90 (1916).

ON THE FOUNDATIONS OF PLANE ANALYSIS SITUS

By Robert L. Moore

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF PENNSYLVANIA

Received by the Academy, March 30, 1916

The notions point, line, plane, order, and congruence are fundamental in Euclidean geometry. Point, line and order (on a line) are fundamental in descriptive geometry. Point, limit-point and regions (of certain types) are fundamental in analysis situs. It seems desirable that each of these doctrines should be founded on (developed from) a set of postulates (axioms) concerning notions that are fundamental for that particular doctrine. Euclidean geometry and descriptive geometry have been so developed.[1] The present paper contains two systems of axioms, Σ_2 and Σ_3, each of which is sufficient for a considerable body of theorems in the domain of plane analysis situs. The axioms of each system are stated in terms of a class, S, of elements called *points* and a class of sub-classes of S called *regions*.

On the basis of Σ_2, *the existence of simple continuous arcs*[2] *is proved as a theorem.*

The system Σ_2 contains an axiom (Axiom 1) which postulates the existence of a countable sequence of regions containing a set of subsequences that close down in a specified way on the points of space. Among other things this axiom implies that the set of all points is separable.[3]

The system Σ_3 is obtained from Σ_2 by replacing Axioms 1, 2, and 4 by three other axioms, Axioms 1′, 2′, and 4′ respectively. Here Axiom 1′ postulates the existence, *for each point P*, of a countable sequence of regions that closes down on P, Axiom 2′ postulates that every two points of a region are the extremities of at least one simple continuous arc that lies in that region, and Axiom 4′ postulates that every region plus its boundary possesses the Heine-Borel property.

An open curve is defined as a closed, connected[4] set of points K such that if P is any point of K then $K-P$ is the sum of two mutually exclusive connected point-sets neither of which contains a limit point[4] of the other one. It is proved that every open curve is unbounded[4] and divides the set of all remaining points into two domains.

Though Σ_3 is a sufficient basis for a very considerable part of plane analysis situs, nevertheless there exist spaces that satisfy Σ_3 but are neither metrical, descriptive[5] nor separable.

It is interesting that no space that satisfies Σ_3 can be potentially descriptive without being also separable and potentially metrical. Indeed if to Σ_3 there be added the axiom that there exists a system of open curves such that through every two points there is one and only one curve of this system, the resulting set of axioms is categorical with respect to *point* and *limit point of a point-set.*

Every space that satisfies Σ_2 satisfies also Σ_3, but not conversely. In every space satisfying Σ_2 there exists infinitely many open curves through any two given points. I have not however determined whether every such space is descriptive.

Definitions.—A point P is said to be a *limit point* of a point-set M if, and only if, every region that contains P contains at least one point of M distinct from P. The *boundary* of a point-set M is the set of all points $[X]$ such that every region that contains X contains at least one point of M and at least one point that does not belong to M. If M is a set of points, M' denotes the point-set composed of M plus its boundary. A set of points K is said to be *bounded* if there exists a finite set of regions $R_1, R_2, R_3 \ldots R_n$ such that K is a sub-set of $(R_1+R_2+R_3+\ldots+R_n)'$. If R is a region the point-set $S-R'$ is called the *exterior* of R. A set of points is said to be *connected* if however it be divided into two mutually exclusive sub-sets, one of them contains a limit point of the other one.

The System Σ_2.—Axiom 1. *There exists an infinite sequence of regions,* $K_1, K_2, K_3, \ldots$ such that (1) if m is an integer and P is a point, there exists an integer n greater than m, such that K_n contains P, (2) if P and $\bar{P}$ are distinct points of a region R, then there exists an integer δ *such that if* $n > \delta$ *and* K_n *contains* P *then* K_n' *is a subset of* $R-\bar{P}$.

Axiom 2. *Every region is a connected set of points.*

Axiom 3. *If* R *is a region,* $S-R'$ *is a connected set of points.*

Axiom 4.[6] *Every infinite set of points lying in a region has at least one limit point.*

Axiom 5. *There exists an infinite set of points that has no limit point.*

Axiom 6.′ *If R is a region and AB is an arc such that AB–A is a subset of R then* $(R+A)-AB$ *is a connected set of points.*

Axiom 7′. *Every boundary point of a region is a limit point of the exterior of that region.*

Axiom 8. *Every simple closed curve is the boundary of at least one region.*

The System Σ_3.—The system Σ_3 is composed of Axioms 1,′ 2,′ 3, 4′, 5, 6,′ 7,′ and 8, where Axioms 1′, 2′, and 4′ are as follows:

Axiom 1′. *If P is a point, there exists an infinite sequence of regions,* R_1, R_2, R_3, . . . *such that* (1) *P is the only point they have in common,* (2) *for every n,* R_{n+1} *is a proper subset of* R_n, (3) *if R is a region containing P then there exists n such that* R_n' is a subset of R.

Axiom 2′. *If A and B are two distinct points of a region R then there exists, in R, at least one simple continuous arc from A to B.*

Axion 4′. *If R is a region, R′ possesses the Heine-Borel property.*

An example of a system satisfying Σ_2 is obtained if in ordinary Euclidean space of two dimensions, the term region is applied to every bounded connected set of points M, of connected exterior, such that every point of M is in the interior of some triangle that lies wholly in M.

Details (including a third system of axioms, the system Σ_1) will appear in *Transactions of the American Mathematical Society*, probably in April, 1916.

---

[1] Cf., among others, D. Hilbert, *The Foundations of Geometry* (translation by E. J. Townsend, Chicago, 1902) and O. Veblen, A system of axioms for geometry, *Trans. Amer. Math. Soc.*, **5**, 343 (1904).

[2] Lennes defines a continuous simple arc connecting two points A and $B, A \neq B$, as a bounded, closed, connected set of points $[A]$ containing A and B such that no proper connected subset of $[A]$ contains A and B. Cf. N. J. Lennes, *Amer. J. Math.*, **33**, 308 (1911).

[3] A set of points M is said to be *separable* if it contains a countable subset K such that every point of M is a limit point of K. See M. Fréchet, Sur quelques points du calcul fonctionnel, *Palermo, Rend. Circ. Mat.*, **22**, 6 (1906).

[4] For definitions of these terms see below.

[5] A space S is said to be *descriptive*, or *potentially descriptive*, if it contains a system of open curves such that through every two points of S there is one and only one curve of this system.

[6] In view of a result due to F. Hausdorff, it is clear that in the presence of the other axioms of Σ_2, Axiom 4 is equivalent to the axiom that if R is a region then R' possesses the Heine-Borel property. See F. Hausdorff, Grundzüge der Mengenlehre, Veit and Comp., Leipzig, 1914.

A GENERAL THEORY OF SURFACES

By Edwin B. Wilson and C. L. E. Moore

DEPARTMENT OF MATHEMATICS, MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Received by the Academy, March 23, 1916

Introduction.—Although the differential geometry of a k-dimensional spread or variety V_k, embedded in an n-dimensional space S_n, has received considerable attention, in particular in the case $k = n-1$, the theory of surfaces V_2 in the general Euclidean hyperspace S_n has been treated extensively by only three authors, Kommerell, Levi, and Segre,[1] of whom the last was interested in projective properties, the first two in ordinary metric relations. The theory of V_2 in S_n does, however, offer points of contact with elementary differential geometry which are fully as illuminating as those developed in the case of V_{n-1} in S_n. We have, therefore, undertaken to provide a general theory of surfaces which shall be independent of the number of dimensions of the containing space.

The first thing to be determined in entering on such an extended study is the method of attack. The available methods were four: (1) The ordinary elementary method of starting with the finite equations of the surface and trying to generalize well-known geometric properties. This was followed by Kommerell. (2) The more advanced method of Levi, which depends upon the finite equations only for calculating certain invariants I of rigid motion and further invariants (covariants) J of transformations of parameters upon the surface. (3) The vectorial method of Gibbs and others which bases the work upon the properties of the linear vector function expressing the relation between an infinitesimal displacement in the surface and the infinitesimal change in the unit normal $(n-2)$-space or the unit tangent plane. (4) The method of Ricci's absolute differential calculus, or Maschke's symbolic invariantive method, which develops the theory from the point of view of the fundamental quadratic differential forms defining the surface.

We adopted the last method because of the possibility of sharply differentiating (1) those properties of surfaces which depend on the first fundamental form, and belong to the surface as any one of the infinite class of mutually applicable surfaces, from (2) the properties which follow from the first and second fundamental forms taken together and which belong to the surface as a rigid surface. The adoption of Ricci's rather than Maschke's form of analysis was further dictated by the fact that Ricci[2] had developed the ordinary theory of surfaces consistently by his method, and all his work upon the first part (1), holding without change in any number of dimensions, could stand without repetition. Our own work therefore need only begin with the study of the second fundamental form. As, however, Ricci's absolute differential calculus is neither well-known nor published in particularly accessible form, it seemed best to prefix to our work an explanation of that calculus.

The second fundamental form.—If $\mathbf{y}\ (u, v)$ be the vector which determines the points on the surface, the second fundamental form is found to be a vector form

$$\Psi = \mathbf{y}_{11} dx_1^2 + 2\,\mathbf{y}_{12}\, dx_1 dx_2 + \mathbf{y}_{22} dx_2^2 = \Sigma_{rs}\, \mathbf{y}_{rs}\, dx_r dx_s,$$

where $\mathbf{y}_{11}$, $\mathbf{y}_{12}$, $\mathbf{y}_{22}$ are the second covariant derivatives of $\mathbf{y}$ with respect to u and v and are vectors normal to the surface. If ds denote a differential of arc and

$$\lambda^{(r)} = dx_r/ds, \qquad \lambda'^{(r)} = dx_r/ds', \qquad r = 1, 2,$$

be the differential equations of a system of curves λ on the surface and of their orthogonal trajectories, the three vectors α, β, μ,

$$\alpha = \Sigma_{rs}\, \lambda^{(r)}\, \lambda^{(s)}\, \mathbf{y}_{rs}, \qquad \beta = \Sigma_{rs}\, \lambda'^{(r)}\, \lambda'^{(s)}\, \mathbf{y}_{rs},$$
$$\mu = \Sigma_{rs}\, \lambda'^{(r)}\, \lambda^{(s)}\, \mathbf{y}_{rs} = \Sigma_{rs}\, \lambda^{(r)}\, \lambda'^{(s)}\, \mathbf{y}_{rs},$$

are invariant of the parameter system on the surface and are associated with a direction λ. Further

$$\mathbf{y}_{rs} = \alpha\, \lambda_r\, \lambda_s + \mu\, (\lambda_r\, \lambda' + \lambda'_r\, \lambda_s) + \beta\, \lambda'\, \lambda',$$

where λ_r is the dual or reciprocal system to $\lambda^{(r)}$ defined by $\lambda_r = \Sigma_s a_{rs}\, \lambda^{(s)}$ if $ds^2 = \Sigma_{rs}\ a_{rs}\ dx_r\ dx_s$ be the first fundamental form.

The geometric interpretation of α is the normal curvature of the surface in the direction λ (i.e., the curvature of the geodesic tangent to λ); of β, the same for the perpendicular direction; of μ, the rate of change of a unit vector drawn in the surface perpendicular to the geodesic tangent to λ. The Gaussian or total curvature G is $G = \alpha \cdot \beta - \mu^2$, the difference of the scalar product of α and β and the square of μ; this is independent of the direction λ and is one of the prime invariants of the surface. There is a vector $\mathbf{h}$ defined by the equation

$$2\mathbf{h} = \alpha + \beta = \Sigma_{rs} a^{(rs)}\, \mathbf{y}_{rs}, \qquad a^{(rs)} = (-1)^{r+s} a_{rs} / \left| a_{rs} \right|,$$

which is also independent of the direction λ and which we call the mean (vector) curvature of the surface at the point considered; the magnitude of $\mathbf{h}$ is another of the prime invariants of the surface.

If $\mathbf{M}$ be a unit (vector) tangent plane to the surface, and $d\mathbf{M}$ its differential, the second fundamental (vector) form Ψ may be written

$\Psi = -(d\mathbf{y}\cdot\mathbf{M})\cdot d\mathbf{M}$, where the dot denotes the inner product,[3] in complete analogy with the expression $-d\mathbf{y}\cdot d\mathbf{n}$, in terms of the unit normal **n**, in ordinary surface theory. Moreover, the square of $d\mathbf{M}$ is $(d\mathbf{M})^2 = -Gds^2 + 2\mathbf{h}\cdot\Psi$, and thus gives what may be called the third fundamental form. (In the three-dimensional case, this is associated with the spherical representation.)

The indicatrix.—If $\delta = \frac{1}{2}(\alpha - \beta)$ and if accents be used to denote quantities corresponding to a direction at an angle θ to λ, we find

$$\mu' = \mu \cos 2\theta - \delta \sin 2\theta, \qquad \delta' = \delta \cos 2\theta + \mu \sin 2\theta.$$

This means that when we turn about a point of the surface the vectors μ', δ' describe an ellipse (with center at the extremity of the mean curvature **h**) of which any two positions of μ', δ' are conjugate radii, the eccentric angle between μ', μ or δ', δ being twice the angle θ. This ellipse we call the indicatrix.[4] The vectors α', β' originating at the surface-point O and terminating in the indicatrix describe a cone of normals, Cone I.

The indicatrix is determinative of a large number of properties connected with the curvature of a surface at a point. As the surface is two-dimensional and the indicatrix with the surface-point O determines a normal three-dimensional space, the properties of curvature are as general for a surface V_2 in a five-dimensional space S_5 as in S_n, and the further developments may be given in the assumption that $n = 5$.

Consecutive normal plane three-spaces N_3 intersect in a line. These normal lines all pass through a point O' lying upon the perpendicular from O, to the plane of the indicatrix and generate a quadric cone, Cone II. The relation between Cones I and II is reciprocal in the sense that each element of either is perpendicular to some tangent plane of the other. Hence if Φ be the linear vector function which occurs in the equation $\mathbf{r}.\Phi.\mathbf{r} = 0$ of Cone II, referred to its vertex, the equation of Cone I, referred to its vertex, is $\mathbf{r}\cdot\Phi^{-1}\cdot\mathbf{r} = 0$.

The function Φ may be written as $\Phi = (\mathbf{hh} - \mu\mu - \delta\delta)/a$, where $a = |a_{rs}|$, in terms of the mean curvature **h** and a pair of conjugate radii of the indicatrix. Φ is the self conjugate part of the function

$$\Omega = \begin{vmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{vmatrix} = \mathbf{y}_{11}\,\mathbf{y}_{22} - \mathbf{y}_{12}\,\mathbf{y}_{21}$$

formed as the determinant (with vector elements) of the second form Ψ. We have

$$\Phi = \tfrac{1}{2}\,\mathbf{y}_{11}\,\mathbf{y}_{22} + \tfrac{1}{2}\,\mathbf{y}_{22}\,\mathbf{y}_{11} - \mathbf{y}_{12}\,\mathbf{y}_{21},$$

and the total curvature G is

$$G = \alpha \cdot \beta - \mu^2 = \Phi_s/a = \Omega_s/a,$$

the ratio of the first scalar invariant of Φ, or of the determinant Ω of the second form Ψ, to the determinant a of the first form.

If the surface V_2 in S_5 be projected successively upon the three spaces S_3 determined by the tangent plane and each of three mutually perpendicular normals issuing from O into the normal space N_3, the three projections have the (vector) sum of their mean curvatures and the (scalar) sum of their total curvatures equal respectively to the mean curvature and total curvature of the given surface at O. There is therefore a cone, Cone III, of normals such that if any element of the cone be chosen as one of the three mutually perpendicular normals, the other two may be chosen in such a way (upon Cone II) that the total curvature of the three projections are respectively G, 0, and 0. The equation of this cone is $\mathbf{r} \cdot (\Phi_s I - \Phi) \cdot \mathbf{r} = 0$, where I is the idemfactor.

Types of surfaces.—For minimal surface $\mathbf{h} = 0$, and the indicatrix everywhere reduces to an ellipse in some normal plane and with its center at the surface-point O.

For the surface formed by the tangents to a twisted curve, the indicatrix reduces to a segment of a line (described twice) reaching from O to the extremity of the vector $2\,\mathbf{h}$. Such a surface is developable, and all ruled developables are of this type.

For any ruled surface the indicatrix lies in a plane passing through the surface-point O, and the ellipse itself passes through O. The total curvature of any real ruled surface (other than developable) is negative.

For a surface of revolution, which is formed by revolving a twisted curve parallel to a plane, the indicatrix reduces to a linear segment (described twice) centered at the extremity of $\mathbf{h}$. There is a large variety of developable surfaces (not ruled) of revolution, of which the simplest is perhaps that obtained by revolving the circular helix parallel to a plane containing its axis and a line perpendicular to the three-space in which the helix lies.

For a developable (non-ruled), the indicatrix is tangent to three mutually perpendicular planes through O, or, if $n = 4$, to two perpendicular lines through O. A developable is not invariant in type under a projective transformation.

The class of surfaces where the indicatrix at each point lies in a plane with O is invariant under projective transformations, and these surfaces have upon them the characteristic lines of Segre.

A special type of surface, which have the indicatrix everywhere a linear segment noncollinear with **h**, is a simple generalization of ordinary surfaces ($n = 3$) in that by a proper choice of parametric curves the first and second fundamental forms reduce simultaneously to sums of squares. The property is not invariant under a projective transformation.

Lines upon surfaces.—Kommerell defines as principal directions those for which the normal curvature is a maximum or minimum, and subsequent authors have followed him. This gives four directions at each point, and they are not simply related to one another. We have chosen to give as a definition of principal directions those for which **h** and μ are perpendicular. There are two such directions at each point, they are orthogonal and have the properties that the differential tangent planes in the two directions are perpendicular and the values of the rate of change $d\mathbf{M}/ds$ of the tangent plane is numerically a maximum or minimum.

Kommerell defines asymptotic lines, and Levi follows him after restricting the definition to a special case. The definition is not satisfactory, and we define asymptotic lines as those for which **h** and α are perpendicular. The rate of turning of the tangent plane along these directions is the square root of the negative of the total curvature. The differential equations of the lines are $\mathbf{h}.\Psi = 0$. The lines are bisected by the principal directions.

Our definitions for principal directions and asymptotic lines become illusory for minimum surfaces.

Segre's characteristics exist when the indicatrix lies in a plane with the surface point O and are then those directions which make α tangent to the indicatrix. The asymptotic directions divide the characteristic directions harmonically.

Development of surfaces.—In the neighborhood of a point a surface may usually be developed in either of two standard forms.

$$z_1 = \tfrac{1}{2}\, h(x^2 + y^2) + e(x^2 - y^2), \qquad z_2 = \tfrac{1}{2}\, f(x_2 - y^2),$$
$$z_3 = \tfrac{1}{2}\, A(x^2 - y^2) + 2\, Bxy, \qquad z_i = 0,\ i > 3,$$
$$\text{or} \quad z_2 = \tfrac{1}{2}\, (Ax^2 + 2\, Bxy + Cy^2), \qquad z_2 = \tfrac{1}{2}\, Dx^2,\ z_3 = \tfrac{1}{2}\, Ey^2.$$

For the first form **h** lies along the axis of z_1, the plane of the indicatrix is parallel to the axis of z_3, and the axis of x and y are along the principal directions. A hyperplane tangent to the surface at O will cut the surface in real, imaginary, or coincident directions according as it cuts the indicatrix in real, imaginary, or coincident points. The second standard form of the surface has the property that the two perpendicular hyperplanes $z_2 = 0$ and $z_3 = 0$ cut the surface in orthogonal coincident directions.

Particular interest attaches to the tangent hyperplane perpendicular to the mean curvature **h**. This cuts the surface in the asymptotic directions and the axes of the (degenerate) conic made up of these two directions are the principal directions. The intersection of the surface and this hyperplane has therefore the fundamental properties of the Dupin indicatrix.

For the proof of the geometric results here stated and for the proofs and statements of a large number of others, many of which are entirely new, some only new statements of the results of Levi, Kommerell, or Segre, reference must be made to our complete memoir 'Differential Geometry of Two-dimensional Surfaces in Hyperspace' which will be published in the *Proceedings of the American Academy*, Boston.

---

[1] Kommerell, Die Krümmung der Zweidimensionalen Gebilde in ebenen Raum von vier Dimensionen, *Dissertation*, Tübingen, 1897, 53 pp; E. E. Levi, Saggio sulla Theoria delle Superficie a due Dimenzioni immersi in un Iperspazio, *Pisa, Ann. R. Scu. Norm*, **10**, 99 pp; C. Segre, Su una Classe di Superficie degl' iperspazî, *Torino, Att. R. Acc. Sci.*, **42**, 1047–1079 (1907).

[2] Ricci, *Lezioni sulla Theoria delle Superficie*, Padova, Drucker, 1898. (Lithographed, edition exhausted.)

[3] The vector analysis used is a modification of the Grassmannian system; see Lewis, *Proc. Amer. Acad. Arts Sci.*, **46**, 165–181 (1910), and Wilson and Lewis, *Ibid.*, **48**, 389–507 (1912).

[4] The corresponding indicatrix for V_2 in S_3 is not Dupin's but the range of points upon the normal described (twice) by the normal curvature vector α.

DYNAMICAL STABILITY OF AEROPLANES

By Jerome C. Hunsaker

U. S. NAVY AND MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Received by the Academy, April 6, 1916

The first rational theory of the dynamical stability of aeroplanes is due to Bryan[1] whose work was extended and applied by Bairstow[2] with wind tunnel tests on models.

The utility of such tests in predicting the aerodynamical properties of a full size aeroplane is now well understood and the validity of this application has been repeatedly demonstrated. The late E. T. Busk of the Physical Staff of the Royal Aircraft Factory, England, applied Bairstow's model tests to the design of an aeroplane and recently succeeding in perfecting an inherently stable machine which could be flown 'hands off.' Neither the details of Busk's experiments nor of the type of aeroplane developed by him have been disclosed by the British War Office.

My present investigation of two different types of aeroplanes, one a standard military tractor with no claims to inherent stability in flight, the other a machine designed to possess some degree of inherent stability while departing as little as possible from standard practice as exemplified by the other, has for its object:

(*a*) To determine the aerodynamical constants for the two aeroplanes by means of model tests in the wind tunnel.[3]

(*b*) To apply the aerodynamical constants so found in the dynamical equations of motion for the full scale aeroplanes in free flight and to examine the stability of the motion.

(*c*) To compare the stability, both lateral and longitudinal of the two chosen types of aeroplane, with a view to tracing to the individual parts of each machine their effects upon the motion.

(*d*) To attempt to formulate general qualitative conclusions which may assist constructors of aeroplanes to avoid instability or to provide any desired degree of stability.

(*e*) To throw light upon the general problem of inherent dynamical stability.

Stability distinguished as statical or dynamical.—An aeroplane in horizontal flight in still air must be driven at such speed and kept at such an inclination of wings to wind that the weight is just sustained. When in this normal attitude the aeroplane, if properly balanced, is in equilibrium. In a statical sense, this equilibrium is stable if righting moments are called into play tending to return the aeroplane to its normal attitude if by any cause it is deviated thereform. In general an aeroplane, if stable in a statical sense, will, when given an initial deviation, take up an oscillation which either may be damped out or may increase in amplitude. The aeroplane is dynamically stable if, and only if, these oscillations die out as time goes on, leaving the aeroplane in its original normal attitude. It is clear that statical stability must first be provided before the dynamical stability of a design can be examined.

The righting moments which tend to restore the aeroplane to its normal attitude are a measure of statical stability. If the statical stability is great, the period of the oscillations will be short and the motion violent, whereas an aeroplane should have a gentle motion of slow period heavily damped. An ocean liner of too great stiffness (large metacentric height) is not suitable for passenger or cattle carrying service; it is preferable in ship-design to give only enough metacentric height to insure that the vessel in never unstable and to damp the roll by generous bilge keels. In a similar manner the theory indicates that, in aeroplane-design, it is preferable to give just enough statical stability to insure that the aeroplane is never unstable and to make the oscillation dynamically stable by the use of generous damping surfaces and large wings.

The investigation is most conveniently discussed in two parts, the first dealing with the 'longitudinal' motion involving pitching of the aeroplane and rising and sinking of its center of gravity combined with change of forward speed, and the second with the 'lateral' motion, which involves side-slipping, or skidding, combined with rolling and yawing to the right and left.

Longitudinal motion.—From the dynamical equations of motion for the full scale aeroplane written down with the aerodynamical coefficients determined by tests on the models, the small oscillations about the equilibrium position are determined by three simultaneous linear differential equations with constant coefficients, of which the solution shows that the motion may be considered as composed of two oscillations one of a period of the order of 2 seconds damped to half amplitude in 0.1 seconds, the other of period from 10 to 30 seconds not very strongly damped. The short oscillation appears never to be of importance in ordinary aeroplanes, but the long oscillation, being only moderately damped, may cause trouble, especially for an aeroplane that flies with a large angle of incidence for the wings.

The calculation of the period and damping of the long oscillation was repeated for several speeds from highest to lowest corresponding to small and large angles of incidence with the results shown in the following table:

Aeroplane*	S	S	S	S	U	U	U	U
Velocity, miles-hour	76.9	53.4	44.6	36.9	79	51.8	47	44.2
Incidence of wings	0°	30°	6°	12°	1°	7°	10°	14°
Period, seconds	34.7	17.6	15.8	10.56	34.0	16.7	13.7	11.5
Damp 50 per cent, seconds	8.1	11.0	13.1	—	11.0	17.7	63.0	—
Double amplitude, seconds	—	—	—	24.7	—	—	—	24.7

* The letters S and U represent respectively the machine designed for inherent stability and a standard military tractor.

Instability at low speed.—It appears that the period becomes much more rapid at low speed, that at some critical speed the damping vanishes, and below this speed both aeroplanes become frankly unstable. This instability at extreme low speed is common to all aeroplanes and the only advantage of our 'stable' aeroplane S is that its longitudinal motion is stable down to about 40 miles per hour while aeroplane U is stable only down to about 47 miles per hour.

A study of the relative magnitudes of the coefficients for these typical aeroplanes leads to the conclusion that longitudinal instability at low speed is due, first, to the decrease in damping of the tail surfaces on account of the low speed and, secondly, to the decrease in rate of change of lifting force with change in attitude for high angles of incidence. The latter has a predominating effect on the damping of the long oscillation. Consequently, if an aeroplane is to be stable and land at a relatively slow speed, it must not operate at too great an angle of incidence. To sustain its weight it should therefore have a comparatively large wing area. The principal difference between aeroplanes S and U is that the former supports a weight of 3.55 pounds per square foot of wing area and the latter 5.2 pounds per square foot.

The following recommendations are made for an aeroplane to have its longitudinal motion damped at lower speeds than is usual in practice: (1) Provided large horizontal surfaces of long arm for damping the pitching. (2) Provide wings of such area that the slow speed does not require a great angle of incidence. Roughly the safe slow speed should not require more than 80 per cent of the maximum lift of the wings. (3) Keep the longitudinal radius of gyration small by concentrating the principal weights.

Slowness in pitching.—It may be imagined that a dynamically stable aeroplane of rapid period might be so violent in its motion that the pilot would be shaken about to such an extent as to be hindered in the performance of his military duties of observation, gun-fire, or bomb dropping. It appears that the expression representing the period of the long oscillation contains certain predominating coefficients, and a consideration of their magnitude leads to the following conclusions: The natural period of pitching is increased by: (1) High speed of flight, (2) Large damping surfaces on the tail, (3) Small angle of incidence, (4) Small righting moments.

Lateral motion.—After measuring the aerodynamical coefficients, and the radii of gyration in roll and yaw, the dynamical equations for the asymmetrical or lateral motion may be set down. For small oscillations these reduce, as in the longitudinal case, to three linear differential equations with constant coefficients. The determinant formed from the coefficients may be factored by use of approximate methods and the motion may be compounded from that represented by each of three factors.

Spiral Dive.—The first factor may correspond either to a damped or to an amplified motion. At high speeds model S shows a subsidence damped to half amplitude in 10.4 seconds. At lower speeds this damping diminishes and at 37 miles per hour the motion becomes a divergence which doubles in amplitude in 7.2 seconds. Aeroplane U is spirally unstable at high speeds. Examination of the preponderating terms in the expression representing the motion shows that the aeroplane starts off on a spiral dive.

A simple relation may be obtained involving four of the aerodynamical coefficients which, if positive, insures that spiral instability of this kind is not present. It appears that spiral instability is caused by too much fin surface to the rear or to too large a rudder, and by not enough fin surface above the center of gravity. A proper adjustment is easily obtained without sacrifice of desirable flying properties. Aeroplane S has a small rudder and wing-tips raised about 1°6; aeroplane U has no rise to wing-tips nor vertical surface above the center of gravity and has a very deep body giving the effect of a rear vertical fin. These differences in design account for the respective stability and instability of the two machines.

Rolling.—The second factor in the equation of motion represents a rolling of the aeroplane which is so heavily damped by the wide spreading wings as to be ordinarily of no consequence. In the extreme case of a 'stalled' aeroplane, the damping of the roll vanishes because the downward moving wing has no more lift than the other. Here we may expect trouble, and frequent accidents to stalled aeroplanes indicate that the pilot's lateral control by ailerons also becomes operative.

Dutch Roll.—The third element in the motion is a yawing to right and left, combined with rolling. The motion is oscillatory of period from 5 to 12 seconds, which may or may not be damped. The analogy to the 'Dutch Roll' or 'Outer Edge' in ice-skating is obvious. If the skater lean too far out on his swings he may fall, and in the same manner if the aeroplane bank too much a slight puff of wind may capsize it.

The motion of the Dutch Roll is stable provided there be sufficient vertical fin surface on the tail and not too much fin surface above the center of gravity. These requirements conflict with those previously stated for spiral stability and a compromise must be made. Over-correction of spiral instability may produce instability in the Dutch Roll and vice versa. Fortunately, the damping of rolling by the wings is helpful in both cases, and it appears possible to obtain that nice adjustment of surfaces which will render both motions stable.

Model S was stable in the Dutch Roll at all speeds, having a period from 6 to 12 seconds, and the initial amplitude damped 50 per cent in from 1.5 to 6 seconds. Model U was stable in this respect except at low speed when it showed a period of 6 seconds and the initial amplitude was doubled in 8 seconds.

The following table summarizes the results obtained for the lateral motion.

Aeroplane	S	S	S	U	U
Rise of wings	1°.63	—	—	0	—
Angle of incidence	0°	6°	12°	1°	15°.5
Velocity, miles	76.9	44.6	36.9	78.9	43.6
Spiral motion					
Damp 50 per cent, seconds	10.4	2.7	—	—	3.3
Double, seconds	—	—	7.2	28.0	—
Dutch roll					
Period, seconds	5.9	10.7	12.0	5.2	5.7
Damp, 50 per cent seconds	1.4	1.3	6.0	1.8	—
Double, seconds	—	—	—	—	7.7

General Conclusions.—It is believed that the majority of modern aeroplanes are spirally unstable but stable in the Dutch Roll. Furthermore it appears to be a simple matter so to adjust surfaces that any aeroplane can be made completely stable without sacrifice in speed or climb. At extreme low speed an aeroplane must be unstable in its longitudinal motion but need not be unstable laterally.

The degree of stability to provide in a given case cannot be determined from mechanical considerations alone. For example, the comfort of the pilot must be a first consideration and for this reason the righting moments giving statical stability should be small; the period of the aeroplane can then be made relatively slow, and if the damping is adequate, the free oscillations will be stable.

The theory is applied here only to flight in still air. Obviously the air is never still, and the aeroplane must finally be judged from its behavior in gusts. An inherently stable aeroplane tends to preserve its normal attitude with relation to the relative wind, and if the velocity and direction of the relative wind change in an irregular manner, the stable aeroplane will tend to follow. The result will be to force on the aeroplane a motion which will be more violent the greater the statical stability. Consequently in rough air an aeroplane very stable statically is unsuitable as a gun platform and for many other military purposes.

Considerations of theory indicate that a slight degree of statical stability combined with the maximum of damping give an aeroplane slow periods of oscillation and a dynamically stable motion, with little ill effect upon performance or controllability.[4]

---

[1] G. H. Bryan, *Stability in Aviation*, Macmillan, 1910.

[2] L. Bairstow, *Technical Report of the Advisory Committee for Aeronautics*, London, 1912–13.

[3] A description of the wind tunnel and the results of some experiments therein may be found in *Smithsonian Inst., Misc. Coll.*, **62**, No. 4, 1–92 (1916).

[4] Full details of this investigation will be offered for publication in a forthcoming number of *Smithsonian Misc. Coll.*

CLIFT ISLANDS IN THE CORAL SEAS

By W. M. Davis

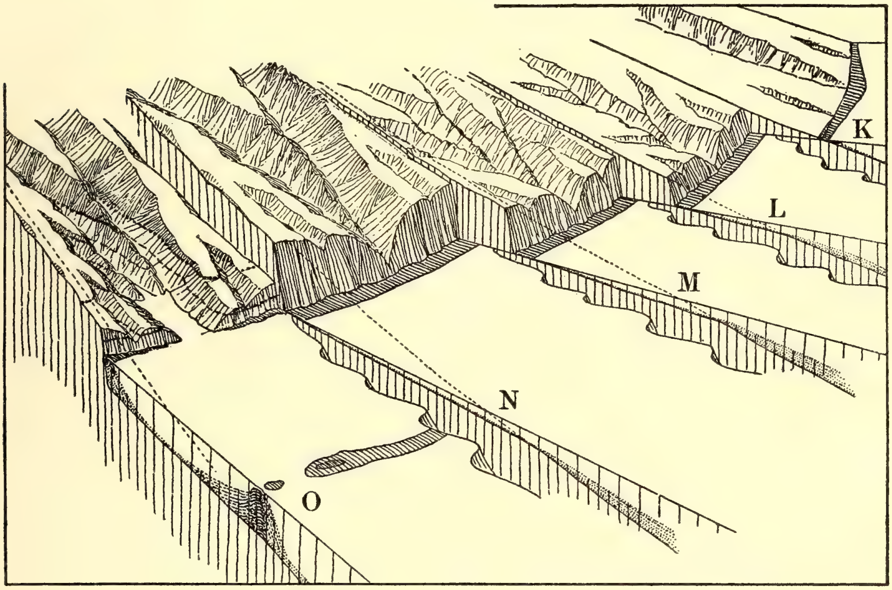
DEPARTMENT OF GEOLOGY AND GEOGRAPHY, HARVARD UNIVERSITY

Received by the Academy, April 8, 1916

The great majority of reef-encircled islands in the Coral Seas are of maturely dissected form, with slopes of moderate declivity that descend to an indented shoreline, where the tapering spurs end in salient points and the widening valleys open into branching bays, thus proclaiming that the islands have suffered partial submergence since they were dissected. It is generally agreed by those who accept this interpretation of insular relief in relation to shoreline pattern that the encircling barrier reefs have grown upwards from their beginning as fringing reefs during the subsidence which gave the shoreline its indented outline. Reasons for this conclusion were given in a previous article 'The Origin of Coral Reefs' in these PROCEEDINGS **1**, 146–152, 1915; they have been more fully stated in 'A Shaler Memorial Study of Coral Reefs' recently published in the *American Journal of Science*, **40,** 223–271, 1915.

The object of the present article is to call attention to certain exceptional reef-encircled islands, Reunion, Tahiti, and New Caledonia, in the Coral Seas, which present clift shore lines, and to suggest an explanation by which their exceptional features may be simply accounted for. If all the islands of the Coral Seas were clift like these three, they would support either the theory of veneering barrier reefs on wave-cut platforms, or the Glacial-control theory of coral reefs. But clift islands are exceptional. Some means of explaining them must be found which will not be in discord with the theory of coral reefs that is supported by the features of non-clift islands, namely, Darwin's theory of intermittent subsidence. It will appear from what follows that the suggested means of explaining the exceptional clift islands of the Coral Seas gives unexpected support to this theory.

Imagine a young volcanic island, from which detritus is washed so abundantly into the sea that corals and the associated reef-making organisms can establish themselves if at all, only in short and discontinuous fringing-reef patches around its shore. So long as the island continues to grow by eruption, its height and perimeter will increase; its earlier fringing reef patches will be buried under newer lava flows, agglomerate beds and ash falls. The completed volcanic island will have a shoreline of simple pattern without pronounced salients and reentrants, because the materials ejected from the summit crater are spread rather equably on all sides of the cone, and thus give it a nearly circular base. The simple shoreline may be at first somewhat prograded by the deltas of the larger streams, sector K of the figure; between the deltas it may be somewhat retrograded and clift by the waves; but an abundant supply of cobbles, gravels and sands from the delta fronts will be swept along the inter-delta shores in the form of beaches that will cover the cliff-base platforms; and the finer sediments will be spread seaward on the submarine flanks of the volcanic mass. The deltas, the beaches and the off-shore sediments will form a detrital sheet, thickest and of coarsest texture on the deltas, of finest texture and for a time thinnest where it slopes into deep water; but as time passes, the fine-textured, off-shore sediments will come to exceed all other parts of the detrital sheet in thickness, for when retrogradation begins and the deltas are cut back in the general retreat of the shoreline, sectors L, M, N, the stream-mouth deposits will be hardly thicker than the cliff-base beaches. Small patches of fringing reef may at first form on the bare unreduced ledges that here and there protrude through the cliff-base beaches; but elsewhere the materials of the detrital sheet are too commonly in movement to permit coral growth.



After volcanic eruptions have definitively ceased and the normal dissection of the cone by weather and streams and the cutting of cliffs around the simple shore by waves are further advanced, it is probable that the first formed reef patches will be overwhelmed by the increasing continuity of the detrital sheet, and if this unfavorable condition for reef development once sets in, it is likely to continue indefinitely, as long as the island stands still. The grinding of the detritus upon itself, as it is shifted by the waves, would soon reduce it to so fine a texture that it would all be swept off the platform into deep water, leaving the platform bare; but the loss thus occasioned is continually made good by a new supply from the valleys and cliffs of the island; hence the detrital sheet is always maintained. The rasping of the detritus on the platform, especially at the time of storms, slowly wears the rock surface down to greater and greater depth, but the degradation thus caused is probably 20 or 40 or more times slower than the retrogradation of the cliffs. The 'edge' at the outer margin of the abraded platform may be somewhat accentuated and stript as it is worn down, because the slow abrasion of its rock surface may be exceeded by the more rapid abrasion of the first deposited detritus next outside of it: here corals and other reef building organisms might grow, but for the constant outward passage of detritus which would smother them. The important matters to note thus far are first, that when the series of normal changes begins around the simple shoreline of the completed island, sector K, the shore waves will at once have to deal with so abundant a supply of detritus washed down by torrents, that a continuous shoreline beach and off-shore detrital sheet will be formed at an earlier stage of shoreline development than is usually the case on the shores of strongly sloping coasts; second, that the detrital sheet, once established, will be thereafter well supplied with waste from the deepening valleys and the retreating cliffs, and will thus be maintained indefinitely, as long as the island stands still; and third, that as long as the detrital sheet exists, the growth of corals is greatly discouraged if not entirely prevented.

Now let it be supposed that when a certain stage of cliff recession is reached, as in sector N, the island subsides by a succession of starts and stops, so that the shoreline crosses the cliffs at mid-height and enters the larger valleys, as in sector O. Several significant changes will follow. The detritus from the larger valleys of the dissected cone will now be pocketed in the valley-mouth embayments; the waves, beating the face of the cliffs without the aid of the boulders and cobbles, with which they had previous attacked the cliff base so successfully, will do little work in the way of cutting the upper half of the cliffs farther back. Under these conditions corals and their associates may attach themselves to the cliff face and form a fringing reef; as soon as such a reef broadens a little, it will hold nearly all the detritus that is washed from the little hanging valleys and weathered from the non-submerged part of the cliff face: thus the detrital sheet on the abraded platform, no longer receiving a renewal of supply from the cliffs or the streams, will be slowly shifted seaward and the rock platform will be at least in part exposed, especially along the 'edge' at its outer margin, where the heavy outside storm waves make their first attack on the bottom: here in particular, the abraded rock of the platform may be laid bare and a favorable site for coral colonies provided; here, if the subsidence has not been too great, a barrier reef may grow up, as in the side section of Sector O.

As soon as the barrier reef is fairly well established, wave work in the lagoon is greatly weakened. If the island now stand still again, the lagoon will be gradually aggraded by overwashed detritus from the reef, by organic deposits formed in the lagoon, and by outwashed detritus from the island: thus a delta-and-reef plain will in time replace the lagoon, and when this stage is approached and reached, the stream-borne volcanic detritus will be again delivered on the reef face, where it will retard the growth of corals and their associates. But if subsidence be renewed and the shore line rise to the broken line of sector O favorable conditions will be re-introduced, and the barrier reef will grow upward again. The cliffs that were cut before subsidence began will in time be completely submerged; the shoreline will then have the pattern shown by a dotted line in sector O, and this pattern will persist as long as subsidence continues, until in the almost-atoll stage the diminishing island is reduced to so small a size that it has no room for valleys; only a central summit will then remain above sea level; in the next stage this disappears, and a perfect atoll results.

Actual examples of islands and reefs should be found in various stages of the sequence of events here deduced, if the scheme by which the deductions are guided is a true counterpart of natural occurrences. Reunion in the western Indian Ocean offers confirmation for sectors K and L. As described, figured and mapped by Maillard, and especially by von Drasche, it has no embayments; part of the shoreline is clift with hanging valley mouths; much of the shoreline is delta-fronted and beached; and coral reefs form only fragmentary fringes along about one-tenth of its circuit.

Tahiti, a volcanic doublet in the Society group of the South Pacific, is clift nearly all around its figure-8 circuit. The cliffs, well described by Agassiz, are highest on the eastern or windward side where some of them rise 1000 feet above the sea; a number of the smaller valleys make mere notches in the cliff top, and the streams cascade down the cliff face; a few of the larger valley mouths are still embayed as all of them once were; hence the island has subsided and the cliff base is submerged Most of the embayments are now filled with deltas, which are indeed advancing into the lagoon, where they form a narrow alluvial plain on which most of the population dwells. A barrier reef rises at just about such a distance off shore as to suggest that its foundation is on the edge of the abraded platform that fronts the submerged base of the cliffs; volcanic detritus is abundant in the enclosed lagoon. Tahiti thus seems to have been clift, about as in sectors L, M or N, and then to have subsided as in sector O; since then it has stood still long enough for abundant delta advance and for partial enclosure by a barrier reef, the further growth of which is now threatened at certain points by the advancing deltas.

New Caledonia is peculiar in being a non-volcanic island, and in being clift only on the northeastern side: hence certain special assumptions must be made in order to bring it under the explanatory scheme here proposed; but the necessary assumptions do not involve any events that are not made probable by what is known of the geological and physiographical history of the island.

The great majority of reef-encircled volcanic islands in the Pacific are volcanic, like Reunion and Tahiti; but they differ from these two submaturely dissected cones in being maturely or more than maturely dissected, so that hardly a trace of their initial conical slopes remains; furthermore, their spurs usually taper to non-clift points and their valleys open into broadening bays, as shown in the dotted high-level shoreline of sector O. If the reef-encircled islands of the Pacific have formerly passed through an introductory, almost reefless stage, like Reunion, and a soon-following reef-encircled stage with partly drowned cliffs and outgrowing deltas, like Tahiti, they have since then continued their subsidence as well as their dissection and their reef growth, so that at present they have no trace of the smooth slopes of their initial conical form, and no trace of the presumable spur-end cliffs of their early reefless stage: they have gained forms that are best explained on the supposition that, whether they past through an introductory reefless stage or not, they have long suffered dissection and intermittent subsidence, while protected from wave attack by an upgrowing, encircling barrier reef.

A theory of coral reefs is certainly commended when a reasonable extension of its postulated conditions and processes enables it to account for the cliffs of exceptional reef-encircled islands, for which no adequate explanation has been previously given.

ON SOME RELATIONS BETWEEN THE PROPER MOTIONS, RADIAL VELOCITIES, AND MAGNITUDES OF STARS OF CLASSES B AND A.

By C. D. Perrine

OBSERVATORIO NACIONAL ARGENTINO, CÓRDOBA

Received by the Academy, November 1, 1915

Attention was called in a former paper[1] to a decrease of proper motion with decreasing brightness in the class B stars, whereas the radial velocities showed an increase. Such a condition was found also in the class A stars, but more pronounced. Owing to lack of proper-motion data for the other classes the matter was not pursued. Since those results were obtained I have seen Kapteyn and Adams' investigation[2] in which they find that the radial velocities increase with increasing proper motion for the spectral classes F, G, K and M. The inference from the magnitude-results for classes B and A had been that the contrary was probably true for these two classes. It seemed that in general large radial velocities would be associated with large proper motions, and *vice versa.*

After seeing Kapteyn and Adams' results I classified the B–B5 and A stars according to proper motion and found that in fact their velocities do appear to decrease slightly with increase of proper motion. As stated, the contrary was found by Kapteyn and Adams for the classes F, G, K and M. The B8–B9 stars were classified separately. These 45 stars appear to follow the later types with respect to proper-motion velocity characteristics if we use the V_1 values, or the earlier B and A types if we use V. It is perhaps significant that the B8–B9 stars have been found not to have a large K term.

The relations of proper motion and radial velocity for classes B–B5, B8–B9 and A are given in Tables I and II. Table III contains the proper motions arranged according to visual magnitude.

Table I reveals an apparent relation of some kind between the magnitudes and proper motions of these three classes. The faintest stars are found *near* (but not in) the stars of very smallest proper motions in all. In both directions from these the magnitudes decrease numerically (stars become brighter). The differences, however, are small and so far as the first group is concerned, particularly in class A, may be accidental, although examination indicates considerable consistency. The V_1 of class B–B5 and V of classes B8–B9 and A show a similar effect for the radial velocities—the largest being *near* (but not in) the very smallest proper motions. Very noticeable is the sudden increase

TABLE I*

Arranged according to proper motion

CLASS	μ	MEAN MAGNITUDE	MEAN V_1 km.	MEAN V km.	NUMBER OF STARS
B–B5......	0″000 to 0″009	4.0	6.2	8.9	38
	0.010 0.019	4.5	7.9	7.6	44
	0.020 0.029	4.2	6.7	8.9	28
	0.030 0.049	3.9	5.5	8.0	49
	0.050 0.119	3.7	4.4	6.0	23
B8–B9.....	0.000 to 0.019	4.2	4.4	3.1	9
	0.020 0.049	4.8	6.7	7.2	16
	0.050 0.099	4.6	7.8	6.8	17
	0.100 0.180	3.0	11.1	6.6	3
A.........	0.000 to 0.020	4.5		10.1	31
	0.021 0.050	4.6		12.4	47
	0.051 0.099	4.5		11.1	60
	0.100 0.199	4.1		10.4	53
	0.200 0.656	3.0		10.1	18

* These tables are based upon Campbell's catalogs of B and A class stars in *Lick. Obs. Bulletin,* Nos. 196 and 211.

TABLE II

Arranged according to radial velocity

CLASS	V_2 km. km.	MEAN MAGNITUDE	MEAN μ	NUMBER OF STARS
B–B5........	0 to 2.9	3.8	0.031	58
	3.0 5.9	4.0	0.024	47
	6.0 9.9	4.2	0.025	42
	10.0 19.9	4.4	0.029	24
	10.0 19.9	4.4	0.025*	23
	20.0 25.6	4.6	0.024	9
A...........	0 to 5.0	4.3	0.083	68
	5.1 9.9	4.2	0.104	49
	10.0 14.9	4.3	0.091	31
	15.0 19.9	4.4	0.112	27
	20.0 29.9	4.1	0.074	27
	20.0 29.9	4.5**	0.066	22
	30.0 35.5	4.7	0.068	8

* Omitting one large value.

** Omitting five bright stars.

TABLE III

	CLASS B–B5		CLASS A	
	Number of stars	Mean μ	Number of stars	Mean μ
1.9 and brighter	11	0″051	8	0″382
2.0 to 2.9	18	0.033	19	0.177
3.0 3.9	38	0.028	26	0.093
4.0 4.9	88	0.024	121	0.075
5.0 and fainter	24	0.025	38	0.066

of velocity in all three classes *near the smallest proper motions.* An examination of the results of Kapteyn and Adams indicates somewhat similar rather sudden increases in velocity in the classes F, G, and K, but in proper motions somewhat larger than in the B and A stars. Such an effect is not noticeable in the M stars. The application of a magnitude velocity correction would tend to reduce the peculiarity slightly.

As these velocities (V_1) for the B stars had been derived by including the constant error K, it was decided to see what the effect might be of neglecting it and using $V_1 = -19.5$ km., the same solar motions as were used in obtaining results for the A stars. The fifth column (V) contains these results. The general decrease of velocity with increasing proper motion is still indicated rather uncertainly but the sudden increase of velocity alluded to has been reversed in the B–B5 stars. This reversal seems to result chiefly from the application of K to the small negative velocities in the two groups.

It seems rather curious that this peculiar maximum in the velocities shifts about with the term K – where a considerable value has been found for it, the maximum and peculiar sudden increase of velocity are shown only if it is used. On the contrary, where no K term resulted, the maximum and sudden increase show with its omission. If confirmed, this indicates a relation of some sort.

The parallactic motion of the Sun is very noticeable in the proper motions of both the B and A classes and it seems not improbable that some systematic effect may have crept into the velocities when classified according to proper motions which have not been freed from the solar motion—that the peculiarities observed may be due to a preference of the faint stars and those of larger velocity for certain regions of sky.

The dependence of velocity upon magnitude is clearly shown in both classes B–B5 and A in the above classifications particularly in the residual radial velocities of table II. The effect of the few apparently abnormal velocities of bright stars in class A is here very marked also.

An examination of table II shows that in B–B5 stars with the possible exception of the smallest radial velocities (very brightest stars) there is little or no connection of the proper motion with the radial velocities. There is, however, a peculiarity in the case of the class A stars—the largest proper motions being associated with velocities of medium value. The largest velocities of all show the smallest proper motions—even after making liberal allowance for a magnitude-velocity effect. Until systematic effects such as magnitude-velocity equation, constant error, etc., are better understood, it seems useless to attempt a discussion of such small peculiarities. It seems probable, however, that the velocity distributions in these two classes may not depart widely from Maxwell's law, but that they are different from those in the spectral classes F, G, K and M.

Conclusions.—1. The radial velocities of the stars of classes B and A appear to decrease in general with increasing proper motion. There are, however, some inexplained anomalies.

2. The velocity distributions of classes B–B5 and A differ from the distributions found for the F, G, K and M classes by Kapteyn and Adams, appearing to be more clearly in accord with Maxwell's law.

3. The effects of the magnitude-velocity equation are clearly shown in the classification of the B and A stars made for this investigation.

4. There are some indications of a small abrupt change in the velocities of both classes in the smaller proper motions. There are also some indications in the classes F, G and K of such a change but in proper motions somewhat larger than in the early classes.

[1] *Astrophys. J.*, **41,** 318.

[2] These Proceedings, **1,** 14.

ASYMMETRY IN THE PROPER MOTIONS AND RADIAL VELOCITIES OF STARS OF CLASS B AND THEIR POSSIBLE RELATION TO A MOTION OF ROTATION

By C. D. Perrine

OBSERVATORIO NACIONAL ARGENTINO, CÓRDOBA

Received by the Academy, April 8, 1916

In the course of other investigations asymmetry in the proper motions of the B stars was suspected. A preliminary investigation has shown a systematic difference between the proper motions in the region about 0^h and that opposite. The results of a comparison of the proper motions in these two regions and also with the regions within 40° of the apex and the antapex of solar motion are given in the table. The average radial velocities of the stars (V_2) are also included.

An examination of the individual values shows a marked consistency in all of the regions. These results depend upon Campbell's catalog of class B stars in *Lick Observatory Bulletin*, No. 196. As is well known these stars have a marked preference for the Milky Way.

The minima corresponding to the apices of the solar motion fall at $6\frac{1}{2}^h$ and 19^h. The two maxima corresponding to the parallactic motion of the Sun fall at 2^h and $13\frac{1}{2}^h$. These two points are almost exactly at right angles to the direction of the Sun's motion.

Region	*Apex*	*Antapex*	22^h–2^h	10^h–14^h	10^h–14^h[1]	10^h–14^h[2]
Magnitude	4.4	4.0	4.3	3.7	4.3	3.7
V_2 in km	7.8	6.2	6.8	5.9	7.1	6.0
Proper Motion	0″.014	0″.013	0″.020	0″.043	0″.038	0″.040
No. of stars	21	44	18	27	20	26

[1] Omitting all stars brighter than $3^{M}0$ to make average magnitudes comparable with region at 22^h–2^h.

[2] Omitting the only apparently abnormal value of $\mu = 0''.119$.

A cursory examination of the right ascensions of Boss' *Preliminary General Catalogue* reveals a consistently smaller number of very small proper motions in the region 10^h to 14^h than in the region 22^h to 2^h. A preliminary examination of the A stars shows a similar (but smaller) effect if we omit the stars having proper motions of 0″.2 and over.

At the time of finding asymmetry in these proper motions the determination by Charlier of the motion of the node of the invariable plane of the solar system on the plane of the Milky Way was unknown to me. The value for the motion derived by Charlier and the preliminary value obtained by me from the B stars are as follows:

Charlier (Boss *P. G. C.*) + 0″.0035 per year,
Perrine (class B) + 0.012 per year.

If the observed phenomenon is in reality a relative motion of the invariable plane with respect to the *Milky Way as a whole* the angular motion as derived from different classes of stars, but especially from stars at widely different distances, should be exactly the same. The above discordances in the values throw doubt on the constancy of that angle.

Preliminary solutions for the apex and velocity of solar motion from northern and southern stars separately of class B and of 110 stars brighter than 3.0 magnitude of all spectral classes give systematically different results. The solar velocities derived in this way have a particular bearing on the above question and are given below.

	Northern Stars *km.*	*Southern Stars* *km.*
Class B, 3.0 and fainter	−17.0	−24.8
2.9 and brighter, all spectral classes	−14.4	−24.5

If there is no motion of rotation the values of the solar velocity derived from these different regions should be the same except for the effects of accidental variations whether the invariable plane is in motion or not. If however, the stellar system is rotating in a retrograde direction about the poles of the Milky Way, exactly such differences as the above should result.

It seems highly probable that the invariable plane may also be in motion.

Conclusions.—1. The stars of class B show systematic differences in the proper motions in the two regions of the Milky Way at right angles to the direction of solar motion.

2. The solar velocities derived from the B stars in the northern and southern hemispheres separately differ, that from the northern stars being the smaller.

3. These conditions appear to be best explained by a general motion of rotation of the system of stars in a retrograde direction about an axis perpendicular to the Milky Way.

The details of these preliminary investigations will be published elsewhere.

THEORY OF AN AEROPLANE ENCOUNTERING GUSTS

By Edwin Bidwell Wilson

DEPARTMENT OF MATHEMATICS, MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Received by the Academy, April 10, 1916

A number of references to the theory of the stability of aeroplanes may be given,[1] but on their behavior in gusts little has been printed,[2] though much is probably known and held secret by foreign governments. Last summer I was asked to develop a theory of the effect of gusts upon (symmetric) aeroplanes, in particular upon a definite machine of which the aerodynamical coefficients were furnished me by Mr. Hunsaker. Two questions were to be answered: First, what is the effect of a gust upon the longitudinal motion in the uncontrolled machine; second, what is the effect when the machine is so constrained by an automatic device as to maintain its axis horizontal? My report, submitted to our National Advisory Committee for Aeronautics, will appear as a part of the *Report of the National Advisory Committee for Aeronautics, 1915* (Senate Document No. 268, 64th Congress, 1st Session). By the courtesy of the Committee I am permitted to give here a brief account of this work.

In treating the motion of an aeroplane the machine is referred to a set of moving axes x, y, z fixed in the body, and directed backward, to the left, and upward in normal horizontal flight with velocity $-U$. Departures from this standard condition of flight are treated by the method of small oscillations. If $U + u$, v, w, p, q, r be the linear and angular velocities in the disturbed motion and if u_1, v_1, w_1, p_1, q_1, r_1 be the linear and rotational velocities of the gusts (taken with the proper sign), the aerodynamic forces acting upon the machine will vary from those acting in the standard conditions by amounts which may be determined by the eighteen aerodynamic constants X_u, X_w, X_q, Z_u, Z_w, Z_q, M_u, M_w, M_q,

Y_v, Y_p, Y_r, L_v, L_p, L_r, N_v, N_p, N_r, of which the last nine refer to the 'lateral' motion which was not discussed.

The equations for the 'longitudinal' motion of the free machine are

$$\begin{aligned} du/dt - g\theta - X_u u - X_w w - X_q q &= X_u u_1 + X_w w_1 + X_q q_1, \\ dw/dt - Uq - Z_u u - Z_w w - Z_q q &= Z_u u_1 + Z_w w_1 + Z_q q_1, \\ B/M.\, dq/dt - M_u u - M_w w - M_q q &= M_u u_1 + M_w w_1 + M_q q_1, \end{aligned} \tag{1}$$

where θ is the inclination of the x-axis, and $B/M = k^2$ is the square of the radius of gyration about the y-axis. For small oscillations these equations are linear with constant coefficients. The solution of (1) may be carried out by the ordinary method. In particular the solution may be cast in such form that a number of different gusts may be discussed very rapidly after a certain amount of preliminary calculation has been made.

The data for the given machine are as follows:

$$\begin{array}{lll} X_u = -0.128, & X_w = 0.162, & X_q = Z_q = M_u = 0, \\ Z_u = -0.557, & Z_w = -3.95, & M_w = 1.74, \\ M_q = -150, & k^2 = 34, & U = -115.5. \end{array}$$

The numerical equation for u is

$$\begin{aligned} (D^4 + 8.49\,D^3 + 24.5\,D^2 + 3.385\,D + 0.917)u = \\ - (0.128\,D^3 + 1.16\,D^2 + 3.385\,D + 0.917)u_1 \\ + (0.162\,D^2 + 0.715\,D + 1.647)\,Dw_1 - (59.37\,D + 560.6)\,q_1, \end{aligned}$$

where $D = d/dt$, with similar results for w and θ. The roots of $D^4 + 8.49\,D^3 + 24.5\,D^2 + 3.385\,D + 0.917 = 0$ are $-4.18 \pm 2.43i$ and $-.0654 \pm .187i$. The solutions are therefore of the form $u = e^{-4.18t}(A \cos 2.43t + B \sin 2.43t) + e^{-.0654t}(C \cos .187\,t + D \sin .187t) + I_u$, where A, B, C, D are constants of integration and I_u is a particular integral obtained from some particular gust u_1, w_1, q_1. There are similar equations for w and θ. Of the twelve constants of integration only four are independent, and the relations between them are independent of the particular integrals I (provided the gust is not tuned to the free oscillation). It is therefore possible to set down, once for all, formulas for the coefficients, A, B, C, D in terms of the initial values of the particular integrals I_u, I_w, I_θ, I'_θ. Where the machine is in normal flight with $u = w = \theta = q = 0$.

The type of gust chosen was $J\ (1 - e^{-rt})$, where J is an intensity-factor. This gust rises from 0 to J in an infinite time, but the greater part of the rise occurs in the time $1/r$ or $2/r$. If we set u_1, w_1, or q_1 equal to $J\ (1 - e^{-rt})$, we obtain a head-gust, up-gust, or rotary gust. The values 0.2, 1.0, 5.0 were successively assigned to r to correspond to mild, moderate, and sharp gusts.

The Uncontrolled Machine.—The result for a head-gust is that, during 15 to 18 seconds after striking the gust, the machine soars aloft to a height from $4J$ to $4\frac{1}{2}J$ above its initial level and subsequently executes a damped oscillation about the level $3\frac{1}{2}J$ above the original. The effect is not seriously affected by the sharpness of the gust (measured by r) but is proportional to its intensity (measured by J). The shock of the gust is small. For a rear-gust the effects are reversed in sign, the machine falls. A severe rear-gust coming upon a machine flying low might therefore produce serious consequences.

The major effect of an up-gust is to carry the aeroplane bodily upward with it. The oscillations are small, but in the case of a sharp gust, the initial acceleration may be considerable and 'bump' the pilot. The effects for down-gusts are reversed in sign.

Little appears to be known in regard to the extent or intensity or permanence of rotary aerial motions, and in the absence of such meteorological information, no satisfactory practical conclusions may be stated as to the effect of these gusts upon flight. The analysis, however, indicates that rotary gusts of relatively small intensity may have relatively large effects on the motion.

The Constrained Machine.—If the machine is so constrained that $\theta = 0$, the equations of motion reduce to the first two of (1). Substitution of the numerical data shows that the motion, when disturbed, is no longer oscillatory, but approaches the original condition asymptotically. The riding of the machine should therefore be steadier. The use of data obtained for lower speeds than $U = -115.5$ feet per second shows, however, that the constrained aeroplane loses its dynamical stability at a higher speed than the free machine, and to that extent is more dangerous in landing.

In a head-gust the constrained machine soars steadily (without oscillation) up to some J feet above its initial level. Thus the upward swoop of $4J$ to $4\frac{1}{2}J$ has been eliminated and the final increase in level is less than one-third as much as before. In a down-gust the effects are reversed in sign.

The up-gust is again important only for its general convective effect upon the machine. In case the gust is sharp, there may, however, be a considerable momentary acceleration or 'bump' at first.

It is interesting to note that if an aeroplane, equipped with the automatic device, is riding on a breeze which has a leisurely periodic gustiness, a habile pilot may 'suck' considerable energy out of the gusts. For he may ride up in the head-gust, with the constraining device thrown off, to a height of some $4J$ and then, with the device working, need drop only about J in the rear-gust. In this manner he can gain the amount $3J$ in altitude during each period. To avoid interference the successive maxima of the gusts should be at least 30 seconds apart for this machine.

[1] G. H. Bryan, *Stability in Aviation,* Macmillan, 1910; L. Bairstow, *Technical Report of the Advisory Committee for Aeronautics,* London, 1912–13; J. C. Hunsaker, these PROCEEDINGS, **2**, 278 (1916).

[2] A general lecture by Glazebrook, *Aeronautical Journal,* 272-301, July, 1914, should be cited.

TERMS OF RELATIONSHIP AND SOCIAL ORGANIZATION

By Truman Michelson

BUREAU OF AMERICAN ETHNOLOGY, WASHINGTON

Received by the Academy, April 17, 1916

Some years ago Kroeber undertook to show that terms of relationship are linguistic and psychological phenomena.[1] Recently Rivers has attempted to overthrow this view completely, and holds that they are sociological phenomena, and consequently that it is entirely possible to infer marriage customs and social organization from these terms.[2] Lowie lately has to a certain extent followed Rivers; he has not followed the latter's survival-theories,[3] and it is doubtful if many American ethnologists will do so.[4]

In this paper I wish to develop Kroeber's thesis from a different angle, and also to make a point on my own account. Long ago Morgan saw that for the most part the terms of relationships are identical in all Algonquian languages with phonetic changes, and consequently for the greater part must go back to the original parent language.[5] Now insofar as this is the case, to this extent terms of relationship are linguistic phenomena. For example, the Fox are organized in exogamic gentes with descent in the male line, and the Plains Cree have no gentile organization at all, yet have at least seventeen terms of relationship in common with the Fox. Again the Delaware who are organized in exogamic clans with descent in the female line have some terms of relationship in common with both the Fox and Cree. Similarly the Shawnee who have an entirely different organization from any of the above mentioned Algonquian tribes,[6] nevertheless have many terms in common with Plains Cree and Fox, and some with Delaware. Accordingly it is obvious that social organization is not the sole factor in terms of relationship. It may be objected that though Plains Cree and Fox possess many terms in common, yet they are not all used with the same meaning, and that the difference in social organization is responsible for this. For example Plains Cree and Fox have a term in common for 'my daughter,' but in Fox a male speaker can apply it to his brother's daughter also, whereas in Plains Cree he cannot. To this we reply that though both the Ojibwa and Fox are organized in exogamic gentes with descent in the male line and possess a number of terms of relationship in common, nevertheless all the terms are not used with the same meaning. An example is the term for 'my father' which is common to both Fox and Ojibwa as well as to numerous other Algonquian tribes, but which can be used in Fox also with the sense of 'my paternal uncle' though it cannot be so used in Ojibwa where an entirely distinct word is used for this latter meaning. So it is clear that difference in social organization by itself will not satisfactorily account for the different usage of the same term in Plains Cree and Fox. When it is further noted that Ojibwa, Ottawa, Algonkin, Potawatomi, and Cree share in common words for 'my brother's son' and 'my brother's daughter,' with a male speaker in each case, as opposed to Sauk, Fox, Kickapoo, Shawnee, Menominee, Peoria and closely allied dialects—all of whom employ the common Algonquian words for 'my son' and 'my daughter' respectively for the above— it becomes clear that we have to deal with linguistic phenomena. For it will be remembered that Cree-Montagnais, Menominee, Sauk, Fox, Kickapoo, and Shawnee, form one sub-division of Central Algonquian dialects and Ojibwa, Ottawa, Algonkin, Potawatomi, Peoria and closely related dialects another such sub-division.[7] Adequately to account for the divergent distribution of the terms in question we must assume that Cree has bodily borrowed them from Ojibwa, and that Peoria and closely related dialects under the influence of Sauk, Fox, etc., have extended the meanings of the terms for 'my son' and 'my daughter' to include also 'my brother's son' and 'my brother's daughter' (with male speaker in each case) respectively. In other words, in the latter case a category has been taken over. The geographical distribution of the tribes mentioned distinctly favors these hypotheses. It is another linguistic question as to which of the two sets of terms represents the hypothetical Algonquian parent-language most closely. The fact that Delaware and Munsee, who are organized in exogamic clans with descent in the female line, and who form a distinct sub-division of the Central Algonquian dialects, and Micmac, a somewhat divergent member of Eastern Algonquian dialects, agree with Sauk, Fox, etc., tends to show that the latter represent the primitive

Algonquian state as opposed to Ojibwa, etc.[8] This is exactly what might have been anticipated, for the sub-division represented by Cree-Montagnais, Menominee, Sauk, Fox, Kickapoo, and Shawnee in many respects linguistically is more archaic than the subdivision represented by Ojibwa, Ottawa, Algonkin, Potawatomi, Peoria and closely related tribes.[7] This applies especially to the first four dialects of the latter sub-division.

I spoke above of the borrowing of categories to explain certain terms of relationship. But this was within a single linguistic stock. Now as some members of both the Iroquoian and Siouan stocks have systems of consanguinity that in certain respects are identical with those of some members of the Algonquian stock, and at the same time geographically are contiguous to them, I do not think it likely that this is merely the result of accident. In short we have borrowing of categories across linguistic stocks. I mention this to show how inadequately social organization by itself can account for the terms of relationship of Algonquian languages.

Again the Piegan, a member of a linguistically divergent major group of Algonquian tribes, who are apparently organized in gentes in the making, have practically no terms of relationship that are the phonetic equivalents of those of the central group. Nor do the categories correspond with theirs. It is difficult to see how this is due to the social organization of the former. It will be borne in mind that the Plains Cree are if anything looser in organization, and yet have the old terms of relationship. It will be recalled that Cree in many respects is an archaic Algonquian language.[7] On the face of it, it looks as if we again had a linguistic problem.

Summing up, we must say that from the point of view of Algonquian tribes terms of relationship are linguistic and disseminative phenomena. But I do not deny that in other cases they may be primarily psychological and sociological. We have no right to generalize from a single stock and apply our conclusions everywhere. When some dozens of stocks shall have been analyzed somewhat on the above plan, it may be possible safely to generalize. Till then we have every reason to believe that all the factors have played a rôle in kinship-terms.

---

[1] *J. R. Anthrop. Inst. G. B. I., London,* **39,** 77–84.

[2] *Kinship and Social Organization,* London, 1914; *The History of Melanesian Society,* Cambridge, England, 1914.

[3] These PROCEEDINGS **1,** 346–349; *Amer. Anthrop., New York,* N. S., **17,** 223–239. However Lowie makes an exception in the case of the Cree terms of relationship which, he thinks,

points to the fact that the Cree once possessed exogamous gentes with descent in the male line. How unnecessary this hypothesis is, is shown in this paper.

[4] See *Amer. Anthrop., New York*, N. S., **17**, 329–340; *Ibid.*, 588–591; per contra, *Ibid.*, 175–177.

[5] *Smithsonian Contributions to Knowledge*, No. 218, p. 217.

[6] According to my unpublished information.

[7] *Smithsonian Inst., Rep. Bur. Amer. Ethn.*, **28**, 221–290b.

[8] We are confirmed in this belief by the fact taat the Atsina, a member of a linguistically divergent major group of Algonquain tribes, organized in non-totemic exogamous gentes with descent in the male line employ the same words for 'my son' and 'my daughter' as they do for for 'my brother's son' and 'my brother's daughter' respectively, with male speaker in the latter cases. The word for 'my daughter' is the common Algonquian one; that for 'my son' apparently is not old.

REPORT OF THE ANNUAL MEETING

Prepared by the Home Secretary

The sessions of the Annual Meeting of the National Academy of Sciences were held in the United States National Museum, Washington, D. C., on April 17, 18, and 19, 1916.

Seventy-two members were present as follows: Messrs. C. G. Abbot, Allen, Ames, Becker, Boltwood, Britton, Bumstead, D. H. Campbell, Cannon, Cattell, W. B. Clark, F. W. Clarke, F. M. Clarke, Comstock, Conklin, Coulter, Cross, Dall, Davenport, Davis, Day, Donaldson, Fewkes, Flexner, Hague, Hale, E. H. Hall, Harper, Hayford, Hillebrand, Holmes, Howell, Iddings, Jennings, Leuschner, Lillie, Loeb, Lusk, Mall, Meltzer, Mendel, Merriam, Merritt, Milliken, More, Morley, H. N. Morse, Moulton, E. L. Nichols, A. A. Noyes, Parker, Pickering, Pupin, Ransome, Reid, Remsen, Rosa, Schuchert, Scott, E. F. Smith, Story, Van Hise, Van Vleck, Walcott, Webster, Welch Wheeler, David White, H. S. White, Wilson, R. W. Wood, Woodward.

BUSINESS SESSIONS

The President announced the following deaths since the last Annual Meeting of the Academy:

John Ulric Nef, died August 13, 1915, elected in 1904.
Frederick Ward Putnam, died August 18, 1915, elected in 1885.
Arthur W. Wright, died December 19, 1915, elected in 1881.
Eugene W. Hilgard, died January 8, 1916, elected in 1872.
Paul Ehrlich, foreign associate, died August 20, 1915.

REPORTS FROM OFFICERS OF THE ACADEMY

The reports of the President and Treasurer for the year 1915 were presented to the Academy in printed form as transmitted to the Senate of the United States by the President of the Academy.

The report of the Home Secretary was presented as follows:

The President of the National Academy of Sciences.

Sir: I have the honor to present the following report on the publications and memberships of the National Academy of Sciences for the year ending April 19, 1916.

The Memoirs of the National Academy of Sciences, volume 12, part 2, entitled, "Variations and Ecological Distribution of the Snails of the Genus Io," by Charles C. Adams, has been published and distributed, as has also the memoir forming volume 12, being, "A Catalogue of the Meteorites of North America," by Oliver C. Farrington. Volume 14, memoir 1, entitled, "Report on Researches on the Chemical and Mineralogical Composition of Meteorites, with Especial Reference to Their Minor Constituents," by George Perkins Merrill, is going through the press and the final proof has been passed. It awaits casting and printing before it is published.

The biographical memoirs of John W. Powell, Miers Fisher Longstreth, Charles Anthony Schott, Peter Lesley, Henry Morton, and Alfred Marshall Mayer have been published, and that of George William Hill, by Ernest W. Brown, has also been published but not distributed.

Four members have died since the last Annual Meeting: John Ulric Nef, on August 13, 1915, elected in 1904; Frederick W. Putnam, on August 18, 1915, elected in 1885; Arthur W. Wright, on December 19, 1915, elected in 1881; and Eugene W. Hilgard, on January 8, 1916, elected in 1872.

One foreign associate, Paul Ehrlich, died on August 20, 1915.

There are 139 active members on the membership list, 1 honorary member, and 38 foreign associates.

Respectfully submitted, Arthur L. Day, Home Secretary.

The report of the Foreign Secretary was presented as follows:

I have the honor to report on the work of the foreign secretary for the year ending April 19, 1916.

An attempt has been made, through correspondence with various academies and societies belonging to the International Association of Academies, to secure a partial continuance of some portions of the Association's work through the period of the war. Although international meetings are obviously not feasible, it was hoped that a temporary transfer of the functions of the leading academy from Berlin to Amsterdam, as suggested by the former body, might serve a useful purpose. Unfortunately, however, certain difficulties of an insuperable nature prevented the proposed transfer, and no further steps can be taken at present.

It was suggested to the Amsterdam Academy by the foreign secretary, also acting in the capacity of secretary of a joint committee of the National Academy and the American Association for the Advancement of Science, that the Accademia dei Lincei be requested to use its good offices to secure the continuation of the work of the Zoological Station at Naples. A favorable reply was received from the President of the Lincei; but the participation of Italy in the war has prevented Dr. Dohrn from retaining the direction of the Station, which is now under an Italian administration.

At the request of the president of the Amsterdam Academy, who is also permanent secretary of the International Geodetic Association, the Secretary of State was asked by the Academy to use his influence to secure the continued participation of the United States in the work of the Association, and the maintenance of the international latitude station at the Ukiah, California. Through the action of the Secretary of State, and the interest of members of Congress, the necessary appropriations have been provided.

George E. Hale, Foreign Secretary.

REPORTS FROM COMMITTEES ON TRUST FUNDS

A report was received from the directors of the Bache Fund, stating that Ira Remsen resigned as Director of the Fund at the annual meeting, 1915, and that the two remaining members of the committee chose Arthur G. Webster as the third member and that Edwin B. Frost was elected chairman. This report contained also an announcement of the research grants made from the Bache Fund during the year ending April 17, 1916; and it stated that on April 7, 1916, the Bache Fund had on hand a cash income balance of $980.62, and an invested income of $2575.

A report was received from the Trustees of the Watson Fund, signed by Edward C. Pickering (chairman) and W. L. Elkin. In the report the trustees recommended that the sum of $500 from the income of the Watson Fund be appropriated to Professor John A. Miller, Director of the Sproul Observatory, for measuring plates already taken for the determination of stellar parallaxes. This is a continuation of a grant awarded last year. It was recommended that the sum of $300 from the income of the Watson Fund be appropriated to Professor Herbert C. Wilson, Director of the Goodsell Observatory, for measurements of the positions of asteroids on photographs already taken. These recommendations were adopted by the Academy.

A report was received from the Committee on the Henry Draper Fund, signed by George E. Hale (chairman) as follows:

> Four members of the committee, without consulting the fifth member (Professor Michelson), recommend that the Henry Draper Gold Medal be awarded to ALBERT A. MICHELSON, of the University of Chicago, for his numerous and important contributions to spectroscopy and astronomical physics.
>
> It is impossible in the brief space of this report even to enumerate Professor Michelson's major services to science. These include the precise determination of the velocity of light; the well-known experiment (with Professor Morley) on ether drift; the measurement of the absolute wave-length of light involved in his determination of the length of the standard meter; the measurement of tides in the body of the earth with new apparatus of extraordinary precision; and the invention of the interferometer, the echelon, and other instruments of prime importance to the student of light. He has also constructed a ruling machine yielding diffraction gratings of the longest size and the highest resolving power yet attained, and carried on a multiplicity of researches of wide range and fundamental significance.
>
> The committee also recommends that a grant of $250 be made to Professor Philip Fox, Director of the Dearborn Observatory, of Northwestern University, Evanston, Illinois, to apply toward the cost of a machine for measuring astronomical photographs.

The recommendations contained in the report were adopted by the Academy.

A report was received from the Committee on the J. Lawrence Smith Fund, signed by Edward W. Morley (chairman). A brief account was given of the researches aided by previous grants. The Committee reported a cash balance of income of $834.77, of which $250 is already appropriated, though not yet paid over. There is also an invested income balance of $1532.50. The Committee unanimously recommended that a further grant of $300 be made to S. A. Mitchell, professor in the University of Virginia, University, Va., to aid in securing observations of paths and of radiants of meteors and in computing orbits where observations are sufficient. This recommendation was adopted by the Academy.

A report was received from the Committee on the Comstock Fund, signed by Edw. L. Nichols (chairman). The Committee reported that the total income from the fund now available is $1661.32. The next award will be made at the end of the five-year period specified in the bequest, namely, at the annual meeting in April, 1918.

A report was received from the Directors of the Walcott Gibbs Fund, signed by the three directors. The report stated that T. W. Richards was elected to fill the vacancy caused by the withdrawal of Ira Remsen from the board. Only one appropriation had been made from the income of the Fund—a grant (No. 6) to Professor Gregory P. Baxter of Cambridge of $300 to provide apparatus, especially of platinum and quartz, and materials for his researches on atomic weights and changes of volume during solution. The unexpended income of the fund was stated to be $90.77. A brief account was given of the progress made on the researches aided by previous grants.

A report was received from the Committee on the Murray Fund, signed by Arnold Hague (chairman). The Committee announced that the unusual expenses due to the designing and striking off of the Agassiz medal, as called for by the terms of the gift, has required all the early income; but that from now on the interest from the fund will be applied for the striking off of the Agassiz medal and contributions to oceanography.

A summarized statement of the grants made from the various trust-funds of the Academy during the year ending April 17, 1916, will be found at the end of this number of the PROCEEDINGS.

REPORT OF THE BOARD OF EDITORS OF THE PROCEEDINGS

The Board of Editors of the PROCEEDINGS reported to the Academy as follows:

> The twelve numbers of the PROCEEDINGS issued since its last annual report contain 157 original papers, the report of the Annual and Autumn Meetings, notices of scientific memoirs, and the report of the Panama Canal Committee. These numbers consist of 641 pages, an average of 53 pages per number, and of about 4 pages per article. The papers are distributed among different sciences as follows: mathematics, 17, astronomy 37, physics 11, chemistry 12, geology, paleontology, and mineralogy 16, botany 3, zoology 13, genetics 17, physiology, pathology, bacteriology, and biochemistry 16, anthropology 13, psychology 2.
>
> The universities or research laboratories that have contributed three or more papers are as follows: Carnegie Institution 32, divided as follows: Mount Wilson Solar Observatory 22, Nutrition Laboratory 4, Marine Biology 3, Terrestrial Magnetism 1, Botanical Research 1, Geophysical Laboratory, 1; University of California 17; Harvard University 16; Johns Hopkins University 14; University of Chicago 13; Yale University 7; Rockefeller Institute 7; Princeton University 6; U. S. Geological Survey 5; U. S. National Museum 5; Station for Experimental Evolution 4; University of Illinois 3; American Museum of Natural History 3.
>
> The edition of the PROCEEDINGS was reduced from 3000 to 2200 copies with the issue of May, 1915, and it is to be further reduced to 1700 copies with the issue of May, 1916, as the distribution of free copies in America is to be now entirely discontinued. At present about 900 copies are mailed to foreign exchanges and about 250 copies to members of the Academy and other American subscribers.

The total expense incurred in connection with the PROCEEDINGS during the year 1915, representing approximately the cost of publication of the first volume, is shown by the report of the treasurer to be $4300.67. Of this sum $2977.37 was expended for printing and distributing, corresponding to a cost of $4.65 per page.

The Board recommends that action be taken by the Academy informing members and others presenting accounts of new researches at the Academy meetings that they are expected to furnish to the Home Secretary at the time of the meeting a brief paper describing these researches in form suitable for publication in the PROCEEDINGS. The Board also suggests that, in order to secure fuller circulation of the PROCEEDINGS in our universities, colleges, and research laboratories members of the Academy be requested to see that their own institutions subscribe to the PROCEEDINGS in case they have not already done so.

GENERAL BUSINESS

A report of the Committee on Bill H. R. 528, discontinuing the use of the Fahrenheit thermometer scale in government publications was adopted as follows:

Your committee for the consideration of Bill H. R. 528, consisting of Messrs. C. G. Abbot, S. W. Stratton and C. M. Marvin, unanimously reports the following resolution, and moves its adoption.

The National Academy of Sciences shares the desire of scientific men in general for international and world-wide uniformity in units of measurement of all kinds, and with this object in view it favors the introduction of the Centigrade scale of temperature, and units of the metric system generally, as standards in the publications of the United States Government.

It must be recognized that considerable initial expense must be incurred by the U. S. Weather Bureau in changing its apparatus to conform to the proposed act. Furthermore, on account of the more open scale of the Centigrade system that Bureau will be subject to a continued increased cost of publication, owing to the necessity of printing the first decimal place in order to maintain the present accuracy. The use of negative temperatures and minus signs entails greater liability to errors, and more clerical labor would be required in checking the accuracy of the reports of coöperative observers of the Weather Bureau, and in computing monthly and other mean temperatures.

Notwithstanding the foregoing, the Academy is in favor of legislation to make the Centigrade scale of temperatures the standard in publications of the United States Government, and funds should be made available by Congress to accomplish the desired result.

The Academy favors Bill H. R. 528, "To discontinue the use of the Fahrenheit Thermometer Scale in Government Publications," but recommends that it be amended by the addition of the following:

"Sec. 4. When in the publication of tables containing several meteorological and climatic elements, the use of data in Centigrade temperatures leads to manifest incongruities, the Chief of the Weather Bureau is directed to publish related data in such units as are necessary to make the tables homogeneous and to secure international uniformity as far as practicable.

"Sec. 5. Nothing in this act shall prevent the use of the absolute Centigrade scale of temperature in publications of the Government."

Upon recommendation of the Council it was voted that in accordance with the request of the chairman of the Committee on Foreign Affairs of the House of Representatives a committee of the Academy be appointed to prepare a report upon the joint Resolution (H. J. Res. 99), "That the President be, and he is hereby, requested to ascertain the views of foreign governments regarding the proposition to appoint an international commission to prepare a universal alphabet," and that the report be submitted to the President of the Academy, who in turn will transmit it to the chairman of the Committee on Foreign Affairs of the House of Representatives, reporting his action in the matter at the next annual meeting of the Academy.

Messrs. Dewey, Bell, Lindgren, Cattell, and Boas were appointed members of this committee.

The Council also recommended to the Academy the appointment of a committee to discuss possible plans of coöperation with a committee of engineers. A committee was appointed, consisting of George E. Hale, chairman; J. S. Ames, John F. Hayford, E. L. Nichols, M. I. Pupin, E. B. Rosa, Elihu Thomson, C. R. Van Hise, C. D. Walcott, and R. S. Woodward.

The President announced that an invitation had been received from the members of the Academy living in Boston that the Academy hold its Autumn Meeting in the year 1916 in that city. The following members were appointed to serve as a local committee of this meeting: William M. Davis, chairman, W. T. Councilman, Arthur A. Noyes, George H. Parker and E. C. Pickering.

ELECTION OF FOREIGN SECRETARY, COUNCILLORS AND NEW MEMBERS

Mr. **George E. Hale** was reelected foreign secretary of the Academy for a term of six years.

R. H. **Chittenden** and M. I. **Pupin** were elected members of the Council for a term of three years.

The following persons were elected to membership in the Academy:

Gilbert Ames Bliss, mathematician, University of Chicago, Chicago, Illinois.
Frank Schlesinger, astronomer, University of Pittsburgh, Pittsburgh, Pa.
Gregory Paul Baxter, chemist, Harvard University, Cambridge, Mass.
Marston Taylor Bogert, chemist, Columbia University, New York City.
Leland Ossian Howard, entomologist, U. S. Department of Agriculture, Washington, D. C.
Alfred Goldsborough Mayer, zoologist, Carnegie Institution, Tortugas, Florida.
Raymond Pearl, biologist, Maine Agricultural Experiment Station, Orono, Maine.
Phoebus Aaron Theodore Levene, physiological chemist, Rockefeller Institute for Medical Research, New York City.
Otto Folin, physiological chemist, Harvard Medical School, Boston, Mass.

SCIENTIFIC SESSIONS

Two public lectures on the **William Ellery Hale Foundation** were given on April 17 and 19 by **Henry Fairfield Osborn**, President of the American Museum of Natural History, on The Origin and Evolution of Life on the Earth.

Four public scientific sessions were held on April 17 and 18 at which the following papers were presented:

SYMPOSIUM ON THE EXPLORATION OF THE PACIFIC. ARRANGED BY W. M. DAVIS

W. M. **Davis**: On exploration of the Pacific.
J. F. **Hayford**: The importance of gravity observations at sea in the Pacific.

L. J. Briggs, President of the Philosophical Society of Washington: A new method of determining gravity at sea.

C. Schuchert: The problem of continental fracturing and diastrophism in Oceanica.

J. P. Iddings: Petrological problems in the Pacific.

G. W. Littlehales: Hydrographic Engineer United States Hydrographic Office: The extent of knowledge of the oceanography of the Pacific.

C. F. Marvin, Chief of the United States Weather Bureau: Marine meteorology and the general circulation of the atmosphere.

Wm. H. Dall: On the distribution of Pacific invertebrates.

H. A. Pilsbury, Academy of Natural Sciences of Philadelphia: Land mollusca of the Pacific.

W. G. Farlow: Marine algae of the Pacific.

D. H. Campbell: Problems of the Pacific floras.

J. W. Fewkes: The Pacific as a field for anthropological investigation.

REGULAR PROGRAM

S. J. Meltzer: On permeability of endothelia.

I. S. Kleiner and S. J. Meltzer: The influence of morphin upon the elimination of intraveneously injected dextrose.

Jacques Loeb: The sex of a parthenogenetic frog.

Edmund B. Wilson: The distribution of the chondrisomes to the spermatozoa in scorpions.

Arthur Keith (introduced by George F. Becker); A new form of metamorphism.

J. P. Iddings and E. W. Morley: Contributions to the Petrology of Japan, Philippine Islands and the Dutch Indies.

Charles R. Stockard, Professor of Anatomy, Cornell University Medical College: Hereditary transmission of defects resulting from alcoholism.

W. B. Cannon: Recent observations on the activity of some glands of internal secretion.

H. H. Donaldson: Studies in the water content of the nervous system.

George E. Hale: Some recent results of solar research.

Charles E. St. John (introduced by G. E. Hale): An investigation of the suggested mutual repulsion of Fraunhofer lines.

Arthur S. King (introduced by G. E. Hale): Anomalous dispersion phenomena in electric furnace spectra.

Walter S. Adams (introduced by G. E. Hale): Illustrations of the new spectroscopic method of measuring stellar distances.

Harlow Shapley (introduced by G. E. Hale): Some results with the new 10-inch photographic telescope.

C. G. Abbot and L. B. Aldrich: The pyranometer, an instrument for the measurement of sky radiation.

G. C. Comstock: Invisible companions of binary stars.

Edwin H. Hall: Theory of electric conduction in metals.

F. R. Moulton: The evolution of the stars.

A. O. Leuschner, Watson Medallist: The minor planets discovered by James C. Watson.

Wm. H. Dall: Biography of Professor Theodore Nicholas Gill (by title).

W. W. Campbell: Biography of Professor Edward Singleton Holden (by title).

W. W. Campbell: Biography of Professor Simon Newcomb (by title).

Fielding H. Garrison: Biography of John Shaw Billings (by title).

Charles R. Van Hise: Report of the work of the committee upon the Panama Canal slides.

H. Fielding Reid: The mechanics of the Panama slides.

Theodore Lyman, Director Jefferson Physical Laboratory, Harvard University: The present state of knowledge of the extreme ultra violet.

Robert A. Millikan: A redetermination of e and N.

Carl L. Alsberg, Chief of the Bureau of Chemistry, United States Department of Agriculture: The relation of investigational work to the enforcement of the food and drugs act.

J. Walter Fewkes: Recent explorations on the Mesa Verde National Park, Colorado.

Erwin F. Smith: Further evidence on the nature of crown gall and cancer and that cancer in plants offers strong presumptive evidence both of the parasitic origin and of the essential unity of the various forms of cancer in man and animals.

AWARD OF MEDALS

At the annual dinner of the Academy held at the Hotel Raleigh on April 18, 1916, Medals for Eminence in the Application of Science to the Public Welfare were awarded to Cleveland Abbe for distinguished public service in establishing and organizing the weather service of the United States, and to Gifford Pinchot for distinguished public service in organizing and directing the movement for the systematic conservation of the natural resources of the United States; and the James Craig Watson medal was awarded to Armin Otto Leuschner for the skill and ability which he has shown in supervising the preparation of tables of the Watson asteroids, involving original methods, and leading to results of much value to celestial mechanics.

RESEARCH GRANTS FROM THE TRUST FUNDS OF THE ACADEMY

During the twelve months preceding the Annual Meeting of the Academy the following grants for the promotion of research were made from the trust funds of the Academy.

GRANTS FROM THE BACHE FUND

No. 188, H. W. Norris, Grinnell College, $100. For assistance in the analysis of the cranial nerves of Coecilians (Herpele and Dermophis).

No. 189, E. J. Werber, Woods Hole, $230. For assistance in experimental studies aiming at the control of defective and monstrous development: (1) the effect of toxic products of metabolism on the developing teleost egg; (2) the effect of experimentally produced diseases of parental metabolism on the offspring of mammals.

No. 190, H. S. Jennings, Johns Hopkins University, $200. For assistance in the study of evolution in a unicellular animal multiplying by fission: heredity, variation, racial differentiation in *Difflugia*.

No. 191, P. W. Bridgman, Harvard University, $500. For mechanical assistance in an investigation of various effects of high hydrostatic pressure, in particular the effect of pressure on electrical resistance of metals. (Supplementary to grant No. 184.)

No. 192, J. P. Iddings, Washington, D. C., $1000. For apparatus and assistance in the microscopical and chemical investigation of igneous rocks for the purpose of extending knowledge regarding petrographical provinces and their bearings on the problem of isostasy.

No. 193, C. A. Kofoid, University of California, $500. For assistance in securing animals in the Indian jungle and in their preparation for study in research on the intestinal protozoa.

No. 194, R. A. Daly, Harvard University, $1000. For the purchase of a thermograph of new design for determining temperatures in the deep sea.

No. 195, R. W. Hegner, University of Michigan, $160. For assistance in the study of the history of the germ cells, especially in hermaphrodite animals, in order to determine the visible changes that take place in their differentiation and the causes of these changes. (Supplementary to grant No. 185).

GRANT FROM THE J. LAWRENCE SMITH FUND

No. 6, S. A. Mitchell, University of Virginia, University, Va., $500. For aid in securing observations of paths and of radiants of meteors and in computing orbits where observations are sufficient.

GRANT FROM THE WOLCOTT GIBBS FUND

No. 6, Gregory P. Baxter, Harvard University, $300. To provide apparatus, especially of platinum and quartz, and materials for his researches on atomic weights and changes of volume during solution.

The following additional grants were authorized by the Academy at the business sessions of April 17–19, 1916.

GRANT FROM THE HENRY DRAPER FUND

Philip Fox, Dearborn University, Northwestern University, $250. To be applied towards the cost of a machine for measuring astronomical photographs.

GRANTS FROM THE WATSON FUND

No. 12, Herbert C. Wilson, Goodsell Observatory, $300. For measurements of the position of asteroids on photographs already taken.

No. 13, John A. Miller, Sproul Observatory, Swarthmore College, $500. For measuring plates already taken for the determination of stellar parallaxes. (Supplementary to grant No. 10.)

GRANTS FROM THE BENJAMIN APTHORP GOULD FUND

S. D. Townley, Lick Observatory, $200. For observing variable stars.

R. W. Wood, Johns Hopkins University, $200. For apparatus for photographing celestial bodies with monochromatic light.

A. Van Maanen, Mount Wilson Solar Observatory, $100. For clerical assistance in the reduction of places of certain propermotion stars.

Edwin B. Frost, Yerkes Observatory, $640. For a measuring machine for measuring parallax plates.

CONTENTS

Page

PHYSICS.—THE HIGH FREQUENCY SPECTRUM OF TUNGSTEN . . . .
By Albert W. Hull and Marion Rice 265

MATHEMATICS.—ON THE FOUNDATIONS OF PLANE ANALYSIS SITUS . . . . . .
By Robert L. Moore 270

MATHEMATICS.—A GENERAL THEORY OF SURFACES . . . .
By Edwin B. Wilson and C. L. E. Moore 273

ENGINEERING.—DYNAMICAL STABILITY OF AEROPLANES . *By Jerome C. Hunsaker* 278

GEOGRAPHY.—CLIFT ISLANDS IN THE CORAL SEAS . . . . *By W. M. Davis* 284

ASTRONOMY.—ON SOME RELATIONS BETWEEN THE PROPER MOTIONS, RADIAL VELOCITIES, AND MAGNITUDES OF STARS OF CLASSES B AND A . *By C. D. Perrine* 289

ASTRONOMY.—ASYMMETRY IN THE PROPER MOTIONS AND RADIAL VELOCITIES OF STARS OF CLASS B AND THEIR POSSIBLE RELATION TO A MOTION OF ROTATION
By C. D. Perrine 292

ENGINEERING.—THEORY OF AN AEROPLANE ENCOUNTERING GUSTS . . . . . .
By Edwin Bidwell Wilson 294

ANTHROPOLOGY.—TERMS OF RELATIONSHIP AND SOCIAL ORGANIZATION . . . . .
By Truman Michelson 297

REPORT OF THE ANNUAL MEETING . . . . *Prepared by the Home Secretary* 300

VOLUME 2 JUNE, 1916 NUMBER 6

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES
OF THE
UNITED STATES OF AMERICA

JUN 24 [illegible]
National Museum

EDITORIAL BOARD

ARTHUR A. NOYES, *Chairman* EDWIN B. WILSON, *Managing Editor*
ARTHUR L. DAY, *Home Secretary* GEORGE E. HALE, *Foreign Secretary*

J. J. ABEL
W. B. CANNON
J. McK. CATTELL
J. M. CLARKE
E. G. CONKLIN
J. M. COULTER
C. B. DAVENPORT
SIMON FLEXNER
E. B. FROST
R. G. HARRISON
W. H. HOLMES
J. P. IDDINGS
R. A. MILLIKAN
E. H. MOORE
ALEXANDER SMITH

Publication Office: Williams & Wilkins Company, Baltimore
Editorial Office: Massachusetts Institute of Technology, Boston
Home Office of the Academy: Washington, D. C.

Entered as second-class matter at the Post Office at Baltimore, Md., under the Act of August 24, 1912

INFORMATION TO CONTRIBUTORS

The Proceedings is the official organ of the Academy for the publication of brief accounts of important current researches of members of the Academy and of other American investigators, and for reports on the meetings and other activities of the Academy. Publication in the Proceedings will supplement, not replace, that in journals devoted to the special branches of science. The Proceedings will aim especially to secure prompt publication of original announcements of discoveries and wide circulation of the results of American research among investigators in other countries and in all branches of science.

Articles should be brief, not to exceed 2500 words or 6 printed pages. Technical details of the work and long tables of data should be reserved for publication in special journals. But authors should be precise in making clear the new results and should give some record of the methods and data upon which they are based. The viewpoint should be comprehensive in giving the relation of the paper to previous publications of the author or of others and in exhibiting where practicable, the significance of the work for other branches of science.

Manuscripts should be prepared with a current number of the Proceedings as a model in matters of form, and should be typewritten in duplicate with double spacing, the author retaining one copy. Illustrations should be confined to text-figures of simple character, though more elaborate illustrations may be allowed in special instances to authors willing to pay for their preparation and insertion. Particular attention should be given to arranging tabular matter in a simple and concise manner.

References to persons should include initials; citations of literature, numbered consecutively, should be placed at the end of the article; and short footnotes should be avoided. Indicate 'subject' for 'Contents.'

Papers by members of the Academy may be sent to Edwin Bidwell Wilson, Managing Editor, Mass. Institute of Technology, Boston, Mass. Papers by non-members should be submitted through some member.

Proof will be sent to authors only on their request, in order to avoid delay in publication. Authors are therefore requested to make final revisions on the typewritten manuscripts. The editors cannot undertake to do more than correct obvious minor errors.

Reprints should be ordered at the time of submission of manuscript. They will be furnished to authors at cost, approximately as follows:

Reprints of - -	2 pp.	4 pp.	6 pp.	8 pp.	Covers extra
Charge for first 100 copies	$1.00	$1.30	$2.20	$2.20	$2.00
Charge for additional 100s	.25	.45	.85	.85	.50

INFORMATION TO SUBSCRIBERS

Subscriptions at the rate of $5.00 per annum should be made payable to the National Academy of Sciences, and sent to Williams & Wilkins Company, Baltimore, or Arthur L. Day, Home Secretary, National Academy of Sciences, Smithsonian Institution, Washington, D. C. Single numbers, $0.50.

Copyright, 1916, by the National Academy of Sciences

Smithsonian Institution
JUN 24 1921
National Museum

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES

Volume 2 JUNE 15, 1916 Number 6

DIFFERENTIAL EQUATIONS AND IMPLICIT FUNCTIONS IN INFINITELY MANY VARIABLES

By William L. Hart

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF CHICAGO

Received by the Academy, April 15, 1916

The paper of which this note gives a brief abstract, has three main divisions. In the first section certain fundamental theorems are developed concerning a type of real-valued functions of infinitely many real variables. In the second section there is considered the problem of infinite systems of ordinary differential equations.

$$\frac{dx_i}{dt} = f_i\,(x_1, x_2, \;.\;\;.\;\;.\;; t), \qquad x_i\,(t_0) = a_i, \qquad i = 1, 2, \;.\;\;.\;\;., \tag{1}$$

in which the f_i are of the type treated in the initial theorems. In the third section of the paper the fundamental problem of implicit function theory in this field is discussed for a system of equations

$$\begin{aligned} &f_i\,(x_1, x_2, \;.\;\;.\;\;.\;; y_1, y_2, \;.\;\;.\;\;.) = 0, \qquad i = 1, 2, \;.\;\;.\;\;., \\ &f_i\,(a_1, a_2, \;.\;\;.\;\;.\;; b_1, b_2, \;.\;\;.\;\;.) = 0, \end{aligned} \tag{2}$$

where the x_j are independent variables and the y_j are to be determined. The results of all three sections of the paper include as special cases the corresponding theorems on functions of a finite number of variables.

The region of real points $\xi = (x_1, x_2, \;.\;\;.\;\;.)$ in the space of infinitely many dimensions in which the functions considered are supposed defined, is of the type specified by

$$R: |x_i - a_i| \leqq r_i, \qquad i = 1, 2, \;.\;\;.\;\;., \qquad 0 < r \leqq r, \qquad r \textit{ finite.}$$

In the first part of the investigation theorems are derived for functions *completely continuous* according to the

Definition 1. *A function* $f(\xi)$ *defined in* R *is completely continuous at a point* $\xi^{(0)}$*—in notation,* $C_1\ (\xi^{(0)})$*—if whenever*

$$\lim_{n=\infty} x_{in} = x_i^{(0)}, \qquad i = 1, 2, \ldots, \tag{3}$$

it follows that

$$\lim_{n=\infty} f(\xi_n) = f(\xi^{(0)}), \qquad \xi_n = (x_{1n}, x_{2n}, \ldots).$$

This concept of complete continuity is the same as that of '*Vollstetigkeit*' which has been much used by Hilbert and his followers. However, in the present paper the functions satisfying Definition (1) are supposed defined for a region of points different from that used by Hilbert and entirely different applications of the concept are made.

A function $f(\xi)$ is said to be C_0 at $\xi^{(0)}$ if it has the weaker continuity resulting from Definition (1) when in (3) it is assumed that the convergence is uniform with respect to i.

In deriving theorems on completely continuous functions important use is made of the following condensation lemma:[1]

Lemma 1. *Let* $S = (\xi_n\colon n = 1, 2, \ldots)$ *be a sequence of points of* R. *Then there exists a point* ξ' *of* R *and an infinite sub-sequence* $\xi'_n\ (n = 1, 2, \ldots)$ *of* S *such that* $\lim_{n=\infty} x'_{in} = x'_i\ (i = 1, 2, \ldots)$.

The theorems obtained, of which the more important ones are given below, are derived by methods similar to those used in obtaining corresponding results in classical analysis.

Theorem 1. *To state that* $f(\xi)$ *is* C_0 *at* $\xi^{(0)}$ *is equivalent to saying that, for every* $e > 0$ *there exists a number* $d_e > 0$ *such that for every* ξ' *of* R *satisfying* $|x'_i - x_i^{(0)}| \leqq d_e$, $i = 1, 2, \ldots$, *there is the inequality* $f(\xi') - f(\xi^{(0)})| \leqq e$.

On taking this equivalent definition of the C_0 property, there is obtained

Theorem 2. *If* $f(\xi)$ *is* C_1 *at every point of* R, *then* $f(\xi)$ *possesses the* C_0 *property uniformly in* R.

The analogues of two fundamental theorems from the theory of continuous functions are found in the next two theorems.

Theorem 3. *If* $f_n(\xi)\ (n = 1, 2, \ldots)$ *are* $C_1(\xi^{(0)})$ *while also* $\lim_{n=\infty} f_n(\xi) = f(\xi)$ *uniformly for* ξ *in* R, *then* f *is* $C_1\ (\xi^{(0)})$.

Theorem 4. *If* $f(\xi)$ *is* C_1 *at all points of* R, *then* $|f(\xi)|$ *is finitely limited and* f *attains its upper and lower bounds in* R.

As a foundation for much of the work in the sections on differential equations and implicit functions, there is established for infinitely many variables an analogue of Taylor's Theorem with n terms and an integral form for the remainder. The 'mean value' case of the general theorem is

Theorem 5. *Suppose that the function* $f(\xi)$ *and its partial derivatives* $\partial f/\partial x_i$ $(i = 1, 2, \ldots)$ *are* C_1 *in* R *and that* $\Sigma_{j=1}^{\infty} r_j \,|\, \partial f(\xi)/\partial x_j \,|$ *converges uniformly for all* ξ *in* R. *Then for* (ξ, ξ') *of* R,

$$f(\xi') - f(\xi) = \sum_{j=1}^{\infty} (x'_j - x_j) \int_0^1 \frac{\partial f[\xi + u(\xi' - \xi)]}{\partial x_j} du.$$

Systems of differential equations of the form (1) were first considered by H. Von Koch[2] who treated a case of an analytic type. A very similar problem was considered later by F. R. Moulton.[3] Infinite systems of a linear form were discussed by E. H. Moore[4] as a special case of a more general investigation made in the sense of Moore's General Analysis.[5]

In the present paper, the problem (1) is treated by a generalized *Picard Approximation* method. The coördinates $x_{ik}(t)$ of the approximations $\xi_k(t)$ to a solution of (1) are defined formally by the equations

$$\begin{aligned} \xi_0(t) &= (a_1, a_2, \ldots), \\ x_{ik}(t) &= x^t{}_0 + \int_0^t f_i[\xi_{k-1}(t), t]\, dt, \qquad k = 1, 2, \ldots. \end{aligned} \tag{4}$$

The existence theorem obtained is

Theorem 6. *Suppose in* (1) *that the* f_i *are defined and* C_1 *in*

$$T: |t - t_0| \leqq r_0; \; R: |x_i - a_i| \leqq r_i \quad (0 < r_i < r; \; r \textit{ finite}), \tag{5}$$

and that there exist positive functions $A_{ij}(t, \xi, \xi')$ *defined and* C_1 *for* t *in* T *and* (ξ, ξ') *in* R. *Assume that*

$$|f_i(\xi, t) - f_i(\xi', t)| \leqq \sum_{j=1}^{\infty} A_{ij}(t, \xi, \xi') \,|x'_j - x_j|. \tag{6}$$

Suppose, moreover, that for $(i = 1, 2, \ldots)$ *and for all admissible values of* (t, ξ, ξ')

$$\sum_{j=1}^{\infty} r_j A_{ij}(t, \xi, \xi') = V_i(t, \xi, \xi')$$

converges uniformly, and that the maxima M_i *of the* $|f_i(\xi, t)|$ *and the maxima* K_i *of the* V_i *satisfy* $M_i \leqq r_i M$, $2K_i \leqq r_i K$, *with* K *and* M *finite. Then the approximations* (4) *exist and converge to a function* $\xi(t)$ *for* $|t - t_0|$ *sufficiently small. Moreover,* $x_i(t)$ *is continuous in* t *and* $\xi(t)$ *is the unique continuous solution of* (1).

The linear system of Moore is not related to the system (1) of the preceding theorem but the results of Moulton, when restricted to reals, are a special case of the conclusion of Theorem 6. In addition to the results of Moulton, however, it follows that the *unique analytic solution* he obtained is, for real values of t, the *only continuous solution of* the system he treated.

Under certain hypotheses in addition to those of Theorem 6, it is proved that the solution $\xi(t)$ has continuation properties which reduce, for the finite case, to the ordinary theorem on the existence of a solution extending to the boundary of the given region of definition of the system.

It is also shown, by the aid of Theorem 5, that the formal hypothesis (6) can be replaced by an assumption concerning the existence and the values of the partial derivatives $\partial f_i/\partial x_j$ $(i, j = 1, 2, \ldots)$.

Infinite systems of equations of the form (2) have been considered by H. Von Koch[6] and R. d'Adhemar.[7] Von Koch treated a system of an analytic type defined in the field of complex numbers and established the existence of an analytic solution. His work, however, is valid only if the sum of the numbers r_i of the region similar to R in which his system is defined converges in a very special manner. R. d'Adhemar treated a special type which arose in a problem he considered in integral equation theory. The results of Von Koch, when restricted to reals, are a special case of the theorem stated below.

In the solution of the system (2), considered in the present paper, infinite sets of linear equations enter in a fundamental fashion. Such hypotheses are imposed that these linear systems come under the theory of infinite systems of linear equations with normal determinants. The method of solution of (2) for the y_j is related to that used by Goursat[9] in his solution of the finite case by a method of successive approximations. An analogue for (2) of the fundamental theorem on implicit functions in the finite case, is obtained in the following form:

Theorem 7. *Suppose that, in* (2), *the functions* f_i *and* $\partial f_i/\partial y_j$ $(i, j = 1, 2, \ldots)$ *are defined and* C_1 *for all points in*

$$S: |x_i - a_i| \leqq r_i;\ T: |y_i - b_i| \leqq r_i,\ i = 1, 2, \ldots, 0 < r_i \leqq r, \quad (7)$$

and that the maxima M_i *of the* $|f_i(\xi, \eta)|$ $(\eta = y_1, y_2, \ldots)$ *satisfy* $M_i \leqq r_i M$ *(M finite). Assume that for all* (ξ, η) *in* (7)

$$\sum_{i,j=1}^{\infty} \left| d_{ij} - \frac{\partial f_i(\xi, \eta)}{\partial y_j} \right|, \qquad d_{ij} = 0,\ i \neq j;\ d_{ii} = 1, \qquad (8)$$

converges uniformly. Suppose that the normal infinite determinant

$$\Delta = \left| \frac{\partial f_i(\alpha, \beta)}{\partial y_j} \right|_{i,j=1,2,\ldots} \neq 0, \qquad \alpha = (a_1, a_2, \ . \ . \ . \ ), \beta = (b_1, b_2, \ . \ . \ . \ ). \quad (9)$$

Suppose that there exists a number B such that

$$\sum_{k=1}^{\infty} \left| D_{ki} \right| \leqq Br_i \qquad i = 1, 2, \ . \ . \ . \ ,$$

where D_{ki} *is the cofactor of the element* $\partial f_k/\partial y_i$ *in* (9). *Let it be assumed that for every* $e > 0$ *there can be found a number* $d_e > 0$ *such that, for* $| x_j - a_j | \leqq d_e$ $(j = 1, 2, \ . \ . \ .)$ *it follows that* $| f_i(\xi, \beta) | \leqq e$ $(i = 1, 2, \ . \ . \ .)$. *Then there exist positive constants* (c, d), $0 < d \leqq 1$, $0 < c \leqq d$, *such that to every* ξ *satisfying* $| x_j - a_j | \leqq cr_j$ *there corresponds one and only one solution of* (2) *in the region defined by* $| y_j - b_j | \leqq dr_j$. *Furthermore, the solution* η $(x_1, x_2, \ . \ . \ .)$ *so determined is* C_1 *for all* ξ *satisfying* $| x_j - a_j | \leqq cr_j$.

The problem of obtaining continuation properties for the solution η (ξ) is not developed in the present paper because the form of the hypotheses used here does not readily lend itself to this generalization.

[1] Cf. an equivalent theorem proved in Bolza's *Vorlesungen über Variationsrechnung*, p. 423, part (b).

[2] *Öfversigt af Kongliga Vetenskaps Akademiens Förhandlingar*, **56**, 395–411 (1899).

[3] *These* PROCEEDINGS, **1**, 350 (1915).

[4] *Atti dei IV Congresso Internazionale dei Mathematici*, **2**, 98 (Roma, 6–11 Aprile, 1908).

[5] Introduction to a Form of General Analysis, *New Haven Mathematical Colloquium*.

[6] *Paris, Bull. Soc. Math.*, **27**, 215 (1899).

[7] *Ibid.*, **36**, 95 (1908).

[8] Cf. Kowalewski, *Einfürung in die Determinanten-Theorie.*

[9] *Paris, Bull. Soc. Math.*, **31**, 184 (1903).

THE SEX OF PARTHENOGENETIC FROGS

By Jacques Loeb

ROCKEFELLER INSTITUTE FOR MEDICAL RESEARCH, NEW YORK

Read before the Academy, April 17, 1916. Received, April 21, 1916

When it was established that larvae could be produced from unfertilized eggs by chemical methods,[1] a number of secondary problems arose. Foremost among these was the question whether or not the organisms thus produced were capable of developing into normal adults. This was at first considered improbable, especially by those who accepted Oscar Hertwig's statement that fertilization consisted in the fusion of the egg and sperm nucleus. Since no such fusion takes place in artificial parthenogenesis, it became obvious that either Hertwig's definition of fertilization was wrong or that artificial parthenogenesis was merely a pathological phenomenon not capable of leading to the formation of a normal organism.

My first experiments on artificial parthenogenesis were carried out on marine invertebrates, sea urchins, starfish, annelids, and molluscs,[2] and since it is difficult to raise the normal larvae of such animals to the adult stage it seemed hopeless to attempt the task in the case of parthenogenetic specimens. Delage[3] had the courage to undertake it on sea urchins and succeeded in raising two parthenogenetic larvae to the adult stage, one of which was far enough advanced to permit the recognition of its sex, which was male.

It seemed more hopeful to make the attempt in vertebrates. Guyer had found that by injecting lymph into the unfertilized egg of a frog development could be induced and Bataillon found later that the mere pricking of the unfertilized egg of the frog with a needle suffices for the purpose; although he believes that with the pricking a blood cell must be introduced in order to induce a normal development of the egg. According to my experience the essential feature in fertilization is an alteration of the surface layer of the egg which in my experiments was brought about by chemical agencies; while in the experiments of Guyer and Bataillon on the egg of the frog it was accomplished mechanically. It may be stated incidentally that this mechanical method has failed in every other form thus far tried in our laboratory.

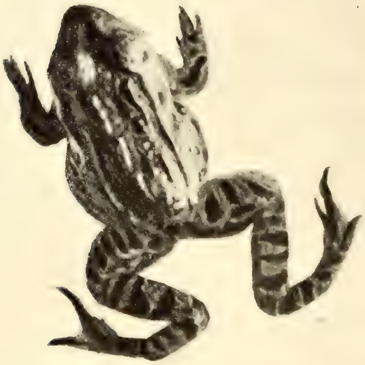


FIG. 1.

I have now seven parthenogenetic frogs, (*Rana pipiens*) over a year old, produced by pricking the unfertilized egg. The growth of these animals was normal and the variations in growth observed were due to differences in taking food. This species of frogs apparently requires two years to become fully mature, and some of these seven parthenogenetic frogs have now grown to more than half their normal size. They are normal in every respect as regards appearance and behavior. One of the frogs became infected and was killed at the age of ten months; figure 1 gives a photograph in natural size. The legs and the abdomen are slightly distended as a consequence of the infection. The egg had been pricked April 17, 1915, the metamorphosis of the tadpole to the frog stage took place August 29. The frog was killed the 26th of February, 1916. Some of the surviving frogs are larger than the one photographed.

These experiments prove that the methods of artificial parthenogenesis can give rise to normal animals, even in forms so high in the scale as the frog; and that these animals are able to live and grow normally.

The second problem connected with the raising of these frogs was to ascertain their sex. Loeb and Bancroft had investigated this problem three years ago on a frog and a tadpole four and a half months old.[4] The investigation met with some difficulty owing to the fact that in young tadpoles and frogs both sexes have eggs in their sex glands. These eggs gradually disintegrate in the testicles, but at the age of four months eggs are still found in the gonads of those which will develop into males. This was the case in the gonads examined by Loeb

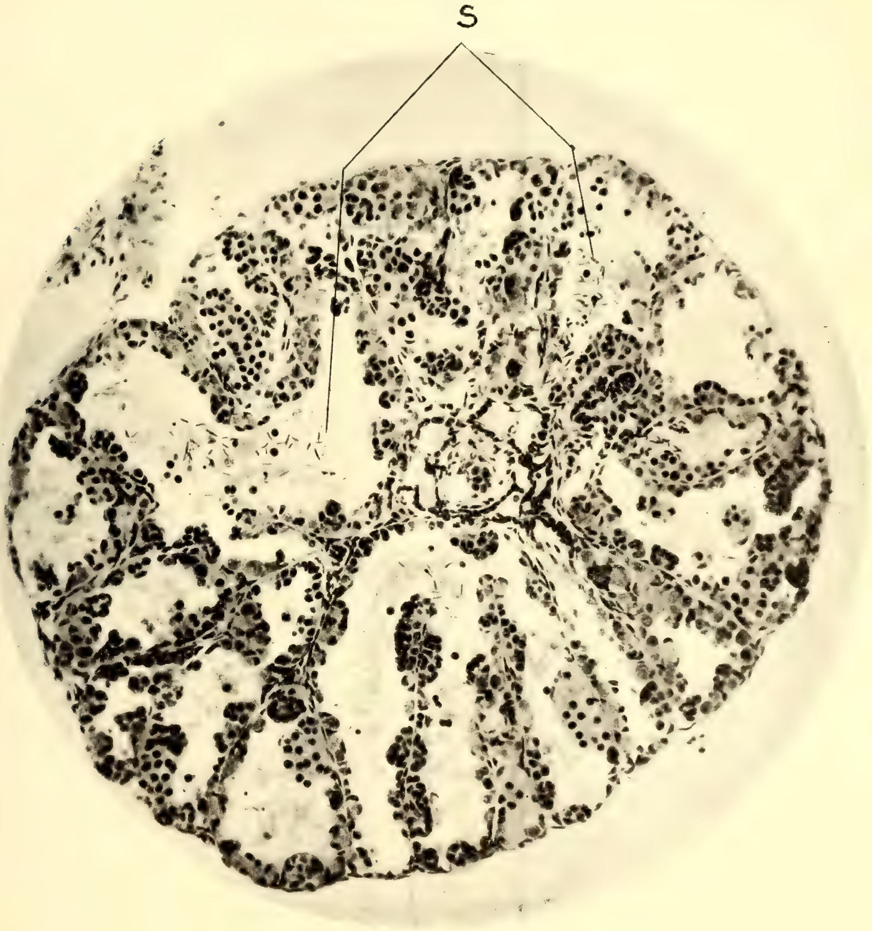


FIG. 2.

and Bancroft, but the eggs were so few in number that it was safe to assume that the two organisms examined would have developed into normal males had they lived; and the other structure of the gonads was such as to support this conclusion. Still it seemed desirable to make sure of this conclusion by examining older specimens of parthenogenetic frogs if they could be obtained. This opportunity offered itself in the case of the ten-months old parthenogenetic frog. Its gonads were hardened in Zenker's fluid[5] and sectioned. It was found that the gonads (figs. 2 and 3) were normal testicles containing normal spermatozoa in large numbers, which leaves no doubt as to the sex of this parthenogenetic frog.

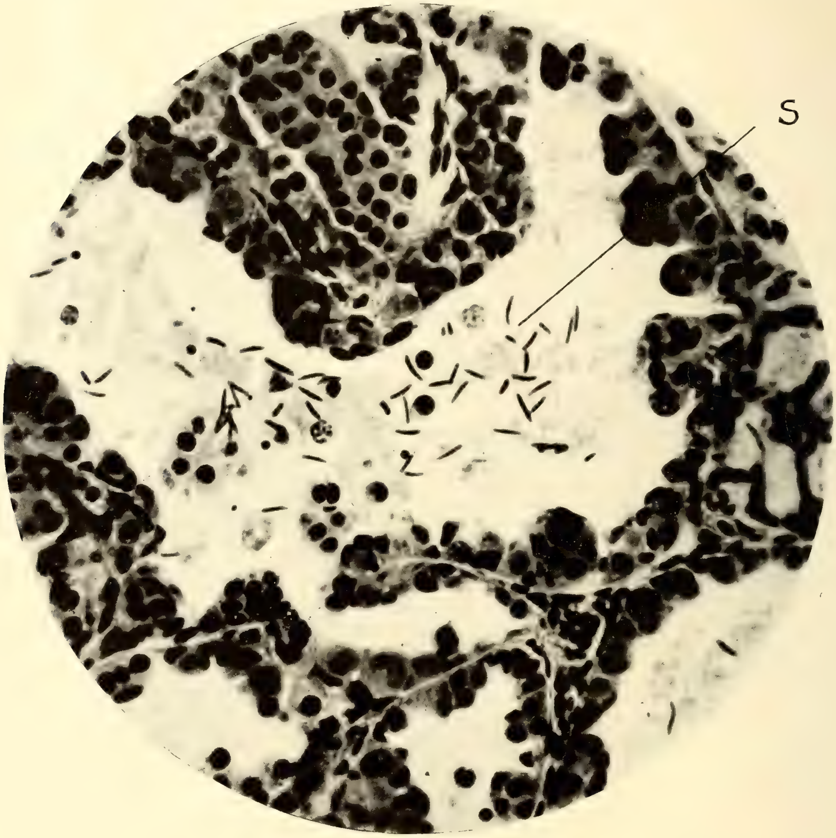


FIG. 3.

According to our present knowledge of the determination of sex it would appear from this that in the frog the male is heterozygous for sex; i.e., that the eggs are all alike and that there are two kinds of spermatozoa, one with and one without a sex chromosome; and that if a spermatozoon of the former type enters an egg a female is produced. Since in artificial parthenogenesis no sex chromosome is introduced the parthenogenetic frogs should be males.

It will be of further interest to find out whether the spermatozoa of such frogs when used for the fertilization of eggs will give rise to normal offspring and to both sexes and it is my intention to carry the experiments if possible to a decision of this question. It is further of interest to study the number and nature of chromosomes in the spermatozoa of the parthenogenetic frogs.

[Since the proof of this paper was read, another of the parthenogenetic frogs died, at the age of thirteen months. It was also a male, possessing well developed testicles of more than 1 mm. in diameter, and the typical pads on the thumb of the forelegs characteristic of the male.]

---

[1] Loeb, J., *Amer. J. Physiol.*, **3**, 135 (1899); **3**, 434 (1900).

[2] Loeb, *Artificial Parthenogenesis and Fertilization*, Chicago, 1913.

[3] Delage, Y., *Paris, C. R. Acad. Sci.*, **148**, 453 (1909).

[4] Loeb, J., and Bancroft, F. W., *J. Exp. Zool.*, **14**, 275 (1913); **15**, 379 (1913).

[5] Dr. Uhlenhuth was kind enough to do this for me.

DE VRIESIAN MUTATION IN THE GARDEN BEAN, PHASEOLUS VULGARIS

By J. Arthur Harris

STATION FOR EXPERIMENTAL EVOLUTION, COLD SPRING HARBOR, N. Y.

Received by the Academy, April 25, 1916

The extensive experience of experimental breeders with *Phaseolus vulgaris* during the past fifteen years has yielded few cases of unquestionable de Vriesian mutation. Such seems the most logical explanation of the origin of a race now under cultivation at the Station for Experimental Evolution.

Among the plants which survived from a lot of 4286 morphological aberrant plants secured in a study of 238,015 seedlings in 1912 were 9 which were distinguished from the remainder by producing only highly abnormal offspring in 1913. Practically without exception the 8000 first, second and third generation offspring produced in 1913, 1914 and 1915 were of a similar morphologically aberrant type. The same is true of a smaller culture of fourth generation plants grown in the greenhouse in 1915.

In this race the whole morphological organization of the seedling has apparently been changed. The new race is also characterized by a high degree of variability.

The axis may be either round and slender throughout or considerably broadened, or even divided. Of the plants about 30% show division of the epicotyl into two or more branches; about 50% are recorded as producing shoots from the axils of the cotyledons.

The race is characterized most specifically by the doubling of the number of cotyledons and primordial leaves. The most frequent number of cotyledons is four, instead of two as is normal in the species.

Leaves.

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	
Cotyledons	2	—	2	11	3	8	5	2	1	1	—	—	—	—	—	33
	3	3	20	69	95	74	36	28	18	4	3	2	—	—	—	352
	4	7	51	114	187	140	97	55	38	24	13	7	—	—	1	734
	5	1	1	7	12	14	20	4	5	3	4	1	1	—	—	73
	6	—	—	—	1	2	—	2	3	—	—	—	1	—	—	9
	7	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
		11	74	201	298	238	158	92	65	32	20	10	2	—	1	1202

The average number is somewhat less than four. The modal number of primordial leaves is also four, but the average number is considerably higher than this number.

The range of variation in number of cotyledons and of primordial leaves is very wide. The table, which gives the frequency of the various combinations of cotyledon and leaf number in several hundred seedlings studied in 1915, shows that plants with from 2 to 7 cotyledons and with from 1 to 14 primordial leaves were observed. These distributions show a variation of about 20% of the mean for number of cotyledons and of about twice that amount for number of leaves. The most frequent combination is that of four cotyledons and four primordial leaves, but the range of combinations is very wide, and correlation in consequence very low.

Variation in the new race is not limited to number of organs, but is conspicuous in the structure of cotyledons and leaves as well as of the axis. The most of the cotyledons are large and typical in form but occasionally those which are small and scale-like are observed. They may be inserted in a regular whorl or widely scattered along the axis. The leaves vary greatly in insertion, size, form and texture. Foliar ascidia are abundant.

History of origin and morphological characteristics are discussed and fully analyzed statistically in a paper appearing in an anniversary volume of the *Memoirs of the New York Botanical Garden.*

STUDIES OF DUCTLESS GLANDS BY THE ELECTRICAL METHOD

By W. B. Cannon

LABORATORY OF PHYSIOLOGY, HARVARD UNIVERSITY

Read before the Academy, April 17, 1916. Received, May 2, 1916

A study of the conditions of activity in the ductless glands, which pass their secretions into the blood stream, is difficult because recognition of the secretion in the blood is uncertain or impossible. It has long been known that physiological activity is accompanied by the development of an electrical difference which may be manifested by connecting an active part with an inactive part through a delicate galvanometer. It seemed possible that by the application of this method important information might be obtained as to the conditions of activity of the ductless glands. This work has been carried on through the cooperation of Mr. McKeen Cattell.

The method was first justified by applying it to the submaxillary gland which has an external secretion. Because an electrical change accompanies the secretion of saliva even though the blood supply is shut off from the gland or the flow through the duct is stopped; and because the change is absent when secretion is absent, although each of the conditions attendant on secretion (such as contraction of blood vessels, relaxation of blood vessels, faster flow of blood, slower flow of blood) may severally be induced, the conclusion is drawn that the electrical change is a manifestation solely of the process of secretion.

The direction of this electrical current of action developed by the submaxillary gland may be reversed although the physiological responses to stimulation remain as usual. Reversal is not, therefore, a certain sign of a reversed physiological process in the gland.

When the action current indicates a maximal activity of the submaxillary gland excited by stimulating the sympathetic nerve in the neck (cat), the electrical response can be augmented by stimulating the chorda tympani nerve and vice versa; sympathetic impulses are ineffective during the height of an effect produced by injected adrenin, and chorda tympani impulses cause no increase of the action current while pilocarpine is strongly operative.

The method thus justified on the submaxillary gland has been applied to the thyroid. Histologists have described nerve fibers leading to the cells of this gland, and anatomists have reported that the fibers going to the thyroid gland arise in the cervical sympathetic ganglia. Previous investigators have shown that severance of its cervical sympathetic nerves causes atrophy of the thyroid, and stimulation of these nerves causes a diminished iodine content of the gland. Severance of the vagus nerve supply has no effect.

If the thyroid gland and neighboring indifferent tissue are connected through a galvanometer, stimulation of the sympathetic strand high in the thorax evokes an action current after a latent period varying usually between 5 and 7 seconds. This effect persists after the superior and the recurrent laryngeal nerves are severed. Experiments have shown that the nerve impulses pass out through both the superior and inferior cervical ganglia.

Simulation of the main trunk of the vagus nerve in a curarized animal, or injection of pilocarpine (which excites vagus endings) has no effect in producing an action current in the thyroid gland.

The influence of sympathetic impulses is not indirect through local anemia of the gland, for when the blood supply is wholly stopped by clamping the blood vessels for a period equal to that of sympathetic stimulation, no noteworthy electrical change is produced.

The conclusion is drawn, therefore, that the nerves distributed to the thyroid cells belong to the sympathetic and not to the vagus supply, and that their effects are not indirect through alterations of blood flow, indeed that they are true secretory nerves.

It is known that the internal secretion of the adrenal gland, or adrenin, will have the same effect in the body as sympathetic nerve impulses. Injection of a small dose of adrenin, 0.1 to 0.2 cc. (1:100,000), evokes a marked action current in the thyroid gland. Also, stimulation of the nerve to the adrenal gland so as to cause its secretion to be poured into the blood stream, will evoke a characteristic electrical change in the thyroid. This electrical change does not occur if the return of blood from the abdomen is prevented, but takes place promptly when the pent blood is released. Furthermore, it fails to appear after stimulating these nerves if the adrenal glands have been previously removed. There is thus definitely established an influence of adrenal secretion on thyroid activity.

Previous studies have shown that the adrenal glands are roused to special activity in times of emotional stress. The thyroid gland is subject to the division of the nervous system which is brought into action in emotional excitement and which causes adrenal secretion. It is probable, therefore, that the thyroid, like the adrenal, has normally functions which are performed in times of critical emergency. It may be that such an emergency function is an exaggerated form of the routine activity of the gland.

The complete account of these researches will be published in the *American Journal of Physiology*, July, 1916.

THE DISTRIBUTION OF THE CHONDRIOSOMES TO THE SPERMATOZOA IN SCORPIONS

By Edmund B. Wilson

DEPARTMENT OF ZOOLOGY, COLUMBIA UNIVERSITY

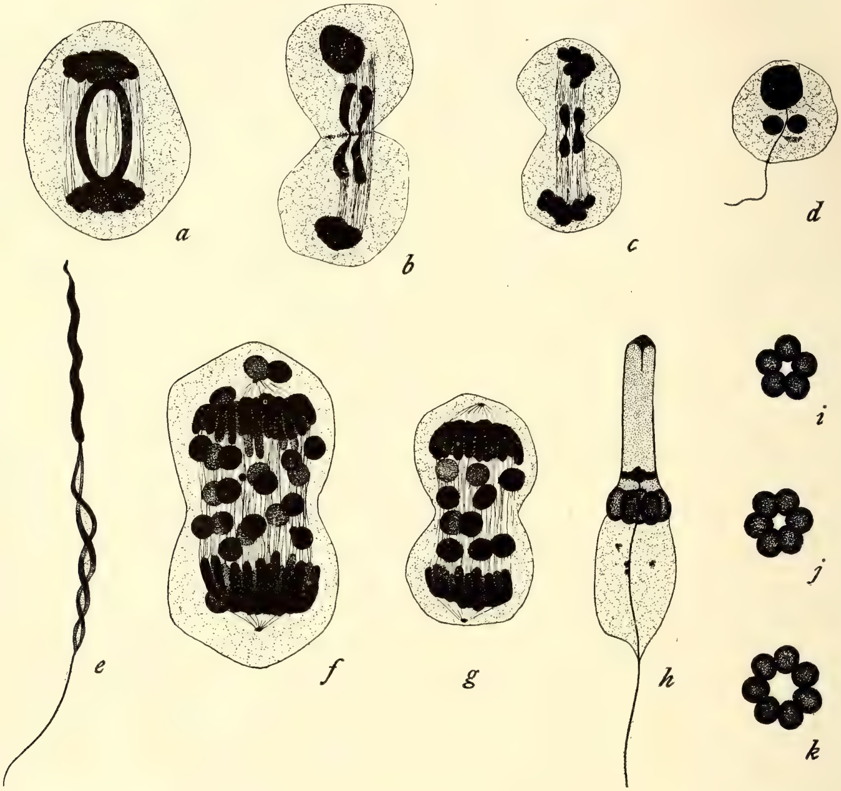
Read before the Academy, April 17, 1916. Received April 29, 1916

The spermatozoon carries into the egg two kinds of bodies both of which have been supposed to play a definite part in heredity; these are the *chromosomes* and the *chondriosomes*, the former belonging to the nucleus, the latter to the protoplasm or cytoplasm. If these bodies are in fact concerned in heredity we should in general expect them to be equally distributed to the germ-cells by means of division or some similar process of equal allotment; and as far as the chromosomes are concerned such is always the fact, except in case of the sex-chromosomes.[1] The chondriosomes, too, are distributed with approximate equality in many cases, but whether they are comparable with the chromosomes in this regard has not yet been certainly determined. I have for some time been engaged with a study of this question in two species of scorpions in both of which the phenomena are displayed with almost unexampled clearness, and which seem to give a conclusive result on this point. It is in some respects a surprising one.

The two species in question are *Opisthacanthus elatus* (Gervais) from Southern California and *Centrurus exilicauda* (Wood) from Southern Arizona.* In the latter, alone among animals hitherto examined, it is possible to conclude with certainty that the chondriosome-material is divided with exact equality among all the spermatozoa. The definiteness of this result is owing to the unique fact that before the maturation-divisions take place the whole of the chondriosome-material in the primary spermatocytes is concentrated in a single ring-shaped body which is then equally divided in such a way that each spermatid receives one-quarter of the products, the process taking place with a precision that is comparable to that seen in the distribution of the chromosome-material. The body in question represents a hitherto undescribed type of chondriosome. It arises from numerous minute mitochondria or chondrioconts which early in the growth-period of the spermatocytes become concentrated to form a large, homogeneous and sharply defined ring, lying in the protoplasm and surrounding the idiozome near one pole of the nucleus. Persisting unchanged throughout the growth-period this body places itself like a heterotype chromosome-ring in a position tangential to the first spermatocyte-spindle, elongates during the anaphases (fig. *a*) and then breaks apart at its polar apices, thus giving rise to two half-rings which quickly straighten out to form two similar and equal rods that lie side by side near and parallel to the spindle. Early in the telophase each rod becomes constricted at its middle point and then divides transversely (fig. *b*) into two equal parts which pass to opposite poles. Each secondary spermatocyte thus receives two equal daughter-rods (each a quarter of the orig-

---

*My thanks are due to Nathan Banks, Esq., for an authoritative identification of the species.



a—e, *Centrurus exilicauda* (Wood)

a, first spermatocyte-anaphase, chromosomes at the poles, chondriosome-ring in typical position to divide; *b*, telophase, showing final division of the ring; *c*, second division telophase, chromosomes at the poles, final division of the chondriosome-rods; *d*, young spermatid with nucleus and double nebenkern; *e*, early stage of elongation and twisting of the double nebenkern to form the spiral envelope of the flagellum.

f—k, *Opisthacanthus elatus* (Gervais)

f, anaphase of first spermatocyte-division, *g*, of second division, showing rod-shaped chromosomes passing to the poles and the rounded chondriosome-spheres scattered about the spindle; 22 of the latter appear in the first case, 10 in the second (complete numbers are 24 and 12 respectively); *h*, early spermatid with ring of chondriosomes (nebenkern-organ) at the base of nucleus; *i*, *j*, *k*, polar views of the nebenkern-organ showing variation in number of chondriosome-spheres.

inal ring) which persist throughout the interkinesis and repeat the process in the second division, taking up a position near the spindle and dividing transversely at their middle points (fig. *c*). Each spermatid receives two of the final products, which give rise to a nebenkern consisting of two equal parts (fig. *d*), the axial filament growing out between them from a blepharoplast situated at the base of the nucleus. It thus comes to pass that each spermatozoon receives exactly two-eighths of the original ring (hence of the original chondrioma of the spermatocyte), the history of the chondriosome-ring suggesting in a general way that of a heterotype chromosome-ring, though its mode of division is very different in detail. In the later stages, which are of great interest, the two halves of the nebenkern assume the form of flattened rods which rapidly elongate backwards, at the same time twisting spirally around the axial filament (fig. *e*) until they finally extend throughout nearly the whole length of the flagellum and are converted into two very fine, closely twisted parallel fibrillae that form the spiral envelope of the tail. Each of these fibrillae, as is proved by its genesis, represents one-eighth of the original ring and hence of the spermatocyte-chondrioma.

Nothing could be more surprising than the contrast to the foregoing mode of chondriosome-distribution seen in *Opisthacanthus*. It should first be emphasized that the two species agree closely in respect to the origin, staining reactions and ultimate fate of the chondriosomes: they are derived from numerous minute chondrioconts; they stain characteristically by Benda's method; they give rise to the nebenkern and ultimately to the tail-envelopes. A most striking difference appears, however, in respect to the remaining phenomena which are in a general way in agreement with those described by Sokolow[2] in the European species *Euscorpius carpathicus*, though some important differences are seen. In place of the ring appear approximately 24 fairly large, separate, hollow spheroidal bodies scattered without discernible order through the protoplasm. These bodies, which may be called *chondriosome-spheres*, do not at any time during the maturation-process show evidence of division nor do they enter into definite relation with the spindle. Scattered irregularly about the latter (figs. *f*, *g*,) they are at each division segregated into two approximately equal groups, half of them passing into each daughter-cell; and during this process they seem to be quite inactive, being passively carried along in the protoplasm as the cell divides. Their total number is thus reduced approximately from 24 to 12 in the first division, and from 12 to 6 in the second. Each spermatid thus receives six chondriosome-spheres as a rule, but sometimes five or seven; and these ultimately arrange themselves in a ring shaped group to form a 'nebenkern-organ' lying at the base of the nucleus and surrounding the blepharoplast (centriole) and the outgrowing axial filament (figs. *h—k*). Two hundred cases thus far tabulated give 73% with six chondriosome-spheres, 16% with five and 11% with seven. Other numbers have not been observed.

The sperm thus passes through a stage in which it possesses a nebenkern closely similar to the definitive middle-piece of certain nemertines, pelecypods and annelids (Retzius); but this is only a temporary condition. Becoming closely applied to the axial filament, the ring of chondriosomes is in later stages progressively drawn out backwards to form the envelope of the flagellum, apparently extending as far as the end-piece of the latter. There are some indications that a remnant of the chrondriosome-ring may finally be cast off along with the general protoplasmic remnant which, as usual, is sloughed off from the elongating flagellum, but this has not yet been clearly established. I have not yet been able to determine whether the envelope develops a spiral structure analogous to that so clearly seen in *Centrurus*.

From the foregoing account it is evident that chondriosome-material having the same origin, fate and (presumably) physiological significance may be distributed to the germ-cells by processes widely different even in nearly related animals. In one of these scorpions the distribution is effected by a definite process of division, in the other by an operation that has at least the aspect of a hit-or-miss segregation, and one that gives only an approximate equality of result. On its face this would seem to indicate that a wide distinction should be drawn between chondriosomes and chromosomes in respect to their power of division and their relation to heredity. It is of course possible that in both the cases here described the chondriosomes may multiply by division at an earlier period (a point now under investigation). It may also be suspected that division of the chrondriosome–ring is not an independent or autonomous act but depends upon the division of other elements with which it is associated. In either case the facts raise interesting questions concerning the power of division on the part of the several cell-components and the relation of this power to the principle of genetic continuity in general.

---

[1] Certain special exceptions, such as the supernumerary chromosomes or the unequal small bivalents of Orthoptera, are here disregarded since they are readily explicable in accordance with the general rule.

[2] *Arch. Zellforschung*, 9, 3, (1913).

NEW DATA ON THE ARCHAEOLOGY OF VENEZUELA

By Herbert J. Spinden

AMERICAN MUSEUM OF NATURAL HISTORY, NEW YORK

Received by the Academy, May 11, 1916

The archaeological reconnaissance of Venezuela made under the auspices of the American Museum of Natural History had for its purpose not so much the study for their own sake of Indian remains in Venezuela, but rather for the light that these remains might cast on certain fundamental problems of American archaeology. The field, although untried, is theoretically of the greatest importance. It is generally recognized as the point of departure for the original culture of the West Indies. Moreover, it is intermediate between the rich and well-known fields of Colombia and Costa Rica on the one hand and of eastern Brazil on the other and might be expected to furnish proofs of cultural connection if such exist. The success attendant upon recent stratigraphic work in the Southwestern States and in Mexico and the great advance in our knowledge of actual chronology in Central America tempt us to widen the recognized horizons of ancient American history whenever this seems possible.

Northern and central Venezuela were visited. The route passed from Maracaibo to Bobures, a port on the southern shore of Lake Maracaibo, and thence across and along the Eastern Andes to Mérida, Trujillo, Tucuyo, and Barquisimeto. Next the rich interior valley running from Valencia to Caracas was examined. From this populous region the road led southward across the llanos to San Fernando de Apure and thence by the Apure and Orinoco rivers to Ciudad Bolivar, the British island of Trinidad and a number of Venezuelan coast ports. Private collections, mostly small, were found in the principal cities. Notes and drawings were made of important specimens in these collections and considerable information obtained from local students. A few important sites were visited.

Space forbids detailed descriptions of archaeological specimens that came to light in Venezuela. Suffice it to say that stone implements, including celts, pestles, etc., vessels and figurines of clay with painted and modeled decorations, personal ornaments of shell, nephrite, jet and serpentine, as well as petroglyphs and pictographs, occur in considerable quantity. Various provinces may be marked off for detailed study, in each of which the ceramic products are sufficiently peculiar to be readily distinguished.

In the Andean region painted pottery is common but elsewhere it is rare. In caves and near sacred lakes on the wind-swept paramo many interesting figurines of men and women have been discovered, the former seated on stools and the latter in a variety of standing and sitting poses. These are seemingly the idols of a primitive agricultural people. By the peculiar style of construction and decoration of these figurines the student of ancient art can clearly demonstrate a cultural bond between Venezuela and Central America. Breast ornaments of shell and serpentine, carved to represent highly conventionalized bats, are common in the Andean province but become rarer as one passes towards central Venezuela.

The shores and islands of Lake Valencia are rich in archaeological remains. The level of this body of water has fallen about twenty feet since the coming of the Spaniards leaving old shore villages high and dry and making possible stratigraphic studies. Irregular earthen mounds containing a wealth of material, broken and entire, are found at a number of sites. Unfortunately for science the most remarkable group of mounds is now being destroyed in a hasty and unguided search for specimens. In this region collars of carved shell beads are often unearthed as well as stone pendants in the form of frogs. Pottery is decorated by modeled designs among which the highly conventionalized bat with outstretched wings is prominent. Figurines that represent human beings, jaguars, frogs, etc., are common and often finely executed. Connection with the Andean region is evident in pottery shapes as well as in the styles of decoration. A development over a long period of time doubtless took place here with a succession of somewhat different types.

Passing towards the east the material available for study falls off in quantity. On the llanos to the south very little collecting has been done although ancient village sites exist along the rivers. The few pieces brought to the attention of the writer show that an ancient sedentary culture of the "archaic school" once flourished here. Archaic pottery is also found at points along the Orinoco and it may be remarked that this ancient ware is very different from the varnished pottery now made by the uncivilized Indians of southern Venezuela. Little is yet known concerning the archaeology of eastern Venezuela. Collections made in Trinidad show a marked change from the types of the central region but not a complete break. West Indian forms are well developed here.

But while regional study shows what might be expected, namely a series of merging types in accordance with the principle of divergent development, there are features of Venezuela archaeology that offer evidence of customs once prevalent over the entire area. Urn burial is such a feature, reported from the island of Aruba, from the vicinity of Maracaibo, Mérida, Valera, Carache, Valencia, Maracay, La Union on the Portuguesa River, San Fernando de Apure, Atures on the Orinoco, etc. The urns are from two to two and a half feet in height, usually with rather narrow mouths closed by an inverted urn or by a shallow bowl. In these urns human remains are encountered in a sitting position with the knees under the chin and with the hands at the side of the face. The small size of the urns raises an interesting question concerning the method of inserting the bodies. It is not unlikely that desiccation preceded burial. These burial urns are sometimes found in caves and sometimes in low mounds but for the most part they are met with at a depth of about two feet below the apparently unmodified surface of the earth. The distribution of this method of burial probably extends beyond the limits of Venezuela and may be continuous over the open lands of the interior to Brazilian Guiana and even to the Island of Marajo in the mouth of the Amazon. On the west urn burial is well known in Nicaragua. The extension of this feature to the West Indies deserves to be studied with care since it is also found in our own southern states.

The statement has already been made that the figurines found in the Eastern Andes resemble closely those of Central America. This might be made stronger and the conclusion brought home that the plastic art of Venezuela is one and the same with the "archaic art" already known in Mexico and Central America. The proof is both objective and subjective. To be sure we must always stand ready to evoke the doctrine of divergent development but with a knowledge of transitional types the very fact that an orderly and systematic change is to be observed makes stronger the proof of cultural dissemination. In Mexico and Central America the archaic art was succeeded by other and higher styles. In Colombia some influence from these later cultures is manifest in pottery and metal work. But in Venezuela no later inflow has been noted and but slight evidence of independent local uplift.

The writer has elsewhere expressed the opinion that the diffusion of ceramic art of the so-called archaic type was contemporaneous with the primary diffusion of the concept of agriculture together with the actual passing of certain cultivated food plants such as maize, beans, and squashes, that are universally known among American Indians on the agricultural plane of life.

As regards Venezuelan archaeology, the question of time should perhaps be held in abeyance. In Mexico and Central America we have reason to believe that the archaic culture gave way to the higher civilization of the Maya at about the time of Christ. It had doubtless lasted a very long time since the deposits of this period are very thick. But once implanted in Venezuela the archaic culture, free from the pressure of higher arts, might have maintained itself till the coming of the Spaniards. There is evidence, however, of considerable pressure of population by wild tribes from the south and the little that is known of Venezuelan ethnology is not in full accord with the archaeology.

Beyond Venezuela we may be permitted to indicate the probable course of ancient empire. There is little doubt in the mind of the writer that the archaic culture—standing everywhere for sedentary agricultural communities, skilful in making pottery and textiles—was once laid down across northern South America and that the remarkable pottery of Marajo, at the mouth of the Amazon will prove to be a distant but congenital relative of the ware from the lowermost stratum of human handicraft in the Valley of Mexico.

The full data resulting from this exploration together with that obtained by further field studies will appear in the *Anthropological Papers* of the American Museum of Natural History.

NOTE ON THE PHOSPHORESCENCE OF URANYL SALTS

By Edward L. Nichols

DEPARTMENT OF PHYSICS, CORNELL UNIVERSITY

Received by the Academy, May 9, 1916

Phosphorescence is commonly regarded simply as the after effect of fluorescence, the emission spectrum immediately after the close of excitation being identical with that immediately before excitation ceases. This has hitherto been only an assumption, since it is thinkable that the process which prepares a substance for phosphorescence might produce emission during excitation differing from that which constitutes phosphorescence and which together with the latter would be present during fluorescence. It is also thinkable although unlikely that the phosphorescence might contain some components requiring a measurable time for development and observable only after an appreciable interval.

This is a matter which it would be very difficult to settle in the cases of phosphorescence hitherto studied because the spectrum of fluorescence and phosphorescence consists of broad bands or complexes of overlapping bands and almost the only criterion of identity is that of color.

It is true that the color of fluorescence is frequently different from that observed during the phosphorescent period but that is rightly ascribed to the existence of two or more bands differing greatly in their rate of decay. This is indeed a phenomenon, common to all the phosphorescent sulphides, which has been extensively studied by Lenard[1] and others and recently by a different method by the present writer.[2]

The uranyl salts on account of their remarkable spectra afford an unusual opportunity for the determination of this question but while the fluorescence of these substances has been examined in great detail little or no attention has been given to their phosphorescence.

A new form of phosphoroscope, the synchrono-phosphoroscope, recently described by the author[3] of this paper is well adapted for the observation of these fleeting phenomena, which have a duration of only a few thousandths of a second, and the study of the phosphorescence of a typical uranyl salt, to be described in the present paper, was one of the first uses to which this instrument was put.

The method, briefly outlined, is as follows. The substance, enclosed in a flat tube of glass about 8 cm. long and 2 cm. wide, is viewed through the rapidly revolving sectored disk of the synchrono-phosphoroscope. It is mounted vertically with its axis at right angles to the radius of the disk as shown in figure 1.

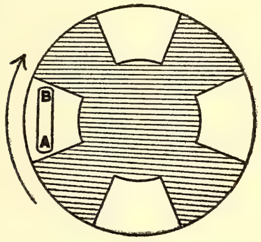


FIG. 1.

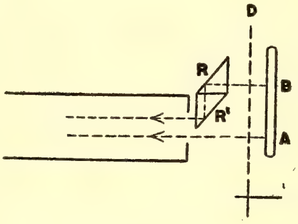


FIG. 2.

It is uniformly excited by zinc sparks 120 times a second while hidden by the closed sectors and is visible for 1/240 of a second during the passage of each of the intervening open sectors.

A phosphorescent substance of slow decay appears under these circumstances to be equally bright from top to bottom but if one of the uranyl salts, such as the double uranyl-ammonio sulphate, which was the substance selected for detailed study, be used, it appears a very bright green at the botton of the tube shading off to bare visibility at the top.

The rate of decay of this substance, and of the other uranyl salts is so rapid that the upper end of the tube, which is seen at the intensity which corresponds approximately to the instant 0.003 second after excitation, has only a small fraction of the brightness of the lower end which is viewed about 0.0005 second after excitation.

The great advantage to be gained from the study of these substances is due to the well known fact that in their spectra the usual broad band of fluorescence or phosphorescence is resolved into a group of seven or more bands which do not overlap and that at low temperatures (−150° to −180°) these are further resolved into complexes of narrow line-like bands often well separated from one another. The particular salt mentioned above was selected because at low temperature each group is unusually well resolved into at least seven such distinct bands. It should therefore be possible by comparing the spectrum during excitation with that immediately after, to detect changes affecting the individual components; something which is impossible in the case of the broad-banded spectra hitherto examined.

For the purpose of such comparisons a pair of right angled prisms was mounted before the slit of a Hilger spectroscope as shown in figure 2.

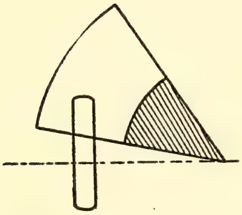


FIG. 3a.

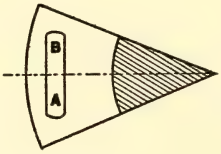


FIG. 3b.

Light from the lower end of the tube *A* enters the lower half of the slit. That from the upper end *B* after two total reflections enters the upper half of the slit and we have two spectra one above the other, coinciding throughout, as to wave length, but separated by a dark line formed by the lower edge of the second prism (R').

To compare fluorescence with phosphorescence the sectored disk was shifted upon its shaft until the lower end of the tube was viewed during excitation, the upper end immediately after (fig. 3 a). To compare the phosphorescence spectrum at an earlier and later stage the disk was so set that its position at the moment of excitation was as shown in fig. 3 b. By means of the reflecting prisms at the slit of the spectroscope, already described, the spectrum of the light emitted from region *A* was compared with that at *B* in each case. At +20° the banded spectra were found to be identical in every respect, except in brightness; and the same was true at low temperatures where it was possible to inspect each of the numerous line-like bands individually.

Of the seven homologous series distinguishable in the fluorescence spectrum all were present in the phosphorescent light, unshifted as to position and not perceptibly enhanced or diminished in relative brightness.

The comparison was less satisfactory as regards minor details in the case of the early and late stages of phosphorescence, some of the fainter bands being invisible but changes such as might be looked for, i.e., those due to the greater persistence of certain series, could scarcely have escaped notice. The significance of these observations is two-fold:

On the one hand we find that for the only examples of luminescence which admit of such detailed inspection, *the spectrum of phosphorescence is identical with that of fluorescence* and since there are no indications to the contrary in the case of other classes of substances thus far studied it is probable that the above statement will apply to all phosphorescent materials.

On the other hand we find that in spite of its great complexity, the luminescence spectrum of a uranyl salt is to be regarded as a unit, all its components decaying at the same rate after the cessation of excitation.

With this instrument the change of intensity of phosphorescence with the time may likewise be readily determined and as no one appears to have studied the uranyl salts in this respect, it seemed of interest to obtain the curve of decay of one of the salts under observation, i.e., the uranyl-ammonio sulphate.

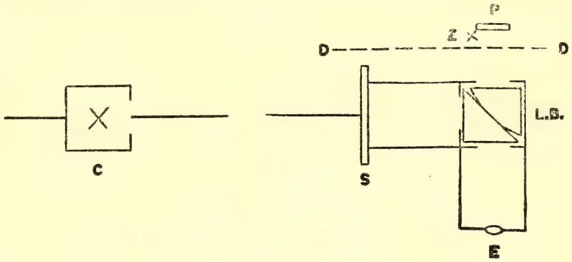


FIG. 4.

To this end a simple form of photometer previously used in a study of the phosphorescence of Kunzite[4] was mounted in front of the sectored disk. A lateral strip of the phosphorescent salt 1 cm. wide was excited by sparks from a single spark gap between zinc terminals and measurements of the brightness were made at various times after the close of excitation. The necessary conditions were attained by shifting the disk successively through small angles so as to vary the interval between excitation and observation. The time could be estimated with sufficient accuracy by noting the instantaneous positions of the disk for each adjustment, as given by the strictly synchronous illumination due to the spark.

The arrangement of the apparatus is shown in figure 4 in which *P* is the phosphorescent surface, *DD* the sectored disk, *L. B.* the Lummer-Brodhun cube of the photometer, *E* the eye-piece, *S* a color screen and matte translucent plate, *C* the comparison lamp which travelled along the track of an optical bench. The cross at *Z* indicates the position of the spark gap.

TABLE I.

T	I	$1/\sqrt{I}$
0.000479	59.49	0.130
0.000637	27.78	0.190
0.000856	16.02	0.250
0.000949	12.62	0.281
0.00110	9.80	0.319
0.00146	5.03	0.446
0.00170	2.03	0.702
0.00193	0.971	1.014
0.00212	0.610	1.280
0.00247	0.296	1.836
0.00287	0.159*	2.524*

* These values are probably somewhat less accurate than the other data given in this table.

In Table I relative intensities I, the reciprocals $1/\sqrt{I}$ and times T after excitation are given. Figure 5 shows the relations between I and T, and $1/\sqrt{I}$ and T respectively in the usual manner.

As appears from the table and curve *ABC* figure 5 this substance exhibits a remarkably rapid decay, falling in the interval between 0.0005 second after close of excitation and 0.003 second to less than three-thousandths of its intensity at the beginning of that interval. To show the degree of accuracy with which the lower intensities were observed the portion of the curve (*BC*) is reproduced with ordinates magnified ten times (*B′C′*). The results are likewise plotted in the customary manner with $1/\sqrt{I}$ as ordinates (curve *DEF*) and this brings out an unusual characteristic hitherto unobserved so far as the writer is aware in studies of phosphorescence except in the solitary case of paraffine recorded by E. H. Kennard.[5] It is usual to find two processes of phosphorescence succeeding one another and represented by the two straight arms of the curve *DE* and *FG* but in all the numerous cases hitherto described, excepting that of paraffine just noted where a very slight upward trend was found, the later process (*FG*) is indicated by a curve of lesser slope. In the case of this uranyl salt, however, *FG* trends very sharply upward showing a greatly accelerated decay. Whether this peculiarity is confined to these salts or is a common property of phosphorescence of exceed-

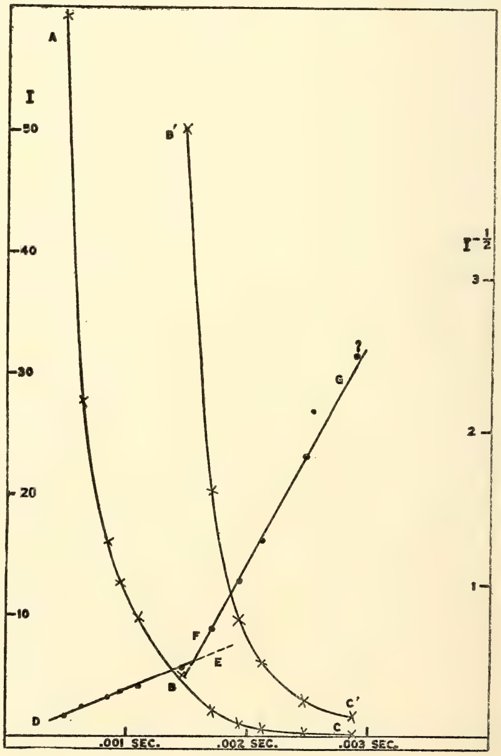


FIG. 5.

ingly rapid rates of attenuation, remains to be determined. With the help of Dr. H. L. Howes, to whose assistance throughout this investigation the author is greatly indebted, this question is now under investigation.

---

[1] Lenard, *Ann. Physik*, ser. 4, 31, 641 (1910).

[2] E. L. Nichols, Paper presented before the Amer. Philos. Soc., April, 1916.

[3] Nichols and Howes, Paper presented before the Amer. Physic. Soc., April, 1916.

[4] Nichols and Howes, *Physic. Rev.*, ser. 2, 4, 19 (1914).

[5] Kennard, *Physic. Rev.*, ser. 2, 4, 278.

THE PYRANOMETER: AN INSTRUMENT FOR MEASURING SKY RADIATION

By C. G. Abbot and L. B. Aldrich

ASTROPHYSICAL OBSERVATORY, SMITHSONIAN INSTITUTION

Read before the Academy, April 18, 1916. Received April 30, 1916

This instrument, as its name (from the Greek *πῦρ*, fire, *ἀνά*, up, *μέτρον*, a measure) indicates, is intended to measure the heat equivalent of radiation received from or going out toward the complete hemisphere above the plane of the measuring surface. We have devised two satisfactory types of the instrument, both derived in principle from the electrical compensation radiation instruments of the late K. Ångström. The full description of the instruments and tests of them will be found in a paper now being published in the *Smithsonian Institution Miscellaneous Collections.* The instruments are adapted to measure direct solar radiation, the total radiation of the sun and sky combined, that of the sky alone, and nocturnal radiation. It is possible to employ screens of selective transmission and thus to limit the measurements to selected spectrum regions. The instruments are of primary standard type, but have been compared with the standardized pyrheliometers of the Smithsonian Institution and found accordant. No auxiliary apparatus other than that employed with the Ångström pyrheliometer is required, and the observations are easy to make.

The simpler form of pyranometer comprises a single blackened manganin strip, 3 mm. wide, 6 mm. long, placed centrally in the plane of the upper surface of a nickel-plated copper disk 75 mm. in diameter. Copper blocks insulated from the rest of the disk, but continuous with it in surface, serve to connect the insulated manganin strip with an electric heating current of adjustable strength. A sensitive thermo-electric couple fastened by means of thin waxed paper to the rear surface of the manganin strip, and embedded at the other end in a recess of the copper disk serves to indicate changes of temperature of the strip. Concentric with the strip is a hollow hemispherical screen of ultra-violet crown glass, 26 mm. in outer diameter and 2 mm. thick. Its purpose is to admit rays of shorter wave-lengths, such as form essentially the whole strength of the direct and scattered solar rays, but to cut off rays of great wave-length proper to the emission of a body at ordinary temperatures. During measurements of nocturnal radiation this glass screen is removed. A nickel-plated hemispherical shell of polished nickel-plated copper encloses this glass, and is removable at pleasure.

If now the shutter is opened diffuse sky-radiation falls upon the strip and warms it, producing a deflection of a moving-coil galvanometer in the circuit of the thermo-couple. The shutter being then closed, an equal deflection may be produced by the electric heating current. As corrected to allow for losses by reflection of the glass and the imperfect absorption by lampblack, the energy dissipated in the strip by the heating current measures the energy of radiation. As constructed the sensitiveness of this instrument is so great that it proves convenient to balance the deflection to zero by means of a potentiometer current in the galvanometer circuit, and so to reduce the operations to the zero method. A defect of this simple form of pyranometer is found to be caused by the slow warming of the glass-covered portion of the copper disk when the shutter is opened, which at other times shades that area of the surface. This warming induces a secondary deflection, because it affects the two differently situated ends of the thermo-couple differently. Experiments have shown, however, that practically the full deflection due to direct heating of the strip occurs in 20 seconds, and that the secondary deflection begins to be sensible after 20 seconds. Accordingly the error is eliminated by balancing the primary deflection by the potentiometer current after exactly 20 seconds, then closing the shutter and waiting two minutes for the secondary heating to subside, before adjusting the heating current.

Fearing that this defect might prove more serious in nocturnal radiation work, we devised a second form of pyranometer. In this form there are *two* blackened manganin strips side by side, each 2 mm. wide, 6 mm. long, separated by a nickel-plated copper bar 2 mm. wide, and both insulated as in the simple form by vertical mica strips coming exactly to the surface of the plate. Thermo-couples connect the two strips at the back, the hot junction behind one strip, the cold junction behind the other. As the two strips absorb radiation equally, there would be an equal rise of temperature, if it were not that one strip is 10 times as thick as the other. Owing to this the conduction to the ends is so much greater for the thick strip that a difference of temperature arises, and a deflection of the galvanometer ensues. The heating current is divided between the two strips, and by suitable resistance coils the circuit is adjusted once for all so that whatever the strength of the heating current it produces equal dissipation of energy in the two strips. If now after closing the shutter the heating current is graduated until the deflection formerly produced by radiation is reproduced by electrical heating, the energy dissipated in either strip is the measure of the absorbed radiation. In the two strip pyranometer the secondary deflection by indirect heating is unimportant, because of the symmetry of the arrangement. However, to avoid this source of error altogether the exposure is limited to 30 seconds, and a full minute is allowed to lapse before introducing electric heating.

Numerous measurements of the sky-radiation have been made from the North Tower of the Smithsonian Institution. On fine days the sky-radiation alone received on a horizontal surface ranges from 0.07 to 0.13 calories per square centimeter per minute. On cloudy days, not thick enough for rain, the values run from 0.20 to 0.30 calories according to the kind of cloudiness prevailing. Measurements were made on the reflection from new fallen snow, and for total solar and sky radiation this proved to be 70%.

In the simpler form the instrument is so sensitive that it could be used in the deep shade of a forest, or with screens of selective transmission, so that it would be suited to botanical as well as meteorological investigations. As in the case of the silver disk pyrheliometer, the Smithsonian Institution may undertake to prepare pyranometers at cost (approximately $150) where valuable investigations may be promoted thereby.

NOTE ON LUCAS' THEOREM

By M. B. Porter

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF TEXAS

Received by the Academy, May 5, 1916

In a recent note[1] I ventured to give a proof, which I thought might be new, of Lucas' Theorem and one of its more immediate generalizations to rational functions. Professor Bôcher has kindly called my attention to the fact that the same proof had previously been given by him[2] and that the extension to rational functions was a special case (in the method of proof as well as in the results obtained) of other of his results.[3] I called attention to the interesting fact that this proof of Bôcher's applies without modification at once to integral functions of class zero. It is the purpose of this second note to show that it also applies without modification to *any* integral function of finite class p whose Weierstrass primary factors are of the normal form

$$f(z) = \Pi_\nu \left(1 - \frac{z}{\alpha_\nu}\right) e^{G_\nu (z)},$$

where

$$G_\nu = \sum_1^p \frac{1}{n} \left(\frac{z}{\alpha_\nu}\right)^n,$$

and where the α's are all real, or more generally to the integral functions

$$\phi(z) = f(z) \exp\left(\gamma_1 z^{p+1} + \gamma_2 z^p\right),$$

where γ_1 and γ_2 are real and γ_1 is negative if p is odd and γ_2 positive or zero if p is even. The essential thing in Bôcher's proof is that the

$$\frac{\phi'}{\phi} = z^p \left[\sum \frac{\alpha_\nu^{-p}}{z - \alpha_\nu} + \gamma_1 + \gamma_2 z^{-1}\right]$$

vectors which occur in the square brackets all lie inside an angle of 180° and hence their sum cannot vanish if z is not on the axis of reals.

Thus we obtain a much more general result than that obtained by Borel[4] or Polya[5] who employed methods that seem applicable only to functions of class zero and one, the theorem at which we have arrived being this: *All the zeros of the derivative of the integral function*

$$e^{\gamma_1 z^{p+1} + \gamma_2 z^p} \Pi \left(1 - \frac{z}{\alpha_\nu}\right) e^{G_\nu (z)}$$

are real, if the α's and γ's are real and γ_1 is negative, if p is odd and γ_2 positive if p is even.[6]

---

[1] These PROCEEDINGS, 2, 247 (1916).

[2] Bôcher, Some Propositions Concerning the Geometric Representation of Imaginaries, *Annals of Mathematics*, 1892.

[3] Bôcher, A Problem in Statics and its Relation to certain Algebraic invariants, *Proc. Amer. Acad. Arts Sci.*, 40, No. 11, 1904. Neither of these papers by Bôcher was listed in the bibliographies of Féjer and Hyashi, which I cited, and hence were overlooked by me.

[4] Fonctions Entières, pp. 32 et seq.

[5] Bemerkung zur Theorie der Ganzen Funktionen, *Jahresber. D. Math-Ver*, October–December, 1916.

[6] The same proof can be further applied to

$$\psi(z) = f(z) \exp. \left(\gamma_0 z^{p+2} + \gamma_1 z^{p+1} + \gamma_2 z^p\right).$$

A VARIABLE SYSTEM OF SEVENS ON TWO TWISTED CUBIC CURVES

By H. S. White

DEPARTMENT OF MATHEMATICS, VASSAR COLLEGE

Received by the Academy, May 15, 1916

Seven points chosen at random on a twisted cubic curve, like six points on a conic in the plane, give rise to a distinctive theorem; for as five points determine a conic, so the twisted cubic is determined by six points. In the case of the conic, this is the theorem of the Pascal hexagon, six points in a definite order leading to a definite line. Conic and line remaining fixed, the hexagon may vary with four degrees of freedom. In the case of the twisted cubic, not a mere sequence of the seven points, but an arrangement of them in seven triads, determines seven planes, and the theorem states that these planes are all osculated by a second twisted cubic curve. So much was established by a direct proof in these PROCEEDINGS in August, 1915; but the question of variability, whether the points and planes are free to move while the two curves remain fixed, was not examined. Now it is found that *the system is variable with one degree of freedom.* Full proof is contained in a paper soon to appear in the *Transactions of the American Mathematical Society.* The following is an outline.

Every twisted cubic C_3 is a rational curve, and the homogeneous coordinates of its points are cubic functions of a variable parameter:

$$x_1 = f_1(\lambda),\ x_2 = f_2(\lambda),\quad \ldots\quad, x_4 = f_4(\lambda).$$

In the same way the osculating planes of any second cubic curve K_3 are represented by cubic functions of a second parameter:

$$u_1 = g_1(\mu),\ u_2 = g_2(\mu),\quad \ldots\quad, u_4 = g_4(\mu).$$

Every point of the first may be put in relation to the three planes of the other that pass through it by the equation

$$u_1 x_1 + u_2 x_2 + u_3 x_3 + u_4 x_4 = 0,$$

or

$$\sum_1^4 f_i(\lambda) \cdot g_i(\mu) \equiv 0,$$

of the third degree in λ and also in μ. Conversely, every bicubical relation, or (3,3) correspondence, may be interpreted as such a point-to-plane relation between points of an arbitrary cubic C_3 and osculating planes of a second. This second curve K_3, however, is completely determined by the first, C_3, and the relation

$$F(\lambda, \mu) = \Sigma f_i(\lambda) \,.\, g_i(\mu) = 0;$$

for there is only one way of expressing a given bicubical function $F(\lambda, \mu)$ linearly in terms of four independent cubics $f_1(\lambda), f_2(\lambda), f_3(\lambda), f_4(\lambda)$.

Such a bicubical function $F(\lambda, \mu)$ contains 15 arbitrary coefficients, besides one multiplicative constant. If it relates cubic curves whose relative situation is like that of the C_3 and K_3 mentioned above, we may say that the relation $F(\lambda, \mu)$ admits a solution of period 7, or briefly, that it admits a Δ_7. A special kind of (3, 3) relation is that which factors into three (1, 1) relations or projectivities:

$$F(\lambda, \mu) \equiv \varphi(\lambda, \mu) \,.\, \psi(\lambda, \mu) \,.\, \chi(\lambda, \mu),$$

the triply bilinear relation. Of this special kind there is a sub-species which admits a Δ_7.

To prove the theorem stated above, viz., *that every* (3, 3) *relation which admits one* Δ_7 *must necessarily admit a simple infinity of* Δ_7*'s*, I proceed by counting the number of free constants in each of the four classes of (3.3) relations which have just now been noticed.

The first class, the general (3, 3) relation, contains 15 constants, including 3 that might have been deducted for linear transformation of either λ or μ. The second class, that admitting one Δ_7 (or more than one), contains the three constants of a linear transformation and apparently 7 others, since according to the former theorem cited above the 7 points on the C_3 can be chosen at random. If however the presence of one Δ_7 should imply ∞^1 others, the number of free constants would reduce to 9:– call the number $10-R$, where $R = 0$ or 1. The third class contains 9 constants, 3 for each of the collineations involved. The use of the fourth class is probably novel, at least in this connection; it contains 3 constants. The proof of this is the essential part of the demonstration.

The argument is now most easily stated geometrically. In a linear (flat) space of 15 dimensions, two contained algebraic varieties or spreads of s and of k dimensions respectively must have in common a spread of *at least* $s + k - 15$ dimensions. Here $s = 10 - R$, $k = 9$, and the common part or intersection is of 3 dimensions. Hence $10-R+9-15 \overset{=}{<} 3$, or $1 \overset{=}{<} R$. But we had $R \overset{=}{<} 1$, therefore $R = 1$, as asserted in the theorem.

THE NEUROMUSCULAR STRUCTURE OF SEA-ANEMONES

By G. H. Parker and E. G. Titus

ZOÖLOGICAL LABORATORY OF THE MUSEUM OF COMPARATIVE ZOÖLOGY AT HARVARD COLLEGE

Received by the Academy, May 6, 1916

Sea-anemones are more or less cylindrical animals that are usually attached to some rock or other fixed object by one end, the pedal disc, and carry on the opposite end or oral disc a single opening, the mouth. This opening, surrounded by tentacles, leads through a short oesophagus into the single internal cavity of the animal, the digestive cavity, and serves not only for the admission of food but also for the discharge of waste. The portion of the animal that connects the pedal disc with the oral disc is the column wall. This wall like that of the oral and of the pedal disc, consists of an outer layer of cells, the ectoderm, and an inner layer next the digestive cavity, the entoderm These two layers are separated by an intermediate layer of secreted material containing cells, the supporting lamella. The entoderm of the column wall is thrown into vertical folds, the mesenteries, which project from the inner face of this wall into the digestive cavity. In the deeper parts of both ectoderm and entoderm are sheets of muscle fibers by whose contraction the whole animal can retract greatly. This form of retraction and the reverse process of expansion are among the commonest activities of the sea-anemone. These operations involve not only the muscle layers just mentioned, but also a primitive nervous mechanism associated with them.

The muscle layers in sea-anemones are not the undifferentiated sheets implied in many of the earlier accounts of the structure of these animals, but fall into fairly well defined separate muscles. In the species of sea-anemone that we studied most fully, Metridium marginatum of the New England shore, there are thirteen differentiated muscles or groups of muscles. The longitudinal muscle of the tentacles is found on the ectodermic surface of these organs. The circular muscle of the tentacles covers their entodermic faces. The radial muscle of the oral disc spreads from the region of the mouth over the ectodermic surface of the disc to its outer edge. The circular muscle of the oral disc covers the entodermic face of this disc. The circular muscle of the oesophagus surrounds this organ on its entodermic side. The circular muscle of the pedal disc is a broad, circular sheet on the entodermic face of this part of the animal. The basilar muscles are radial muscles attached to the mesenteries where these join the pedal disc. The longitudinal muscles of the mesenteries extend in the mesenteries from the pedal disc to the oral disc. The transverse muscles of the mesenteries are at right angles to the longitudinals and extend from the outer wall of the sea-anemone to the inner free edge of the mesentery or to the oseophagus when the mesentery unites with that organ. The parietal muscles of the mesenteries are longitudinal strands in the mesenteries at the region of attachment of these organs to the column wall. The circular muscle of the column covers the entodermic face of the column wall. The sphincter is a specialized band in the circular muscle of the column which it surrounds at a level close to the oral disc. The longitudinal muscles of the acontia are extremely tenuous muscles in these filamentous organs.

The nervous system of sea-anemones consists of sense cells said to be in the entoderm as well as in the ectoderm whose deep ends form a nervous network in close proximity to the muscles. This network includes in its meshes ganglion cells. In 1879 the Hertwigs described a concentration of nervous material in the oral disc of sea-anemones and believed this to be the beginnings of a central nervous organ. Grosely claimed that the nervous centralization is in the oesophagus. Many recent workers, however, have declared the nervous system to be diffuse and not centralized at all.

According to most investigators the ectodermic nervous network connects with the entodermic one only in the region of the mouth where these two layers are confluent being separated in other places by the supporting lamella. Havet, however, in 1901 claimed that nervous tissue could be traced through the supporting lamella thus connecting ectoderm and entoderm directly. We have found evidence of this both histological and physiological. By special staining methods we have confirmed Havet's statement that the supporting lamella contains nervous elements and by experiment we have shown that these elements connect the ectoderm *directly* with the longitudinal muscles of the mesenteries (entoderm), that is, without passing through the mouth region. If a small area on the ectoderm of the column wall is stimulated mechanically or chemically, the sea-amenone will retract the oral disc through the action of the longitudinal muscles of the mesenteries. If this area is partially isolated by making a circular incision around it and completely through the column wall so that it is attached to the animal only by the deep-lying mesenteries, the longitudinal muscles in these organs will regularly contract on stimulating its ectodermic face. Thus there must be direct nervous connections between the ectoderm of the column wall and the longitudinal muscles of the mesenteries, and this connection appears to consist of a relatively complex but diffuse nervous metwork.

A second type of neuromuscular structure is seen in the outer layer of the tentacles of Metridium. Here ectodermic sense cells connect directly with the underlying longitudinal muscle fibers and thus these fibers are brought into action without the intervention of so extensive a network as in the former instance.

What seems to be a third type of neuromuscular mechanism is seen in the circular muscle of the column of Metridium. If the exterior of the column of this animal is stimulated mechanically, retraction generally follows. If the spot stimulated is anesthetized by allowing a few crystals of magnesium sulphate to dissolve on it and it is then stimulated, general retraction does not result but in the course of a minute or so a band formed by a local contraction in the circular muscle of the column appears and gradually spreads around the column. This band of contraction then slowly disappears. The reaction is apparently due to the direct stimulation of the circular muscle of the column, a muscle which from other grounds is known to be open to indirect stimulation probably through the nervous network.

A fourth type of muscular activity is seen in the longitudinal muscles of the acontia. If these filamentous organs are detached from a Mertidium, they will live hours in ordinary sea-water moving about in tortuous lines by means of their cilia. If such free acontia are stimulated mechanically, they slowly tangle themselves up and afterwards slowly untangle. If now they are subjected to what would be thorough anesthetization with chloretone and are again mechanically stimulated, they tangle themselves up as before. Apparently their muscle is entirely independent of nerves and under ordinary conditions is brought into action by direct stimulation.

These four types of muscle action are of phylogenetic significance. The last mentioned, that of the longitudinal muscle of the acontium, is the most primitive and resembles the type found in sponges in that it is slow in action and not under nervous control. The next in advancing complexity is the circular muscle of the column partly independent and partly under nervous influence. The third, the longitudinal system of the tentacle, is a well defined union of sense cells and muscle fibers and is relatively quick in action. Finally the most complex type is the sense cell, complex nervous network, and muscle fiber as first described, a quickly responding, most highly differentiated example. These four types show that the neuromuscular mechanism of sea anemones is by no means so simple and uniform as was originally supposed but embraces a variety of structural conditions which serve different purposes and probably represent evolutionary steps.

The full paper will be published in the *Journal of Experimental Zoölogy.*

CHANGE OF THE IONIZATION OF SALTS IN ALCOHOLIC SOLVENTS WITH THE CONCENTRATION

By Frederick G. Keyes and W. J. Winninghoff
RESEARCH LABORATORY OF PHYSICAL CHEMISTRY, MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Received by the Academy, May 8, 1916

The deviation of largely ionized substances, even in fairly dilute solution, from the mass-action law, and in general from the behavior of perfect solutes, has been the subject of extended discussion among physico-chemical investigators. Evidence is accumulating that at sufficiently small ion-concentrations these substances become normal in their behavior. Thus Kraus and Bray[1] showed that uniunivalent salts dissolved in liquid ammonia and in certain organic solvents conform to the mass-action law when the concentration of the ions in the solution lies below about 0.0001 normal in ammonia, 0.0005 normal in organic solvents; and Arrhenius[2] has made recalculations with the data of Kohlrausch and Maltby on the conductance of aqueous solutions of sodium chloride and nitrate which indicate that these salts behave as perfect solutes in water at concentrations between 0.00002 and 0.0002 normal.

A great variety of expressions have been proposed to account for the deviations at higher concentrations. Of these the one which is most generally applicable is that proposed by Kraus and discussed fully by Kraus and Bray.[3] Kraus and Bray have shown that the conductance of almost any uniunivalent solute in any solvent from the concentration zero up to a fairly high concentration (one where the change in the viscosity of the solution becomes an important factor) can be expressed by an equation of the form:

$$\frac{(c\gamma)^2}{c\,(1-\gamma)} = K + D\,(c\gamma)^m.$$

In this equation K, D, and m are empirical constants which vary with the nature of the solute and of the solvent and with the temperature, and γ represents the equivalent-conductance ratio Λ/Λ_0. This expression evidently requires that any solute in any solvent conform to the mass-action law at sufficiently small concentrations, when the term $D(c\gamma)^m$ becomes negligible.

The present investigation on the conductance of sodium iodide and ammonium iodide in isoamyl alcohol and of sodium iodide in propyl alcohol was undertaken for two purposes: primarily, in order to determine whether in these solvents, somewhat similar in nature to water, salts conform to the mass-action law at very small concentrations; and secondarily, to test further the applicability of Kraus' empirical equation throughout the fairly wide range of concentration employed in our work. Dutoit and Duperthuis[4] have already studied solutions in these alcohols, but it seemed desirable ro extend and confirm their results.

For the conductance measurements a new form of cell was constructed which enabled them to be made in a vacuum, entirely out of contact with air, which exerts a slow oxidizing action. The specific conductance of the purified alcohols employed was about 2×10^{-8}.

Several series of measurements at 25° were made with each kind of solution. The equivalent conductances were plotted against the logarithms of the concentrations on a large-scale plot, a representative curve was drawn through them, and values of the equivalent conductance were accurately read off at various round concentrations. These values are presented in Table 1 under the heading 'Λ obs;' the concentrations in milliequivalents per liter of solution being given under the heading 10^3c.

The values of the conductance at zero concentration given in the table and the values of the constants of the Kraus equation were determined by plotting the results in various ways. The constants were found to have the following values, when the concentration is expressed in equivalents per liter:

For NaI in isoamyl alcohol: $10^4K = 5.85$; $D = 0.374$; $m = 1.20$.
For NH_4I in isoamyl alcohol: $10^4K = 6.93$; $D = 0.324$; $m = 1.17$.
For NaI in propyl alcohol: $10^4K = 38.3$; $D = 0.208$; $m = 0.75$.

With the help of these constants the equivalent conductances at round concentrations were calculated. These are given in the table under the heading 'Λ calc.'

In the columns of the table which are headed '% diff.' are given the percentage differences between the calculated and observed values, showing the degree of conformity of the results with the requirements of the equation. In the most accurate series, that with sodium iodide in isoamyl alcohol, the maximum difference, even though the conductance passes through a minimum (at 0.1 normal), will be seen to be 1.4% except at the highest concentration (0.400 normal), where the calculated values begin to deviate widely from the observed ones.

The values of $1/\Lambda$ were plotted against those of $c\Lambda$, in order to test their conformity at very small concentrations with the mass-action law, which requires that such a plot be a straight line. The results showed that the mass-action law holds true within the experimental

TABLE 1

OBSERVED AND CALCULATED VALUES OF THE EQUIVALENT CONDUCTANCE

$10^3 c.$	*NaI in Isoamyl Alcohol*			*NH_4I in Isoamyl Alcohol*			*NaI in Propyl Alcohol*		
	Λ obs.	Λ calc.	% diff.	Λ obs.	Λ calc.	% diff.	Λ obs.	Λ calc.	% diff.
0.00	7.79			7.92			20.10		
0.05	7.182	7.194	+0.17	7.396	7.476	+1.08	(20.11)		
0.10	6.636	6.658	+0.33	7.032	7.105	+1.04	19.65	19.53	−0.61
0.20	6.115	6.142	+0.43	6.256	6.352	+1.54	19.17	19.13	−0.21
0.40	5.307	5.335	+0.53	5.464	5.598	+2.46	18.42	18.49	+0.38
0.80	4.451	4.460	+0.20	4.672	4.794	+2.62	17.38	17.56	+1.03
1.00	4.184	4.183	−0.03	4.408	4.552	+3.03	17.00	17.21	+1.24
2.0	3.394	3.374	−0.61	3.588	3.663	+2.23	15.73	16.03	+1.90
5.0	2.560	2.534	−1.02	2.656	2.676	+0.75	13.62	13.56	−0.44
10.0	2.024	1.995	−1.43	2.168	2.164	−0.19	12.04	11.80	−1.99
20.0	1.649	1.635	−0.69	1.776	1.763	−0.73	10.44	10.31	−1.25
40.0	1.408	1.404	−0.28	1.448	1.456	+0.55	8.90	8.82	−0.90
100	1.294	1.290	−0.31	1.336	1.339	+0.22	7.33	7.26	−0.95
150	1.312	1.309	+0.023	1.332	1.335	+0.22	6.85	6.79	−0.88
200	1.339	1.336	−0.22						
300	1.393	1.400	+0.50						
400	1.413	1.459	+3.20						
500	1.396								

error between the concentrations 0.00005 and 0.0004 normal. At the latter concentration the correction term $D(c\gamma)^m$ in the Kraus equation amounts to only 3.5% of the mass-action constant K in the case of sodium iodide in isoamyl alcohol.

A full description of this work will appear in the June number of the *Journal of the American Chemical Society.*

This investigation was carried out with the aid of a grant made to Prof. A. A. Noyes by the Carnegie Institution of Washington, for which we wish to express our great indebtedness. We also wish to thank Prof. C. A. Kraus and Prof. A. A. Noyes for many helpful suggestions.

[1] Kraus and Bray, *J. Amer. Chem. Soc.*, 35, 1354, 1384 (1913).

[2] Arrhenius, *Meddelanden fr. K. Vetenskapakademiens Nobel-Institut*, 2, No. 42, 10 (1913).

[3] Kraus and Bray, *Loc. cit.*, p. 1319.

[4] Dutoit and Duperthuis, *J. chim. phys.*, 6, 699 (1908).

CONTENTS

Page

MATHEMATICS.—DIFFERENTIAL EQUATIONS AND IMPLICIT FUNCTIONS IN INFINITELY MANY VARIABLES . . . . *By William L. Hart* 309

ZOOLOGY.—THE SEX OF PARTHENOGENETIC FROGS . . . . *By Jacques Loeb* 313

GENETICS.—DE VRIESIAN MUTATION IN THE GARDEN BEAN, PHASEOLUS VULGARIS *By J. Arthur Harris* 317

PHYSIOLOGY.—STUDIES OF DUCTLESS GLANDS BY THE ELECTRICAL METHOD . . . *By W. B. Cannon* 319

ZOOLOGY.—THE DISTRIBUTION OF THE CHONDRIOSOMES TO THE SPERMATOZOA IN SCORPIONS . . . . *By Edmund B. Wilson* 321

ANTHROPOLOGY.—NEW DATA ON THE ARCHAEOLOGY OF VENEZUELA . . . . *By Herbert J. Spinden* 325

PHYSICS.—NOTE ON THE PHOSPHORESCENCE OF URANYL SALTS . *By Edward L. Nichols* 328

ASTRONOMY.—THE PYRANOMETER: AN INSTRUMENT FOR MEASURING SKY RADIATION *By C. G. Abbot and L. B. Aldrich* 333

MATHEMATICS.—NOTE ON LUCAS' THEOREM . . . . *By M. B. Porter* 335

MATHEMATICS.—A VARIABLE SYSTEM OF SEVENS ON TWO TWISTED CUBIC CURVES *By H. S. White* 337

ZOOLOGY.—THE NEUROMUSCULAR STRUCTURE OF SEA-ANEMONES . . . . *By G. H. Parker and E. G. Titus* 339

CHEMISTRY.—CHANGE OF THE IONIZATION OF SALTS IN ALCOHOLIC SOLVENTS WITH THE CONCENTRATION . . . . *By Frederick G. Keyes and W. J. Winninghoff* 342

VOLUME 2 JULY, 1916 NUMBER 7

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES
OF THE
UNITED STATES OF AMERICA

Smithsonian Institution
National Museum.

EDITORIAL BOARD

Arthur A. Noyes, *Chairman*
Edwin B. Wilson, *Managing Editor*
Arthur L. Day, *Home Secretary*
George E. Hale, *Foreign Secretary*

J. J. Abel
W. B. Cannon
J. McK. Cattell
J. M. Clarke
E. G. Conklin
J. M. Coulter
C. B. Davenport
Simon Flexner
E. B. Frost
R. G. Harrison
W. H. Holmes
J. P. Iddings
R. A. Millikan
E. H. Moore
Alexander Smith

Publication Office: Williams & Wilkins Company, Baltimore
Editorial Office: Massachusetts Institute of Technology, Boston
Home Office of the Academy: Washington, D. C.

Entered as second-class matter at the Post Office at Baltimore, Md., under the Act of August 24, 1912

INFORMATION TO CONTRIBUTORS

THE PROCEEDINGS is the official organ of the Academy for the publication of brief accounts of important current researches of members of the Academy and of other American investigators, and for reports on the meetings and other activities of the Academy. Publication in the Proceedings will supplement, not replace, that in journals devoted to the special branches of science. The Proceedings will aim especially to secure prompt publication of original announcements of discoveries and wide circulation of the results of American research among investigators in other countries and in all branches of science.

ARTICLES should be brief, not to exceed 2500 words or 6 printed pages. Technical details of the work and long tables of data should be reserved for publication in special journals. But authors should be precise in making clear the new results and should give some record of the methods and data upon which they are based. The viewpoint should be comprehensive in giving the relation of the paper to previous publications of the author or of others and in exhibiting where practicable, the significance of the work for other branches of science.

MANUSCRIPTS should be prepared with a current number of the Proceedings as a model in matters of form, and should be typewritten in duplicate with double spacing, the author retaining one copy. Illustrations should be confined to text-figures of simple character, though more elaborate illustrations may be allowed in special instances to authors willing to pay for their preparation and insertion. Particular attention should be given to arranging tabular matter in a simple and concise manner.

REFERENCES to persons should include initials; citations of literature, numbered consecutively, should be placed at the end of the article; and short footnotes should be avoided. Indicate 'subject' for 'Contents.'

PAPERS by members of the Academy may be sent to Edwin Bidwell Wilson, Managing Editor, Mass. Institute of Technology, Boston, Mass. Papers by non-members should be submitted through some member.

PROOF will be sent to authors only on their request; in order to avoid delay in publication. Authors are therefore requested to make final revisions on the typewritten manuscripts. The editors cannot undertake to do more than correct obvious minor errors.

REPRINTS should be ordered at the time of submission of manuscript. They will be furnished to authors at cost, approximately as follows:

Reprints of - -	2 pp.	4 pp.	6 pp.	8 pp.	Covers extra
Charge for first 100 copies	$1.00	$1.30	$2.20	$2.20	$2.00
Charge for additional 100s	.25	.45	.85	.85	.50

INFORMATION TO SUBSCRIBERS

SUBSCRIPTIONS at the rate of $5.00 per annum should be made payable to the National Academy of Sciences, and sent to Williams & Wilkins Company, Baltimore, or Arthur L. Day, Home Secretary, National Academy of Sciences, Smithsonian Institution, Washington, D. C. Single numbers, $0.50.

Copyright, 1916, by the National Academy of Sciences

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES
Volume 2 JULY 15, 1916 Number 7

ON THE MOBILITIES OF GAS IONS IN HIGH ELECTRIC FIELDS

By Leonard B. Loeb
RYERSON PHYSICAL LABORATORY, UNIVERSITY OF CHICAGO

Received by the Academy, May 20, 1916

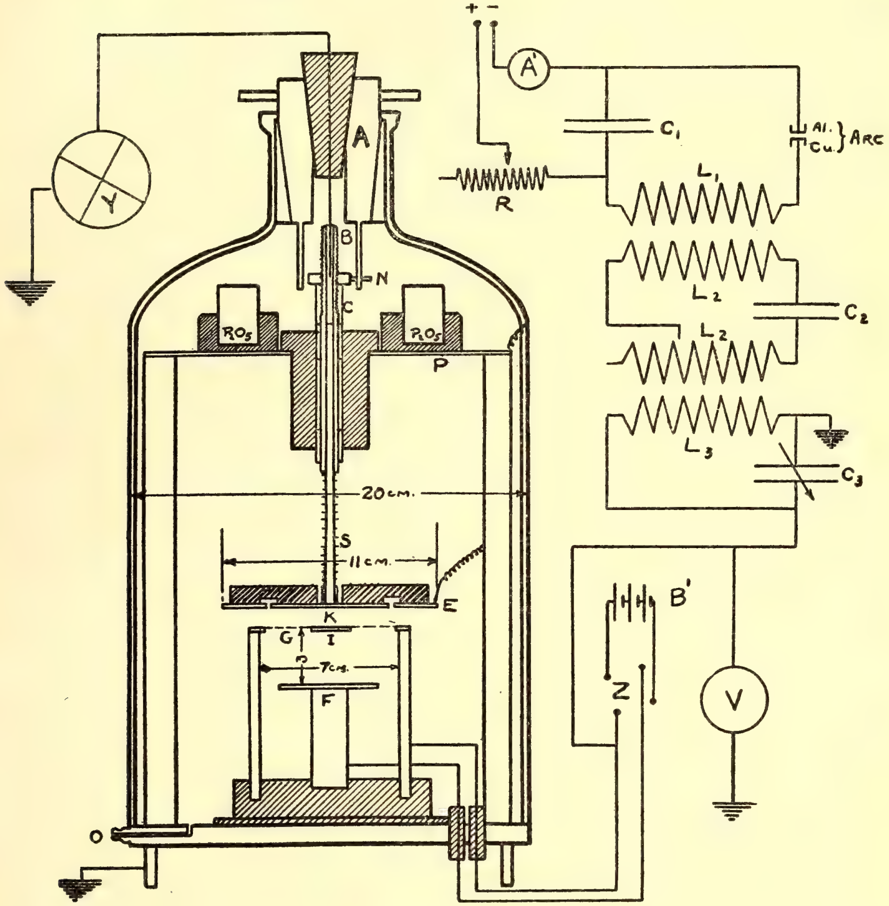
Up to the present two theories have been advanced to explain the low order of magnitude of the mobility of the ordinary gaseous ion. The first theory known as the 'cluster' theory is due to Langevin.[1] It assumes that the ion consists of a cluster of neutral molecules surrounding an electron or a positive atomion. The second theory was proposed by Wellisch[2] and is known as the 'small ion' theory. This assumes that both positive and negative ions consist of single charged molecules. In its path through the gas such an ion might be retarded abnormally due to the fact that its charge, acting on neutral gas molecules, drags in a greater number of collisions than would an uncharged molecule.

On the 'cluster' theory it would be expected that if the ion acquired sufficient kinetic energy, it would begin to break up. Such energy might be acquired in traversing a mean free path under a sufficiently high field. The break up would then be indicated by an abnormally high mobility. One would certainly expect such a breaking up of the cluster before field strengths great enough to cause ionisation by collision were reached. The other theory would expect no such break up. It is, however, conceivable on the 'small ion' theory that when the kinetic energy reaches a very high value, near that at which ionisation by collision begins, the negative ion might lose its electron. Experiments which were performed by a number of observers on the gas ions at low pressures (i.e., where with a small value of the field the ion can pick up considerable energy on a long mean free path), indicated that for the negative ions such a break up did occur. It was consequently thought of importance to see if such a breaking up, or abnormal increase of mobility, of either ion could take place in high electric fields at ordinary pressures. It was considered particularly important to test this point for the case of the positive ion. If in this case an abnormally high mobility were found, only one conclusion could be drawn as to its significance, namely, that the positive ion consists of a cluster. In the case of an abnormally high mobility of the negative ion, the presence of the free electron might make the result ambiguous. It was with this end in view that the work was undertaken at the suggestion of Prof. R. A. Millikan.

Measurements were made by means of the Rutherford[3] alternating current method. With this method the problem resolves itself into one of obtaining high field strengths, e.g., 5000 volts per cm., with alternating fields of frequencies of about 5000 cycles per second. Such frequencies are demanded by the high fields in order to make the distance between the plate and the gauze (i.e., the distance that the ions traverse in one-half cycle), small enough to keep the field between the plate and gauze uniform. These high frequencies were obtained from the resonance circuit of a Chaffee[9] arc. This arc gives nearly undamped sine waves, and is comparatively easy to operate. Using a tertiary circuit as seen in the diagram (fig. 1), an alternating potential difference of 4400 volts at a frequency of 7670 cycles per second was obtained from the arc. The frequency was determined by photographing the spark in a revolving mirror. The mobility-measuring apparatus is similar, as the diagram shows, to that used by other observers. The ions were generated by ionium on a disc I attached to the bottom of the gauze. The whole apparatus was placed inside a silvered bell jar which could be exhausted. Since neither the potential difference nor the frequency could be conveniently varied, the measurements were taken by varying the distance between the plate and the gauze. To do this, the apparatus was so arranged that the upper or electrometer plate could be raised or lowered from the outside of the jar. The distances between the plate and the gauze were plotted as abscissae, and the electrometer deflections as ordinates. The point where this curve intersected the axis of abscissae was taken as the critical distance which the ions could just traverse in the time of one-half period of oscillation. From the value of this distance d, the mobility U of the ion is obtained at once with the aid of the equation $U = \pi N d^2/E$, in which N is the frequency and E the maximum value of the potential.

The measurements were made in air, which was carefully dried with phosphorous pentoxide in the later measurements. The variations of the values of the mobilities as shown in the table were in a large measure attributable to insufficient drying in the earlier determinations. Fluctuations in the operation of the arc, together with slight uncertainties as to the value of the constants of the alternating potential difference, might possibly have caused an error of ten per cent in the determination of the absolute value of the mobilities. The actual error, however, is probably much less than this. By means of check determinations, using the 120 volt 60 cycle city alternating potential, which gave strictly normal values of the mobilities, the possibility of any instrumental errors was precluded.

FIG. I



The results obtained are shown in Table 1. In the first column are the frequencies of alternation in cycles per second. In the next two columns the critical distances in centimeters are given. The next two give the mobilities. The next column gives the pressure in millimeters of mercury. The next columns give the values of the mobilities reduced to 760 mm. pressure. The last two columns give the field strengths for the maximum values of the fields in volts per centimeter.

The measurements were carried up to fields of 12,450 volts per centimeter in air at atmospheric pressure, and to somewhat lower field strengths at pressures as low as 300 mm. of mercury. The results could not be extended to higher field strengths nor lower pressures, because of the fact that irregularities in the operation of the arc caused serious sparking beyond these limits. It can be clearly seen that for a range of field strengths extending from 90 volts per centimeter to 12,450 per centimeter in air at atmospheric pressure, *there is neither for the positive nor the negative ion any marked increase in the absolute value of the mobility. It is also to be noticed that the mobility of the negative ion shows no abnormal increase relative to that of the positive ion.* In other words, in fields of nearly half the value of the sparking field strength in air (30,000 volts per centimeter at 760 mm. pressure), and in fields where occasional sparking did actually occur (due to irregularities of

TABLE I

IONIC MOBILITIES OBTAINED WITH CHAFFEE ARC

N	d_+	d_-	u_+	u_-	p	U_+	U_-	X_+	X_-
					mm.				
60	0.99	1.38	1.10	2.12	760	1.10	2.12	121	87
60	1.00	1.26	1.13	1.78	756	1.12	1.76	120	96
*60	1.00	1.28	1.13	1.85	748	1.12	1.84	118	93
60	1.33	1.56	1.95	2.70	578	1.48	2.05	91	77
7670	0.50		0.97		750	0.98		12,450	
7670	0.55	0.64	1.17	1.59	746	1.19	1.62	11.460	9,750
7670	0.54	0.61	1.07	1.37	747	1.09	1.40	11,550	10,200
7670	0.50	0.65	1.25	1.80	735	1.28	1.86	12,450	9,600
7670	0.66	0.84	1.70	2.74	534	1.27	1.95	9,580	7,430
*7670	0.78	1.02	2.36	4.04	436	1.36	2.32	8,000	6,000
*7670	0.85	1.01	2.72	3.88	430	1.54	2.18	7,346	6,160
7670	0.82	1.10	2.58	4.29	416	1.41	2.29	7,610	5,670
7670	0.85	0.90	2.67	3.07	384	1.35	1.53	7,340	6,910
7670	0.80	0.86	2.51	2.90	382	1.26	1.46	7,800	7,260
†7670	1.11	1.22	4.78	5.80	304	1.96	2.32	5,610	5,160

Mean Mobilities at 760 mm. 1.33 1.89

* These readings were taken with very dry air.

† The value of the + mobility only estimated here, not determined.

the arc), one *finds that the mobilities are absolutely normal.* Even at pressures as low as 300 mm., where the potential fall per mean free path is larger still, there is no abnormal mobility.

These results as regards the mobility of the positive ion for high values of the product, field strength times mean free path, seem to be in direct contradiction to the results of Todd,[4] who, working at very low pressures and small field strengths, got an apparent abnormal increase in the mobility of the positive ion. The value of this product may be expressed as the ratio X/p, to which it is proportional, where X is field strength in volts per centimeter and p is the pressure in millimeters of mercury. The value of X/p for which Todd[4] observed the abnormality of the positive ions is 2.6. The values of X/p at which I worked are as high as 18.0. The latter's results agree, however, with those of Wellisch[5] recently obtained at pressures as low as 0.05 mm. of mercury. Wellisch[5] found no abnormal mobilities of the positive ions for values of X/p as high as 34.5. This value of X/p is close to that for ionisation by collision at this pressure.

The results obtained by me with negative ions for high pressure for values of X/p near where ionisation by collision must begin, seem to be contrary to the results of most of the observers who worked at low pressures with much smaller values of X/p. Townsend,[6] basing his assertions on the work of Lattey,[7] states that for low pressures the abnormal mobility, i.e., the breaking up of the cluster, should begin at $X/p = 0.1$. This appears to be in direct contradiction to the results of the writer obtained with air at pressures as low as 300 mm., and with $X/p = 16.0$, where strictly normal negative mobilities are found. In view of this contradiction, I worked over the results of Kovarick,[8] who obtained abnormally high negative mobilities at low pressures, to see whether the appearance of the abnormally high mobilities in his work was a function of X/p. The initiation of these abnormal mobilities was found to be a function of p rather than of X/p, which indicated, since my pressures were much higher than Kovarick's,[8] that the two sets of results were not necessarily in conflict.

All of the above facts are in accord with the recent results of Wellisch[5] on the mobilities of negative ions at low pressures. He found that, even at the lowest pressures at which he worked, the negative carriers were in part at least perfectly normal negative ions. This was the case for values of X/p close to the value of X/p for ionisation by collision at those pressures. He also found increasingly great numbers of free electrons in air as the pressure was reduced below 8 cm., but no intermediate negative ions. These free electrons could not, according to Wellisch, be detected by the other observers because of their low frequencies of alternation. The result was that the curves of these observers gave them apparent abnormal increases in the mobility of the negative ion, which increased in value with decreasing pressures. In my experiments these electrons were of course absent, and so no apparent abnormal increase was obtained.

The conclusion to be drawn from these results seems to be that the 'cluster' theory, which has until now been most generally accepted, is not correct. This forces us to accept the 'small ion' theory in some form or other.

Summary.—1. The mobilities of positive ions have been determined in electric fields very nearly strong enough to cause ionisation by collision at atmospheric pressures and have been found to be perfectly normal within the limits of error of the measurement.

2. The mobilities of the negative ions have also been determined, under the same circumstances, with the result that they not only showed no relative abnormal increase in value over those of the positive ion, but also showed a perfectly normal absolute value of the mobility.

3. These results, though at variance with those of most observers at low pressures for the negative ions, are in good agreement with recent results of Wellisch,[5] and likewise lead to the conclusion that the 'cluster' theory is no longer tenable.

[1] Langevin, Thesis, *Ann. Chim. Phys., Paris,* 1902.

[2] Wellisch, *London, Phil. Trans. R. Soc.,* A, **209** (1909).

[3] Rutherford, *Cambridge, Proc. Phil. Soc.,* 1898.

[4] Todd, *Phil. Mag., London,* June, 1913.

[5] Wellisch, *Amer. J. Sci., New Haven,* May, 1915.

[6] Townsend, *Electricity in Gases,* 1914.

[7] Lattey, *London, Proc. R. Soc.,* A, **84,** 1910.

[8] Kovarick, *Physic. Rev., Ithaca,* 1911.

[9] Chaffee, *Boston, Proc. Amer. Acad.,* **47,** No. 9, 1911.

THE RELATION OF MYELIN TO THE LOSS OF WATER IN THE MAMMALIAN NERVOUS SYSTEM WITH ADVANCING AGE

By Henry H. Donaldson

WISTER INSTITUTE OF ANATOMY AND BIOLOGY, PHILADELPHIA

Read before the Academy, April 17, 1916. Received, June 6, 1916

Starting from birth, the water-content of the mammalian body diminishes with age, and the same statement holds for the several anatomical systems which compose the body (Lowrey[1]). My own studies have been made on the albino rat in which the changes in the water-content of the central nervous system have been followed as a phenomenon of growth (Donaldson[2, 3, 4, 5]).

The progressive loss in the percentage of water in the brain and in the spinal cord is closely linked with age, although it can be slightly modified by nutritive conditions. The age of the animal becomes therefore the most important datum when a determination of the percentage of water is to be made.

In Chart 1 the change, on age, of the percentage of water in the brain and in the spinal cord of the male albino rat is shown by graphs. The observed mean values for groups of cases are indicated by small dots, for the brain, and by small triangles, for the cord. The respective graphs were determined by formulas devised by Dr. S. Hatai, and based on the observed group values (Donaldson[6]). Along the graph for the brain are seen four large dots. These represent observations on the human brain—taken from Koch and Mann, and from Weisbach.[7, 8]

Table 1 gives the percentage of water and the age of the human brains here used and also the equivalent rat ages.

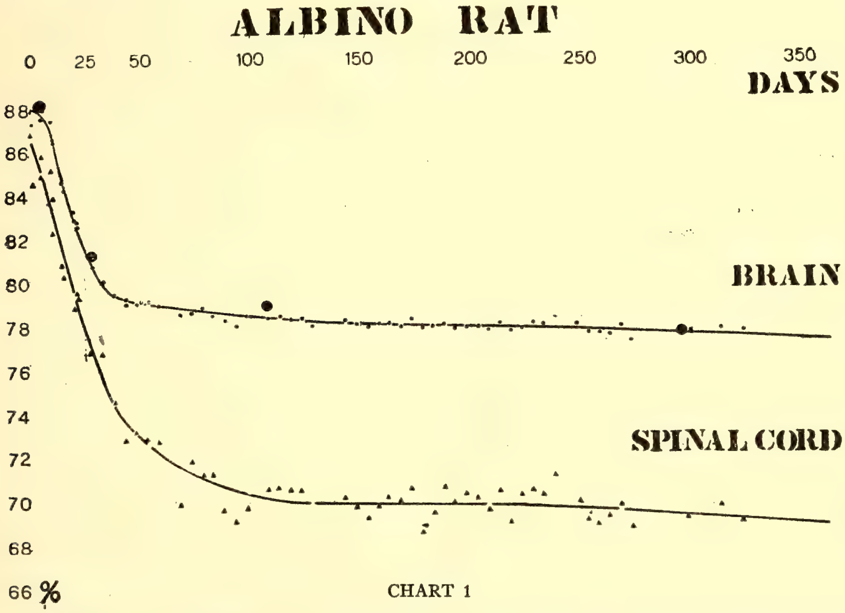


CHART 1

Showing the percentage of water, on age, in the central nervous system of the albino rat. The upper graph gives the values for the water in the brain as determined by the formulas (Hatai—in 'The Rat,' Donaldson,[9]—1915). The lower graph gives the corresponding values for the spinal cord, determined in the same way.

The small black dots indicate the observed values for the several age groups for the brain—and form the data for the formulas. The small black triangles have a like value in relation to the spinal cord.

The large black dots along the graph for the brain represent the human data as given in Table 1.

TABLE 1

Percentages of water in the human brain

Koch and Mann, (K); Weisbach, (W)

HUMAN AGE, YEARS	OBSERVER	PERCENTAGE OF WATER	EQUIVALENT RAT AGE DAYS
Birth	(W)	88.3	Birth
2.0	(K)	81.1	26
9.5	(W)	79.2	115
25.0	(W) (K)	(77.0) (77.8)	290

We find that the span of life for man is about thirty times as long as that for the rat and hence by reducing any observed human age to one-thirtieth of its value we obtain the corresponding rat-age to which it is equivalent. These equivalent ages are given in Table 1. As is seen from the chart, when the water determinations for man are entered in accordance with this age relation they agree well with those for the rat. We conclude, therefore, that essentially the same process is occurring in the brains of both man and the rat—only in the rat it is proceeding thirty times faster than in man. These relations have been pointed out here in order to show that the conclusions reached at the end of this paper will probably apply to man, although directly based on the rat.

The fact that there is a progressive loss of water in the brain and in the spinal cord with advancing age having been established, it is desirable to consider the organs themselves and to find how this loss is distributed within them. The following analysis applies both to the brain and to the spinal cord—but, for convenience, it will be given as if for the brain alone. The brain is very largely composed of nerve cells or neurons. When reduced to its simplest terms a neuron consists of a cell body—with a principal outgrowth—the axon. All axons are free from visible myelin sheaths when young—but as they mature many, but not all, acquire rather thick sheaths of myelin. Myelin is a morphological term used to designate these sheaths, which are composed mainly of lipoids and which give the nerve fibers a white color. Broadly speaking, there is little or no visible myelin present in the brain at birth—while with advancing age it increases rapidly, and at maturity forms a considerable portion of the entire brain.

It is my purpose to determine by this study whether the loss of water, which occurs in the brain between birth and maturity, takes place equally in the cell body and its axon, on the one hand, and in the myelin sheaths, on the other, or whether this loss is unequally distributed.

By way of introduction the following data from the literature are presented (de Regibus[10]).

In man at birth the brain has about 88% of water and both the gray substance (in which cell bodies are abundant), and masses of fibers alone (later to become 'white substance'), have the same percentage. At maturity, however, the case is very different, as will be seen from Table 2.

TABLE 2

Percentage of water. Human brain

	CORTEX (GRAY)	CALLOSUM (WHITE)
	per cent	*per cent*
At Birth	88.0	88.0
At Maturity	86.0	70.4

According to this table the (gray) cortex has lost 2 points and the (white) callosum, 17.6 points in the process of maturing.

It is never possible to obtain at maturity the cortex, or any other gray mass, without an admixture of some myelinated fibers, and I have therefore credited one point of the loss, noted in the water content of the cortex, to the presence of such fibers.

According to this assumption the mature gray substance, when the myelin of the myelinated fibers is excluded, contains 87% of water.

In the computations which follow, the neurons (= cell bodies and axons), without myelin, are assumed to have 87% of water.

The fact that the fibers without myelin (see Table 2) have at birth a high percentage of water (88%) while at maturity, after myelination, they have lost 17.6 points, indicates that either axons of this type have a peculiar capacity for losing water, or that the accumulation of myelin has caused the reduction observed.

To obtain a notion of the approximate distribution of the water between the myelin and neurons proper, it is necessary to have data on the relative abundance of these two constituents of the brain.

In 1913 W. Koch and M. L. Koch[11] made a study of the chemical composition of the brain of the albino rat at six ages, between birth and maturity, and of the spinal cord, at one age. The data thus obtained are those which will be utilized here. The authors determined seven fractions:—proteins, organic extractives and inorganic constituents, which three taken together, we shall designate, protein (or non-lipoid); and phosphatides, cerebrosides, sulphatides and cholesterol, which four taken together, we shall designate, lipoid.

These data give us at each age, therefore, the protein and the lipoid present in the brain, or to be a little more exact, we should say the lipoid and the non-lipoid fractions. The lipoid (in part) represents the myelin sheaths, while the protein, with part of the lipoid, represents the cell bodies and their unsheathed axons.

With the exception of the one day group, the ages for which analyses were made are given in Table 3.

At one day—or practically birth—it is found that the lipoid is present to the extent of 0.31 or nearly one-third of the weight of the protein. There is, however, no visible myelin at this age, so it is concluded that this proportion of the total lipoid is normally associated with the protein and is not to be included in the lipoid which forms the myelin sheaths. We have treated the data for the later age groups in accordance with this relation—and in each case have taken from the total lipoid found an amount equal to 0.31 of the protein found. The remaining amount of lipoid is assumed to be that used for the sheaths.

In Table 3, the column (2) headed 'Corrected Protein' gives the observed protein (non-lipoid) plus 0.31 of itself—and the column (3) headed 'Corrected Lipoid' gives the observed lipoid less the amount of lipoid added to the protein.

TABLE 3

To show for five brains of the Albino rat and for one spinal cord the percentage of water in the myelin as computed according to the method described. The protein and lipoid are given in percentages of the total dry substance (Based on Table 2. Koch and Koch)[11]

(1)	(2)	(3)	(4)	(5)	(6)	(7)
				PERCENTAGE OF WATER		
AGE IN DAYS	CORRECTED PROTEIN. PER CENT	CORRECTED LIPOID. PER CENT	PROPORTION OF LIPOID FOR DIFFERENT AGES. LIPOID AT 20 DAYS = 1	Entire brain observed.	In neurons = Protein (C) (assumed).	In myelin = lipoids (C) (computed).
			Brain			
10	93.80	6.2		86.5	88	63.8*
20	88.88	11.12	1.	82.5	87	46.5†
40	82.86	17.14	1.5	79.4	87	42.7
120	75.11	24.84	2.2	78.4	87	52.4
210	76.36	23.63	2.1	78.1	87	47.4
Average of 20 to 210 days						47.8
			Spinal Cord			
120	52.92	47.08	4.2	70.4	87	51.0

* First traces of myelin. † Myelin well shown.

In Table 3 the data are given in five age groups for the brain and in one age group for the spinal cord. It is to be noted that the 10 day brain group—which stands just at the beginning of the myelin formation—is here excluded from the discussion and we begin the comparisons which are to be made, with the 20 day brain group.

In the brain series (with one exception) the corrected protein diminishes and the corrected lipoid increases with advancing age. Between 20 and 210 days the proportion of the lipoid doubles. We have, in column (5), the observed percentage of water in the brain as a whole. It is assumed, as previously noted, that the corrected protein (neurons, in the strict sense: = both cell-bodies and axons) have 87% of water. From these several data we can compute the percentage of water to be assigned to the corrected lipoid, which represents the myelin.

The method of computation may be illustrated by the data for the 20 day group. Reference to Table 3 shows that, at this age, there is 1 part of lipoid (11.12%) to 8 parts of protein (88.88%). This gives 9 parts, representing the entire brain and having 82.5% of water. The product, 9 × 82.5 = 742.5. We assume that the 8 parts of protein have 87% of water. The product, 8 × 87 = 696. The 1 part of lipoid, representing the myelin, will then have a percentage of water equal to the difference of these products: 742.5 – 696 = 46.5%. The values thus obtained are entered in column (7), and, taken together, the four entries give a mean value of 47.8% of water for the myelin.

In this connection it should be noted that the spinal cord, which has about twice as much lipoid as the brain at the same age, gives also a similarly low value for the water in the myelin—51%.

The significance of these results lies not in the particular percentage of water here determined for the myelin—as that depends on the percentage of water assumed for the protein—but in the similarity of the values found in all the five cases examined.

We conclude from these results that there is no evidence that the cell bodies and their unsheathed axons suffer any significant loss of water between birth and maturity, and that the progressive diminution in the water content of the entire brain and spinal cord is mainly due to the accumulation of myelin—with a water content of about 50%. Moreover, the myelin must be regarded as a more or less extraneous substance, having but little significance for the characteristic activities of the neurons. As the diminution in the percentage of water in the central nervous system is preëminently a function of age, and as it appears to be due almost entirely to the formation of the myelin, it follows that the myelin formation is also a function of age. A glance at the graphs in Chart 1, and at column (4) in Table 3, will show that the most active production of myelin, as indicated by the rapid loss in the percentage of water, occurs early, i.e., during the first twentieth of the life span in both the rat and man.

[1] Lowrey, L. G., *Anat. Rec.*, 7, 143–168 (1913).

[2] Donaldson, H. H., *J. Comp. Neur. Psychol.*, 20, 119–144 (1910).

[3] Donaldson, H. H., *Ibid.*, **21**, 129–137 (1911).

[4] Donaldson, H. H., *Ibid.*, **21**, 139–145 (1911).

[5] Donaldson, H. H., *Ibid.*, **21**, 161–176 (1911).

[6] Donaldson, H. H., The Rat, *Mem. Wistar Inst. Anat. Biol., Philadelphia*, No. 6 (1915).

[7] Koch, W. and Mann, S. A., *Arch. Neur. Psychiat.* (Mott), **4**, 201–204 (1909).

[8] Weisbach, A., *Med. Jahrbücher* **16**, Nos. 4 & 5, 1–76 (1868).

[9] Donaldson, H. H., The Rat, *Mem. Wistar Inst, Philadelphia*, No. 6 (1915).

[10] De Regibus, C., *Torino, Atti. R. Acc. Med.* **6**, 323–328 (1884).

[11] Koch, W. and Koch, M. L., *J. Biol. Chem.*, **15**, No. 3, 423–448 (1913).

DIFFERENTIAL MITOSES IN THE GERM-CELL CYCLE OF DINEUTES NIGRIOR

By R. W. Hegner and C. P. Russell

ZOÖLOGICAL LABORATORY, UNIVERSITY OF MICHIGAN

Received by the Academy, May 26, 1916

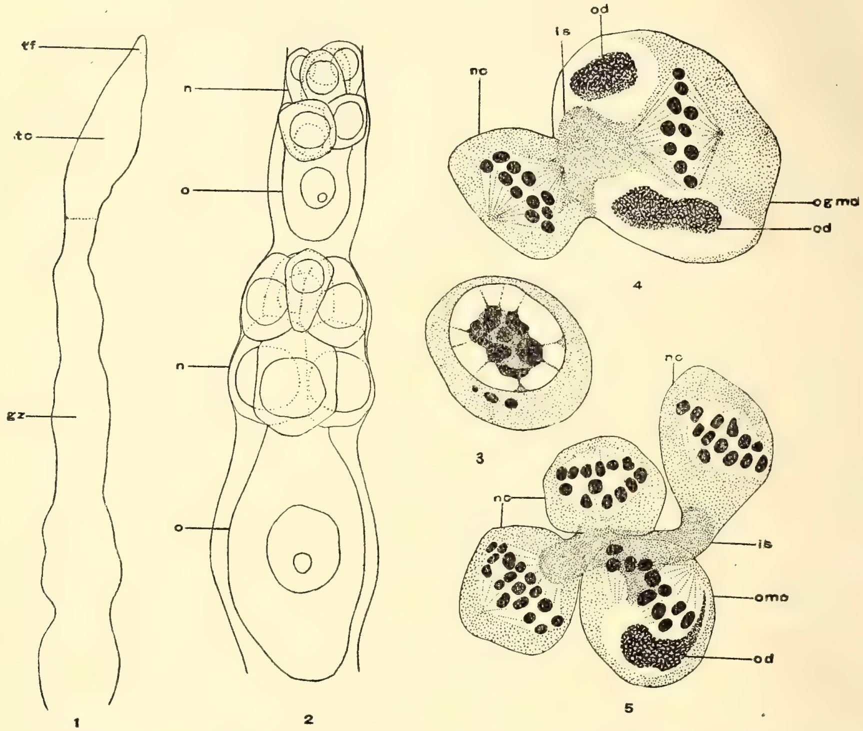
One of the most interesting and important periods in the germ-cell cycle of certain insects is that during which oögonia give rise to nurse cells and oöcytes. While such a period does not occur in certain insects, such as the paedogenetic fly, *Miastor*, in which the nurse cells are of mesodermal origin (Kahle, 1908; Hegner, 1914), perhaps in the majority of the members of this class, the growth of the egg is preceded by the formation of nurse cells from which the oöcyte derives most of its contents.

Many investigators have studied the origin and history of the cellular elements within the ovaries of insects, but in only one family, the Dytiscidae, have clearly defined visible differences been discovered between the nurse cells and the oöcytes at the time of their origin from the same mother cells. In the diving beetle, *Dytiscus marginalis*, Giardina (1901) discovered true differential mitoses which result in the derivation of one oöcyte and fifteen nurse cells from each ultimate oögonium. In this case there is a series of four mitoses during each of which one cell divides unequally; the larger daughter cell is characterized by the presence of an extra-nuclear 'chromatic ring' and leads to the formation of the oöcyte; the other gives rise only to nurse cells. Giardina supposes that this peculiar chromatic ring consists of part of the chromatin contained in the nucleus of the ultimate oögonium. He found that a chromatic network remains in the nucleus after the chromosomes are formed and that the ring results from the condensation of this network. Günthert (1909) repeated Giardina's work and confirmed it in almost every respect. Regarding the origin of the chromatic ring, he believes that it may consist of chromatin that has separated from the chromosomes. Debaiseaux (1909) also studied these differential mitoses in *Dytiscus marginalis* but added very little to Giardina's account. His most interesting conclusion is that the material of the chromatic ring may really be nucleolar in nature.

Several writers had noted peculiarities in the ovarian cells of beetles before Giardina published the results of his investigation but did not work out their history. Thus Will (1886) described and figured the origin of the nurse cells in the beetle, *Colymbetes fuscus*, but his work is lacking in detail, and Korschelt (1886) actually observed the chromatic ring is *Dytiscus marginalis* but failed to determine its real significance. Since Giardina's investigations were published many attempts have been made to find differential mitosis of a similar sort in other insects, but thus far without any success. Kern (1912) found a group of granules in the oögonia of the ground beetle, *Carabus nemoralis*, and a similar group in the oöcyte of *C. glabratus* but none of the intervening stages were seen and the connection between these granules and differential mitoses is therefore doubtful.

It seems particularly fortunate, considering these many failures, that a species belonging to another family of insects has at last been found in which visible substances can be observed during the differential mitoses. The material on which the following report is based consists of over thirty ovaries from the whirl-i-gig beetle, *Dineutes nigrior*.[1] These were obtained by the senior author[2] at Woods Hole, Massachusetts, on August 28, 1915. Some of them were fixed in Meves' modification of Fleming's solution, some in Bouin's picro-formol solution, and some in Carnoy's acetic-alcohol-chloroform-sublimate mixture. Longitudinal sections were cut 5 microns thick and stained with Heidenhain's iron-haemotoxylin. The general relations of the nurse cells and oöcytes were obtained from *in toto* preparations.

One of the ovarioles from an ovary of *Dineutes nigrior* is shown in longitudinal section in figure 1. It consists of three general zones, the terminal filament (*tf*), the terminal chamber (*tc*) and the growth zone (*gz*). The anterior end of each ovariole is attached to the dorsal body wall by a long terminal filament made up of epithelial cells. The oögonia multiply and differentiate into nurse cells and oöcytes within the terminal chamber. After differentiation has occurred, the oöcytes and their accompanying nurse cells lie at the anterior end of the zone of growth. Each oöcyte is nourished by seven nurse cells which lie in a nurse chamber anterior to it. The arrangement of these nurse cells is shown in figure 2, *n*.



1. Outline of an ovariole. *tf* = terminal filament, *tc* = terminal chamber containing undifferentiated oögonia; *gz* = zone of growth. (× 140)

2. A portion of growth zone, drawn from in toto preparation, showing eight cell stages. *n* = 7 nurse cells arranged in nurse chamber; *o* = oöcyte connected with nurse chamber. (× 225)

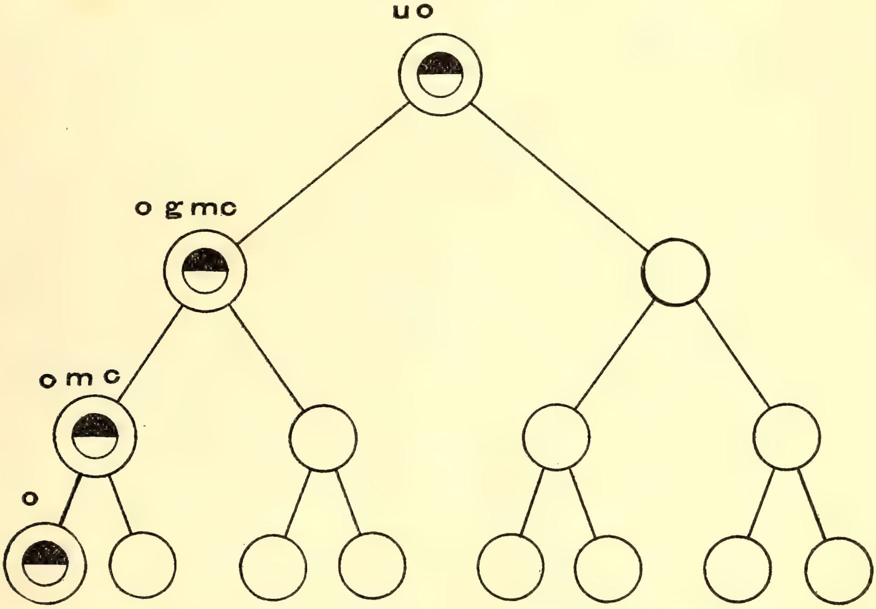
3. A single ultimate oögonium from anterior end of terminal chamber. (× 2000)

4. Two-cell stage. Metaphase of second division. *nc* = nurse cell; *ogmc* = oöcyte grandmother cell containing oöcyte determinant *od*; *is* = intercellular strand. (× 2000)

5. Four-cell stage. Metaphase of third division. *nc* = nurse cells, *omc* = oöcyte mother cell, containing oöcyte determinant *od*; *is* = intercellular strand. (× 2000)

The large terminal chamber contains at its distal end many oögonia ready to undergo differentiation. These ultimate oögonia (fig. 3) are characterized by a nucleus containing a deeply staining central mass of chromatin from which many fine strands radiate to the nuclear membrane. A few small darkly staining bodies are sometimes present in the cytoplasm. The two daughter cells of such an ultimate oögonium are shown in figure 4 preparing for division. One of these cells (*nc*) is smaller than the other and gives rise to nurse cells only. The other, the oöcyte grandmother cell (*ogmc*) is larger and contains within its cytoplasm portions of a deeply staining body that we do not hesitate to homologize with the 'anello cromatico' of Giardina and which we may call the 'oöcyte determinant.' Between the two cells is an intercellular strand (*is*) which represents the remains of the spindle fibres of the previous mitotic division.

A four-cell stage such as is illustrated in figure 5 proves that the division of the oöcyte grandmother cell (fig. 4, *ogmc*) is differential since one cell, the oöcyte mother cell (fig. 5, *omc*) has received all of the oöcyte determinant (*od*), whereas the other three (*nc*) are smaller and lack this body, and are destined to become nurse cells. The intercellular strands are quite conspicuous here and leave no doubt in the mind of the observer as to the origin of the four cells from a single cell, the ultimate oögonium. Each of these four cells undergoes a single mitotic division resulting in the formation of seven nurse cells and one oöcyte, the latter containing the oöcyte determinant.



6. Diagrammatic representation of oöcyte differentiation in Dineutes nigrior. *uo* = ultimate oögonium, containing oöcyte determinant within its nucleus; *ogmc* = oöcyte grandmother cell having similar nuclear characteristics; *omc* = oöcyte mother cell, *o* = oöcyte. Plain circles indicate nurse cells.

The most conspicuous difference that we have discovered between the origin of the oöcyte in *Dineutes nigrior* and in *Dytiscus* is in the number of differential mitoses; in *Dineutes nigrior* there are only three (fig. 6) whereas in *Dytiscus* there are four.

As a result the oöcyte of *Dineutes nigrior* is accompanied by only seven nurse cells, that of *Dytiscus* by fifteen. The origin and history of the oöcyte determinant are of considerable interest and importance because of their bearing on the subject of the physical basis of heredity. A more detailed study of our material is now being made and the ovaries of *Dineutes nigrior* and of several other species promise to furnish a means of clearing up the details of differential mitoses in these insects.

Debaisieux, P., Les débuts de l'ovogenese dans le Dytiscus marginalis, *La Cellule*, **25** (1909).

Giardina, A., Origine dell' oocite e delle cellule nutrici nel Dytiscus, *Intern. Monatschr. Anat., Leipzig*, **18** (1901).

Govaerts, P., Recherches sur la structure de l'ovaire des insects, *Arch. biol., Paris-Bruxelles*, **28** (1913).

Günthert, T., Die Eibildung der Dytisciden, *Zool. Jahrb., Jena*, **30** (1910).

Hegner, R. W., Studies on Germ Cells, I and II, *J. Morph., Boston*, **25** (1914).

Kahle, W., Die Paedogenese der Cedidomyiden, *Zoologica, Stuttgart*, **21** (1908).

Kern, P., Ueber die Fortpflanzung und Eibildung bei einigen Caraben, *Zool. Anz., Leipzig*, **40** (1912).

Korschelt, E., Ueber die Entstehung und Bedeutung der verschiedenen Elemente des Insectenovariums, *Zs. wiss. Zool., Leipzig*, **43** (1886).

Maziarski, S., Sur la persistance de résidus fusoriaux pendant les nombreuses générations cellularies du cours de l'ovogenesese de Vespa vulgaris, *Arch. Zellforsch.*, **10** (1913).

Will, L., Oogenetische Studien, I. Die Entstehung des Eies von Colymbetes fuscus, *Zs. wiss. Zool.*, **43** (1886).

[1] We are indebted to Mr. A. W. Andrews for the identification of this species.

[2] Mr. R. W. Hegner is indebted to the Bache Fund for assistance in preparing the material on which this report is based.

SOME MINERALS FROM THE FLUORITE-BARITE VEIN NEAR WAGON WHEEL GAP, COLORADO

By Esper S. Larsen and Roger C. Wells

U. S. GEOLOGICAL SURVEY, WASHINGTON, D. C.

Received by the Academy, May 27, 1916

In the summer of 1912 Messrs. W. H. Emmons and E. S. Larsen[1] made a hasty examination of the fluorite-barite vein which passes through the hot springs near Wagon Wheel Gap, Colorado. Since that time this vein has been extensively prospected and a considerable amount of fluorite has been produced. In the summer of 1915, in the course of the reconnaissance geologic study of this region under the direction of Dr. Whitman Cross, Mr. Larsen made another brief visit to the deposit and collected two specimens of material not before recognized in the deposit.

One of these specimens proved to be the unusual mineral gearksutite, and the other was made up of a peculiar kaolinite and a new fluoride-sulphate, *creedite.* It is the purpose of the present paper[2] to describe these minerals.

The vein is entirely in Tertiary volcanic rocks made up of lava flows and tuff beds of rhyolite and quartz latite, with subordinate andesite. It can be followed for perhaps half a mile and is variable in width but is not uncommonly wide enough to make the mining of the fluorite profitable. Much of the vein filling is nearly pure fluorite but locally barite is abundant. The wall rock shows considerable alteration and commonly carries much disseminated pyrite.

Gearksutite.—The lower tunnel of the mine, whose portal is about 100 feet east of one of the hot springs, follows the vein and some distance from the portal it passes through a large body of soft, highly altered rock, giving rise to considerable caving. This soft material is a highly altered rhyolite or quartz latite, probably a tuff, which carries very abundant balls, up to several inches across, of snow white, powdery gearksutite. These balls are easily separated from the altered rock and resemble a very fragile chalk or kaolinite. When wet they become plastic and resemble ordinary dough and are without grit.

A microscopic examination shows that the material is homogeneous and is made up of an aggregate of shreds so minute as to be recognized only with a high magnification. It has a mean index of refraction of 1.454 ± 0.003 and a moderate birefringence. The specific gravity measured by the pycnometer method is 2.768.

This agrees with the properties of gearksutite from Greenland,[3] which "occurs in white, chalky aggregates of minute fibers or prisms which are optically negative and have a moderate axial angle. X is normal to the fibers and Y makes a large angle with them.

The mineral is probably monoclinic with X = b.

$\alpha = 1.448 \pm 0.003, \quad \beta = 1.454 \pm 0.003, \quad \gamma = 1.456 \pm 0.003.$"

A chemical analysis of the material from Wagon Wheel Gap shows its identity with gearksutite. In the following table Analysis 1, by R. C. Wells, is of gearksutite from Wagon Wheel Gap, Colorado. Analysis 2, by G. Lindström, is of gearksutite from Ivigtut, Greenland, and Analysis 3, by Hillebrand, is of that mineral from St. Peters Dome, Colorado. Column 4 gives the theoretical composition,[4] on the basis of the formula $CaF_2.Al(F,OH)_3.H_2O$. with F:OH = 2:1.

Analyses of gearksutite

	1*	2	3	4
Al	15.11	15.37	15.20	15.1
Fe	tr.	0.30		
Ca	22.41	21.02	22.30	22.4
Mg	tr.	0.16		
Na	0.04	1.06	0.10	
K	0.07	0.23	0.04	
Cl		0.20		
F	41.00	41.81	42.07	42.9
O	5.09	4.82	4.83	4.5
H_2O-	0.44			
H_2O+	15.20	15.03	15.46	15.1
	99.36	100.00	100.00	100.0

* SiO_2 none, Al_2O_3 28.49, Fe_2O_3 tr., CaO 31.37, MgO tr., Na_2O .05, K_2O .08, F 41.00, H_2O- .44, H_2O+ 15.20, less O for F 17.27, total 99.36.

Although the direct determination of fluorine, in which the fluorine was weighed as CaF_2, yielded 41.0 per cent in Analysis 1, no great accuracy is claimed for this determination. The agreement of the other constituents in the three analyses, however, shows the practical identity of the minerals, the chief difference being that a small amount of Ca is replaced by Na in the Greenland mineral. All four analyses agree very well with the formula $CaF_2Al(F,OH)_3.H_2O$. with $F:OH = 2:1$.

Heretofore gearksutite has been described only as an alteration product of cryolite, but at Wagon Wheel Gap no cryolite has been recognized and the gearksutite is believed to have been formed by a metasomatic alteration of the rhyolitic wall rock much as kaolinite or sericite is formed. Indeed, on the first visit to the deposits the gearksutite was mistaken for kaolinite. It seems probable that the gearksutite was developed in the wall rock by the ascending hot solutions which deposited the fluorite in the vein.

Creedite, a New Mineral.—A specimen collected from the dump of one of the upper workings of the fluorite mine proved to be made up in large part of a new mineral. The name 'Creedite' is proposed for this mineral from its occurrence near the center of the Creede quadrangle of the United States Geological Survey.

Creedite makes up about two-thirds of the specimen and is in white to colorless grains and poorly developed crystals up to 5 millimeters across, imbedded in a dull white kaolinite, with a very little barite. The kaolinite is believed to be derived from the alteration of some unknown mineral.

Creedite is nearly colorless, has a hardness of about $3\frac{1}{2}$, intumesces before the blow pipe, showing the calcium flame, and finally fuses to a white enamel. It is somewhat slowly but completely soluble in acid. Its specific gravity as measured on the powder by the pycnometer method is 2.730.

Most of the grains show no good crystal development, but a few are stout prismatic crystals with an acute rhombic cross section. One crystal showed curved and striated pyramidal faces. The faces are dull, curved, and imperfect, and not suitable for accurate measurements. The angle between the prism faces measured under the microscope on the cross section of a prism gave 68° and is believed to be accurate to within about 1°. However, a measurement on a second prism with distinctly curved faces gave only 59°. The mineral has a perfect cleavage parallel to the elongation of the prisms and bisecting the obtuse angle of the prismatic faces.

Creedite has distinctive optical properties and was first recognized by them. Extinction angles can not be measured accurately on account of the imperfect development of the crystals. However, cleavage fragments show sensibly parallel extinction and the emergence of an optic axis very nearly normal to the cleavage plane. Prismatic sections normal to the cleavage (bisecting the acute angle of the faces) show extinction angles of about 41° ($Z \wedge$ elongation), the emergence of **Y**, sensibly normal, and irregular twinning. Crystals lying on the crystal face show extinction angles ($Z \wedge c$) of $42\frac{1}{2}° \pm 1°$. The mineral is therefore believed to be monoclinic with the faces (110), perfect cleavage (100), $Y = b$, and $Z \wedge c = 41°$; (100) is a twinning plane.

The mineral is optically negative. The axial angle as measured on the axial angle apparatus is:

$$2V_{li} = 64° \, 30' \pm 10', \qquad 2V_{na} = 64° \, 22' \pm 10', \qquad 2V_{Tl}, \, 64° \, 20' \pm 10'.$$

Other less satisfactory sections gave

$$2V_{na} = 64° \, 18' \pm 20' \qquad \text{and} \qquad 2V_{na} = 64° \, 50' \pm 30'.$$

The dispersion was perceptible only on one optic axis.

The indices of refraction were measured by the immersion method by matching in a liquid and immediately measuring the index of refraction of the liquid at the same temperature; the probable error should be less than ± 0.001. All of the grains give the same value showing that the mineral is homogeneous.

$$\alpha = 1.461, \qquad \beta = 1.478, \qquad \gamma = 1.485.$$

The axial angle computed from the indices of refraction is 65° which agrees very well with the measured values.

Material for a chemical analysis was carefully purified by heavy solution (bromoform) and a careful microscopic examination showed only a very little kaolinite present. The analyses by R. C. Wells gave the following results:

Analysis and molecular ratios of Creedite

					2.
Al	11.58	427	210	2 × 105	11.0
Ca	23.98	599	295	3 × 98	24.4
SO_4	18.32	191	94	1 × 94	19.5
O*	3.97	248	122	1 × 122	3.2
H_2O-	0.72				
H_2O	11.08	615	303	3 × 101	11.0
F^1*	30.35	1591	784	8 × 98	30.9
	100.00				100.00

* Calculated on the basis of a summation of 100%.

This leads to the formula $CaSO_4.2CaF_2.2Al\ (F,OH)_3.2H_2O$ with F:OH 2:1 and the mineral has the composition of gearksutite with one molecule of $CaSO_4$ added. The composition corresponding to this formula is given in column 2. Creedite is one of the very few minerals described that contain both sulphate and fluoride as essential constituents.

Isotropic Kaolinite-Like Mineral.—The creedite is imbedded in a small amount of a kaolinite-like mineral with rather unusual properties, which is believed to have been derived from some unknown mineral. In the hand specimen it is a white powder, in the thin sections it is clear, in large part sensibly isotropic, and can be distinguished from the Canada balsam in which it is imbedded, only by testing its index of refraction. Locally it shows birefringence in wavy streaks, probably due to strain in the soft mineral or to incipient crystallization. It has an index of refraction of 1.557 ± 0.003. When immersed in a liquid with that index of refraction it is at first clouded but the oil rapidly penetrates its pores, the mineral becomes clear, and the grains can be found only by the colored borders formed by the Schroeder van der Kolk test for the index of refraction. The specific gravity of the powder measured by the pycnometer method is 2.548.

A partial analysis by Roger C. Wells gave:

SiO_2	Al_2O_3	CaO	MgO	F	Ign.	Total
44.2	40.2	0.3	tr.	none	15.5	100.2

Imbedded in this mineral and like it derived from some mineral that has disappeared, are spherulites of an undetermined mineral with rather strong birefringence, positive elongation, and a mean index of refraction of about 1.50.

---

[1] Emmons and Larsen, The hot springs and the mineral deposits of Wagon Wheel Gap, Colorado, *Economic Geology*, 8, 235 (1913).

[2] Published with permission of the Director of the United States Geological Survey.

[3] Unpublished manuscript by Esper S. Larsen.

[4] Dana, *A System of Mineralogy*, sixth edition, p. 181, 1892.

THE PROCESSES TAKING PLACE IN THE BODY BY WHICH THE NUMBER OF ERYTHROCYTES PER UNIT VOLUME OF BLOOD IS INCREASED IN ACUTE EXPERIMENTAL POLYCYTHAEMIA

By Paul D. Lamson

PHARMACOLOGICAL LABORATORY, JOHNS HOPKINS UNIVERSITY

Received by the Academy, June 9, 1916

In a previous communication to the PROCEEDINGS[1] I reviewed the work of the past on polycythaemia, and reported a series of experiments carried out in the hope of determining if possible the process by which the number of erythrocytes per unit volume of blood is increased in acute experimental polycythaemia. Since then further experiments have been undertaken, some of which have already been reported.[2] The results of these experiments, and others about to be published, are collected here in a brief summary of this work up to the present time.

This complicated problem may be divided into four main parts:

(1) The causes capable of producing an increase in number of erythrocytes per unit volume of blood.

(2) Localization of these processes.

(3) The manner in which the number of erythrocytes is increased.

(4) The mechanism by which the red corpuscle content of the blood is controlled.

As previously pointed out, the causes of polycythaemia are very numerous, and have received a great deal of attention, but little work has been done concerning the localization and manner in which this increase in number of erythrocytes is brought about.

From experiments previously reported, the present author concluded that the liver is the organ which is responsible for the changes in number of erythrocytes produced by the intravenous injection of epinephrin.[3] This conclusion has been confirmed by the following experiments here summarized.

(1) Epinephrin when injected into a dog or cat intravenously in doses of 0.9 mg. per kilo causes an increase in number of erythrocytes of 1.5 to 2.0 millions per cubic millimeter of blood in from five to ten minutes, lasting about one half hour and then gradually returning to normal.

(2) Removal of the stomach, intestine, mesentery, omentum, pancreas, and spleen, either singly or all together has no appreciable effect on this reaction to epinephrin.

(3) A head-thorax animal does not respond to epinephrin.

(4) The addition to the head-thorax animal of the liver, supplied by arterial blood only, gives an animal which responds to epinephrin.

(5) Removal of the liver from the circulation by ligation of the hepatic artery and shunting the portal bood around the liver by means of an Eck fistula, gives an animal in all respects normal except for the absence of the liver. Epinephrin in such an animal causes no increase in number of erythrocytes per unit volume of blood.

(6) In an animal in which all arterial blood supply to the liver is shut off by ligation of the hepatic artery (the portal circulation being left intact), epinephrin causes no increase in number of erythrocytes. But if one half hour or later, after the injection of epinephrin, arterial blood is again allowed to reach the liver by removal of this ligature, the number of erythrocytes is immediately increased as if a fresh dose of epinephrin had been injected into the animal.

(7) In an animal in which the arterial blood supply to the liver is shut off by ligation of the hepatic artery the injection of epinephrin into the portal vein causes an immediate increase in number of erythrocytes.

From these experiments it is evident that it is necessary for the liver to be present in the circulation for polycythaemia to take place. If epinephrin does not reach the liver as is the case when the hepatic artery is ligated (the epinephrin being probably adsorbed to a great extent in its passage through the other portal organs) no polycythaemia takes place. But if in this condition, epinephrin is allowed to reach the liver by injecting it into the portal vein, polycythaemia takes place at once.

As regards the manner in which the number of erythrocytes per unit volume of blood is increased following the injection of epinephrin we know that it must take place in one or more of the following ways.

(1) A decrease in plasma volume.

(2) An absolute increase in number of erythrocytes by (*a*) a formation of new erythrocytes, (*b*) a division of erythrocytes (a theoretical consideration), (*c*) bringing erythrocytes into the circulating blood from a reservoir or reservoirs in the body.

The following facts have been experimentally demonstrated.

(1) The number of erythrocytes per unit volume of blood is increased.
(2) An absolute increase in number of erythrocytes takes place.
(3) Plasma volume is decreased.
(4) Total blood volume is decreased.
(5) The average volume of the individual erythrocyte is decreased.
(6) The haemoglobin content of the average erythrocyte is decreased.
(7) No evidence of newly formed erythrocytes has been obtained.
(8) No evidence of the division of erythrocytes has been obtained.
(9) Proof that erythrocytes temporarily stored somewhere in the body, are brought into the circulation, has been obtained by deduction from the above facts, and a condition in which erythrocytes are stored has been experimentally demonstrated as follows.

If the hepatic artery is ligated and epinephrin intravenously injected, the number of erythrocytes per unit volume of blood remains unchanged, but at the same time the plasma volume is markedly decreased.[4] This is theoretically impossible without a destruction or a storing of erythrocytes somewhere in the body. No evidence of their destruction has been found. There is no haemolysis, and no haemoglobinuria. Furthermore polycythaemia may be repeatedly produced in the same animal.

The localization of this reservoir of erythrocytes was experimentally carried out in the following manner. As shown above, the liver is responsible for the increase in number of erythrocytes which follows the injection of epinephrin, and a certain portion of this increase is due to cells brought forth from some reservoir. In the experiments in which the hepatic artery is ligated and no increase in number of erythrocytes follows the injection of epinephrin, but where a storing of erythrocytes must take place, opening the hepatic artery is immediately followed by an increase in number of erythrocytes in the generally circulating blood. Or if in this same condition where the hepatic artery is ligated, epinephrin is injected into the portal vein, causing a constriction of the liver capillaries, and a decrease in liver volume, an immediate increase in number of erythrocytes occurs.

It has thus been experimentally shown, that the polycythaemia following the intravenous injection of epinephrin is due to some action of the liver. A condition has been brought about by ligation of the arterial blood supply to this organ and the intravenous injection of epinephrin, in which erythrocytes must be stored away. By returning the arterial blood supply to the liver, or by causing a constriction of the liver capillaries by the injection of epinephrin into the portal vein, both of which procedures offers mechanical conditions favorable for forcing out erythrocytes lying dormant in the capillaries of this organ, the red blood corpuscles known to have left the general circulation immediately appear in the blood stream causing an increase in number of erythrocytes per unit volume of blood. From these experiments it is concluded that the liver acts as a reservoir for erythrocytes.

The process by which the liver increases the number of erythrocytes is thought to be a loss of plasma from the liver capillaries, together with a constriction of these vessels driving the erythrocytes on into the blood stream. It has been experimentally shown that the liver capillaries constrict with epinephrin, causing a sudden diminution in size of the liver. This does not occur except to a slight extent when the hepatic artery is tied, and epinephrin injected intravenously, in which condition no increase in number of erythrocytes takes place. It is also of interest to note, as more fully described in a previous paper,[5] that Mautner and Pick[6] have shown that the livers of cats and dogs constrict very markedly when epinephrin is perfused through them or intravenously injected into these animals. Rabbits however do not respond in this way to epinephrin, the liver capillaries not contracting. They also show no increase in number of erythrocytes per unit volume of blood when epinephrin is intravenously injected. Thus we have two types of animals, one the cat, whose liver responds to epinephrin by constriction of its capillaries and gives a polycythaemia, the other the rabbit, whose liver does not respond to epinephrin and in which this substance produces no polycythaemia.

Concerning the mechanism which normally controls the red corpuscle content of the blood, my view was previously expressed that the adrenal glands play an important part in this process. Certain experiments were carried out in which the polycythaemia occurring in the normal animal did not occur in animals after the removal of the adrenals. Two additional experiments have been made which have strengthened the belief in this view.

As previously shown, certain nervous phenomena are capable of producing a polycythaemia. This increase in number of erythrocytes must occur either by some entirely different process from that caused by the injection of epinephrin, or by some nervous influence directly or indirectly affecting the liver. Experiments in which the nerves going to the liver were stimulated, showed no increase in number of erythrocytes. Furthermore in the experiments above reported, in which epinephrin is shown to have no effect on the number of erythrocytes per unit volume of blood in the rabbit, nervous phenomena known to cause polycythaemia in cats (which respond to epinephrin) caused no increase in number of erythrocytes in these animals. If polycythaemia is due to some direct nervous influence it might be expected that in these animals in which the nervous condition is intact, nervous stimuli as fright, would cause an increase in number of red corpuscles.

As the direct stimulation of the nerves to the liver causes no increase in number of erythrocytes, and as in animals in which epinephrin causes no polycythaemia nervous stimuli also cause no increase in number of red corpuscles, it appears probable that the polycythaemia following the stimulation of certain nerves is not due to a direct nervous influence, but to a reflex stimulation of the adrenals and a secondary action of epinephrin on the liver.

---

[1] Lamson, P. D., These PROCEEDINGS, **1**, 521-525 (1915).

[2] Lamson, P. D., *J. Pharm. Exp. Therap.*, **7**, 169-224 (1915).

[3] The terms red count, number of red corpuscles, etc., will be used occasionally for the sake of brevity, instead of the more exact term, number of erythrocytes per unit volume of blood.

[4] Lamson, P. D., and Keith, N. M., *J. Pharm. Exp. Therap.*, **8**, 247-251 (1916).

[5] Lamson, P. D., *J. Pharm. Exp. Therap.*, **8**, 167-173 (1916).

[6] Mautner, H., and Pick, E. P., *Münchener med. Wochenschr.*, Nr. 34, S. 1141, 1915.

THE INFLUENCE OF MORPHIN UPON THE ELIMINATION OF INTRAVENOUSLY INJECTED DEXTROSE IN DOGS

By I. S. Kleiner and S. J. Meltzer

DEPARTMENT OF PHYSIOLOGY AND PHARMACOLOGY,
ROCKEFELLER INSTITUTE FOR MEDICAL RESEARCH

Read before the Academy, April 17, 1916. Received June 9, 1916

The sugar content of the circulating blood is practically constant. This constancy is upheld by some stable mechanism which controls the supply of sugar to and its elimination from the circulation. Some time ago[1] we communicated the instructive fact that after the intravenous injection of four grams of sugar per kilo of body weight the sugar content of the blood returns to nearly normal again in about 90 minutes. These animals had ether anesthesia during the operation and afterwards a subcutaneous injection of morphin. The question presented itself, whether the anesthetics which were used influenced the rate of elimination of the added sugar through the kidneys and the blood capillaries, or, to express it in the terms of the hypothesis which was uppermost in our minds, whether the anesthetics affected the *physiological permeability* of the endothelia and epithelia which are concerned in the process of elimination.

We tested first the action of morphin alone. Two series of experiments were made. Eight dogs received morphin, about ten milligrams per kilo body weight. Ten dogs were operated under local anesthesia produced by cocain or ethylchloride; no anesthesia was needed during the infusion of the sugar solution. In all cases of both series four grams of dextrose per kilo body weight was given, the duration of the intravenous infusion being in all cases about one hour. The dextrose was given in a 20% solution, the solvent being water, except in four non-morphinized dogs in which the dextrose was dissolved in an M/4 solution of sodium sulphate—for reasons which will be stated later.

The resulting differences were striking and instructive. We shall give first the urinary results. In the morphinized dogs the elimination of the sugar through the urine was considerable. In eight experiments the average quantity of sugar in the urine secreted in two hours and a half (that is from the beginning of the injection to one hour and a half after the end of it) amounted to 63% of the injected sugar, 80% being the largest and 52% the lowest quantity. The average quantity of sugar in the urine of six non-morphinized dogs (those which received the dextrose in water), in two hours and a half, amounted only to about 17% of the injected sugar, about 30% being the highest and about 4% the lowest quantity.

There was, however, a proportionate difference between the two series of dogs also in the volume of the secreted urine. In the morphinized dogs the average of the injected dextrose solution was 137 cc. and of the secreted urine—197 cc.; that is, the quantity of the secreted urine exceeded that of the injected sugar solution, while in the non-morphinized dogs the average of the injected dextrose solution was 187 cc. and of the secreted urine only 83 cc.; that is, the quantity of the secreted urine was much smaller than that of the injected sugar solution. It could then perhaps be assumed that the morphinized dogs eliminated more sugar merely because they have secreted more urine. On this account experiments were made on four non-morphinized dogs in which the dextrose was dissolved in 1/4M solution of sodium sulphate. That produced a greater diuresis; here the relation of the volume of the secreted urine to the volume of the injected dextrose was reversed: 212 cc. of dextrose solution injected and 281 cc. of urine secreted. Nevertheless, the relation of the elimination of sugar in the urine was not reversed. In fact, in those four experiments with non-morphinized dogs the elimination in the urine of sugar was even diminished; it amounted on the average only to 9.2% of the injected sugar, about 13% being the highest and 7% the lowest quantity.

These three series of experiments show, then, that morphin increases considerably the elimination through the kidney of intravenously injected sugar, and increases moderately the volume of the secreted urine.

In these various series of experiments also the sugar content of the blood was tested and compared with one another. Tests were made before the intravenous injection of sugar and at the end of it, and thirty minutes, sixty minutes and ninety minutes after the end of the injection. The results were in a certain sense opposite to those observed for the urine. The averages for six morphinized dogs were as follows: 0.24% before the injection, 0.99% at the end, 0.57% after thirty, 0.41% after sixty and 0.32% ninety minutes after the end of the injection. The average for six non-morphinized dogs (dextrose in water) were: 0.16% before the injection, 0.6% at the end, 0.19% after thirty, 0.16% after sixty and 0.16% ninety minutes after the end of the injection. These data mean that in morphinized dogs, after intravenous injection of dextrose, the sugar content of the blood returns to the normal level much slower than in non-morphinized dogs.

However, in consideration of the fact that morphin alone causes a moderate hyperglycaemia (with a very slight glycosuria) two experiments were made on non-morphinized dogs which had at the start a hyperglycaemia. These animals received intravenously one gram of dextrose per kilo, which brought the glycaemia to 0.35%; then four grams more per kilo were injected. At the end of this injection the hyperglycaemia was 0.45%; thirty minutes after the end, about 0.16%; after sixty minutes, 0.15%; and after ninety minutes, 0.15%. This shows that without morphin the sugar of the blood returns more rapidly to the normal level even when the intravenous injection of four grams of sugar per kilo is started while the sugar content of the blood is temporarily on a level higher than that due to morphin. The elimination of sugar in the urine amounted to about 18.5% of the entire quantity of the injected sugar.

Our experiments show the following facts. 1. Morphin, in the dose employed in our experiments, increases considerably the elimination through the kidneys of intravenously injected dextrose, although the increase in the volume of secreted urine is comparatively moderate. Without morphin the elimination of sugar is comparatively small even if the volume of the secreted urine is considerably increased by the addition of sodium sulphate to the intravenously injected dextrose. 2. Morphin retards the return of the sugar content of the blood to its previous level. The originally slightly higher level of glycaemia which is produced by morphin itself does not seem to be responsible for either the slow return of the glycaemia or for the greater elimination through the kidney.

If we assume with Heidenhain and others, as most physiologists now do, that urine is produced by a 'vital' activity of the cells of the glomeruli and of the convoluted tubules, we may perhaps express our findings by the statement that morphin increases the 'physiological permeability' of the kidney cells while it decreases the same kind of permeability of the endothelia of the capillaries of other tissues of the body.

[1] Kleiner and Meltzer, *Amer. J. Physiol.*, **33**, 17 (1914); also Kleiner, *J. Exp. Medicine*, **23**, 507 (1916).

THE WORK OF THE AMERICAN METEOR SOCIETY IN 1914 AND 1915

By Charles P. Olivier

LEANDER McCORMICK OBSERVATORY, UNIVERSITY OF VIRGINIA

Received by the Academy, May 29, 1916

The year 1915 saw a very great increase in the interest in the study of meteors, which was evidenced by the large number of observations made by members of the American Meteor Society. This gratifying increase became largely possible on account of a grant to Dr. S. A. Mitchell of the Leander McCormick Observatory from the J. Lawrence Smith fund of the National Academy of Sciences. This appropriation, which was made in April, 1915, permitted the work of the Meteor Society to obtain wider publicity by the publicaion and distribution of bulletins, maps and blanks to prospective members.

As a consequence, it is believed that the largest amount of systematic work ever done in one year in America, was sent in; the results of these observations have been prepared for publication and are now awaiting printing. Briefly, this publication will contain the results from 540 observations made by 4 persons in 1914 and from 5003 observations made by 36 persons in 1915. While most of these 36 persons are amateurs, five have had astronomical training, one is a colonel in the U. S. Army, one is an observer of meteors in the U. S. Weather Bureau of wide experience, three are students in astronomy at the University of Virginia, and several others are trained in various scientific lines which would make their work the more valuable. The observers were stationed in 17 states, 2 provinces of Canada, and one in the Argentine Republic. It might be added that the Meteor Society has members in several foreign countries and several dozen more in America from whom no reports have yet been received, while a week rarely passes without a new person applying for membership.

From the 5543 observations of meteors mentioned, we have been able to deduce 139 radiants of sufficient accuracy to calculate parabolic orbits for the meteor streams they represent. These orbits are contained in full in one of the tables. Following this is a table containing 81 less certain, but probably existant radiants, for which as yet no orbits are calculated. Other tables contain analyzed data of the distribution of meteors as to magnitudes and average durations of their times of visibility. There is also a table containing a few real heights, which were obtained in August, 1915, between Richmond, Va., University of Virginia, and Washington, D. C. This latter work we hope to repeat, on a larger scale and under better conditions, during the summer of 1916. The text of the publication contains details as to the organization, plans and methods of reduction of the work. It further contains full explanations as to the derivation and use of the figures found in the tables. Actual directions to the members were omitted since these had been printed at great length both in *Popular Astronomy* and also in Bulletins 2–5 of the Meteor Society, which were distributed to all members and applicants.

The present publication and the two similar ones previously prepared by me will bring up the number of results to 440 parabolic orbits of meteor streams, based on about 14,000 meteors. The peculiar value of this contribution lies in the fact that a fairly uniform plan has been followed by all the observers, and that the results were computed and deduced by one person using the methods and care with which other astronomical work is handled in all regular observatories.

It may be of interest to mention the various methods used to reach amateurs who might care to join in the work. At the very organization of this society, the coöperation of the members of the Meteor Section of the Society for Practical Astronomy was secured by the appointment of the writer as director of the latter. Then last spring a number of articles were published by Dr. S. A. Mitchell and myself calling attention to the desirability of coöperation on the part of all amateurs. These articles appeared in the *Scientific American*, *Journal of the Royal Astronomical Society of Canada*, and *Popular Astronomy;* by reprinting them in part or in whole, the press of the country assisted in bringing our work to the notice of amateur observers and, as a result, wide publicity was secured. Not less than 200 people have written letters on the subject to date, over half desiring to join in making observations. It goes without saying that many have never been heard from again, but so many have worked with real enthusiasm and success, that it would seem that the future of meteoric astronomy is brighter at the present time than ever before, so far as America is concerned.

This is peculiarly fortunate at such an epoch when similar societies in Europe must be greatly reduced in membership and activity.

Having secured the approbation and support of the National Academy of Sciences for the coming year, through a further grant from the J. Lawrence Smith fund, it is hoped that the results for 1916 will surpass those for the previous year, and indeed a good start has been made in that direction. We still need and desire the help of other persons interested in such work and a cordial invitation is again extended to them.

THE LIGHT EXCITATION BY SLOW POSITIVE AND NEUTRAL PARTICLES

By A. J. Dempster

RYERSON PHYSICAL LABORATORY, UNIVERSITY OF CHICAGO

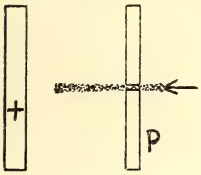
Received by the Academy, June 12, 1916

The speed at which an electron begins to excite light, and the character of the light emitted by slow electrons has formed the subject of several recent papers;[1] the corresponding problem for slow positive rays has not been carefully investigated. The only papers on the subject are those by Stark[2] and Wehnelt[3] who detected a luminosity probably due to positive rays with potential differences as low as 50 volts on the tube.

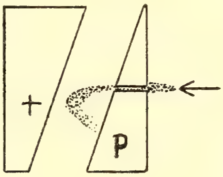
The light excitation by positive and neutral rays of greater energy is included in the many papers on the light emission of canal rays. These particles have energies corresponding to a fall of potential of from 500 volts up, depending on the pressure, the low speeds necessarily being at a relatively high pressure. The Doppler effect in these canal rays in hydrogen shows a dark space between the displaced and undisplaced lines, and this may be explained by assuming that those particles in the rays which have less than a certain speed (corresponding to 50–80 volts fall of potential for the hydrogen atom) are unable to excite light.

The method used in the present experiments was to ionize hydrogen by electrons from a Wehnelt cathode and to allow the positives thus made to pass through a slit in a plate behind the cathode (P in the figures) into a second chamber where their light emission and the deflection of their path by electric fields could be studied undisturbed by the light and the electric fields in the main tube. In one tube (*A*) a plate was placed in the second chamber at right angles to the beam of rays so that by charging it positively the positive rays could be stopped and turned back; in a second arrangement (*B*) the rays were allowed to enter an oblique retarding field so that the paths of the positive particles would be parabolas; and in a third experiment (*C*) the particles passed between the plates of a condenser which would deflect any charged particles sideways out of the beam.

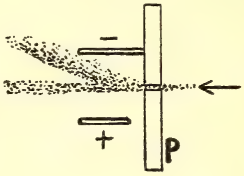
The phenomena depended greatly on the pressure of the gas in the tube. If the pressure were very low so that the positives made practically no collisions with the molecules of the gas, the rays, which were followed by the luminosity they make in the gas, could be stopped entirely by the directly opposing field or bent into a parabolic path by the inclined field or deflected sideways by the cross electric field. *A* and *B* indicate the appearance with high vacua. The rays could be detected with potentials as low as 30 volts on the tube, although they were then very faint. To stop the rays, or rather to reduce their speed so that they are unable to cause light, opposing potentials were required which were practically the same as those which originally gave the particles their speed. We conclude that *very slow positive rays are still able to excite light, certainly with a speed corresponding to less than 5 volts.*



A



B



C

This fact needs to be considered in connection with the above mentioned experiments on the lower limit at which electrons excite light.

If the pressure of hydrogen is taken a little higher so that some of the positives make collisions, the phenomena are more complicated. If the positives are then deflected out by a cross field, there is an undeflected bundle of light left, due to the neutral rays which have been formed from the positives. *C* indicates the appearance at about 0.005 mm. of mercury pressure when the two bundles are of approximately equal intensity. With the arrangements *A* and *B* there is superimposed on the deflected beam a luminosity continuing up to the opposing electrode. If the pressure is taken higher still, the positive bundle becomes fainter till finally the neutral bundle alone is present. Two things are of special interest. First, *the neutral rays can also excite light at very slow speeds.* The lowest speed at which the neutral bundle has been detected is 50 volts, and it is possible that that is a lower limit at which the neutrals begin to excite light. With potentials of 1500 volts, when the light is much stronger, photographs of the spectra of the positive and neutral bundles showed the hydrogen series lines in both.

The second point is that there is a sharp separation between the two bundles at the proper pressure. This shows that light is not emitted only during the return of a corpuscle to a positive centre, as very often assumed, for the neutrals cause light in a region where all free corpuscles have been swept out by the field. Also the sharp separation of the bundles shows that at these pressures changes take place slowly, so that if a positive were formed in the neutral bundle it would be deflected out before becoming neutralized again. We are thus led to conclude that *light excitation may occur directly because of the collision of a neutral particle with a neutral molecule of the gas.* We may regard the light emission as taking place during the rearrangement of the electrons in the atom after one has been detached by the collision (Stark's theory), or we may retain the picture underlying Bohr's theory, if we regard the displaced electron as not leaving entirely the centre to which it is attached.

[1] E. Gehrcke and R. Seeliger, *Verh. D. Physik. Ges.*, **15**, 897 (1913); H. Rau, *Ber. Phys.-med. Ges., Würzburg*, Feb. 1914; J. Frank and G. Herz, *Verh. D. Physik. Ges.*, **16**, 512 (1914); J. C. MacLennan and J. P. Henderson, *London, Proc. R. Soc.*, A., **91**, 485 (1915).

[2] J. Stark, *Leipzig, Ann. Physik.*, **13**, 390 (1904).

[3] A. Wehnelt, *Ibid.*, **14**, 464 (1904).

AN APPARENT DEPENDENCE OF THE APEX AND VELOCITY OF SOLAR MOTION, AS DETERMINED FROM RADIAL VELOCITIES, UPON PROPER MOTION

By C. D. Perrine

OBSERVATORIO NACIONAL, ARGENTINO, CÓRDOBA

Received by the Academy, June 3, 1916

The position of the solar apex which had been derived from the stars of class B both from proper motions and radial velocities differed so much from the apex derived from radial velocities of A stars, that an investigation was undertaken of the radial velocities of all of the spectra classes. The discordances were not only between the spectral classes as a whole but appeared to be between the results from northern and southern stars as well.

In the course of the work it was observed that frequently there appeared to be marked discordances between the radial velocities of stars whose proper motions also differed in size and sign. This, and the desire to test the effect of differences of distance as evidenced by proper motion, led to the separation of the stars in the classes A, F, G, and K according to proper motion in right ascension into four classes—0 to $\overset{s}{.}0049$, $\overset{s}{.}0050$ to $\overset{s}{.}0099$, and $\overset{s}{.}0100$ and over, designated respectively in the table as small, medium, and large, and the stars having 'contrary' parallactic signs, i.e., negative signs in the first and fourth quadrants of right ascension and positive signs in the second and third quadrants.

The sky was divided into areas approximately 30° square and the mean of the stars in each region treated as one observation. The results of least-squares solutions are given in the table. A and D are the right ascension and declination respectively, of the solar apex and $V_\odot$ the velocity of the solar system. As the stars of class B practically all fall in the small proper motion division, the results from them are included in that division in the table.

μ_α	SPECTRAL TYPE	NORTH			SOUTH		
		A	D	$V_\odot$	A	D	$V_\odot$
				km.			km.
Large	A	242°	+ 7°	−14.2	306°	+14°	−20.7
	F	254	+ 5	−23.2	238	−11	−13.1
	G	268	− 4	−36.4	287	− 6	−13.9
	K	277	+18	−26.3	298	+19	−30.0
		260	+ 6	−25.0	282	+ 4	−19.4
Medium	A	260	− 4	−22.6	305	+20	−14.5
	F	253	−17	−25.5	307	+19	−19.9
	G	272	+ 7	−37.6	246	− 6	−44.1
	K	262	−34	−25.2	232	+59	−21.3
		262	−12	−27.7	272	+23	−25.0
Small	A	260	+35	−17.7	261	+ 3	−20.2
	F	270	+37	−15.0	264	+32	−20.7
	G	271	+27	−14.6	242	+28	−12.4
	K	265	+32	−19.6	258	+39	−22.4
		266	+33	−16.7	256	+26	−18.9
	B	254	+27	−17.0	288	+31	−24.8
Contrary	A	209	+37	−12.2	240	+18	−18.9
	F	292	+58	−13.3	204	+32	− 8.4
	G	207	+80	−19.0	147	+ 1	− 6.4
	K	288	+44	−14.2	292	+60	−20.3
		249	+55	−14.7	221	+28	−13.5
	Omitting G	263	+46	−13.2	245	+37	−15.9

The principal conclusion from this investigation is that the position of the solar apex and the solar velocity appear to vary with the proper motions of the stars used in the determination, at least for the fainter naked-eye stars of the northern sky, the declination approaching the well-known ellipsoidal axis and the solar velocity increasing as we go from the stars of small and 'contrary' proper motion to those of medium and large proper motion.

Such variations of the position of the apex although apparently connected with distance or other conditions, would seem to point ultimately to some form of rotary or spiral motion among the stars themselves.

The details of the investigation, which is based upon Campbell's well-known catalogs of about 1300 radial velocities, will be published in the *Astrophysical Journal.*

CHANNELED GRATING SPECTRA, OBTAINED IN SUCCESSIVE DIFFRACTIONS

By C. Barus

DEPARTMENT OF PHYSICS, BROWN UNIVERSITY

Received by the Academy, June 13, 1916

The great variety of channelled spectra obtained, when white light is successively diffracted by two gratings, are referrable to the fringes of the diffraction of homogeneous light, observed *outside* the principal focal plane, on a spectrometer. In other words, if light of a given pure color (sodium, mercury) is used, a single grating suffices. Each line of the spectrum is resolved into well defined groups of fringes, if it is observed either in front of or behind the principal focal plane. The arrangement of fringes varies in marked degree with the distance of the plane observed, from the latter. If reflecting gratings are used, there is no other possible source of interferences; but reflecting and transmitting gratings show the phenomenon equally well.

After finding how easily the Fresnellian interferences of two virtual slits could be reproduced in the telescope and observed on either side of (before or behind) the plane of the sharp slit images, it seemed reasonable to suppose that the diffraction of a slit could also be produced and exhibited in this way; but the availability of this anticipation is attended with much greater difficulty. The image of a very distant slit does indeed show separated diffraction fringes on either side of the principal focal plane in the observing telescope. But they move right and left with the eye, in the same direction and in this respect do not at once recall the phenomena under consideration. Usually the blurred image, out of focus, is stringy, without definite structure.

To obtain sharp stationary fringes from an image of the slit, this image must be produced by the diffraction of a grating, having a dispersing power above a certain minimum. Thus in a grating of less than 7,000 lines to the inch, the undeviated slit image and the image of the first order are not clearly resolved, unless the slit is very fine. In the second and higher orders, however, the resolution is very pronounced and the fringes stationary.

The resolution of fringes is equally manifest in front of, or behind the principal focal plane, so that if a weak convex lens is added to the objective of the telescope, the succession of fringes is found with an outgoing ocular; if a weak concave lens is added to the objective, with an ingoing ocular, starting in each case near the principal focus. As the fringes increase in size they in turn subdivide, as if each fringe were a new slit image, capable of undergoing secondary diffraction. Beyond these secondary fringes no further resolution was detected.

Returning to the work with two successive gratings and white light, the channelled spectra obtained are too complicated for concise description. A very interesting result, however, is the passage of the fringes across the stationary sodium line, when the grating is moved, fore and aft, in a direction normal to its plane. The region of the *D* line is thus alternately dark and bright. The direction of these rays remains unaltered while the illumined strip is shifted horizontally across the ruled space of the second grating. It is sometimes difficult to see the *D* line in the focal plane of the fringes. When homogeneous light is used this fiducial mark is necessarily absent and the cross hairs of the ocular must be supposed to replace it. The shift of the fringes is then equally obvious and sometimes (sodium light) different groups seem to travel in opposite directions while the grating moves in one direction. In case of homogeneous light and two gratings, moreover, the fringes seem to be of minimum size in the conjugate focal plane of the gratings. They increase in size and in turn split up, in focal planes before and behind this.

An insight into these occurrences was finally obtained in observation with homogeneous light, on the spectrometer, by shifting the grating (transmitting) in its own plane, right and left. The fringes in such a case move *bodily* across the field of the telescope, new groups entering on one side for those which leave on the other. These fringes, even if quite distinct, are differently arranged in the coarse and fine series and are frequently accompanied by dark or bright bands. If the ocular is drawn out and set outward from the principal focal plane (at which the slit image is quite sharp) into a different position, the fringes move in a direction opposite to the grating. If the ocular is set inward from the principal focal plane, they move in the same direction as the grating.

This would not be unexpected; but secondary fringes, or something else in the field, seem to remain stationary. Successive fields may be quite different as to arrangement of fine and coarse lines, but all plane gratings exhibit the same phenomena. Thus it is obvious that the fringes of the present paper result from a residual irregularity in the rulings of the grating. Micrometrically, the successive strips of a slit image, however fine, are of unequal intensity. Between these there is diffraction as may be tested by examining the clear glass at the edge of the ruled space.

It is obvious that in the otherwise indistinguishable images of a slit in homogeneous light, however sharp or however narrow, in its own focal plane the nature of its origin still persists and may be detected by observations outside of the principal focal plane. A fine slit is in all cases presupposed and all the phenomena vanish for a wide slit. On the other hand the width of the pencils of parallel rays may be far greater than is necessary to show the strong Fraunhofer lines.

A fuller report of this work has been presented to the Carnegie Institution of Washington, D. C.

THE EFFECT OF PARENTAL ALCOHOLISM (AND CERTAIN OTHER DRUG INTOXICATIONS) UPON THE PROGENY IN THE DOMESTIC FOWL

By Raymond Pearl

BIOLOGICAL LABORATORY, MAINE AGRICULTURAL EXPERIMENT STATION

Received by the Academy, June 7, 1916

The investigation here reported deals with the general problem of the origin and causation of new, heritable variations. That this is one of the most fundamental problems of genetics admits of no doubt. The method by which this general problem is attacked in the present investigation is that of exposing systematically the germ-cells of an animal to something unusual or abnormal in the surrounding conditions, and then analyzing, so far as may be, not only the new heritable variations themselves (provided any such appear), but also the factors which underlie their causation.

The specific problems with which this investigation deals are these:

1. Does the continued administration of ethyl alcohol (or similar narcotic poisons) to the domestic fowl induce precise and specific changes in the germinal material, such as to lead to new, heritable, somatic variations?

2. Failing a specific effect is there a general effect upon the germinal material leading to general degeneracy of the progeny?

3. What in general are the effects upon the soma of the treated individual of the continued administration of such poisons?

4. Are the somatic effects upon the treated individuals of a sort to give any clue to the probable origin, or mechanism of the germinal changes?

The present paper reports, in brief abstract, the results obtained from the beginning of the experiment in September, 1914, to February 1, 1916. A complete report is now in process of publication in another place. In that report the data will be presented in detail, with probable errors, etc.[1]

The foundation stock used in these experiments came from pedigreed strains of two breeds of poultry, Black Hamburgs and Barred Plymouth Rocks. Both of the strains used have been so long pedigree-bred by the writer, and used in such a variety of Mendelian experiments, that they may be regarded as 'reagent strains,' whose genetic behavior under ordinary circumstances may be predicted with a degree of probability amounting practically to complete certainty. Furthermore, the results of crossing these two breeds reciprocally have been thoroughly studied by me.

Three different series of birds were started. To the birds in one series was administrated 95% ethyl alcohol. To those in the second series was administered methyl alcohol, and to those in the third series, ether.

The method followed in these experiments for the administration of the poisons was essentially that which has been used by Stockard,[2] namely the method of inhalation.

The number of treated birds used in the experiments to the date covered in this report is 19. The number of untreated control brothers and sisters is 58.

Results in the treated individuals.—Before entering upon any discussion of the effect of the alcohol treatment on the progeny it seems desirable to examine the effects, both structural and physiological, upon the treated individuals themselves. The main results are summarized in Table I.

The plan of this table is as follows: In the last column of the table a plus sign denotes that, with reference to the particular character discussed, the alcoholists have been favorably affected; a minus sign that they have been unfavorably affected as compared with untreated

TABLE I

Showing in Summary Form the Effect of Continued Administration of Alcohol (Ethyl and Methyl) and Ether, by the Inhalation Method, upon the Treated Individuals Themselves

CHARACTER OR QUALITY STUDIED	TREATED INDIVIDUALS	UNTREATED CONTROLS	NET RESULTS ON ALCOHOLISTS
1. Mean number per bird of consecutive days of treatment	344.2	0	
2. Net percentage mortality (to Feb. 1, 1916) exclusive of birds accidentally killed	*0*	41.0	+
3. Mean body weight of females (in gms.)	3266	*2953*	–
4. Mean egg production per bird, 14 months	183.97	180.80	0
5. General activity	Reduced	*Normal*	–
6. Sexual activity	Reduced	*Normal*	–

controls. A zero indicates that no effect of the treatment, one way or the other, has beeen detected.

From these summarized data it is possible to gain a tolerably clear comprehension of the objective happenings in these experiments so far. The treated animals themselves are not conspicuously worse or better than their untreated control sisters or brothers except in respect to mortality rate. This is much lower in the treated than in the concontrols.

Regarding egg production the following details may be mentioned.

The egg production of the treated birds and the untreated controls was entirely normal in respect of its seasonal distribution, as well as in regard to its amount.

There has been *no significant difference* in the egg production of the treated birds and their untreated control sisters, either in the total average number of eggs produced per bird, nor in the seasonal distribution of this production. The only conclusion which can be drawn from the statistically insignificant differences which appear between treated and control birds is that the inhalation treatment has not affected the egg production of the birds, either favorably or adversely.

Results in the F_1 progeny of treated individuals.—The general plan of the matings in 1915 was to breed a treated male of each of the three classes, ethyl, methyl, and ether with (*a*) untreated control females, and with (*b*) treated females of his own class (i.e., ethyl ♂ × ethyl ♀, methyl ♂ × methyl ♀, ether ♂ × ether ♀). In addition to these matings an untreated control male was mated with (*a*) untreated control females (*b*) ethyl females, (*c*) methyl females, and (*d*) ether females.

All of the matings were of the type Black Hamburg ♂ × Barred

Plymouth Rock ♀. There were produced 234 chicks from matings wherein one or both parents were treated.

The following reasoning has been used in devising a numerical expression of the dosage, so far as concerns the progeny. Two germ cells, a sperm and an ovum, unite to form the zygote of each progeny individual. It is proposed to designate as the 'total germ dosage index' the total number of days during which the two gametes making the offspring zygote have been exposed to alcoholic influence while sojourning in the body of the treated individuals.

The total germ dosage index for the F_1 progeny in these experiments ranges from 130 days to 354 days, with the matings for the different substances used well scattered over the range. A summarized statement of the effects on the progeny is given in Table II.

TABLE II

SHOWING IN SUMMARY FORM THE EFFECT OF CONTINUED ADMINISTRATION OF ALCOHOL (ETHYL AND METHYL) AND ETHER BY THE INHALATION METHOD, UPON THE PROGENY

CHARACTER STUDIED	OFFSPRING OF				NET RESULT ON ALCOHOL OFF-SPRING
	Treated ♂♂ × un-treated ♀♀	Treated ♂♂ × treated ♀♀	All treated parents	Un-treated controls	
1. Mean germ dosage index	137.8	299.0	210.35	0	
2. Percentage of infertile eggs (i.e., eggs in which no zygote was formed)	25.2	59.2	41.7	*25.3*	–
3a. Percentage of embryos dying in shell	36.6	*26.9*	33.3	42.2	+
3b. Percentage of fertile eggs (i.e., zygotes) which hatched	63.0	*72.3*	66.7	57.8	+
4. Percentage of all eggs which hatched	47.1	29.4	38.6	*44.4*	–
5. Percentage mortality under 180 days of age	21.1	*10*.6	17.6	36.9	+
6. Percentage mortality over 180 days of age	*5.9*	13.6	10.3	15.3	+
7. Sex ratio: per cent ♂♂	48.9	45.5	47.7	50.0	0
8. Mean hatching weight per bird, males	34.91	*36.97*		34.24	+
9. Mean hatching weight per bird, females	35.04	*37.17*		34.73	+
10. Mean adult weight per bird, males	2669	*2815*		2392	+
11. Mean adult weight per bird, females	2020	*2063*		1928	+
12. Percentage of weak or deformed chicks	0.7	*0*	0.4	1.0	+
13. Abnormalities of Mendelian inheritance	0	0	0	0	0

An examination of this table shows that *out of 12 different characters for which we have exact quantitative data, the offspring of treated parents taken as a group are superior to the offspring of untreated parents in 8 characters.* The offspring of untreated parents are superior to those of the alcoholists in respect of but two characters, and these are characters which are quite highly correlated with each other and really should be counted as but one single character. Finally with respect to two character groups there is no difference between the alcoholists and the non-alcoholists.

We may evaluate our results in general terms as follows:

1. There is no evidence that *specific* germinal changes have been induced by the alcoholic treatment, at least in those germ cells which produced zygotes.

2. There is no evidence that the germ cells *which produced zygotes* have in any respect been injured or deleteriously affected.

3. The results with poultry are in *apparent* contradiction to the results of Stockard and others with mammals. This contradiction is, however, probably only apparent and not real, paradoxical as such a statement may appear.

4. The results with poultry are in a number of important respects in essentially complete agreement with those of Elderton and Pearson[3] on parental alcoholism in man, and of Nice[4] in mice.

The interpretation of these results which seems to account best for all the facts is that the apparent discrepancy between avian and mammalian results is fundamentally due to a difference in degree of resistance of the germ cells to alcohol. Given the existence of variation in the vigor or resisting power of germ cells, which is certainly a fact, we have the necessary basis for the action of a selective agent. The hypothesis which we wish to suggest is that alcohol acts as such a selective agent upon the germ cells of alcoholized animals, eliminating the weak and permitting the survival of the vigorous and highly resistant.

Detailed evidence in support of this hypothesis is presented in the complete paper. The experiments are being continued.

---

[1] Some account of this work has already appeared. Cf. Pearl, R., On the effect of continued administration of certain poisons to the domestic fowl, with special reference to the progeny, *Proc. Amer. Phil. Soc.*, **55**, 243–258 (1916).

[2] Cf. for summary and bibliography of earlier papers Stockard, C. R., and Papanicolaou, G., A further analysis of the hereditary transmission of degeneracy and deformities by the descendants of alcoholized mammals, *Amer. Nat.*, **50**, 65–88, 144–177 (1916).

[3] Elderton, E. M. and Pearson, K., A first study of the influence of parental alcoholism on the physique and ability of the offspring, *Eugenics Lab. Mem.*, **10**, 1–46 (1910). (Second edition.)

[4] Nice, L. B., Comparative studies on the effects of alcohol, nicotine, tobacco smoke, and caffeine on white mice. I. Effects on reproduction and growth, *J. Exper. Zöol.*, **13**, 133–153 (1912).

THE EFFECTORS OF SEA-ANEMONES

By G. H. Parker

ZOÖLOGICAL LABORATORY OF THE MUSEUM OF COMPARATIVE ZOÖLOGY AT HARVARD COLLEGE

Received by the Academy, May 6, 1916

Sea-anemones, like other animals, have special sets of organs, effectors, by which they respond to changes in the environment. In these animals there are at least four sets of such organs: the mucous glands, the nematocysts, the cilia, and the muscles.

The mucous glands are unicellular glands found on almost all surfaces of the body and concerned partly in protecting the surface against insult and partly in rendering it sticky whereby the animal as a whole adheres to a rock or other fixed object or foreign bodies adhere to the animal as in the collection of food by the tentacles.

The nematocysts are the well-known organs of defense and offense. They are most numerous and best developed on the tentacles and on the acontia. By their abundant discharge large animals may be stung and driven away and small ones killed and appropriated for food.

Cilia are means of generating currents and of transporting small bodies over surfaces and through tubes. They are best developed on the tentacles, the acontia, and the oesophagus. The oesophageal cilia ordinarily beat outward, but in the presence of food they reverse temporarily and beat inward thus carrying the food into the digestive cavity.

The mucous glands, the nematocysts, and the cilia have all been supposed to be under nervous influence. In all cases their action in reference to the stimulus appears to be strictly local; that is, they become active only over the exact region stimulated. Moreover if sea-anemones are subjected to anesthetics, such as chloretone, or magnesium sulphate, their nervous activitites can be completely abolished but without interfering in the least with the secretion of mucus, the activity of nematocysts or of cilia including their reversal. For these reasons it is concluded that the three types of effectors just mentioned are independent of nervous control and respond to direct stimulation.

Muscles on the other hand are commonly under the influence of nerves, an influence that disappears entirely when the sea-anemone is fully anesthetized. Some muscles, however, such as the longitudinal muscles of the acontia, have been shown to be independent effectors. They are slow in response often requiring a minute or more between the time of stimulation and that of contraction, and are not influenced by ordinary anesthetics.

The majority of the muscles in a sea-anemone respond quickly under the influence of the nerve-net and enter into a state of enduring contraction (tonicity). This is as characteristic of a small fragment of an animal, provided it contains a nerve-net as well as muscle, as it is of a whole animal. So striking is this excessive tonicity in the muscle of sea-anemones that it has been assumed to be their exclusive function and they have recently been regarded (Jordan) as animals incapable of ordinary reflexes. Such a conclusion, however, seems to be too sweeping.

If a specimen of Metridium is allowed to expand fully and a small piece of meat is placed on its tentacles, the mouth and oesophagus soon open and the sides of the column are marked by a few pronounced vertical grooves. After the food has been swallowed the grooves disappear and the oesophagus closes. The opening of the oesophagus is brought about by the contraction of the transverse muscles of the mesenteries whose action is so precisely associated with the appropriate stimulation of the tentacles that it carries with it all the signs of a reflex. It therefore seems clear that among the muscles in sea-anemones there are not only independent effectors, and tonus muscles associated with nerve-nets, but neuromuscular combinations that exhibit true reflex action.

The detailed paper will be published in the *Journal of Experimental Zoology.*

PRELIMINARY EVIDENCE OF INTERNAL MOTION IN THE SPIRAL NEBULA MESSIER 101

By A. van Maanen

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, June 14, 1916

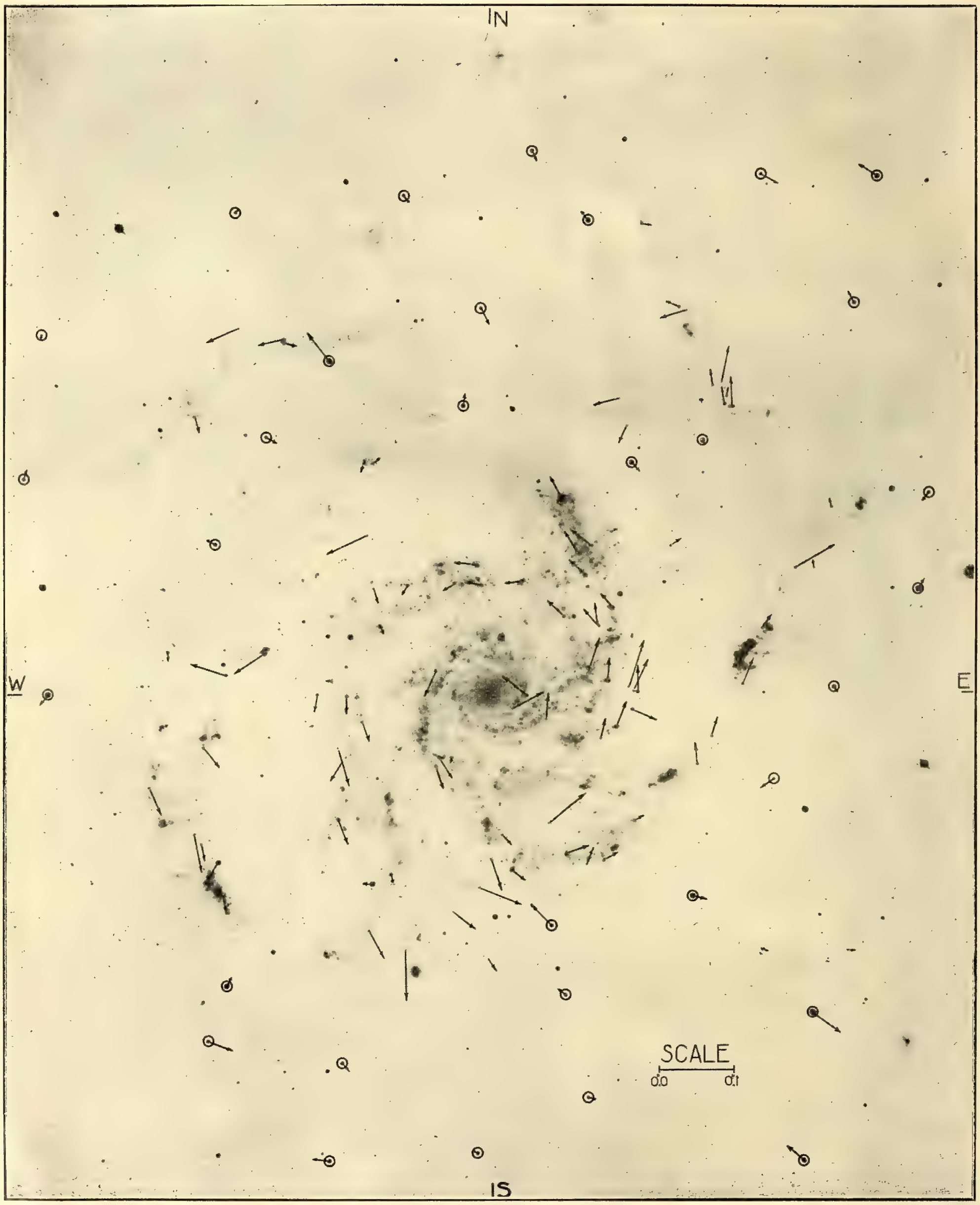
Inasmuch as data for the proper motions of stars, determined by photographic methods, have been rapidly accumulating since the beginning of the century, it may seem strange that the first results for nebulae should have been published only in 1915. It must not be forgotten, however, that photographs of nebulae require much longer exposures, and that, even with the best plates, the measures are more difficult and less accurate, because of the unsymmetrical character of the points and condensations upon which settings must be made, than is the case with the round images of stars. A given point in a nebula may be bisected quite differently on different plates and the measures will therefore fail to reveal the proper motion with anything like the precision attainable in a star cluster, for instance. The difficulty of bisection may, however, be largely overcome by measuring corresponding points on different plates in immediate succession.

For such measures the monocular arrangement of the stereocomparator is an admirable instrument, and that it is capable of yielding very accurate results has already been pointed out.[1] When therefore Mr. Ritchey placed at my disposal for measurement two excellent plates of the spiral nebula Messier 101, taken in 1910 and 1915, there was no question but that the stereocomparator was the best instrument for the purpose.

Although the results showed striking evidence of internal motion, the necessity of additional plates was strongly felt. At my request Dr. Curtis kindly placed at my disposal three photographs made with the Crossley reflector of the Lick Observatory, one by Keeler in 1899, one by Perrine in 1908, and one by Curtis himself in 1914.

The pair taken by Mr. Ritchey was completely measured twice, the Lick pairs 1914–1899 and 1908–1899 once each; each pair was measured in four positions, with east, west, north and south, respectively, in the direction of increasing readings of the micrometer screw. On the Mount Wilson plates 87 nebulous points were measured; on the first Lick pair 46, and on the second 69 points, while on all the pairs the same 32 stars were used for comparison purposes. The measures and reductions, which will be published in full in the *Astrophysical Journal*, were made substantially in the manner described in my recent paper on the determination of stellar parallaxes,[2] the principal difference being that the quadratic terms could not here be neglected and were accordingly included in the reductions.

The results showed that to each pair of plates could be given the same weight, and the direct mean of the values found for the proper motions of each point is therefore used in the discussion. The resulting motions, which are those relative to the mean of the 32 comparison stars, are due partly to a motion of translation of the nebula as a whole, and partly to a possible internal motion. The annual motion of translation, which was derived by three different methods of reduction, was found to be: $\mu_\alpha = +0\overset{''}{.}005$, $\mu_\delta = -0\overset{''}{.}013$. Subtracting these from the total motions, the results are what may be called the internal motions. The accompanying plate shows these mean internal motions for each of the 87 points of the nebula. The individual motions for the comparison stars, which are surrounded by circles are also shown. The scale for the annual motions is given at the bottom of the plate. The



INTERNAL MOTIONS IN MESSIER 101

The arrows indicate the direction and magnitude of the mean annual motions. Their scale (0″.1) is indicated on the plate. The scale of the nebula is 1 mm. = 10″.5. The comparison stars are enclosed in circles.

density of the center of the nebula has been reduced to show the motions more clearly.

If the results as illustrated on the plate could be taken at their face value, they would certainly indicate a motion of rotation, or possibly motion along the arms of the spiral. Without expressing a final opinion as to the character of the motion, which must be determined by future work, it may be of interest to examine the evidence afforded by the existing material. Comparing the motions with the directions of the branches of the spirals, we find from 52 points in which this direction can be specified with fair accuracy, that the mean divergence of the motions is 7° ± 4° toward the concave side of the spirals.

To discuss the internal motions from the standpoint of rotation, they were analyzed into two components, along and perpendicular to the radius, the latter for convenience being spoken of as the rotational component.

The results are as follows:

78 points have a left-hand motion (N. W. S. E.), only 9 moving right-handedly; 58 points appear to be moving outward, while 28 show motion inward. The rotational motion is the larger in the majority of cases, viz., 63 points. The mean rotational motion is 0″.022 left-handed; the mean radial motion is 0″.007 outward. The probable reality of the result is indicated by the satisfactory agreement of the pairs of plates, as shown by the following summary.

$\mu_{\text{rot.}}$	$\mu_{\text{rad.}}$		
+0″.021	+0″.004	Ritchey	1910 and 1915
+0.032	+0.012	Ritchey	1910 and 1915
+0.012	+0.007	Keeler and Curtis	1899 and 1914
+0.017	+0.006	Keeler and Perrine	1899 and 1908

The measures indicate a small but scarcely reliable decrease of rotational motion with increasing distance from the center, as shown by the following table:

Distance from Center	$\mu_{\text{rot.}}$	No. of Points
<3′.1	0″.024	19
3.1 to 5.0	0.028	29
5.1 to 7.0	0.014	18
>7.0	0.019	21

The annual rotational component of 0.022 at the mean distance from the center of 5′ corresponds to a rotational period of about 85,000 years. If we knew the parallax of the nebula, and if we could assume that the motions and the distances of the points from the center are mean values for elliptical orbits, the central mass could be calculated. Even though this assumption may be far from the truth it seems worth while to accept certain more or less hypothetical values of the parallax in order to get any idea of the order of the masses with which we are concerned.

Two estimates as to the parallax can be made, (1) by comparing the average motion of translation of 66 spiral nebulae, as given by Curtis,[3] with those of the stars; (2) by comparing these cross-motions with the observed radial velocities of the few spirals for which such results are known. In the first case we derive a parallax of $0''.005$; in the second, of $0''.0003$. The corresponding central masses are in both cases very large, viz., 30,000 and 140,000,000 times that of the sun. Various objections to the acceptance of these results, even as rough guesses can be made, but they are the best we have at the moment. The corresponding orbital motions would be 21 and 345 km./sec. These quantities do not seem absurd if we remember that Wolf by spectroscopic methods found a rotational component of ± 100 km./sec. in Messier 81.[4]

As the detailed results will soon be published I will only mention two more points which seem to confirm the reality of the motions. Mr. Nicholson very kindly spent much time in making check-measures on the plates taken by Mr. Ritchey, both with the stereocomparator and with another measuring machine in which two microscopes were mounted in such a way that they were directed toward corresponding points on the two plates, mounted on the same plate-carrier and moved by the same micrometer screw. His measures give satisfactory confirmation of my own results. Further, I have measured two plates of Messier 81, taken by Mr. Ritchey in 1910 and 1916, which show motion similar to that found above for Messier 101. It seems hardly necessary to suggest the importance of internal motions, such as are indicated here, in connection with the Chamberlin-Moulton hypothesis as to the origin of spiral nebulae, and it is to be hoped that material will soon be available for a fuller discussion.

[1] *Astronomical Journal*, **27**, 140 (1912).

[2] *Mt. Wilson Contr.*, No. 111, 4 seq., 1916.

[3] *Publications Astronomical Society Pacific*, **27**, 216 (1915).

[4] *Vierteljahrsschrift Astronomischen Gesellschaft*, **49**, 162 (1914).

THE EXPLORATION OF THE PACIFIC

By W. M. Davis

DEPARTMENT OF GEOLOGY AND GEOGRAPHY, HARVARD UNIVERSITY

Read before the Academy, April 17, 1916. Received May 25, 1916

A voyage across the Pacific, such as the Shaler Memorial voyage that I made two years ago, gives a traveller time on the free days between groups of islands to think of many problems, old and new; and thoughts of unsolved problems tend to run on into dreams about their possible solution. Even after returning home, the dreams continue in intervals of rest between spells of work on the results of the voyage; for just as the unrecorded facts that one so often finds while in the broad ocean impress their observer with the great volume of work still to be done, so the records of earlier work which one reads in a home library too frequently disappoint him by their incompleteness, their insufficience; thus at home and abroad, the wish of a fuller accomplishment is repeatedly present.

The insufficience of earlier work and hence the abundant opportunity for new work was first borne in upon me in connection with the old problem of coral reefs, to the investigation of which my own voyage was directed. This does not mean that a later traveller is ungrateful for what his predecessors have done, but that he need not carry gratitude to the point of being thankful to them for having left so rich and ripe a harvest to be gathered by their successors. Nor would a reference to the insufficience of earlier work be just, if the insufficiency concerned matters or methods unknown in earlier years. But when their insufficiency concerns truths so well established as the geological principle of unconformity and the physiographic principle of embayed shores, known and understood, one for a century, the other for half a century past, and directly pertinent in the problem of coral reefs, one must wonder in how many other directions the work thus far performed in the Pacific is incomplete. A student of coral reefs today cannot avoid regret at the repeated failure of earlier observers to note matters so manifest as the embayed shore lines of the central islands within barrier reefs, and the unconformable contacts of many fringing and elevated reefs with their deeply eroded foundations, inasmuch as these simple and evident relations immediately demonstrate submergence in association with reef formation, and thus contribute valiantly to the solution of the old coral-reef problem. With the regretful discovery of such oversights with respect to one's own special subject in the records of the most famous Pacific expeditions, one cannot avoid suspecting the occurrence of similar oversights in other subjects, and one must thereupon experience a great wish to see thorough and comprehensive work instituted in all subjects with the least possible delay.

For example, following the belief of the eminent Austrian geologist, Suess, regarding changes of ocean level, two responsible students of coral reefs, who must certainly have been regarded as competent investigators by the institutions that sent them forth on their travels, have independently announced that certain high-standing atolls owe their altitude to a subsidence of the ocean—the entire ocean—while these atolls, little specks in the vast ocean, stood fixed: yet it may be easily shown that that explanation is preposterously inadequate to account for the total facts. Hence, if inadequate treatment has been given to so fundamental a problem as this, is it too much to say that the more recondite problems of the Pacific cannot be solved by the methods heretofore in use? Even subjects of so large dimensions as the circulation of the atmosphere over the Pacific and of the Pacific waters in their basin are known to us only in the most general way; like many other subjects, our knowledge of them comes from records made by observers little trained in special fields. It cannot be doubted that large rewards will follow from exploration systematically carried on by trained specialists.

Adventurous voyages of discovery sufficed in the eighteenth century, when the method of exploration may be described as discontinuous and local. Less adventurous and more scientific voyages of investigation in the nineteenth century made brave attack on many problems by a method that may be described as continuous and linear. Now, so exacting have the demands of science become that nothing less than an areal survey of the Pacific will satisfy them; that is, a survey in which all the islands shall be included, and in which the successive routes of linear observation on the ocean shall be so closely interwoven that, like the work of the magnetic survey of the Pacific by the Carnegie Institution of Washington, the results gained may be reasonably regarded as applicable to the intermediate spaces.

The exploration of the Pacific should not only be continuous in the areal sense, but continued in the time sense. Many problems will call for 'one voyage to learn,' however careful the preparation before setting out. Exploration should therefore be continued through a number of years under one administration, so that the voyaging staff may gain in the early voyages the expertness needed for the solution of their difficult tasks in the later voyages. Many returns must be made to critical points, where first observations regarding air, water, land, or life will hardly do more than open up the more complicated subjects for investigation; for Pacific exploration should not only be continuous in the areal sense, and continued in the time sense, but comprehensive in every sense. It should reach from the depths of the sea to the heights of the air; from the cold waters of the North, narrowly limited in their connection with the Arctic ocean, across the torrid zone, to the cold waters of the South, broadly continuous with the Antarctic; from the little broken shores of continental America to the repeatedly interrupted shores of half-drowned Australasia; it should include all the islands of the open ocean, and all forms of life from bacteria to man.

The exploration of the Pacific is not a new theme. The voyages of Magellan, Cook, and others gave it early fame. The United States Exploring Expedition under Captain, later Commodore, Wilkes made a great advance 80 years ago. Forty years afterwards, the 'Challenger' did forty-years better; and yet we now learn from the refined studies of Norwegian hydrographers that the errors of certain instruments in the 'Challenger' outfit were greater than the variations of fact which those instruments were intended to measure. An interesting but sporadic and not long-lasting effort at Pacific exploration was made some 40 years ago by the Museum Godeffroy, founded at Hamburg by a firm of merchants: the Dutch Government has carried on scientific investigations of many kinds in its East Indian possessions, bordering the Pacific; much excellent work is now in process at the Bishop Museum, centrally located at Honolulu, where the great galleries of collections open to the public are backed by a large building devoted to research laboratories. The Hawaiian Islands seem, indeed, to be a center of inspiration on our subject, for a resident there, Prof. W. A. Bryan, elaborated a general scheme of Pacific exploration several years ago, and a recent visitor, Prof. R. A. Daly, has lately discussed the same problem; and in Hawaii as well as in the Philippines our governmental bureaus are prosecuting cartographic and other surveys. Specialists cross the Pacific from time to time on particular quests; the following papers will tell something of such work by Messrs. Briggs, Iddings, Pilsbry and Campbell. But the Pacific is vast. Discontinuous, local or linear, invidual work, economically conducted, cannot, however excellent, compass the immense extent and the infinite variety of that great water hemisphere. Thorough-going Pacific exploration will demand most munificent support.

It has been urged by some of those with whom I have talked on the financial side of the scheme that the present troublous times are not propitious for its launching. True; and we are not launching it now; we are only laying its keel. But these troublous times cannot last indefinitely; they will be past before we have had time—two, three, or four years—for the deliberate development of our plans. It is therefore with no idea of immediate action that the dreams of certain spare hours and days in the last two years have here been written down in brief outline; nor is it with the least intention or expectation on my part of sharing in the work of exploration, if the dreams are realized, that the outline is here presented. The work must be done by men of middle age or less, and they must be selected and directed by whomsoever brings the dreams into execution. If some antipodal Croesus rise to the occasion, the plan is his and welcome; but I believe that, if the plan is carried out at all, it will be by Americans, to whom the scientific conquest of the Pacific may make strong appeal. Even after the war is over, European munificence, if any of it survive, will be heavily burdened with home duties: and while the war lasts, even American capital may be so largely invested in commercial enterprises that little of it will be diverted to science: but when peace comes it is by no means beyond the limit of possibilities that our plan may arouse the interest of an American patron; for America's outlook upon the Pacific is large. Hence, in scientific as well as in national affairs, preparedness may well be our motto; and the first step in preparedness is—not a precipitate plunge into uncorrelated action—but the careful consideration of a comprehensive plan.

All that can be accomplished today is, to summarize a few of the broad problems that have been opened but not closed in the greatest of the world's oceans; and this will be done by the following speakers, who have generously responded to my appeal. Each one will touch briefly on certain topics—merely a few of many—yet representative, each one of all. I shall at some later time ask again if the whole subject, of which a few parts now are to be set before us, is not worthy of further consideration by the Academy, in the hope—not a vain hope, I believe—that the preparation of a well developed plan of investigation may be the prelude to a grand undertaking and a superb accomplishment.

THE IMPORTANCE OF GRAVITY OBSERVATIONS AT SEA ON THE PACIFIC

By John F. Hayford

COLLEGE OF ENGINEERING, NORTHWESTERN UNIVERSITY

Read before the Academy, April 17, 1916. Received, May 24, 1916

The time is ripe for decided advances in our knowledge of geodesy and geology to be made by a study of observed values of gravity. Observations for this purpose are needed at sea, especially—rather than on land. On the seas a given amount of observing will produce most progress if that observing is done on the Pacific.

In geodesy, gravity determinations furnish the most powerful, the most accurate, known method of measuring the flattening of the earth, and thereby furnish the most severe, and therefore the most valuable, single test of the reliability of conclusions drawn from the deflections of the vertical which are put in evidence by triangulation and astronomic observations.

So also when conclusions have been reached as to the completeness and location of isostatic compensation on the basis of observed deflections of the vertical the most valuable single test of those conclusions is furnished by observed values of gravity.

The geodetic evidence as to the completeness and location of isostatic compensation furnishes effective tests of the validity of an important group of the fundamental ideas of geology. These tests are being applied by the geologists more frequently and more energetically each year.

Moreover, it has recently been shown by Mr. William Bowie,[1] that after observed values of gravity have been corrected for isostatic compensation the remaining anomalies, indicating outstanding excesses or deficiencies of density beneath the surface of the earth are, in some cases at least, related to the geological history of the region. It is probable that in due time geodesy will, in this line, furnish additional help to geology.

In general geodesy furnishes the most powerful known means of investigating the distribution of density beneath the earth's surface, to a moderate depth, say 200 miles. Hence any geological premise which depends on assumptions as to the distribution of densities, within that 200 mile zone—and there are many such premises—finds a severe test in the geodetic evidence. So geodesy may, and will, help the progress of geology.

Suppose it is granted that it is important to secure additional gravity observations. Why is it especially important to secure the additional observations at sea rather than on land?

Good determinations of gravity have already been made at 3000 widely scattered stations on the one-quarter of the earth's surface which is land. No reliable observations of the necessary degree of accuracy have been made on the three-fourths of the earth's surface which is covered by water.

As soon as it becomes possible to determine gravity satisfactorily on a moving ship at sea, it will be possible to secure observations so rapidly at very widely scattered stations that the new observations made within a single year may furnish a more accurate value of the flattening of the earth than has yet been obtained from all the work of the past. Let us make this more definite and concrete. Assume that it becomes possible to take a series of observations on a moving ship which will determine the force of gravitation at the point of observation with a probable error of ±0.008 dyne (about 4 times the probable error of a land determination). Past experience indicates that the probable error in one such result due to all causes, including the anomalous part of the distribution of densities beneath the surface, will be less than ±0.020 dyne. Six hundred such observations could be secured in a single year scattered from latitude 60°N. to latitude 60°S. From these observations alone the value of the polar flattening of the earth could be computed more accurately than it has yet been computed.

Of course if the best observations that can be made at sea are of less accuracy than *this* their value will be less.

I am making this statement on the assumption that all such observations would be corrected for topography and isostatic compensation by the method now in use in the Coast and Geodetic Survey. Such corrections are essential to reliability and serve to increase greatly the accuracy of the computation of the flattening. I have just compared a very recent computation[2] by F. R. Helmert of the flattening of the earth with an older computation[3] of the flattening by William Bowie. Both used gravity determinations. Helmert used 700 widely scattered selected observations from among the 3000 available over the whole world. Bowie used 122 out of the 124 gravity observations in the United States alone. Helmert made no corrections for topography and isostatic compensation. Bowie applied such corrections. I am convinced from a study of the evidence, including the evidence of systematic errors, that Bowie's value of the flattening derived from 122 gravity observations in a small region is more accurate and reliable than Helmert's value from 700 carefully selected and widely scattered observations.

Observed values of gravity in the United States after correction for topography and isostatic compensation show no relation to the topography. On the other hand without such corrections, as Helmert's recent investigation again shows clearly, observations along the coasts stand in a class by themselves, observations in low interior regions in another class, those in mountainous regions in another class, and those on small oceanic islands in still another class, and each class is subject to its own peculiar systematic errors which are large.

It should be evident that it is extremely desirable to extend to the open oceans the proof, which is now conclusive for land areas, that the application of corrections for topography and compensation makes the corrected results independent of topographic effects and eliminates a large part of the systematic error otherwise inherent in the results. I am confident that good observations at sea will promptly furnish such a proof. They would certainly give a very severe and therefore very valuable test of the conclusions as to isostasy which have been drawn from the observations on land.

It is obviously important to determine as well as may be the prevailing depth at which masses of abnormal density lie. Are they ordinarily within 10 miles of the surface, or are they as frequently more than 40 miles down? The nearer to the surface such an abnormal mass lies the more rapid will be the space-change of gravity as an observer approaches, passes over, and recedes from the region on the surface which lies above the abnormal mass. For this purpose, therefore, for indicating the approximate depth of abnormal masses, it should be evident that closely spaced continuous lines of gravity observations at sea such as could be easily secured would be much more effective than are stations on land under ordinary conditions.

To what is the permanency, or semi-permanency, of the great oceanic depressions due? Adequate gravity observations at sea would establish conclusively the extent to which the rocks underlying the oceans are more dense than those under the land, and thus furnish a conclusive partial answer to the question.

The present indications, from a few gravity stations on such islands, is that gravity is in excess on oceanic islands, such as the Hawaiian Islands, where vulcanism is active. How far does said excess extend out to sea? Observations of gravity at sea would answer that question and in doing so might contribute much to our knowledge of the nature and cause of vulcanism.

What is the nature of the deep troughs that occur at various places in the oceans and which show a suggestive tendency to be located near and parallel to an elongated land area or a mountain chain? Gravity observations at sea may throw a light upon this question by showing the density of the rocks below such troughs.

I have indicated why I believe it to be especially important to secure gravity observations at sea. If such observations are to be made, why is the Pacific the ocean upon which they will be most effective?

Of course it occurs to one at once that the Pacific is the greatest ocean and that therefore the largest blanks in which there are now no gravity observations are there. That will on examination prove to be a more weighty consideration in favor of the Pacific than appears at the first glance.

Two other considerations also combine with this to indicate strongly that the Pacific is the most effective place to make gravity observations at sea in order to advance our knowledge of geodesy and geology.

First, it is important to get observations at sea so far from any continent as to be certainly free from any continental effect.

It is difficult to get 2000 miles from all continents on the Atlantic. That distance is not sufficient. On the Pacific there is a considerable area 3000 miles from any continent. The Pacific ocean occupies the water hemisphere.

The second consideration in favor of the Pacific is that it offers an unequaled variety of special opportunities to study special questions under extreme and contrasting conditions. In it there are small oceanic islands of volcanic origin far from land, and equally small oceanic islands far from land apparently not of volcanic origin. There are several troughs more than 8000 meters deep lying adjacent to large islands in some cases and in others far from any but very small islands. There are flat bottomed areas of more than 6000 meters depth of various sizes, some covering several square degrees, and in various relations to land. There are several areas of less than 200 meters depth, some of which are far from any land, except very small islands. No other ocean offers an equal variety of equally favorable opportunities for special studies based on gravity observations at sea.

Adequate observations of gravity, at sea, on the Pacific Ocean would contribute greatly to progress in geodesy and geology.

---

[1] Special Publication No. 10 of the Coast and Goedetic Survey, pp. 113–117 and Special Publication No. 12, pp. 18–21. Both these publications are under the title Effect of Topography and Isostatic Compensation upon the Intensity of Gravity.

[2] "Neue Formeln für den Verlauf der Schwerkraft im Meeresniveau beim Festlande," von F. R. Helmert; Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften, 1915, XLI, Gesamtsitsung vom 21 Oktober.

[3] Special Publication No. 12 of the Coast and Geodetic Survey, pp. 24–26.

A NEW METHOD OF MEASURING THE ACCELERATION OF GRAVITY AT SEA

By Lyman J. Briggs

BUREAU OF PLANT INDUSTRY, WASHINGTON, D. C.

Read before the Academy, April 17, 1916. Received June 19, 1916

Introduction.—The desirability of more extensive and more accurate measurements of the acceleration of gravity at sea has recently been emphasized by Bauer[1], Bowie[2], and Hayford.[3] Such measurements are essential in the accurate determination of the figure of the earth. Ocean gravity measurements now available are due almost wholly to Hecker,[4] who employed the mercurial barometer-hypsometer method in an elaborate and extended series of measurements in the Atlantic, Pacific, and Indian oceans. In this method the outstanding difference at a given station between the atmospheric pressure as computed from the boiling point of water and directly observed with the mercurial barometer is attributed to the difference in the gravitational force acting on the mercurial column at the given station and at the standard station (Lat. 45°, sea level).

There are certain difficulties inherent in the barometer-hypsometer method which greatly lessen its usefulness. (1) The atmospheric pressure must be determined in absolute measure by each method in order that the determinations may be comparable, so that systematic errors are serious; (2) the boiling point determinations must be carried out with the highest degree of refinement in order to secure even moderate precision in the determination of g. The observed boiling points are not simply differential measurements. The true temperature interval between the melting point of ice and the observed boiling point must be known in terms of the hydrogen-scale before reference can be made to vapor pressure tables for the determination of the atmospheric pressure.

An independent measure of the accuracy attainable in the determination of atmospheric pressure by the boiling point method is afforded by measurements of the 'fundamental interval' of standard mercurial thermometers. Waidner and Dickinson[5] found in a study of the standard mercurial thermometers of the Bureau of Standards that the fundamental intervals varied through a range of 0.015C. during the ten-day period covered by the measurements. This variation they attribute in part to the sticking of the meniscus with resulting variation in the capillary pressure.

The probable error of a fundamental interval determination in Waidner and Dickinson's measurements is about ±0°.003C. The measurements were carried out with a refinement which is probably unattainable at sea, and may be taken to represent the limit of accuracy attainable in such determinations on board ship. We may now consider the effect of such an error in the determination of g. A probable error of ±0°.003C. in the true boiling point temperature would correspond in atmospheric pressure to ±0.083 mm. of mercury and to a probable error of ±0.11 cm. per sec. per sec. in the value of g. The uncertainties due to the boiling-point measurements alone under the most favorable conditions would, therefore, result in a probable error of 1 part in 10,000 in the value of g. This is 13 times the limit of error set by Hayford for ocean gravity measurements. To this must be added the errors in the barometric observations. Errors in the boiling-point temperature alone would justify Bowie's statement that Hecker's gravity measurements at sea "are subject to uncertainties as large as the largest of the new-method anomalies of gravity in the United States, that is, between 0.05 dyne and 0.10 dyne."

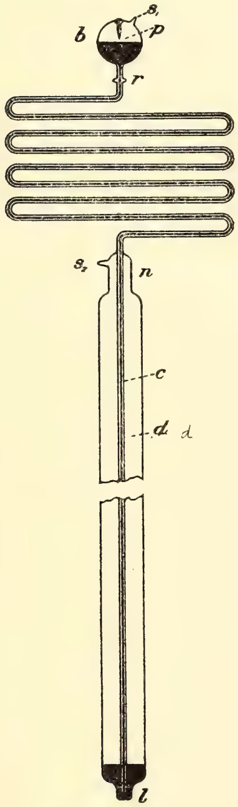


FIG. 1.

Description of the New Method.—The new method of measuring g at sea described in this paper is based upon the change in height of a barometric column sealed off from communication with the atmosphere.[6] All boiling-point determinations are thus avoided. The apparatus is kept at constant temperature in a bath of melting ice. The determination only of the position of the upper end of the barometric column is necessary. The design of the instrument is such that in making this setting the enclosed gas mass is automatically reduced to a constant volume; and since temperature is constant, the measurements are made at constant pressure.

A sketch of the glass part of the apparatus is shown in figure 1. The mercurial column is contained in the capillary c (bore 0.6–0.7 mm.) the lower end of which opens beneath mercury in the bottom of the gas chamber d. This capillary is sealed to the wall of the gas-chamber where it passes through the upper end. The upper part of the capillary is bent into a flexible zig-zag, and expands into the spherical bulb b (diameter 2 cm.) The bulb contains a fixed iron point p sealed to the inside of the bulb by means of an inserted platinum wire and extending vertically downward, so that the point is approximately at the center of the bulb. The length of the mercurial column is about 74 cm.

The flexible capillary permits a slight vertical movement of the observing bulb with respect to the gas chamber. This movement is determined by a micrometer screw of 1 mm. pitch which controls the motion of a carriage in which the observing bulb is rigidly mounted. The carriage slides on parallel rods mounted on a base which is rigidly cemented to the neck *n* of the gas chamber, so that the position of the bulb relative to the gas chamber is definitely determined by the screw.

After the apparatus has been thoroughly cleaned and dried, pure mercury is introduced in excess of the amount required to fill the capillary and observing bulb. The apparatus is then highly evacuated through s_1 and s_2 and the observing bulb sealed at s_1. Dry nitrogen is introduced into the gas chamber in excess of atmospheric pressure through a stop-cock temporarily sealed to the chamber at s_2. The micrometer head is then mounted, the protective casing of the gas chamber adjusted, and the whole apparatus placed in the ice-tank in a vertical position and surrounded with melting ice. A large U-type vacuum manometer is connected with the stop-cock of the gas chamber and the air exhausted from the connecting tube. After temperature equilibrium is attained, the stop-cock of the gas chamber is opened and the pressure of the nitrogen slowly reduced until the mercury stands in contact with the fixed point in the observing bulb when the carriage is near the middle of its range. The pressure in the gas chamber at this time is determined by reading the difference in level of the mercury surfaces in the manometer by means of a cathetometer. The observed pressure, corrected to mercury at 0°C., gives the difference in height of the two mercury surfaces in the gravity apparatus. The length of the column, less the scale reading, gives the 'constant' of the instrument, which is added to subsequent micrometer readings to obtain the true height of the mercurial column.

When the above determination is made, the gas chamber connection is sealed off with a blow-pipe. The apparatus is now perfectly gas-tight, since it is entirely free from ground joints, stop-cocks and sealed-in platinum connections, and the original openings at s_1 and s_2 have both been sealed with the blow-pipe. The apparatus is also readily transportable, since at ordinary temperatures the pressure in the gas chamber is sufficient to fill the observing bulb with mercury. It can then be tipped in any position.

In observing, the apparatus is supported in a vertical position in the cork-covered ice tank and completely covered with fragments of melting ice 1 to 3 cm. in thickness, dropped loosely into the tank. The fixed point is observed through a glass tube introduced through the ice and is illuminated through a similar tube on the opposite side. The observing tube is equipped with a low power lens. The ice tank is 25 cm. in diameter and is covered with cork lagging 7.5 cm. thick.

At sea the tank is swung from gimbals, the outer gimbal ring being suspended from 4 spiral steel springs hung from the four corners of a frame of 1-inch galvanized pipe, the four legs of which are screwed to the deck. Movable weights on the bottom of the tank serve to adjust the apparatus to a vertical position, which is determined by means of a sensitive level on the head of the instrument. When the apparatus is mounted on the open deck, an effective wind shield is essential, the supports of which must be entirely independent of the apparatus.

Theory.—Let us determine through a series of observations the height of the mercurial column at some station where g is accurately known. Let us now consider the apparatus to be transported to another station, where g is greater. The mercurial column will be depressed, compressing the gas. The observing bulb is accordingly lowered until the fixed point is again in grazing contact with the mercury surface. The volume of mercury in the bulb and capillary is now the same as at the first station, since the slight flexure of the capillary produces no appreciable change in its volume. Therefore the volume of mercury in the gas chamber is also the same as in the first observation. In other words, the volume of the gas is constant when the upper mercury surface is in grazing contact with the fixed point, and is independent of the position of the bulb. Since the temperature is constant, the pressure p of the gas is always the same at the time of taking an observation. We have then

$$p = \rho g h = g_1 h_1 \qquad (1)$$

in which g and g_1 represent the acceleration of gravity at the two stations, h and h_1 the corresponding heights of the mercury column, and ρ the density of mercury. The latter is constant, since the mercury is always at the temperature of melting ice. Equation (1) then reduces to

$$g / g_1 = h_1 / h \qquad (2)$$

or, the height of the column is inversely proportional to the acceleration of gravity. If the height h is measured at some station where g is known, the acceleration of gravity at any other station may therefore be determined simply by measuring h_1.

Let $h - h_1 = \Delta h$ represent the difference in the height of the column at the two stations. On substituting this value of h_1 in equation (2) we have

$$g \;/\; g_1 = 1 - \Delta h \;/\; h \tag{3}$$

in which Δh represents simply the difference of the micrometer readings at the two stations, and does not involve the absolute height of the column. Since h is at least 200 times as large as Δh, the absolute height of the column does not, in this method, need to be determined with great precision.

Discussion of errors.—The apparatus was designed with the object of obtaining, if possible, an accuracy of 1 part in 100,000 in the measurements. The discussion of the various sources of error will therefore be made on this basis.

Temperature variation.—During the measurements the whole apparatus was surrounded with melting ice, so that the effect of slight variations in temperature need be considered only in connection with the temperature of the gas. Artificial ice was employed and all ice that was not perfectly clear and crystalline was discarded. The purity was systematically checked by measuring the electrical conductivity of the tap water. The impurities were never sufficient to depress the theoretical freezing point more than 0°.001C., whereas a variation of ±0°.003C. would be necessary to produce a change of 1 in 100,000 in the gas pressure. The drip water was allowed to escape from the ice tank through a small trap near the bottom in order to insure the ice extending below the bottom of the gas chamber.[7]

Error in setting point in contact with mercury surface.—Lord Rayleigh[8] found in his investigations with the micromanometer that the fixed ground glass point could be set in contact with the mercury surface with an error not exceeding ±0.0015 mm. An accuracy of 1 part in 100,000 in gravity observations necessitates a probable error in the micrometer observations not greater than ±0.008 mm., which is readily attainable from a series of readings at sea under favorable conditions. The writer has found that in measurements at sea a metallic point is superior to a glass point, due to the fact that the latter becomes electrified through the motion of the mercury surface and this influences the readings.

Error in the determination of the instrument constant.—Reference to equation (3) will show that since the height of the column is at least 200 times the total range (Δh), the uncertainty in h may be 100 times that permissible in Δh. The constant of the instrument (i.e., the vertical distance from the lower mercury surface to the zero on the scale) therefore does not need to be known with an accuracy greater than 0.5 mm. Determinationsof the constant agreeing to 0.1 mm. can be obtained with the auxiliary manometer to which reference has already been made.

Thermal hysteresis of the glass envelop.—This phenomenon is generally recognized in precision thermometry, and results in the so-called depression of the zero. This effect would tend to introduce an uncertainty in the determination of the difference in the acceleration of gravity at two stations where it is necessary to remove the apparatus from the ice in proceeding from one station to the other. A slight change of this kind (2 parts in 100,000) was observed at Balboa, following the dismantling of the apparatus and its transportation across the isthmus by rail, owing to the slides in the canal. These instruments were made of German glass. Waidner and Dickinson[9] state that thermometers made of the best boro-silicate glass show a depression of the zero of 0°.03. This would correspond to a change of 1 part in 200,000 in the volume of the bulb. Consequently if boro-silicate glass were used in the construction of the glass parts of the apparatus, the error due to thermal hysteresis would fall within the limit set in this discussion.

Disturbances arising from the motion of the ship.—Three classes of disturbances are encountered on board ship:

1. *Tremors*, due to the engines or auxiliary machinery or to the impact of waves. The effect of such disturbances can be greatly reduced by suspending the ice tank from spiral springs. This form of support does not eliminate all vibration. Slight tremors however appear to be advantageous rather than otherwise, as they help to bring the mercury surface to its true position in the bulb.

2. *Horizontal translation*, due chiefly to rolling, which tends to swing the apparatus from its vertical position. Such translations produce two effects which can be made to counteract each other to some extent. (a) The vertical component of the column is shortened by an amount proportional to the sine of the deflection from the vertical. Such deflections therefore tend to increase the length of the column. (b) The centrifugal force resulting from the deflection tends to depress the column when the latter is mainly below the point of support. This can be eliminated by mounting the column so that it is bisected by the gimbal plane. By suitably adjusting the position of the column, the two effects can thus be made to compensate in part.[10]

3. *Vertical motion*, due to rolling and pitching or to the rise and fall of the ship as a whole in a heavy sea. This is the most serious of all the disturbances to contend with, for the motion is accelerated, and is superimposed on the gravitational acceleration. Extensive damping through the use of the capillary column has been employed in the apparatus here described to reduce

the effect of vertical motion. The expansion of the top of the column into a bulb having a cross-sectional area 500–800 times that of the capillary reduces proportionally the change in level in comparison with the actual movement of mercury in the capillary. The rate of change in the acceleration of gravity with latitude is so slight that extensive damping is permissible from this standpoint, although the time required to secure an observation is of course correspondingly increased.

Correction for the course and speed of the ship.—Eötvös[11] has shown the necessity of applying a correction for the easterly or westerly motion of the ship, due to the fact that the ship's motion modifies the angular velocity of revolution of the apparatus about the earth's axis. The centrifugal force acting on the mercurial column when on board a ship moving east or west is therefore not the same as when the ship is at rest or moving north or south. The correction may be as great as 1 part in 10,000, but can be accurately computed if the course, speed, and approximate latitude of the ship are known.

The probable error of the observations.—In 1914, observations were made from Sydney, Australia, to San Francisco by way of Wellington, N. Z.; and in 1915, two instruments were taken from New York to San Francisco via Panama. The following table shows the results of gravity determinations on board ship in various harbors during the two voyages, and where pendulum observations are available near the stations they are appended for comparison. The 1915 observations show a mean probable error in the harbor determinations of 13 parts in 1,000,000. The readings of the two instruments at sea were for the most part as consistent as at the harbor stations. The ocean measurements will be discussed in a later paper.

The method is by no means to be considered as perfected. One of the instruments used in 1915 gave results consistently lower than the other. This indicates a systematic error which must be located. It is also highly desirable that new instruments constructed of boro-silicate glass should be carried over the same course several times with different sea conditions in order to determine whether systematic errors are introduced into measurements made on a rough sea.

Acknowledgments.—In the first apparatus constructed, the gas chamber and capillary were made of steel. This apparatus developed a leak on the voyage to Sydney, and repeated attempts to repair it in the Fiji Islands and at Sydney met with failure. This disappointment was however more than offset by the kindness of Dr. J. A. Pollock, Professor of Physics in the University of Sydney, who placed the facilities of his laboratory and the services of his glass-blower and mechanician

TABLE I.

GRAVITY DETERMINATIONS ON BOARD SHIP AT HARBOR STATIONS

	$\frac{\text{cm.}}{(\text{sec.})^2}$
1914	
With San Francisco as base,	
g at Wellington, N. Z.,	
observed, instrument No. 1	980.316
by pendulum (Wright)	980.292
	+0.024
g at Sydney, N. S. W.,	
observed, instrument No. 1	979.68
by pendulum (Smithsonian tables)	979.69
	−0.01
1915	
With New York as base,	
g at Colon, Panama,	
observed, instrument No. 2	978.196
instrument No. 5	978.236
	978.216±0.013
g at Balboa, Panama,	
observed, instrument No. 2	978.175
instrument No. 5	978.195
	978.185±0.007
g at San Francisco,	
observed, instrument No. 2	979.940
instrument No. 5	979.995
	979.968±0.019
by pendulum (Smithsonian tables)	979.98
	0.012±0.019

at my disposal in the construction of a new apparatus of glass, which was used on the return voyage.

I am also deeply indebted to Dr. C. G. Abbot, Director of the Astrophysical Observatory of the Smithsonian Institution, for his assistance in connection with the steel apparatus, and for valued suggestions regarding the construction of the later form.

Lieut. R. S. Wright, R.E., has kindly supplied the value of *g* at Wellington from his unpublished pendulum observations made in connection with the Scott Antarctic expedition.

The voyage in 1914 was made with grants from the Australian and New Zealand governments, in connection with the Australian meeting of the British Association for the Advancement of Science. I desire to express my obligation to these governments and also to the American Association for the Advancement of Science for a grant used in connection with the 1915 measurements.

[1] Bauer, L. A., On gravity determinations at sea, *Amer. J. Sci.*, **31**, 1 (1911). Hecker's remarks on ocean gravity observations, *Amer. J. Sci.* **33**, 245 (1912).

[2] Bowie, Wm., Isostasy and the size and shape of the earth, *Science*, **39**, 705 (1914).

[3] Hayford, J. F., these PROCEEDINGS, **2**, 394 (1916).

[4] For a description of the apparatus employed by Hecker and a summary of his ocean measurements, see Hecker, O., Bestimmung der Schwerkraft auf dem Schwarzen Meere und, an dessen Kuste sowie neue Ausgleichung der Schwerkraftmessungen aud dem Atlantischen, Indischen, und Groszen Ozean, *Zentralbur. Internat. Erdmessung, Veroffentlichungen Berlin,* N. F., Nr. 20 (1910).

[5] *Washington, D. C., Bull. Bur. Standards,* **3**, 663 (1907).

[6] This principle has already been employed by Mascart, [*Paris, C. R. Acad. Sci.*, **95**, 631, 1882], who used a sealed-off barometer of the U-tube type. The temperature was not controlled, and volume and pressure were both variable. The gas volume had therefore to be measured, the pressure corresponding to the observed temperature and volume calculated, and finally reduced to standard conditions.

[7] This procedure now seems undesirable, since Pernet has observed a departure of 0°.01C. from the true zero when the bulb of a thermometer is surrounded by artificial ice freely drained, due to the ice being undercooled. (See Pernet, J., Sur les moyens d'eliminer l'influence de la variation des points fixes des thermometres a mercure. *Travaux. Bur., International des Poids et Mesures,* 1881, second partie.) In the writer's measurements, ice was added in small quantities at frequent intervals so as to keep the tanks completely filled, and remained in the apparatus for several hours before melting sufficiently to sink to the level of the gas chamber. It is consequently doubtful whether the effect observed by Pernet influenced the measurements, but it is a possible source of error which can be avoided by keeping the interstitial spaces filled with water.

[8] *Phil. Trans. R. Soc., London,* **196**, 205 (1901).

[9] *Loc. cit.*

[10] The above procedure was not followed strictly in the trials of the apparatus which have so far been made at sea, due to the difficulty of following the observing tube with the eye when the ship is rolling in a heavy sea. Experience shows however that such an arrangement is necessary if observations are to be made in rough weather, and the difficulty in observing can apparently be met by a modification of the viewing apparatus.

[11] See Hecker, *loq. cit.*

THE PROBLEM OF CONTINENTAL FRACTURING AND DIASTROPHISM IN OCEANICA

By Charles Schuchert

DEPARTMENT OF GEOLOGY, YALE UNIVERSITY

Read before the Academy, April 17, 1916. Received, June 1, 1916

Paleogeographic studies during the past thirty years have been developing the hypothesis that the ancient continental platforms were arranged latitudinally rather than longitudinally as they are now, and, further, that their areal extent, including their emergent and submerged portions, was greater than at present. It appears that vast land-masses have been fractured, broken up, and more or less permanently taken possession of by the oceans, a history which none exhibits better than the Australia-New Zealand region.

We have learned from the several deep-sea expeditions something of the rare and strange life of the oceanic abysses. An analysis of these organisms shows that no Paleozoic forms occur among them, and very little of the life indeed is ancestrally traceable even to the stocks of Triassic times. It is with the Jurassic and later life that the organisms of the abysses have their affinities. This seems to indicate that the oceans have been progressively deepened only since the Triassic. As one of the most marked crustal deformations, however, began in the Coal Measures of the Paleozoic and continued, though with pauses, well into the Triassic, it therefore appears that the oceans have been progressively enlarged and deepened ever since Permian time. This is in keeping with the theory that the earth's radius has been gradually diminishing and that the compensation therefor has been greatest in the oceanic basins, the areas of greatest rock densities.

It is now more than fifty years since James D. Dana began to teach that the rising continents and the sinking oceanic basins have been, in the main, permanent features of the earth's surface. He did not mean, however, that the continents have always had essentially the same shape, elevation, and areal extent that they have today. Still, Dana did not fully appreciate the amount of continental fragmenting that has taken place in the course of geologic time, though he clearly pointed out the foundering of Australasia, speaking of it in his famous *Manual of Geology* (p. 797) as "a fragment of the Triassic world." The teachings of Dana as to the permanency of continents and oceanic basins have been accepted in some form by all geologists, and lie at the basis of all zoögeography and evolution as well. Geologists are holding more and more to the hypothesis that the earth periodically shrinks, and each time it does so some parts or all of the continents may rise, but that in the main there is subsidence of the ocean bottoms, that the water of the hydrosphere is constantly increasing in amount, and that even though the continents are in the main permanent, yet they are partially breaking down into the oceanic basins.

From this we conclude that the enlarging oceanic basins are the most permanent features of the earth's surface. On the other hand, along with the progressive subsidence, the bottom of the Pacific is also built up into many local volcanic cones by outpourings of lava, and, further, it rises into more or less long mountain ridges. Some of these elevations of the bottom appear at the surface of the ocean as groups or lines of dead or active volcanoes. Another general conclusion is that most of the "deeps" of the Pacific Ocean situated between 18,000 and 31,800 feet beneath the surface occur near the continents that exist now or existed formerly, or that they are located on the outer or oceanic side of mountain chains. These, the "foredeeps" of Suess, are striking tectonic features of the lithosphere. As for the true limits of the Pacific Ocean, Suess states that they are seen in the trends of long mountain folds. "So it is from New Zealand and New Caledonia to the borders of eastern Asia, to the Aleutians, and all along the western coast of both Americas."[1]

So far we have been considering the problem of crustal depressions essentially from the standpoint of hypothesis; now let us see what is actually known as to the topography of the Pacific Ocean and the geologic history of the Australasian region. An excellent summary of the present geography of the Pacific Ocean and the topography of its bottom is shown on the splendid map by Max Groll, recently published by the Institut für Meereskunde of the University of Berlin (1912). This map is based on Lambert's equal-area azimuthal projection, with a replotting of all geographic and bathymetric data ascertained up to January 1912, and is therefore more up-to-date and far better than any heretofore published. Groll states that he considered at least 15,000 soundings, made in all the oceans, and that yet there are many areas in the Pacific, hundreds of miles across, without a single one. It is therefore natural for him to add: "The greater part of the Pacific Ocean is still unexplored. . . . One is actually frightened at the little that is yet known of the bottom relief of the oceans and at the few data on which our representation of it is based. . . . Even in so relatively well known an area as the East Australasian seas, there are rarely more than from four to six deep-sea soundings to each five-degree field." Our detailed knowledge of the actual configuration of the bottom of the Pacific is therefore seen to be very slight indeed.

Let us now review the larger features resulting from the ancient cycles of aërial erosion and marine deposition through which has been determined the paleogeography of Australasia. An analysis of this history since the Cambrian seems to show that at least two northeasterly trending troughs of sedimentary accumulation began to form early in the Paleozoic. The western one, which may be known as the Tasman trough, almost wholly of Paleozoic development, is now partially elevated into the plains and mountains of eastern Australia, while the rest of it has sunk deep into the present sea, and with it considerable of what was formerly western New Zealand. The other, or eastern trough, which also appeared early in the Paleozoic, maintained itself after this era in diminished extent throughout the Mesozoic and even into Pliocene time. This may be known as the New Zealand trough, a far narrower but much longer one than that of Australia; the shorter southern portion has now risen into the mountains of New Zealand, while the far longer northern part has apparently subsided to a depth of not more than 9000 feet, forming a submerged plateau upon which stand the volcanic islands of the Kermadecs and the Tongas.

In the New Zealand trough there appear to be, according to Park,[2] no less than 45,000 feet of Paleozoic and 11,000 feet of Mesozoic sediments, all of which are apparently of marine origin. These are coarse in grain and have much interbedded igneous material, which indicates that the adjacent lands were unstable and repeatedly reëlevated into high lands. There were at least four times when the New Zealand trough was markedly subject to folding and uplift; these were toward the close of the Silurian, Devonian, Jurassic, and Cretaceous periods. During the Tertiary, the New Zealand trough also appears to have been in continuous subsidence from late Eocene into Pliocene time, when about 9000 feet of marine sediments had been laid down along the eastern sinking margin. Late in the Pliocene there was marked vertical uplift, probably as much as 4500 and possibly even 6000 feet. The nearly horizontal Tertiary strata are now found in places at an elevation of 3000 feet, having been depressed 1500 feet during the time of Pleistocene glaciation. The high condition of New Zealand at this time united into a greater New Zealand all of the present outlying islands of the New Zealand plateau, no part of which is now submerged more than 3000 feet.

In Australia there is no evidence of the Tasman sea during Cambrian time, for the marine invasions at first are from the south and later across the entire medial portion of the continent. The trough begins to appear as a sea-way in the Ordovician (?5000 feet of deposits, according to Süssmilch[3]), with the greatest time of subsidence during the Devonian (27,000 feet); it continued with some interruptions throughout the Carboniferous and Permian (36,000 feet). During the Paleozoic, about 70,000 feet of essentially coarse sediments and interbedded volcanics were laid down in New South Wales, though smaller thicknesses seem to prevail elsewhere in eastern Australia. Here again we see the geologic results of high adjacent and often rejuvenated western lands. The record also shows that there were in Paleozoic time at least three periods of decided crustal folding (Ordovician, Silurian, and Devonian), and one of vertical uplift with faulting (during the close of Permian time). Following the Permian deformation, the continent was repeatedly lifted above the embrace of the Tasman sea, and most markedly so in the Pliocene, when all of eastern Australia was vertically elevated and block-faulted between 1500 and 7300 feet above sea-level (during the 'Kosciusko epoch'). In compensation for this elevation the Tasman sea sank, as now there are great depths close to the continent, in one place going down to 18,500 feet.

Australasia has been the most remarkable asylum among the continents for the preservation to this day of living examples of the plants and animals of the medieval world. Among these in great variety are the marsupial or pouch mammals, and the far less diversified, more primitive, but more remarkable egg-laying monotremes. The marsupials were at their culmination in the Pliocene, when forms existed larger than any living rhinoceros (*Diprotodon australis*). From the chronogenesis of these stocks and their diverse evolution in Australia, we learn that they must have been on that continent long before, and that they had been free from all Asiatic invasions and therefore escaped destruction by the higher, more intelligent carnivorous placental mammals. We must therefore conclude that Australia has been an island continent at least since late Eocene time, for it is since then that the placental mammals have elsewhere dominated all other land life.

The question next arises, When was Australia severed from Asia? From the paleogeography as now deciphered, we learn that Asia and Australasia were in complete connection throughout the early Paleozoic to the close of the Devonian. In Lower Carboniferous time, however, southeastern Asia began to be invaded by the Indian and Pacific oceans in the region of what are now the East Indian Islands. A greater subsidence here and in New Guinea, New Caledonia, and elsewhere in Australasia began in the Jurassic and probably persisted well into Cretaceous time. However, from the fact that carnivorous dinosaurs—land reptiles that arose either late in the Permian or shortly afterward—are known in the Triassic of Australia (none at all occur in New Zealand), we must conclude that there were still at this time intermittent land-bridges connecting this continent with Asia. The time of complete severance apparently came in the Jurassic, and the trough of separation seems to be the present Molucca-Banda sea, which has depths varying between 4650 and 21,100 feet.

Finally we must ask, When did the thousands of oceanic islands—the Oceanides—arise? They are probably in the main of volcanic origin and occur singly, in groups, and most abundantly in linear arrangement. The isolated and the grouped islands probably all represent great volcanic cones that have built themselves up from the ocean bottom through the eruption of rock material. What is the origin, however, of those that are arranged in linear series? Are they ranges of volcanoes that have likewise grown from the depths but are situated on lines of fracture in the lithosphere, or do they rest on the crests of great arches or foldings of the ocean bottoms? Equally important questions are: What is their geological history, and have they simultaneous or successive origins? So far as known, none of the smaller oceanic islands reveals fossils older than the later Tertiary, a condition that appears to be in harmony with the theory that the sum of their movements is negative and thus in keeping with the idea that the oceanic bottoms are subsiding areas. We have as yet little to show when they originated, and still on the basis of the periodically recurring diastrophism it would seem that none can be older than the Permian, a time of intense and world-wide crustal deformation. Others may have originated during the late Cretaceous crustal movements, and all may have again been reëlevated and stirred into volcanic activity with the world-wide crustal readjustments that began in the Miocene and continued into late Pliocene time.

The views just presented are those of most paleontologists, but there are geologists and zoögeographers who do not accept the idea of continental fragmenting taking place on so large a scale as is here indicated. They hold firmly to the theory of the permanency of continents and ocean basins, believing that these positive and negative elements of the earth's surface have always retained the forms they now have. In their eyes, the physical evidence in the areas of fragmentation, and especially in the southern hemisphere, is not of a nature to compel the view that large lands formerly existed here, and they say, further, that there is no process in the mechanics of the earth known to them that would account for such down-breaking of the lithosphere.

As for the ancient life found in Australia, those who hold the above view say that we are still too ignorant of the world's organisms and their histories to conclude from them that their asylums were formerly connected with other land-masses, or they hold that the animals reached these places by accidental dispersal, through the air or by being rafted across the intervening water areas. Hence this conflict of views marks one of the greatest outstanding problems of geology and paleontology. The writer, however, is overwhelmed by the facts revealed in the geographic distribution of ancient and modern animals, and is compelled to dissent from the rigid view of the permanency of continents.

To sum up, in conclusion, we may say that the bottom of the Pacific Ocean in the region of greater Australasia seemingly became more and more mobile with the Lower Carboniferous and especially during the Jurassic and Cretaceous. During this very long time, the eastern half of the continent, a land about 1800 miles east and west and 2200 miles north and south, nearly all went down more and more beneath the level of the sea to a maximum depth of about four miles and an average depth of between one and two and a half miles. Further, the entire area of the Oceanides also subsided, and possibly to an equally great depth; while this was taking place the bottom was apparently folded and built up by volcanic material into many more or less parallel ridges, a series of arcs extending over an area of about 3500 miles east and west and the same distance north and south. Finally, we may add that the entire western half of the Pacific bottom appears to be as mobile as any of the continents of the northern hemisphere, with the difference that the sum of the continental movements is upward, while that of the ocean bottoms is downward. This paper will be published at greater length and with illustrations in the *American Journal of Science.*

[1] Suess, *Natural Science*, **2**, 180 (1893).

[2] Park, *Geology of New Zealand*, 1910.

[3] Süssmilch, *Geology of New South Wales*, 1911.

THE PETROLOGY OF SOME SOUTH PACIFIC ISLANDS AND ITS SIGNIFICANCE

By Joseph P. Iddings

BRINKLOW, MARYLAND

Read before the Academy, April 17, 1916. Received, May 12, 1916

Petrology as a more comprehensive term than petrography embraces all the phenomena and material characters of rocks, as well as the theories regarding their origin and the relations between the rocks of the earth and the problems of geodynamics. Knowledge of the composition, mode of occurrence, and distribution of igneous rocks should contribute materially to the elucidation of those problems in geology which are concerned with the constitution and behavior of the outer and inner portions of the earth.

For many years some geologists and petrologists have been convinced that the various kinds of igneous rocks in different parts of the world, both volcanic outflows and intruded bodies, are so intimately related to one another within each region that they must have been derived from some parent lava, or rock magma, by processes of physico-chemical differentiation; and further, that in different regions of the earth the series of igneous rocks in each possess chemical and mineral characteristics which distinguish them from series of rocks in other regions. In short, that the series of igneous rocks in various parts of the world are not exactly alike, though some may be.

Recently I have called attention to the apparent relation between regional characteristics of igneous rocks and the broad demands of the theory of isostasy. The igneous rocks of the North American continent, both as a whole and when considered in regional groups, such as those of the Pacific cordilleras, the Colorado plateau, or the Atlantic coast, average lighter than the igneous rocks of islands in the deep oceans such as Hawaii in the Pacific, and Réunion in the Indian Ocean. The relative densities of a completely crystallized average magma of the Pacific Coast rocks, and those of average magmas for Hawaii and Réunion are to one another as 2.85 to 3.12, for each of the island groups of rocks. This appears to be in accord with the demands of isostasy, if igneous rocks represent the material forming deep portions of the lithosphere, or material immediately below it, or at its base. However, much remains to be done in studying the igneous rocks of all parts of the earth, especially of the little known islands of the Pacific Ocean.

It was with the hope of contributing something to the solution of the problem of isostasy from the petrographic side that I undertook recently a reconnaissance of some volcanic islands in the South Pacific, namely: Tahiti, Moorea, the Leeward Islands: Huahine, Raiatea, Tahaa and Bora Bora; besides the two largest islands of the Marquesas, Hiva-oa and Nukahiva.

So long ago as 1849 James D. Dana published an excellent description of Tahiti, and mentioned the occurrence of a variety of syenite among the basalts of this volcano, which he considered to be "only a feldspathic variety of the same igneous rocks that constitute the island;" a perfectly correct assumption. Darwin visited Tahiti a few years earlier than Dana and noted the basaltic character of the rocks. In a conversation with the writer 30 years ago Dana remarked "If you want to study a dissected volcano, go to Tahiti." At that time it seemed a difficult thing to do. Today the island is on a direct mail route between San Francisco and Wellington, N. Z.

In 1910 Lacroix published a description of Tahitian rocks which were in the Museum of National History in Paris. Besides abundant basalts the collection embraced coarsely crystallized gabbros, syenites and lavas that are characterized by nephelite and haüynite, minerals generally considered absent from igneous rocks occurring in the region of the Pacific Ocean, but already found to some extent in the Hawaiian and Samoan Islands.

Professor Marshall, of Dunedin, N. Z., has described nephelite-bearing rocks from the Cook Islands and from some of the Leeward Islands in the Society group; and in 1915 he published an account of his journey to the center of the island of Tahiti. From these descriptions it was known that all of these islands are basaltic and that some of them contain trachytes and phonolites, but the relative abundance of these rocks and the more specific characters of the basaltic lavas were not definitely known, and no approach to an estimate of the average magmas of the several islands could be gained from the fragmentary nature of the information at hand.

A reconnaisance of the islands was thought to be worth the effort, and has been productive of valuable results. However, a great deal remains to be done in the way of more accurate detail exploration of the islands than I was able to carry on in the few months at my disposal, and with the limited means at my command.

Tahiti, the largest island in this region, is an extinct volcano, deeply eroded by streams that have cut canyon-like valleys, which radiate from a circular range of high mountains, surrounding the deep central basin heading Papenoo valley, which drains northward. This central basin is 5 or 6 miles across from crests of the encircling range with its peaks of 4000, 5000, 6000, and in Orofena of 7000 feet in altitude.

In the geometrical center of the island, which is in the western head of the Papenoo Valley, there is a low conical hill, Ahititera, composed of coarsely crystallized rocks, gabbros, peridotites with subordinate amounts of nephelite-syenite and other rocks. The surrounding rocks, so far as seen through the forest of tropical vegetation, are basaltic tuff breccias and basaltic lava flows, the latter preponderating in the upper parts of the mountains and in the outlying spurs and slopes of the ancient volcano.

The top of the central hill, Ahititera, is about 2500 feet above sea level, and may be assumed to be about 7000 feet below what was at one time the bottom of the final crater of the volcano, if the mountain had the same profile as the great modern volcanoes of Hawaii. The valley bottoms are nearly level for miles inland from the coast, and their heads lie deep below the summits of the mountain ridges between them. Their sides are very steep, in many places almost vertical walls, thinly covered with clinging ferns and vines.

The lavas composing the Tahitian volcano are basalts rich in ferromagnesian minerals, which commonly appear as prominent crystals. Feldspars are seldom seen, but are prominent in some varieties of the basaltic lavas. There are very few lavas with a trachytic appearance on the main island of Tahiti, though they are common on the lesser volcano of Taiarapu which is connected with Tahiti by a narrow isthmus.

The preponderating lavas of the Taiarapu volcano are basalts like those of Tahiti proper. The smaller volcano has been deeply eroded, and a central valley reveals a core of coarse-grained rocks: gabbros and peridotites, which represent the lavas that solidified in the conduit of this volcano when it became extinct. No syenites were found among the rocks, although trachytes and phonolites are common among the latest lavas in the upper, or marginal, parts of this volcano. Such alkalic feldspathic lavas are clearly differentiates of the basaltic magma, the complimentary varieties being peridotites and peridotitic lavas which are characteristic of these and neighboring volcanoes.

Of the chemical analyses of Tahitian rocks already published by Lacroix two are of preponderating varieties of basalt, and two are of preponderating varieties of gabbro from the central core in Papenoo Valley. The four analyses agree closely with one another and their average may be considered a first approximation to what was probably the parent magma from which all the lavas of Tahiti were derived. The estimated specific gravity of a completely crystallized rock of such a composition is 3.18. The average of the relative densitites of 4 specimens of gabbros and peridotites from the Papenoo Valley is 3.18, and of 4 similar rocks from the central valley of Taiarapu is 3.16. These estimates leave out of account the syenites, trachytes and phonolites of the island, for the amount of these rocks when compared with the bulk of the basaltic volcanoes is almost negligible.

This preliminary estimate of the relative density of the solidified magmas of these two volcanoes may be slightly too high, and is somewhat greater than those calculated for the solidified magmas of Hawaii and Réunion. It clearly indicates that the magma from which the Tahitian volcanoes sprang is sufficiently heavy to correspond to the great depth of the Pacific Ocean in this region according to the theory of isostasy.

Moorea, or Eimeo, is an island volcano ten miles northwest of Tahiti, which has been greatly eroded and partly submerged under the sea. The central portion has been reduced to a deep valley surrounded by high precipitous mountains 2500 to 3500 feet in height. The upper portions are composed of horizontal layers of basalt lava. The central basin is drained northward by two valleys, drowned by the sea, forming bays with a high mountain ridge between. Small, radial, valleys cut the main ridge into short spurs which pitch steeply to the coast. The exposed lava sheets are seen to dip away from the center of the island toward the ocean, and show that the volcano originally was a flat-topped dome, the upper portion of which was mostly solid flows of lava, while the lower middle part of the volcano was largely tuffs and breccia.

No core, or conduit, of coarsely crystallized rocks has been exposed by erosion, which is an indication that the volcano of Moorea was not as high above the sea as those of Tahiti and Taiarapu. The deep bays in Moorea show that this volcano has been flooded by the sea to a considerable depth, while the absence of bays on Tahiti indicates that this island and Taiarapu have not sunken sufficiently to submerge the deeply eroded valleys, although the lower portions of the stream channels are only a few feet above the surface of the sea. The principal lavas of Moorea are basalts like those of Tahiti, but there are subordinate though large bodies of trachytic and phonolitic lavas, and small bodies of peridotitic rocks.

The Leeward Islands, or the Society Group, 100 miles northwest of Tahiti, are similar in structure and in the character of their lavas. Huahine, the most easterly, is a volcano so greatly eroded and flooded that a narrow, shallow, strait separates the northern from the southern portion, and connects the heads of east and west bays. The eroded center of the volcano is submerged in the head of the east bay. The mountains and ridges, with spurs sloping seaward, consist of lava flows nearly horizontal in the central peaks, but dipping down the spurs and ridges in all directions toward the coast. The basaltic lavas have the same general composition as those of Tahiti, but there are mountain masses of phonolite and trachyte in several parts of the island.

Raiatea, the largest island of this group, consists of basaltic breccias at its center, with basaltic lavas in sheets which are horizontal in the upper parts of the central mountains, but slope toward the coast in the spurs. A great sheet of trachytic lava caps the long ridge forming the middle of the northern portion of the island, and reaches the shore at the ends of the northwest spurs. There are also several mountain masses of phonolite. The volcanic center of the island has been eroded to a deep valley draining east and flooded by the sea. There are several other drowned valleys on Raiatea.

Tahaa, which is within the same lagoon and barrier reef as those surrounding Raiatea, is another dissected volcano having a deeply indented coast with bays extending to some distance inland, and having lava flows dipping down the spurs to the sea, and more or less horizontal in the central portion.

Farther west, the smaller island, Bora Bora, is much more reduced by erosion, and consists of a central mountain about 2500 feet in height, with a vertical escarpment of fully a thousand feet which exposes horizontal basaltic lavas. The marginal ridges surrounding two deep bays are the remnants of volcanic slopes in which the lava sheets dip outward toward the surrounding lagoon.

The still smaller island of Maupiti is the remains of a basaltic volcano almost completely submerged in the ocean. The encircling lagoon is relatively larger than those about the less submerged islands farther east. In fact there is a progressive relation between the width and depth of the lagoons around these islands and the apparent submergence of the volcanoes. The most deeply submerged islands and relatively largest lagoons are in the west or northwest, that is, in the Leeward islands, and in the northwestern part of these.

There is less submergence and the lagoons are less well developed at Moorea and Tahiti. However, in this, the Georgian group, Tetiaroa, 26 miles north of Tahiti, is only an atoll, while Mehetia, 60 miles east of Taiarapu is said to be an extinct crater 435 meters high, having a peak to the north and a gentle slope to the south, and having a difficult coast.

The islands of the Marquesan group, 600 miles northeast of Tahiti, are extinct and eroded volcanoes consisting of heavy basalts similar to those of Tahiti and the Leeward Islands, with quite subordinate amounts of trachytic lavas. Erosion has cut deeply into the volcanic mountains in places and has carved steeply walled valleys, surrounded by precipitous mountain ridges, with some commanding peaks, as at Traitors Bay and the Bay of Hanaiapa on Hiva-oa, and along the south coast of Nukahiva. This coast is very rugged with cliffs 1000 feet high, topped by hanging valleys.

The coasts of Nukahiva and of Hiva-oa plunge steeply into the sea which in places is a thousand feet deep within a few hundred feet from shore. No coral reefs surround these islands, though a few fragments of corals are found on the beaches which occur at intervals along the coast. Small reefs of coral are said to occur in some localities in this region. The northwestern side of Nukahiva is a long, gentle volcanic slope reaching sea level.

In the Marquesan Islands, so far as my observations go, sea erosion appears to have progressed more rapidly than stream erosion in places where sea cliffs rise to great heights, with hanging valleys and with waterfalls plunging into the sea. However, there are great differences in the topographic relief in neighboring parts of the same island, as on Hiva-oa, where in the western portion deep valleys have been cut with mountainous walls and sharply serrated dividing ridges, while in the middle portion of the island there are broad expanses of rolling highlands which are drained by shallow channels, or short steep gulches.

The high massive cliffs along parts of the coasts and around some of the largest valleys are probably due to the fact that the upper thousand feet of the ancient volcanoes consisted of superimposed sheets of dense lava, beneath which are breccias and tuffs which yield much more readily to erosion and permit the overlying massive portions to be undermined. From the great depth of the sea off the coast and within the bays, as well as from the absence of coral reefs or of extensive sea shelves, it appears probable that these islands have been submerged to a considerable depth within comparatively recent times.

IN RELATION TO THE EXTENT OF KNOWLEDGE CONCERNING THE OCEANOGRAPHY OF THE PACIFIC

By G. W. Littlehales

U. S. HYDROGRAPHIC OFFICE, WASHINGTON, D. C.

Read before the Academy, April 17, 1916. Received, May 24, 1916.

The body of water whose oceanography is under discussion is of an extent so vast that its area exceeds by 10,000,000 square miles the total land surface of the globe, and its cubical content is estimated to be seven-fold greater than all the land above sea-level. The indications are that throughout nine-tenths of its expanse the depths are greater than one mile, and throughout three-fourths of its expanse the depths are greater than two miles. It is the field of the interplay of many different forces exercising an important influence in terrestrial physics, and presents a realm of unsurpassed promise for the fruits of investigation.

The accumulated oceanographical observations in the Pacific relate principally to the surface and the bottom. Even these are deficient, and the intermediate depths have been much less investigated. The materials from centuries of voyaging and from the expeditions for sounding the ocean sent forth since the last quarter of the nineteenth century, when deep-sea soundings first began to be taken in the Pacific, have provided information of the general distribution of barometric pressure and winds over this vast tract and also of the general aspects of surface circulation, temperature, and salinity; but the details of these matters have scarcely been touched. Until the tides have been gauged in the open ocean away from the land, it is not likely that a clear solution of the tidal problem will be completed. Despite the tidal observations hitherto gathered on the islands of the Pacific and on the coasts of the countries bordering this ocean, the means for the construction of the cotidal lines are either wanting or deficient in many parts.

The manuscript sheets of the Bathymetrical Chart prepared by the Hydrographic Office of the Navy Department, containing all the authentic deep-sea soundings, show the small extent to which the basin has been sounded and the distribution of bottom deposits made known, and prove the inadequacy of existing measurements to define the general contours of configuration. Even the continental shoulder is imperfectly known around most of the ocean circuit. In the North Pacific there is a tract twice as large as the United States which has been crossed by only a single line of bottom soundings about 250 miles apart; and in different parts of the Pacific other tracts as large as the continent of Australia remain entirely unfathomed. The majority of soundings are grouped in two lanes running respectively from the United States to Australia and from the Hawaiian Islands to the Philippines and Japan. No trustworthy contours of equal depth can be drawn beyond the continental shelf, because of insufficient information. Where knowledge is so imperfect, surprises should be in store for those who have accepted the representations appearing in depth-maps of the Pacific in which the isobathic contours have been courageously supplied. The soundings have generally been spaced too far apart to admit of conclusions in relation to even the grosser aspects of the orography, and hence the shape of nearly one-half of the earth is little known.

The deposits on the floor of the ocean have been reached at relatively few points and have generally been penetrated only to the depth of a few inches. The dearth of knowledge of their thickness and stratification makes it desirable to devise methods of bringing up a core several feet long. Although it is extremely doubtful whether these deposits have any analogues in the terrestrial rocks, the interest in the evidences of their stratification arises on other grounds. For instance, if a layer of red clay in great depths should be found overlying a layer of globigerina ooze, the induction would be that the floor of the ocean at this station had once occupied a position at a less depth below the surface of the ocean than the one in which the red clay was deposited; and this would bear strongly on great problems of crustal deformation.

The few records of serial temperatures and of the analyses of water taken from various depths, show that the Pacific bears a close resemblance to the other oceans in the general distribution of temperature, salinity, and density. That is to say, the waters have a gradual increase in density from the surface to the bottom, a gradual decrease in salinity from the surface to the bottom, and a gradual decrease in temperature from the surface to the bottom. But the details of the distribution of these quantities is unknown; of their variations from season to season and from year to year few observations have been made in the depths of the Pacific, and there is as yet very limited knowledge of the import of such changes upon the variations of climate and of physical and biological oceanography.

The observational foundation for investigating the ocean from the standpoint of thermodynamics requires the study in detail of definite stations occupied in concert and periodically revisited for the purpose of observing, as nearly as possible at the same time, such physical conditions, at certain depths, as the temperature, the salinity, the gas-content, and the currents; and in this way affording the means of presenting in the form of synoptic charts the changing network of lines of equal values of the physical elements in their distribution in the depths.

To sum up our thoughts, we may fix attention (1) upon the basin, of which no model can be at present constructed; (2) upon the deposits, whose thickness and stratification still remain unrevealed; (3) upon the waters, whose variations in physical conditions have not yet been sufficiently observed to explain the inner mechanism by which they operate to produce their effects in the economy of the earth.

MARINE METEOROLOGY AND THE GENERAL CIRCULATION OF THE ATMOSPHERE

By Charles F. Marvin

U. S. WEATHER BUREAU, WASHINGTON, D. C.

Read before the Academy, April 17, 1916. Received June 13, 1916

Stations for the surface observation of meteorological conditions are now numerous in the principal civilized countries of the world, and within the past twenty-five or thirty years explorations and investigations of the free upper air have been actively conducted at a number of places on land. In a few instances soundings of the air have been made on the North Atlantic ocean and in waters adjacent to Europe under the initiative of the late A. Lawrence Rotch, Teisserenc de Bort, and others. As a whole, however, little is known in detail of the meteorology of the oceans, except as revealed by the simple observations of weather and wind which many merchant and naval vessels have been accustomed to report for a number of years while plying their regular courses. Observers at sea on such occasions have but scant opportunity or incentive to engage in serious scientific observations, and difficult investigations are impossible. Consequently, what has been obtained in the past through these opportunities, and what may possibly be thus obtained in the future, is limited and restricted in every way, that is, limited as regards the kind and quantity of data that may be obtained, and restricted as to the region or locality to which it applies.

A proposal to launch and equip an expedition to make a scientific exploration of the Pacific Ocean is, therefore, hailed by the meteorologists with enthusiasm.

The aerological investigations now being made at a very few continental stations by means of kites and balloons constitute but a fragment of the evidence and data needed to exhibit the more detailed features of the circulation of the atmosphere. Data from and over the ocean can be obtained only by means of *vessels devoted exclusively* to scientific investigations, and the vast stretches of the Pacific, dotted with its occasional islands that afford useful vantage points as bases of reference, offer a field for such explorations unsurpassed elsewhere.

To state the proposal and indicate the objects of the exploration seem to be all that is necessary to enlist the fullest support thereof. It offers to meteorology the only opportunity possible to obtain full and complete observational data prepared by experienced and competent observers qualified to conduct the difficult exploration of the free air now so much needed. With the few exceptions previously alluded to, upper air explorations have been made only at a small number of continental stations, located at a few points in England, France, Italy, Belgium and Germany. These, in a sense, constitute only a small group in the aggregate and are supplemented by a few additional detached points of observations in the United States.

Both the surface and the free air observations from the ocean are valuable in themselves as supplementing the corresponding continental data, but the opportunity presented by a special expedition moving from point to point in both latitude and longitude on the greatest of all oceans permits of extending observations by the same standard methods to those regions from which the information is most valuable and most needed.

Progress in the development of our knowledge of the upper air and its general circulation awaits accurate observations by the aid of kites and balloons. The scanty observations from a few stations on limited and scattered continental areas show that the atmosphere arranges itself in two or more well-defined layers of different characteristics and extent. Little is known and much is as yet conjectured as to the real boundaries, dimensions, and characteristics of these several strata, even over land areas. Only an expedition like that proposed can suffice to extend such studies to our greatest ocean and over which there is every reason to suppose the atmosphere disposes itself in its best defined, simplest and most orderly arrangement because an ocean represents an almost unlimited extent of level and uniform surface conditions, accompanied by stable and uniform gradients of temperature and other meteorological conditions. The modifying effects experienced at land stations due to their elevation above the sea, their local topographic environment, and other disturbing causes are wholly absent or inappreciable over the ocean, whence marine observation may be expected to supply, not only a kind of data the meteorologist greatly needs, but the best data of that particular kind.

By the aid of suitable recording instruments carried aloft by balloons we may obtain, if the instruments are recovered, a record of the temperature, pressure, and moisture of the air, the sunshine, and possibly some other conditions. However, as a free balloon simply drifts along with the air strata through which it rises and falls, no record of the *motion* of these strata can be procured from the balloon itself. To ascertain the *motions* we must continuously triangulate the successive positions of the balloons. This calls for two or more observers, with appropriate theodolites, located at the ends of a suitable base line. Such observations are often not even attempted as a part of aerological work, but they are indispensable in studies of the circulation of the upper air. No other observations are more urgently needed in meteorology at the present time than these, and it is difficult to conceive of a better field for conducting such observations on a broad and all-inclusive plan and scale than the Pacific Ocean. These motions of the free air, in conjunction with the pressure and temperature thereof, are the data most needed to verify or disprove, or rather, if possible, to adapt and apply Ferrel's general theories to the motions of the air as they may be actually observed on an expedition of the kind proposed.

Emphasis has been laid upon the aerological work which a Pacific exploration could perform, because the need for this is greater, perhaps, than for any other phase of meteorological investigation, and also because the expedition provides such exceptionally favorable opportunities for its prosecution. Nevertheless, the complete study of surface conditions, the observation of clouds, fogs, waterspouts, auroras and lightning, including the photographing of such phenomena, and the intensive study of the inception, development, progress and decline of typhoons and similar marked storms constitute a program of work that claims the careful attention of those who plan the expedition, and can not fail to command the interest and approval of men of science generally.

ON THE DISTRIBUTION OF PACIFIC INVERTEBRATES

By Wm. H. Dall

SMITHSONIAN INSTITUTION, WASHINGTON, D. C.

Read before the Academy, April 17, 1916. Received, May 24, 1916

The distribution of Marine invertebrates is important, as one of the keys to the former distribution of land masses, and to our very imperfect knowledge of their distribution in the Pacific. Certain species, usually those inhabiting the reefs and comparatively shallow water, are very widely distributed over the region usually referred to as Indo-Pacific; but when a careful collection of the species belonging to any isolated island or group is available it becomes evident that a large proportion of them are local and combine to form a local fauna. A knowledge of these faunas is necessary before any satisfactory discussion can be had of the presumably Tertiary fossiliferous deposits which are found fringing the more elevated Pacific islands. The landshells of the Hawaiian and Tahitian groups indicate a high antiquity for their isolation according to Pilsbry, the most eminent student of these animals. The facies of the Tertiary fossils obtained by Ochsner on the Galapagos Islands indicates a derivation from the American rather than the Indo-Pacific fauna, with which the recent invertebrates are commingled. These facts indicate the interest which attaches to a wider knowledge of the Pacific faunas.

THE MARINE ALGAE OF THE PACIFIC

By W. G. Farlow

DEPARTMENT OF BOTANY, HARVARD UNIVERSITY

Read before the Academy, April 17, 1916. Received, May 24, 1916

In considering the desirability of an exploration of the Pacific Islands the following points relating to our knowledge of the marine algae of the Pacific may be mentioned. Our present knowledge is so fragmentary that it is not possible as yet to suggest any special problem of a general nature whose solution would be aided by a well arranged expedition. What the important general questions are we cannot tell until after a more thorough exploration has given us a more detailed knowledge of the species which inhabit the islands of different groups. It may be well therefore to give a general review of what we know from previous explorations of the different portions of the Pacific and at the same time to distinguish between those regions whose marine flora is now being studied successfully either by native algologists or by already organized bodies of collectors and explorers and those regions which are still in need of exploration by some specially organized expedition.

In this connection what is said must be understood to apply to the alga flora proper which is littoral and does not extend to any great depth and of which very few or no representatives are to be expected in deep soundings. In regard to the pelagic or plankton flora it may be assumed that no expedition would fail to make collections wherever and whenever possible.

Our knowledge of the algae of the Pacific has been obtained in part from the collections of the earlier national expeditions, few of which were accompanied by expert algologists, or from collections of a more recent date made by a very small number of residents of different islands who were interested in algae and by botanical travellers stopping for brief periods on their journeys across the Pacific. Islands lying on the trades routes have naturally been more frequently explored than others. In the earlier expeditions, even those whose object was ostensibly the exploration of the Pacific, we notice that the greater part of the time was spent in exploring the outlying islands as Hawaii, Japan, and New Zealand. In the Wilkes Expedition, which may be taken as a type of the old national expeditions, the algae collected were largely from Brazil, the West Coast of America, New South Wales and New Zealand, while hardly fifty species were obtained at Hawaii, the Fiji Islands and Tongatabu, with half a dozen, not including Diatoms, from the Philippines. In the majority of expeditions Hawaii, the Fiji Islands, and the Friendly Islands, were visited. Our knowledge of the Eastern Polynesian groups, as the Society Islands, the Marquesas Islands, and the Low Archipelago is derived largely from the early French expeditions, as that of the Bonite, that of D'Urville, Au Pol Sud, and that of Jardin to the Marquesas. Tahiti, however, was visited by numerous expeditions and it has been a favorite resort of botanical travellers in recent years. On the other hand, of such remote, small islands, as Pitcairn's and the Gambier Islands we have only the scantiest information. We are not to assume, however, that because very little has been found on the smaller islands that there is not much more which may be found.

The Hawaiian Islands have been visited by more expeditions than any other of the Pacific Islands, and their marine flora has lately been studied by competent algologists who have made large collections during a residence there; hence a new expedition will not spend more than a comparatively short time there. The case of the Philippines is peculiar. Although many expeditions have stopped there, the collections have been scanty and probably they do not fairly represent the marine flora of those large islands. Nevertheless it does not seem to be advisable to send a special expedition there for the study of algae because there is now in the Philippines a number of resident botanists who are able to collect year in and year out; and more is to be expected of them than from an expedition moving from one part of the Pacific to another. On the other hand the relation of the flora of the Philippines to that of the Caroline and Ladrone Islands ought to be closely studied. We have a limited knowledge of the species of those two groups obtained from various expeditions; and quite recently the Japanese algologist, Okamura, has published a paper on the Caroline algae. Other regions the relation of whose marine flora should be compared with that of the Pacific Islands are the Bismarck Archipelago and adjoining regions of which there are numerous, short scattered notices and the more eastern islands of the Dutch East Indies where collections have been recently made by the Siboga Expedition. It is hardly necessary to send a new expedition to those regions whose flora is being well studied by competent experts.

After this statement of the regions to which it is inadvisable to send an expedition for the reasons given above, we may turn to a more exact consideration of the regions more in need of exploration. As has been said, the islands lying in or near the trade routes from Australia and New Zealand to North America have not infrequently been visited by botanists. These islands are the Fiji, the Samoa and the Friendly Islands groups. One of the more important papers on these groups is that of Grunow on the algae collected by Von Graeffe, including about 120 species. The distinguished algologist, W. H. Harvey, also issued a set of Friendly Island algae. From these and other papers we know the main features of these islands, but there must be much more to be learned about them. We would advise a new exploring expedition to make either Fiji or Samoa a base of operations. Starting thence as a centre, a year could be well spent in a careful study of the whole group of Polynesian Islands, the Friendly Islands to the Marquesas on the northeast and Pitcairn's Island on the southeast. Another year, or better more, should be spent in traversing the Solomon Islands to the Caroline and Ladrone groups returning by the Gilbert and Marshall Islands to Samoa and thence to Hawaii on the return voyage.

As will be seen, I have assumed that the exploration should last several years. A hurried trip with short stops at a few remote points is not enough at the present day. What we wish is a more detailed knowledge so that the fragmentary facts which we now possess may be fused into a more complete picture of the Pacific Marine Flora. Then and not until then shall we be able to discuss intelligently the question of the distribution and general character of the insular floras and their relation to those of the nearest continents.

The exploration of the marine flora is of course only one of the numerous objects of an expedition to the Pacific, but it is an important object when one considers the great extent of the coast to be studied. The work in this important field should be intrusted to an expert well trained in the study of algae and not left to the chance work of a general collector who, although he might bring back valuable material, could not be expected to recognize the smaller species which are quite as interesting, often more interesting than the larger species.

THE PACIFIC AS A FIELD FOR ETHNOLOGICAL AND ARCHAEOLOGICAL INVESTIGATION

By J. Walter Fewkes

BUREAU OF AMERICAN ETHNOLOGY, WASHINGTON, D.C.

Read before the Academy, April 17, 1916 Received, May 24, 1916

In the following suggestions regarding the Pacific as a field for anthropological investigation the valuable work already accomplished in this branch of study has not been overlooked. The contributions which, from time to time, have been published by local students have been most important, and the admirable reports of expeditions sent out by the different governments of Europe and by local museums, have revealed a wealth of material on the aboriginal inhabitants of the Pacific.

But the field for anthropological investigation in the Pacific is so vast that many additional laborers are needed to gather the plenteous harvest it offers. There is an urgent call for coöperation of many more specialists so that this work may be carried on as part of a coördinated plan, systematically followed. This necessitates more observations on the migrations of the Polynesians, Melanesians, and dark races of the Pacific, to determine their center of origin and diffusion from island to island, a study directly connected with geographical phenomena. Investigations of the colonization of the Pacific islands, of the migration of man from island to island, and of the effect of insular environment on human culture, are very attractive fields of research. Much remains to be investigated in this line of study. Many direct observations are desirable to advance our knowledge of the Melanesians, Papuans, and especially those Polynesians, if any, that are still uncontaminated by civilization and missionaries, with a view to discover ways of primitive thought. We need a comprehensive study of the linguistics of the Pacific islands by linguists trained in modern intensive methods, in order to determine the relationship of the Melanesians and Polynesians. We have no collections of texts, few myths and only a fragmentary knowledge of the material culture of these peoples.

One of the most important lines of work among Pacific islanders is to gather from natives data of this kind which is rapidly being modified and will be lost forever if not gathered in the immediate future. Much remains to be done on the sociology of the Polynesians and Melanesians, notwithstanding the important observations that have already been made. There is an urgent call to collect material on terms of family relationship as a contribution to primitive society. The observations already made should be verified and augmented by new facts regarding the sociology of these fast disappearing natives.

There are many islands which offer facilities for determining the antiquity of man on the Pacific islands. Throughout Polynesia are scattered many prehistoric ruins of which comparatively little is known. Megalithic monuments occur not only on Easter Island, but also on Tahiti, the Marquesas, Tonga, the Carolines and Ladrones. Although Pitcairn island was uninhabited when discovered, huge carved pillars, massive walls and images, indicate the former existence of a characteristic culture. The great stone temple at Atahura in Tahiti, said to be 270 feet long, by 94 feet wide and 50 feet high, was approached by a flight of steps hewn out of coral and basalt rock, showing artistic skill beyond that of the present inhabitants. There are ancient stone platforms in the Marquesas constructed by people of mixed Polynesian origin which rank among the largest in Oceania. On Ponape of the Carolines carved stone images over eight feet high are reported; and at sacred Tonga there is a trilithon monument, consisting of two massive upright stone blocks into which is morticed a sculptured transverse megalith on which is a circular stone basin. Our knowledge of these monuments is very limited. Field work on them is very desirable.

Mr. Christian has published an entertaining account of the so-called Pacific Venice, an architectural marvel, situated in the Caroline Islands. This monument consists of ancient platforms and massive walls made of hewn basalt. It measures 185 feet in length, and from 20 to 40 feet in height. Within its enclosure on a raised inner terrace is a stone mortuary chamber in which ancient chiefs of the island are said to have been buried. The work thus far done by archaeologists on these great monuments is comparatively superficial. Further investigations promise much additional material.

The islands of the Pacific present most instructive problems in physical anthropology. There is every evidence that renewed study in this line would reveal much new material.

The strongest appeal that the anthropologist can make for additional field work on the prehistoric inhabitants of the Pacific and their culture is their bearing on the unity or plurality of origin of man. Culturally the aborigines of America and those of the Pacific islands were in the Stone age when discovered. We naturally look to the Pacific for the cultural kin of the American race. It is desirable that extended observations be made on the Polynesians to supplement what is known of the Stone age of the continents, especially America.

I would naturally lay great stress on a systematic survey of the aboriginal monuments in the Pacific islands to discover their history. The first step would be to make an archaeological reconnaissance to determine the distribution and character of antiquities. Having determined in this way which one of the many sites of human occupation shows superficial evidences of the greatest age, excavation should be made upon it to ascertain its age, history and relation of former inhabitants, as shown by skeletal material, remains of architecture, minor artifacts and other archaeological material.

MID-PACIFIC LAND SNAIL FAUNAS

By H. A. Pilsbry

ACADEMY OF NATURAL SCIENCES OF PHILADELPHIA

Read before the National Academy, April 17, 1916. Received, June 7, 1916

It may seem presumptuous to infer from the distribution of little land snails on scattered Pacific islands, that large Pacific land areas have disappeared in Tertiary time; but if we accept the general principle that similar organisms have evolved from a common ancestry, this bold inference seems justified. Briefly, the reasons are as follows:

The land snails of the Pacific islands, as far as now known, may be divided into two groups: those living on low islands and on the shore zone of high islands, and those inhabiting the forests of high islands. Many of the first group have a wide distribution suggestive of dispersal by human or other adventitious agency and are therefore not significant in the present problem.

The most striking features of the second group in Polynesia and Hawaii are: (*a*) The absence of all the highly evolved continental groups of land snails. There are no Helicidae, no slugs, no carnivorous snails, none of the operculate Cyclophorid snails—in a word, none of the groups that have constituted the bulk of all known continental faunas since the Eocene. (*b*) The prevalence of groups that belong, by their relatively primitive anatomical structure, to early branches of the phyletic tree of the land snails. Some, such as the Amber snails (Succinea), and the Endodonts, have relatives in Mesozoic continental deposits, and still form a small part of the modern continental faunas. Some, like the Helicinidae, are presumed from their structure to be ancient groups, early terrestrial adaptations derived from the most primitive group of marine Prosobranch snails. (*c*) Another series of families, the Achatinellidae, Amastridae, Partulidae, Tornatellinidae and Pupillidae, belong to a primitive group having pallial organs like the aquatic snails Lymnaea and Auricula. They form a primary division of the land snails called Orthurethra. One may compare their relation to the dominant groups of continental snails to the relation Monotremes bear to the placental mammals.

This low group is represented on the continents by such minute forms as the Pupillidae and Valloniidae, which were present in essentially their modern forms in the Eocene, and still seem to exist by grace of their insignificance, occupying a station which no higher group has been modified to fill.

In the island world, this primitive group has had a wonderful evolution. It comprises all of the large snails, and outnumbers all other groups. Shell forms have been evolved paralleling most of those of continental snails, except that up to this time no shell-less, sluglike form has been found. There are adaptations to all varieties of terrestrial and arboreal stations.[1]

The greatest variety of Orthurethra is in the mid Pacific; westward a few forms extend into islands also inhabited by some distinctively continental families of snails. Fiji is the extreme outpost of the latter. There they stop abruptly. The presence of this border of continental snails in the southwest Pacific has been looked upon by Hedley and others as evidence of former continental extension, while some zoogeographers attribute it to over-sea drift. The suggestion may be entertained that here a later fauna has been superposed upon an earlier.

Where shall we look for the source of this unique Pacific fauna? It is known that on the continents the chief existing families of land snails were already established in the Eocene; and that many of the existing genera were represented in the Oligocene. If Pacific faunas were derived as has often been claimed, from waifs drifted thither on 'natural rafts' or carried by birds from the continents, such migration must have stopped as an effective factor in colonization before Tertiary times, for otherwise it seems unaccountable that *all* dominant Tertiary snails of the continents are absent, as with our present knowledge, they appear to be. There is small reason for believing that supposed means of mollusk transportation which have failed during the three or four million years of Cenozoic time, were any more efficient before that. It appears that here, as on the continents, their dispersal has been chiefly by their own known powers of locomotion.

It is difficult to escape the conviction that the archipelagoes of today are superposed upon folds of vastly greater extension, now lost by subsidence, upon which the Pacific snails were evolved, and which served as paths for their migrations. It may be inferred that in the Mesozoic or earlier, continental connections, or close proximity, gave the snails of that time access to the Pacific lands. It is not necessarily inferred that at any one time an area of continental proportions existed. It is of course heterodox to doubt the absolute permanence of oceanic basins, but I must confess that my faith in the good old doctrine has been shaken. It seems conceivable that the doctrine of isostatic balance is not incompatable with the idea that some former areas of low gravity, perhaps intermediate between that of the continents and the sea floors, have been depressed in late geological times.

The limits of my space do not allow discussion of much evidence favoring the distribution of mid-Pacific land snails by former land extensions. The great amount of differentiation among the primitive Pacific snails indicates a very long period of evolution—far longer than any estimate of the duration of the present volcanic islands. The affinities of island faunas are highly suggestive, even though so imperfectly known, of distribution along certain trends. The trade winds have always blown from eastward, yet there is absolutely no snail of a distinctively American group known in the Pacific faunas.

Some light may be thrown upon the subject of inter-island distribution or migration by a brief view at close range of a limited area. In the Hawaiian Islands, where alone I have had personal experience, no large species of land snail is found living on more than one island, but adjacent islands, such as Molokai, Maui and Lanai, have closely related species. The tree snails and allied ground snails inhabit mountain forests, mainly above 1000 feet, but before the destruction of the lower forests they came down to less than half that elevation, though not into the shore zone. Each species has its definite territory, sometimes one, sometimes several adjacent ridges. They bear living young. If their dispersal were effected to any extent by birds, one would expect to find sporadic colonies, at least in a few cases, instead of the logical and definite distribution which is actually found.[2] Inter-island drift upon trees, here as throughout Polynesia, is excluded by the absence of large streams to transport such materials to the shore. The present currents and trades would not drift from island to island. Finally, the conditions on the shore zone are such that a forest snail thrown upon the strand could not exist.

The Pleistocene deposits of land shells, though only now beginning to be explored, show that even then the island fannas were more closely related than at present. Numerous relatively large species were common to two or more islands, while at the present day there are none. All the facts are explicable on the theory that the islands have been successively isolated by subsidence. Otherwise the distribution of the snails seems enigmatic.

It is obvious that speculation upon the origin and migrations of the Pacific faunas, such as I have been allowing myself to indulge in, will have a vastly broader basis when we have adequate knowledge of the faunas of all the principal high islands. Hypotheses which now seem reasonable may be confirmed, or they may be definitely excluded from further consideration. All the high islands of the Caroline, Austral and Hervey groups should be investigated. In Samoa and the Marquesas we certainly know only a fragment of the fauna. Outlying islands, such as Rapa, specially need examination. On the American side, Juan Fernandez and Mas-a-fuera call for investigation.

At present it may be said that we have a good, though by no means complete knowledge of the snails of the Hawaiian Islands, which resident naturalists are zealously working on. In the Society Islands, through the work of Andrew Garrett, we have a fair knowledge. In Fiji, Tonga and scattered islands of other groups, it is likely that most of the large forms are known, the small and minute scarcely at all. Work on land snails has been sporadic, in large part done by chance visits of naturalists whose chief interest was in other subjects. Everywhere material must be preserved for anatomical study, as the affinities of many land snails can be worked out only from the soft parts, and at present we are guessing at the taxonomy in some of the families. Outside of Hawaii but little material has been collected for anatomical study.

A word as to the 'low islands.' A group of small or minute land snails has been adapted to atolls and the shore zone of high islands.

Many of the species have a wide distribution on different islands or different groups. To be found in any coconut grove, hiding under leaves or wood to which they stick fast in dry weather, they might easily be carried about by human agency. Some groups of small operculate snails, such as Truncatella, live only near the sea, where the salt breeze blows. These seem particularly adapted to distribution by flotsam, yet even they fail on the more outlying islands, some of which have rich faunas of forest snails. The atoll faunas are poor. Thus, Funafuti has 7 species, Palmyra 1; none being peculiar to these islands. Contrast Hawaii, with over 800 species on the 8 islands, or the far less explored Society group with over 100.

Except in Hawaii, such investigation as is needed in the Pacific is beyond the reach of the individual naturalist. It must be done soon, or much of the data will surely be lost forever. The introduction of continental plants and insects, particularly the all-devouring ants, of goats and cattle, of snails and slugs from Europe, Java, Japan and China, has already made serious inroads upon the native faunas and floras. Every year sees the passing of some island species, perhaps a link in the chain of evidence which cannot be replaced. On some small islands, such as Kahoolawe (south of Maui, area 69 square miles), the destruction has already been completed. The native fauna and flora, with whatever they had to tell us, has disappeared in the last 50 years or less, except as we can recover fragments from the aeolian deposits in ravines. Over many miles the very humus has blown away, and desolation like a lunar landscape meets the eye. Such a fate awaits many a Pacific island, we cannot doubt. A comprehensive scientific exploration of these unique faunas must be made while yet it can be done, before they are mutilated beyond restoration.

[1] The snails of the group Orthurethra best known to the general zoological reader are the Achatinellidae, which formed the basis of Gulick's contributions to evolution doctrinex, brought together in his *Evolution, Racial and Habitudinal*, published by the Carnegie Institution of Washington, 1905. Also the Partulidae, which many travelers in Polynesia and Micronesia have noticed.

[2] The data may be found in the writer's *Manual of Conchology*, vols. 21–23.

SOME PROBLEMS OF THE PACIFIC FLORAS

By Douglas H. Campbell

DEPARTMENT OF BOTANY, LELAND STANFORD UNIVERSITY

Read before the Academy, April 17, 1916. Received, July 6, 1916

No botanist, who is at all acquainted with the rich and varied floras of the different regions of the great Pacific area, can fail to be impressed with the numerous important problems that present themselves—many of which have as yet received but scant attention. Perhaps most important of all are those dealing with the origin and distribution of these rich floras.

First, and perhaps foremost, is the collecting of data for a study of the relationships existing between the floras of the great Continental areas of the Southern Hemisphere, viz.: Australasia, South America, and South Africa. These areas, so completely separate at present, nevertheless show such similarities as to suggest a common origin for many of their elements. Such a study involves a consideration of the existing floras, which are pretty well known, but also an investigation of the fossil floras, in regard to which we are by no means so well informed as might be wished. Perhaps the fossils collected in the recent Antarctic Expeditions may help us, and, of course, the investigations of the geologists in regard to possible former land connections must be taken into account.

Second, a study of the floras of the innumerable islands of the Pacific should afford much material for many important generalizations. While our knowledge of the special floras—e.g., of the Galapagos Islands and Hawaii, is reasonably complete, there remains room for work concerning the origin of these floras and their later evolution.

The Hawaiian Islands, especially, offer what is probably the most favorable field in the world for the study of the evolution of plant forms. Hillebrand, in his excellent *Flora of the Hawaiian Islands* gives in the introduction (unfortunately left uncompleted by his untimely death), some most valuable suggestions of the possibilities afforded by a study of the flora of Hawaii. It is possible that further work has been done in regard to this matter since Hillebrand's, but if so, I am not acquainted with it. It is generally believed that these islands have always been thus isolated, and while this is not universally admitted, at any rate any former land connections must have been of a very remote period.

A most striking feature of Hawaii is the different ages of the islands which have been formed in succession, beginning with Kauai at the northern end of the group. The next and largest island, Hawaii, is still in process of formation.

Hillebrand noted the extremely interesting fact that Kauai, although much smaller than the southern islands, Maui and Hawaii, and with much less range of elevation, is botanically much richer than either of those islands, and contains a much larger proportion of endemic species. It would seem then, that a study of these endemic plants and their distribution in the islands ought to throw much light upon the processes of species formation.

Some of the factors concerned in this remarkable degree of endemism are evident—notably the extreme isolation of the islands; but the agencies that have acted in the islands themselves are not all so clear. The very large percentage of endemic species which, according to Hillebrand, comprises about 80% of the higher plants, suggested to him the possibility of the islands being a centre of special creative energy.

It might be asked perhaps, whether, in newly formed areas, especially those of volcanic origin, there is some factor present which tends to produce variation in plants introduced to them. This is a topic which, as far as I know, has never received attention. It is notorious that volcanic soils are extremely fertile, and some of the richest floras of the world are found in volcanic regions. May it not be possible (this suggestion is offered with some diffidence), that new elements in recent volcanic soils may have an effect, stimulating or otherwise, upon newly introduced plants, causing changes which, fixed by time, result in new species. This is a topic which the numerous volcanic islands of the Pacific should help to elucidate, and one which it seems would be quite worth while investigating.

This question was suggested by observations which I made a few years ago while collecting liverworts in the Malayan region. In Java, which is entirely volcanic in its formation, the liverworts, as well as other types of vegetation reach an astonishing development, both in number and variety, and a comparison of other regions where these plants abound, shows that most of them are volcanic—e.g., Hawaii, New Zealand, the Philippines, Sumatra, and Japan. On the other hand, the granitic formations of the Malay Peninsula and North Borneo, climatically quite similar to Japan and Sumatra, are noticeably deficient in liverworts.

Third, the agencies by which plants reach such remote regions as the Hawaiian Islands, for example, have been repeatedly studied, but very much can still be done, and apparently the amount of accurate experimental work in this direction is not large. Hillebrand thinks a most important agency is that of certain birds. He mentions specially, a species of plover, and the tropic bird. As is well known, Darwin brings out this point, having examined the mud adherent to the feet of certain migratory birds, in which be found seeds of many kinds. A complete investigation of this topic would probably be worth while. The rôle of air and water currents as agencies in plant distribution, still affords a large field for investigation.

In connection with the study of the island floras, especially those of the volcanic islands, it would be very desirable to have careful observations made on the reëstablishment of the vegetation on areas covered by recent volcanic ash or lava. Such observations, for example, as those made upon Krakatoa after the great eruption of 1883. It would probably not be possible to find so complete a sterilization of a whole island as was then the case, but if by chance any similar occasion should arise, the opportunity should certainly not be lost.

The study of the reëstablishment of the vegetation on such a sterilized area would be of great scientific value; and probably results quite worth while might be had from an investigation of the development of vegetation on less extensive new lava flows, or areas covered with fresh volcanic ash. By such studies it might be possible to determine what effects, if any, are produced upon plants growing in lava or ash, of different chemical or physical properties.

The Pacific Coast of America, both north and south, offers many interesting problems, dealing with various phases of plant origin and distribution, and probably California would prove the richest field for such studies.

Aside from the question of variation in the native plants, and the reasons for the high percentage of endemism involving the question of mutation, etc., there are various other points to be considered. If California, like Hawaii, is to be looked upon as a centre of 'formative activity,' the reasons for this need to be carefully and thoroughly investigated.

California has received additions to its flora both from the north and from the south; and there is much to be learned as to the means by which these immigrants reached California, and just where was the home whence they came. Thus, of the northern elements of the flora there is a small infusion of Asiatic and Eurasian types, such as the Tan-bark Oak, the Giant Equisetum, species of Fritillaria, the Sitka Spruce, Madroño (Arbutus), etc. Probably a thorough study of the fossil plants of the State will throw light on some of these questions, and a beginning has already been made in this direction.

Finally, and this applies to all the regions involved, there is to be taken into account the many species, weeds and cultivated plants, which have been introduced through the agency of man. A study of these immigrants—the origin of most of which can be fairly well ascertained, especially when they have been long established in their new habitat—ought to be one of the best means of determining the effects, if any, of a changed environment. The vexed question of the inheritance of such acquired changes can thus be studied on a large scale and under normal conditions. It is surprising that so little attention has been paid to this great field by the many students of mutation and genetics who set the fashions in biology just now.

These are but a few of the many problems which might be undertaken by the botanist in this vast region which it is proposed to explore. It is perhaps presumptuous for one whose work has been mainly in other directions, to suggest what are the most important lines of work in fields in which he is very much of an amateur.

NERVOUS TRANSMISSION IN SEA-ANEMONES

By G. H Parker

ZOÖLOGICAL LABORATORY OF THE MUSEUM OF COMPARATIVE ZOÖLOGY AT HARVARD COLLEGE

Received by the Academy, June 1, 1916

The retraction of a sea-anemone like Metridium can be accomplished by stimulating mechanically almost any point on its exterior. Hence there must be nervous connections between all points on the surface open to stimulation and the longitudinal muscles of the mesenteries by which retraction is brought about. To ascertain how extensive these connections are, the following experiments were tried. The column wall of a Metridium was completely girdled by a cut deep enough to penetrate the wall but not sever the mesenteries. Stimulation of either the oral or pedal portion of the partly divided animal resulted in general retraction. Complete removal of the oral half left the pedal half capable of retraction. Tongues of column wall cut in any direction and as long as five centimeters transmit when stimulated at their free ends to the longitudinal muscles thus causing retraction. If a Metridium is cut through vertically except for the pedal disc or a piece of the column or the oral disc, the stimulation of one piece will cause the whole animal to retract. If an oblong outline is cut on the column wall so that the encircled area remains attached to the rest of the animal only by the mesenteries, stimulation of this area is followed by the retraction of the animal as a whole. It thus appears that the connections between the surface of the sea-anemone and the deep seated muscles concerned with retraction are so numerous and devious that a nervous network is the only basis of explanation. That this nervous network is not equally developed in all parts of the animal's body is seen from the fact that when a sea-anemone is cut vertically in two except for the lips, it is very difficult to get a retraction in one half of the body when the stimulus is applied to the other half. The lips are poor means of transmission compared with other parts of the body. Notwithstanding the generally diffuse condition of the transmission system in Metridium, there is evidence also for a certain degree of specialization in the parts concerned. Stimulation of the tentacles by mussel juice calls forth a gaping of the oesophagus (contraction of the transverse mesenteric muscles) and by weak hydrochloric acid a retraction of the oral disc (contraction of the longitudinal mesenteric muscles). These two forms of response afford good ground not only for the assumption of independent receptors but of relatively independent transmission tracts, a first step in the kind of differentiation so characteristic of the nervous organization in the higher animals.

The extended paper will be published in the *Journal of Experimental Zoölogy.*

THE RESPONSES OF THE TENTACLES OF SEA-ANEMONES

By G. H. Parker

ZOÖLOGICAL LABORATORY OF THE MUSEUM OF COMPARATIVE ZOÖLOGY
AT HARVARD COLLEGE

Received by the Academy, June 1, 1916

As long ago as 1879 von Heider announced that tentacles severed from a sea-anemone were capable of much the same range of activities that these organs exhibit when normally attached to the animal. This statement has been variously accepted or questioned by subsequent workers. Favorable material for testing its validity was found in the Bermudian sea-anemone Condylactis. The tentacles of this form may measure as much as 15 cm. in length and may have a basal diameter of 1.5 cm. Severed tentacles from Condylactis contract and remain so for some time. They can be brought to a state of least disturbance by suspending them on a metal hook in seawater. Under such circumstances they can be inflated by gently running seawater into them till they attain about two-thirds their ordinary length. In this condition they are under a pressure of not over a few millimeters of water. If this internal pressure is much increased, they will contract vigorously and discharge more or less of their contents. The slightly contracted state of the excised tentacle is therefore not due to lack of pressure. It is also not due to the absence of inhibitory influences from the rest of the animal, for, if an attached tentacle is partly cut into at its base, it exhibits the same partial contraction that the excised tentacle does. It is highly probable that the incomplete expansion of severed tentacles is due to the operative complications which are involved in cutting it and which induces a heightened tonicity in its musculature.

When an excised suspended tentacle is stimulated mechanically or by the application of weak acetic acid, mussel juice, or a weak solution of quinine, appropriate responses are called forth that differ from those of the attached tentacles only in that they are feebler and less precise. As this modified form of response is also noticeable in attached tentacles which by previous stimulation have been partly contracted before the particular stimulus was applied, it is believed that the feebleness and lack of precision in the reactions of the excised tentacles is due to their partly contracted condition and not, for instance, to the loss of central influences. Since the excised tentacles exhibit a range of responses like those of the normal ones except in so far as operative conditions modify them, it seems clear that the tentacles of sea-anemones must contain within themselves the neuromuscular mechanism essential to their activities as originally suggested by von Heider.

When various stimuli are applied to the ectoderm of an excised tentacle, they are followed quickly by a muscular response. When they are applied to the entoderm of a tentacle by being injected into its interior, they are also followed by the same form of response but only very slowly. This slowness in reaction is believed to be due to the non-receptive character of the entodermal surface and to the necessity for the stimulating material to the wall of the tentacle before it can reach the receptive ectoderm on the outside. Such a transfusion has been demonstrated in the case of weak acetic acid.

The tentacles of Condylactis like those of other sea-anemones, exhibit polarity. Their cilia beat always from the base of the tentacle towards its tip. If the tentacle is stimulated mechanically at a particular spot, the resultant muscular activity spreads from the stimulated spot almost exclusively toward the base of the tentacle. The ciliary polarity is unaffected by anesthetics, but that of the muscles largely disappears under such treatment. The muscular polarity seems, therefore, to depend upon a nervous condition and is probably due to the fact that the majority of nerve-fibers from the ectodermal sense-cells extend toward the base of the tentacle, not toward its tip, and thus deliver impulses proximally rather than distally along the tentacle.

From these various observations, it is concluded that the tentacles of sea-anemones, in contradistinction to such appendages as those of the arthropods and of the vertebrates, must be regarded as containing within themselves a complete neuromuscular mechanism by which their responses can be carried out quite independently of the rest of the animal. These organs thus possess great autonomy and act in harmony and unison more because of simultaneous stimulation than of subordination to a common nervous centre.

The full paper will be published in the *Journal of Experimental Zoölogy.*

CONTENTS

Page

Physics.—On The Mobilities of Gas Ions in High Electric Fields . . . . . *By Leonard B. Loeb* 345

Anatomy.—The Relation of Myelin to the Loss of Water in the Mammalian Nervous System with Advancing Age . . . . *By Henry H. Donaldson* 350

Zoölogy.—Differential Mitoses in the Germ-Cell Cycle of Dineutes Nigrior *By R. W. Hegner and C. P. Russell* 356

Mineralogy.—Some Minerals from the Fluorite-Barite Vein Near Wagon Wheel Gap, Colorado . . . . . . *By Esper S. Larsen and Roger C. Wells* 360

Physiology.—The Processes Taking Place in the Body by Which the Number of Erythrocytes Per Unit Volume of Blood is Increased in Acute Experimental Polycythaemia . . . . *By Paul D. Lamson* 365

Physiology.—The Influence of Morphin Upon the Elimination of Intravenously Injected Dextrose in Dogs . . *By I. S. Kleiner and S. J. Meltzer* 369

Astronomy.—The Work of the American Meteor Society in 1914 and 1915 . . *By Charles P. Olivier* 372

Physics.—The Light Excitation by Slow Positive and Neutral Particles . . *By A. J. Dempster* 374

Astronomy.—An Apparent Dependence of the Apex and Velocity of Solar Motion, as Determined from Radial Velocities, Upon Proper Motion . . *By C. D. Perrine* 376

Physics.—Channeled Grating Spectra, Obtained in Successive Diffractions . *By C. Barus* 378

Genetics.—The Effect of Parental Alcoholism (and Certain Other Drug Intoxications) Upon the Progeny in the Domestic Fowl. *By Raymond Pearl* 380

Zoölogy.—The Effectors of Sea-Anemones . . . . *By G. H. Parker* 385

Astronomy.—Preliminary Evidence of Internal Motion in the Spiral Nebula Messier 101 . . . . *By A. van Maanen* 386

Symposium on the Exploration of the Pacific:

- The Exploration of the Pacific . . . . *By W. M. Davis* 391
- The Importance of Gravity Observations at Sea on the Pacific . . . . . *By John F. Hayford* 394
- A New Method of Measuring the Acceleration of Gravity at Sea . . . *By Lyman J. Briggs* 399
- The Problem of Continental Fracturing and Diastrophism in Oceanica *By Charles Schuchert* 407
- The Petrology of Some South Pacific Islands and its Significance . . . *By Joseph P. Iddings* 413
- In Relation to the Extent of Knowledge Concerning the Oceanography of the Pacific . . . . *By G. W. Littlehales* 419
- Marine Meteorology and the General Circulation of the Atmosphere . *By Charles F. Marvin* 421
- On the Distribution of Pacific Invertebrates . . . . . . *By Wm. H. Dall* 424
- The Marine Algae of the Pacific . . . . *By W. G. Farlow* 424
- The Pacific as a Field for Ethnological and Archaeological Investigation *By J. Walter Fewkes* 427
- Mid-Pacific Land Snail Faunas . . . . *By H. A. Pilsbry* 429
- Some Problems of the Pacific Floras. . . . . *By Douglas H. Campbell* 434

Zoölogy.—Nervous Transmission in Sea-Anemones. . . . . *By G. H. Parker* 437

Zoölogy.—The Responses of the Tentacles of Sea-Anemones. *By G. H. Parker* 438

VOLUME 2 AUGUST, 1916 NUMBER 8

PROCEEDINGS

OF THE

NATIONAL ACADEMY OF SCIENCES

OF THE

UNITED STATES OF AMERICA

FOUNDED 1863

JUN 24 [illegible]
National Museum

EDITORIAL BOARD

Arthur A. Noyes, *Chairman*
Edwin B. Wilson, *Managing Editor*
Arthur L. Day, *Home Secretary*
George E. Hale, *Foreign Secretary*

J. J. Abel
W. B. Cannon
J. McK. Cattell
J. M. Clarke
E. G. Conklin
J. M. Coulter
C. B. Davenport
Simon Flexner
E. B. Frost
R. G. Harrison
W. H. Holmes
J. P. Iddings
R. A. Millikan
E. H. Moore
Alexander Smith

Publication Office: Williams & Wilkins Company, Baltimore
Editorial Office: Massachusetts Institute of Technology, Boston
Home Office of the Academy: Washington, D. C.

Entered as second-class matter at the Post Office at Baltimore, Md., under the Act of August 24, 1912

INFORMATION TO CONTRIBUTORS

The Proceedings is the official organ of the Academy for the publication of brief accounts of important current researches of members of the Academy and of other American investigators, and for reports on the meetings and other activities of the Academy. Publication in the Proceedings will supplement, not replace, that in journals devoted to the special branches of science. The Proceedings will aim especially to secure prompt publication of original announcements of discoveries and wide circulation of the results of American research among investigators in other countries and in all branches of science.

Articles should be brief, not to exceed 2500 words or 6 printed pages. Technical details of the work and long tables of data should be reserved for publication in special journals. But authors should be precise in making clear the new results and should give some record of the methods and data upon which they are based. The viewpoint should be comprehensive in giving the relation of the paper to previous publications of the author or of others and in exhibiting where practicable, the significance of the work for other branches of science.

Manuscripts should be prepared with a current number of the Proceedings as a model in matters of form, and should be typewritten in duplicate with double spacing, the author retaining one copy. Illustrations should be confined to text-figures of simple character, though more elaborate illustrations may be allowed in special instances to authors willing to pay for their preparation and insertion. Particular attention should be given to arranging tabular matter in a simple and concise manner.

References to persons should include initials; citations of literature, numbered consecutively, should be placed at the end of the article; and short footnotes should be avoided. Indicate 'subject' for 'Contents.'

Papers by members of the Academy may be sent to Edwin Bidwell Wilson, Managing Editor, Mass. Institute of Technology, Boston, Mass. Papers by non-members should be submitted through some member.

Proof will be sent to authors only on their request, in order to avoid delay in publication. Authors are therefore requested to make final revisions on the typewritten manuscripts. The editors cannot undertake to do more than correct obvious minor errors.

Reprints should be ordered at the time of submission of manuscript. They will be furnished to authors at cost, approximately as follows:

Reprints of - -	2 pp.	4 pp.	6 pp.	8 pp.	Covers extra
Charge for first 100 copies	$1.00	$1.30	$2.20	$2.20	$2.00
Charge for additional 100s	.25	.45	.85	.85	.50

INFORMATION TO SUBSCRIBERS

Subscriptions at the rate of $5.00 per annum should be made payable to the National Academy of Sciences, and sent to Williams & Wilkins Company, Baltimore, or Arthur L. Day, Home Secretary, National Academy of Sciences, Smithsonian Institution, Washington, D. C. Single numbers, $0.50.

Copyright, 1916, by the National Academy of Sciences

JUN 24 1921
National Museum

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES
Volume 2 AUGUST 15, 1916 Number 8

THE ABSORPTION COEFFICIENTS OF SOFT X RAYS

By C. D. Miller
RYERSON PHYSICAL LABORATORY, UNIVERSITY OF CHICAGO

Received by the Academy, June 14, 1916

The X rays studied were produced at potentials ranging from 2500 to 10,000 volts. The tube (see fig. 1) in which they were generated was provided with a hot cathode, *C*, consisting of a flat spiral of tungsten wire, similar to the cathode in a Coolidge tube. The anode was of iron, but this has no bearing on the work reported here, as the characteristic radiation of iron did not appear in noticeable quantity. The rays escaped through an aluminum window, *W*, of 0.001823 grams per square centimeter, or of 0.000675 centimeters thickness, supported on wire gauze of 180 meshes to the inch. The rays, after passing through the sheets of absorbing material, entered the chamber of the gold leaf electroscope, by means of which their ionizing effect was measured.

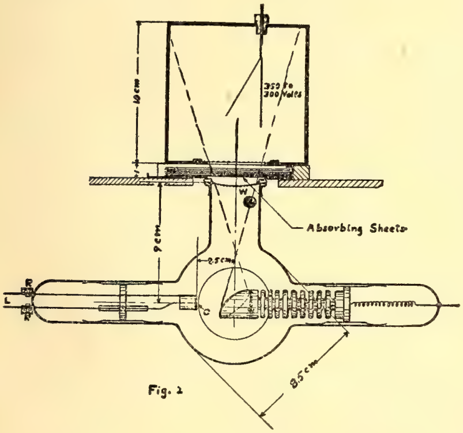

10cm
350 to
300 Volts
W
Absorbing Sheets
9cm
2.5cm
R
L
R
C
8.5cm

Fig. 1

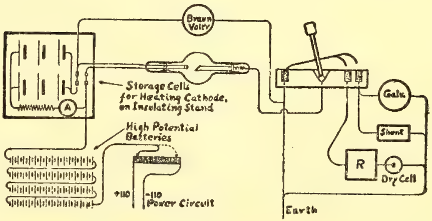

Brown Voltr
Storage Cells for Heating Cathode, on Insulating Stand
A
High Potential Batteries
+110
-110
Power Circuit
Galv.
Shunt
R
Dry Cell
Earth

Fig. 2

The electrical connections are shown in figure 2. The cathode current from the tube was balanced by the current from the dry cell, when the resistance R was properly adjusted, by throwing the three point key to the right, and the galvanometer indicated when a balance was obtained.

The absorbing material consisted of sheets of gelatin, celluloid, aluminum foil and ordinary sheet aluminum. As the ratios between the absorption coefficients in these substances were found to be independent of the nature of the X rays, the observed absorption coefficients in gelatin and in celluloid have been reduced to equivalent absorption coefficients in aluminum, and the thickness of absorbing material through which the X rays past before the absorption coefficient was observed, has been similarly reduced to the equivalent thickness of aluminum. The ratios used for this reduction are

$$\frac{(\mu/\rho)\text{ celluloid}}{(\mu/\rho)\text{ aluminum}} = 0.171, \qquad \frac{(\mu/\rho)\text{ gelatin}}{(\mu/\rho)\text{ aluminum}} = 0.162.$$

μ represents the absorption coefficient and ρ represents density, hence μ/ρ is the mass-absorption coefficient.

The results of the investigation are shown graphically in figure 3. The values of μ/ρ for the six higher potentials manifestly come to a steady value with increasing thickness of absorbing material.

In figure 4 are plotted log V and log μ/ρ in which the values of μ/ρ are the limiting values, for the higher potentials, and the lowest values obtained, for the lower potentials. The points for the six higher potentials lie very close to a straight line, the slope of which indicates

$$\frac{\mu}{\rho} V^{2.77} = \text{constant}.$$

The other points do not represent the limiting values of μ/ρ, and hence would be expected to lie above this line. The mean value of the constant is 4.24×10^{12}; hence

$$\frac{\mu}{\rho} = \frac{4.24 \times 10^{12}}{V^{2.77}}. \qquad (1)$$

The six values of μ/ρ as given by this formula agree to within less than one per cent with the observed values.

If the 'end' radiation, which is homogeneous according to the absorption test, has a definite frequency given by the quantum theory relationship

$$Ve = h\nu, \tag{2}$$

then

$$\frac{\mu}{\rho} = 19.4\lambda^{2.77}. \tag{3}$$

The mass per square centimeter of aluminum, ρd, which was found necessary to render the X rays approximately homogeneous, is given, as nearly as could be observed, by

$$\rho d = 10^{-12} \times V^{2.77}. \tag{4}$$

It has been known for some time that an equation of the form of (3) would represent roughly the relationship between absorption coefficients and wave lengths, but the exponent has been estimated by Bragg,[1] who is perhaps the foremost observer, at from 2.5 to 3.0, whereas these results indicate it to be 2.77 ± 0.03. Bragg's estimate is based on the absorption coefficients of monochromatic rays obtained by the use of the X ray spectrometer. It has also been known[2] that X rays produced at steady potentials become homogeneous after passage through a sufficient thickness of absorbing material. However, no formula such as (4) has been given by which to estimate the necessary thickness. Data similar to those on which equation (1) is based have been used by Duane and Hunt[3] and by Rutherford, Barnes and Richardson[4] to calculate wave lengths by means of relationships similar to (3),

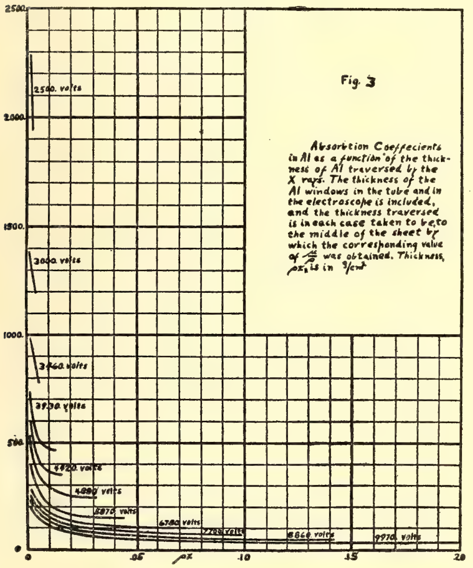


Fig. 3

Absorbtion Coeffecients in Al as a function of the thickness of Al traversed by the X rays. The thickness of the Al windows in the tube and in the electroscope is included, and the thickness traversed is in each case taken to be, to the middle of the sheet by which the corresponding value of $\frac{\mu}{\rho}$ was obtained. Thickness, ρx, is in g/cm^2

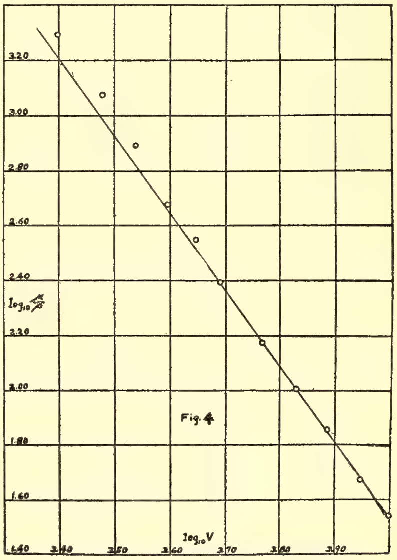


Fig. 4

although the constants in this equation were not accurately known, and thus to test the quantum relationship (2). The results of Hull[5] indicate clearly that the quantum relationship (2) holds for the maximum or limiting frequency of the X rays produced at high potentials, but although the range of potentials used by Rutherford (13,200 to 175,000 volts) was sufficient for the accurate determination of the constants of (1) or (3), they cannot be determined from his results, either because such relationships do not hold for very high potentials, or because the X rays had not passed through a sufficient thickness of absorbing material to give the limiting values of the absorption coefficients. The X rays produced at the potentials used by Rutherford and by Duane are of such penetrating power that no special window such as W, figure 1, was necessary. X rays produced at the potentials used in this investigation (2,500 to 10,000 volts) are of such low penetrating power that it was necessary to make the window W as thin as could be made to withstand atmospheric pressure, and to use the thinnest sheets of absorbing material that could be obtained, and still have them of uniform thickness.

Summary.—A relationship such as (3) has been known for some time to hold approximately for X rays of the ordinary range of penetrating power, but the constants have not previously been accurately determined. These results indicate that such a relationship holds for very much softer X rays, and values of the constants have been determined with considerable accuracy. These values are in agreement with those which hold for the higher potentials, as nearly as the latter have been determined.

---

[1] W. H. Bragg and W. L. Bragg, *X Rays and Crystal Structure*, p. 180. W. H. Bragg *Phil. Mag.*, March, 1915.

[2] Wm. R. Ham, *Physic. Rev.*, January, 1910.

[3] Duane and Hunt, Proc. Amer. Phys. Soc., *Physic. Rev.*, August, 1915.

[4] Rutherford, Barnes and Richardson, *Phil. Mag.*, September, 1915.

[5] Albert W. Hull, Proc. Amer. Phys. Soc., *Physic. Rev.*, January, 1916.

FURTHER EVIDENCE AS TO THE RELATION BETWEEN CROWN GALL AND CANCER

By Erwin F. Smith

LABORATORY OF PLANT PATHOLOGY, UNITED STATES DEPARTMENT OF AGRICULTURE

Read before the Academy, April 18, 1916. Received, June 15, 1916

1. *Fundamental concepts.*—It is my belief that a diligent study of tumors in plants will help to solve the cancer problem. Expressed in other words, my text is this: Cancer occurs in many kinds of plants in which it passes through an essentially parallel course of development

to that of cancer in man and animals, some allowance, of course, being made for differences in the structure and development of plants. My concept is this, that fundamentally plants and animals are alike, that physical and chemical laws apply equally, that is uniformly, to all living things, and hence that discoveries relative to the fundamental cell-mechanics of animals apply equally to plants, and vice versa.[1] If cancer occurs in plants and is due to a parasite, as I maintain, then cancer in man must also be due to a parasite, and since I show that several kinds of plant cancers are due to the same organism, the differences being due solely to unlike tissue reactions, we may anticipate the same thing to be true in man, and search for one parasite rather than many.

2. *Human and animal tumors for which no cause has been discovered.*—These tumors fall readily into two groups which intergrade more or less. These are (*a*) encapsuled benign tumors, and (*b*) free-growing malignant tumors or cancers. I will here consider only the malignant tumors. There are several types of these; the simplest and the commonest being proliferations (1) of the connective tissues, resulting in fleshy tumors known as *sarcomas*, and (2) of the various kinds of epithelium, resulting in eroding tumors known as *carcinomas*. In addition to tumors of the brain and nervous system, which are sometimes malignant but which may here be left out of account, there are (3) very curious tumors containing mixed tissue elements of two germ layers (connective tissues, cartilage, striped and plain muscle-fibres, glandular tissue, etc.) some of which are benign while others are extremely malignant, and finally, (4) embryonal teratomas which, in addition to the cancerous element, contain numerous foetal fragments, derived from all three of the primitive germ layers, often a great variety of tissue-fragments, variously fused and jumbled, but all embryonic and never coming to maturity.

The leading characteristics of these cancerous tumors are (1) Unlimited cell proliferation emancipated from bodily control; (2) Appearance of destructive secondary tumors, often in vital organs, which tumors are derived from the primary tumor either by development from dislodged creeping and floating tumor cells, or by uninterrupted growth from a chain of tumor cells originating in the primary tumor. These secondary tumors wherever they are developed repeat more or less closely the structure of the parent tumor. Add, to these manifestations, (3) a destructive action on surrounding tissues, which are crushed, absorbed or poisoned, and we have in brief the main phenomena of cancerous growth. The other phenomena are subsidiary and may be left out of account in this brief abstract.

3. *Earlier discoveries in plants.*—In 1907 the author and his colleagues (Townsend and Brown) proved that a common hyperplasia of various cultivated plants which had been for many years under observation in this country and in Europe and for which various causes had been inconclusively assigned (frosts, physiological disturbances, mites, myxomycetes, etc.) was really due to a polar flagellate schizomycete, named by us *Bacterium tumefaciens.* This was first cultivated from tumors on the Paris daisy in 1906, but afterwards it was obtained from various other plants, in several distinct strains (daisy strain, hop strain, etc.) and shown to be cross-inoculable on a variety of plants with resultant tumors in which no bacterial cavities or bacterial occupation of cells, vessels, or intercellular spaces could be seen in fresh material or demonstrated in serial sections by stains, and yet from which the causal organism was readily cultivable in small numbers by the common methods of the bacteriologist.

4. *Further discoveries.*—In 1911 I discovered in inoculated Paris daisies a parenchymatous tumor-strand connecting the primary stem tumors with deep-seated secondary tumors which had developed in the leaves some weeks after the stem inoculations. I also showed these secondary leaf-tumors to be possessed of stem structure, i.e., to have the structure of the primary tumor, thus following the law of cancer in man and animals. These discoveries were fully illustrated and described at that time in United States Department of Agriculture bulletins[2] and elsewhere. From the origin, structure and general appearance of these tumors I likened them to sarcomata.

5. *Other resemblances of crown gall to cancer in man and animals.*—Without entering into details which may be found elsewhere a few other resemblances are here summarized: (1) No parasite visible; (2) Embryonic character and rapid multiplication of the proliferating cells; (3) Loss of polarity in the proliferating tissues; (4) Invasive surface growth of the tumor and absence of a capsule; (5) Non-granulomatous development of secondary tumors; (6) Growth in autonomous species from grafts; (7) Presence of degenerative changes in the proliferating cells as shown by behavior toward stains, by lobed and cleft nuclei, polynuclear cells, etc.; (8) Destruction of surrounding tissues; (9) Defective vascularization and early central necrosis with invasion of the open wounds by secondary parasites and by saprophytes; (10) Frequent return after excision; (11) Presence of atrophy and cachexia.

6. *Possibility of the existence of carcinomas and of mixed tumors in plants.*—Slides were shown suggesting that it is experimentally possible to induce the proliferation of glandular tissue and the epidermis (skin) of plants by bacterial inoculations, but so far no metastases have been obtained. This whole subject, however, is yet in its infancy and further reports will be made from time to time as results are obtained.

7. *Production of embryonal teratomata.*—Last winter I discovered that when growing plants are inoculated in the vicinity of dormant buds a new type of tumor is produced. This tumor bears on its surface diminutive abortive shoots (vegetative or floral), and in its interior, along with the cancer cells, numerous fragments of embryonic tissues, variously fused and oriented, often upside down and curiously jumbled. These tumors have never been seen by the writer in nature or at least if seen not recognized as crown gall tumors, but undoubtedly we shall now find them.[3] These tumors have all been produced with *Bacterium tumefaciens* plated from an ordinary sarcoma-like tumor of the hop received from the Pacific Coast 8 years ago. The plants chiefly experimented on have been Pelargonium, Nicotiana, Lycopersicum, Citrus, and Ricinus. All of these and some others (Mangifera, Allamanda, etc.) have yielded teratoid tumors from inoculations in leaf axils.

The inoculated tobacco also developed secondary teratoid tumors at a distance from the primary tumor in both stems and leaves, i.e., tumors bearing numerous diminutive and abortive shoots, which at longest have lived but a few months. In other words, like the secondary tumors on the Paris daisy they have repeated the structure of the parent tumor but this time that which is repeated is not a simple sarcoma developed out of fundamental tissues but a complex tumor comparable to the embryonal teratomata or atypical teratoids occurring in man. Many attempts have been made in recent years to produce such tumors in animals but hitherto all experimenters have failed. Monsters, it is true, have been produced repeatedly by fragmenting eggs, and in other ways, but always they have been *typical teratoids*, never cancers.

However, the most striking result I have obtained is yet to be told. All the embryomas previously mentioned were obtained by making my bacterial inoculations in the vicinity of dormant buds (totipotent anlage) which were then apparently torn by the rapid growth of the tumor, the fragments being variously distributed in the tumor and there stimulated to develop, the result being embryonic growths of extremely variable size but all referable to the original disturbed shoot-anlage. If, however, inoculations are made in regions not known hitherto to contain totipotent cells, the same results can be obtained, e.g., I have obtained on the tobacco a shoot-bearing tumor from the middle of an internode, and numerous such tumors from various parts of leaves, twenty-seven on a single plant, seven on a single leaf, and too many it would seem to be explained on Cohnheim's theory,[4] i.e., as growths due to the development of embryonic 'cell rests,' which are conceived to be fragments displaced from the embryo in early stages of its growth and enclosed in other tissues where they remain dormant until a cancer stimulus sets them growing.

8. *Bearing of these discoveries on the cancer problem.*—It has been pointed out repeatedly that embryomas are the crux of the whole cancer problem. Hitherto they have remained unexplained. Now I have succeeded in bringing them into correlation with the simpler forms of cancer by showing that they also are specific tissue responses to the stimulus of a specific schizomycete. If one set of plant tissues is inoculated sarcoma develops; if another set, carcinoma (?); if a third set (and this time it must be a complex anlage containing totipotent or nearly totipotent cells), an embryoma develops. To a biologist, therefore, the conclusion is almost irresistible that human cancer must be due to a parasite, and that one parasite may well be the cause of the most diverse forms, as we have seen to be the case in plants. Those who consider plants and animals so different that no conclusion can be drawn from the one to the other will see no interrelation, but I am hoping the recorded facts will make a strong appeal to those who know something about cell metabolism and the similar response of living plants and animals to a great variety of stimuli.

---

[1] See *J. Cancer Res.*, **1**, No. 2, April, 1916, and *Science*, N. S., 43, 871, where I have elaborated this idea and also other ideas only touched upon here.

[2] *Bureau of Plant Industry, Bulletins* No. 213 and No. 255.

[3] As this goes to press I have had confirmation of this belief, having received from a florist in Massachusetts a crown gall of the rose showing abortive shoots growing out of stem tumor.

[4] For illustrations of teratomas produced in tobacco leaves by direct bacterial inoculation consult *J. Agric. Res.*, April 24. 1916, Plate 23.

LOCOMOTION OF SEA-ANEMONES

By G. H. Parker

ZOÖLOGICAL LABORATORY OF THE MUSEUM OF COMPARATIVE ZOÖLOGY AT HARVARD COLLEGE

Received by the Academy, June 15, 1916

Although sea-anemones belong to a group of animals characterised by radial symmetry, they have long been known to exhibit a bilateral arrangement in their mesenteries and adjacent parts. Most bilateral animals move in definite relations to their axis of symmetry. Is this true of sea-anemones?

Single specimens of Actinia or of Sagartia will creep now in the direction of their planes of symmetry, now at any angle to those planes and thus demonstrate the entire independence of the direction of locomotion and the axis of symmetry. This is especially clearly seen in Sagartia which will creep away from a source of light irrespective of the relation momentarily called forth between the direction of locomotion and the animal's structural axis. Locomotion in sea-anemones is therefore a radial operation performed by the radial pedal disc and not necessarily associated with the more or less bilateral oral disc.

Locomotion is accomplished by a wave-like movement that progresses over the pedal disc in the direction of locomotion. In a specimen of Sagartia with a pedal disc of about 4 mm. diameter, the locomotor wave coursed over the disc in an average time of 1.65 minutes and with each wave the animal progressed on the average 1.2 mm. In a large sea-anemone, Condylactis, with a pedal disc 130 by 80 mm. the passage of a locomotor wave required on the average three minutes and the animal progressed for each wave on the average 11.4 mm.

In the locomotion of sea-anemones each part of the pedal disc is successively raised from the substratum, moved forward, and put down. The attachment to the substratum is due chiefly to adhesion heightened by the secretion of a thick slime rather than to a sucker-like action of the pedal disc. The mechanism of locomotion consists of the circular muscle of the pedal disc, the basilar muscles, and the longitudinal muscles of the mesenteries all of which act on the fluid-filled spaces in the pedal region. The pressure thus generated is not above 6 cm. of water.

Specimens of Sagartia from which the oral disc has been cut off will creep in an essentially normal manner, for instance, away from a source of light. Hence the pedal portion of a sea-anemone, like its tentacles, must contain a neuromuscular mechanism sufficient for the activity of that part of its body.

The extended publication will appear in the *Journal of Experimental Zoölogy.*

THE BEHAVIOR OF SEA-ANEMONES

By G. H. Parker

ZOÖLOGICAL LABORATORY OF THE MUSEUM OF COMPARATIVE ZOÖLOGY AT HARVARD COLLEGE

Received by the Academy, June 15, 1916

The older students of sea-anemones, such as Gosse and von Lendenfeld, believed that these animals were endowed with mental traits not unlike those of man. Later workers, such as Loeb, Jordan, and others, regard these forms as finely adjusted machines devoid of psychic attributes. To discover something of the nervous nature of these animals two forms of their behavior were studied in detail, the appropriation of food, and general retraction. The work was carried out for the most part on Metridium and Sagartia.

In the appropriation of food, the parts chiefly concerned are the five following: the tentacular gland cells whereby the tentacles are rendered adhesive for food, the tentacular muscles by which the tentacles are pointed toward the mouth, the tentacular cilia by which the food is delivered from the tip of the tentacle to the mouth, the transverse mesenteric muscles whose action opens the mouth, and the oral cilia (lips and oesophagus) whose reversal in the presence of food carries this material into the digestive cavity of the sea-anemone. Of these five parts the mucous cells, the tentacular cilia, and the transverse mesenteric muscles are so uniform in their action that they need no further consideration. The oral cilia and the tentacular muscles on the other hand are much more open to variation and hence may serve to indicate to some extent the condition of the animal as a whole.

The oral cilia after having reversed their effective stroke in the presence of food a number of times, eventually cease to show this change, a condition supposed to be due to altered metabolism as a result of feeding. But this same cessation occurs when the oral membranes are cut from the animal and worked with separately. It is strictly local in its appearance and probably a pure fatigue effect.

The feeding movements of the tentacles, though also modified by fatigue, have been supposed to show changes of a more significant kind. If the tentacles on one side of the animal are much exercised, those of the other side decrease in responsiveness. This change has also been supposed to be due to changed metabolism, but, since it appears quickly and before a general metabolic change can have occurred, it is probably that this too is the result of fatigue in that the food juices from the cavity of the tentacles transfuse the walls of these organs and thus reduce the sensitiveness of their exterior. Thus none of the elements in the feeding responses of sea-anemones imply that these animals are organisms that respond as firmly united wholes.

Retraction, whereby the more delicate parts of the sea-anemone are drawn in and covered, is the commonest protective act of this animal. Expansion is the reverse of retraction and puts the animal in form for full activity. Vigorous mechanical stimulation, most chemical stimuli, strong light, and high temperature induce retraction. The presence of food in the adjacent water, and water currents induce expansion. Oxygen as such seems to have little effect on these reactions.

Sagartia retracts when left dry by the tide and expands when it is again covered by the returning water. Metridium retracts in bright daylight and expands at night. The tidal rhythm of Sagartia and the nychthemeral rhythm of Metridium are not retained after the rhythmic stimulus is removed as has been claimed for European species by Bohn. There is also no evidence of an anticipatory reaction to the tides as maintained by Piéron. The retraction and expansion of sea-anemones, therefore, give no support to the view that these animals act under highly specialized nervous states.

The form of response which more than any other involves a sea-anemone as a whole is creeping. But even this form of activity can be accomplished by the pedal half of the animal. To repeated stimuli sea-anemones quickly adjust themselves rather by a process of adaptation then by one of exhaustion. Yet they have been found to show no evidence of associative capacity. They are animals whose momentary conditions are dependent upon the combined stimuli of their immediate surroundings rather than forms that are greatly influenced by their past history. And in consequence of this their unity is not of a pronounced type. They are more in the nature of a sum of parts than they are organic units such as we are familiar with among most of the higher animals.

The extended paper will be published in the *Journal of Experimental Zoölogy.*

A CONTRIBUTION TO THE PETROGRAPHY OF JAPAN

By J. P. Iddings and E. W. Morley

BRINKLOW, MARYLAND, AND WEST HARTFORD, CONNECTICUT

Read before the Academy, April 17, 1916. Received, June 17, 1916

It has been known for many years that the volcanic lavas of the Japanese islands are chiefly pyroxene-andesites, and that other lavas are more siliceous and some less so, but the actual chemical composition of most of them has long remained unknown, though chemical analyses of a few varieties have been published. This was the case to within a few years, when some of the principal volcanoes were visited by one of us in order to study various forms of craters of a few of the many active volcanoes of this region, and to collect material for chemical and microscopical study. Since that time a number of excellent chemical analyses of igneous rocks have been made in the laboratory of the Imperial Geological Survey of Japan, some of which have been published in volume 2 of *Igneous Rock* by J. P. Iddings.

The accompanying seventeen analyses, by E. W. Morley, furnish new chemical data relating to lavas of several of the most prominent Japanese volcanoes, and form a series from the basalts, or basaltic andesites, of Fuji yama and Aso to dacites and rhyolites of less well-known localities.

The habit of the pyroxene-andesites is much the same in all the varieties collected, except that some are compact rocks, others vesicular. They have abundant small phenocrysts, that is, they are sempatic and seriate minophyric. The most abundant phenocrysts are strongly calcic feldspars. Pyroxenes are inconspicuous and are in large part hypersthene, the relative amounts of augite and hypersthene varying considerably. Olivine occurs as phenocrysts in few of the rocks analyzed, and quartz phenocrysts in a few. Magnetite is abundant in most of the andesites, chiefly as microscopic constituents of the groundmass together with microlites of feldspar and pyroxene. It also forms small phenocrysts.

While the lavas at each volcano vary somewhat in composition, the variation is within narrow limits so far as observed. The material analyzed represents characteristic rock from each locality, but not all that occurs there. Analysis 1 is of the rock from the Hoyei crater on the east slope of Fuji yama. The rock is basalt with andesitic habit, and contains abundant olivine and is the lowest in silica of the rocks analyzed. Nevertheless, the calculated norm contains a small amount of quartz with considerable normative hypersthene. The normative feldspar is strongly calcic.

Recent lava from the north rim of the central crater of the volcano Aso, is pyroxene-andesite, anal. 2, with nearly the same chemical composition as the basalt from Fuji yama, and with slightly more normative quartz, but no modal olivine, so far as observed. The normative feldspar is less calcic and there is less of the femic components. The chemical resemblance between the two rocks is well shown by the symbols and magmatic names.

The lava of Niokiba crater, Nazu volcano, which was erupted in 1881, anal. 3, is typical bandaite, that is, it is a pyroxene-andesite with laboradorite feldspars and a notable amount of normative quartz, some of which appears as quartz phenocrysts. The next six rocks, anals. 4 to 9, are pyroxene-andesites, which are transitional bandaites, since the normative feldspars are andesine-laboradorite. The lavas occur at widely scattered volcanoes, Zao San, Asama yama and Azuma yama on the main island, Hondo; Ohachi, and Shimasaki, a quarry near Kumamoto, on Kyushu; and Komagatake, on Hokkaido.

The lava erupted from Tarumai, Hokkaido, in 1909, anal. 10, is chemically similar to the preceding, but the feldspars are slightly less calcic. It is a transitional variety of pyroxene-andesite related to bandaite. The andesite from the rim of the central crater of Iwakisan, in northern Hondo, anal. 11, is closely similar to bandaite, with somewhat less pyroxene. The rock from the southern summit of Sakurajima, Kyushu, anal. 12, which was collected in 1910, before the last great eruption of this volcano, is a glassy pyroxene-andesite, having the chemical composition of an andesine-dacite, or shastaite. The hornblende-mica-andesite of Mae yama, near Shimabara, anal. 13, and the spherulitic dacite from Kosaka, Hondo, anal. 14, are chemically similar, though modally quite unlike. They may be classed as shastaites, because of the character of their normative feldspars. The glassy dacite, anal. 15, from near the Shinyu Inn at the west base of Aso, Kyushu, is a variety of ungaite, judging by the norm. The normative plagioclase is oligoclase, with which is subordinate normative orthoclase. It may be classed as an oligoclase-dacite transitional to dellenite.

Analysis 16 is of a garnet-bearing rhyolitic obsidian, from Hiwashima, and analysis 17 is of a spherulitic rhyloitic obsidian said to occur on the sea coast north of Kagoshima and Sakarajima. The seventeen analyses show that there is a strong chemical likeness among the rocks collected, which is well brought out by the norms and magmatic symbols.

TABLE OF CHEMICAL ANALYSES AND NORMS OF JAPANESE LAVAS

	1	2	3	4	5	6	7	8
SiO_2	50.61	53.38	58.93	57.04	57.75	59.27	59.79	60.95
Al_2O_3	17.84	18.15	16.56	15.78	17.50	16.66	17.41	16.30
Fe_2O_3	2.97	2.27	2.82	2.36	3.45	1.56	1.84	2.47
FeO	7.12	6.21	4.36	5.37	3.54	4.79	4.27	4.05
MgO	5.99	3.60	3.81	4.44	3.87	3.97	3.19	3.27
CaO	10.14	9.60	7.84	7.54	7.45	7.77	7.34	7.29
Na_2O	2.48	2.89	2.34	2.74	2.83	2.92	2.71	2.50
K_2O	0.74	1.75	1.00	1.31	1.33	1.03	1.85	1.68
H_2O-	0.00	0.00	0.25	0.39	0.17	0.00	0.00	0.12
H_2O+	0.39	0.14	0.71	1.32	0.44	0.16	0.11	0.32
TiO_2	0.66	0.86	0.59	0.79	0.67	0.81	0.63	0.60
P_2O_5	0.34	0.29	0.15	0.19	0.26	0.55	0.33	0.07
MnO	0.80	0.43	0.44	0.44	0.53	0.35	0.15	0.19
ZrO_2	0.00	0.00	0.00	0.02	0.01	0.01	0.01	0.00
CO_2	0.00	0.01	0.03	0.00	0.01	0.01	0.00	0.00
Cl	0.08	0.05	0.07	0.12	0.06	0.06	0.14	0.09
F	0.04	0.10	0.08	0.08	0.03	0.02	0.07	0.02
S	0.03	0.02	0.02	0.05	0.08	0.06	0.08	0.05
Cr_2O_3	0.02	0.00	0.01	0.01	0.01	0.00	0.00	0.00
BaO	0.03	0.04	0.02	0.08	0.04	0.03	0.08	0.04
SrO	0.02	0.03	0.00	0.01	0.02	0.04	0.04	0.02
	100.30	99.82	100.02	100.08	100.05	100.07	100.04	100.03

Norms

q	1.38	3.66	18.42	12.42	14.52	15.42	15.60	19.32
or	4.45	10.56	5.56	7.78	7.78	6.12	11.12	10.01
ab	20.96	24.63	19.91	23.06	24.10	24.63	23.06	20.96
an	35.31	31.14	31.97	26.97	31.14	29.47	29.75	28.36
c								
di	10.63	11.77	4.73	7.94	3.13	4.54	3.86	6.31
hy	20.64	12.10	12.62	14.61	11.64	14.54	11.65	9.37
mt	4.41	3.25	4.18	3.48	5.10	2.32	2.55	3.71
hm								
il	1.37	1.67	1.22	1.52	1.22	1.52	1.22	1.22
ap	0.67	0.67	0.34	0.34	0.67	1.34	0.67	
etc	0.61	0.38	1.16	2.08	0.86	0.38	0.53	0.66
	100.43	99.83	100.11	100.20	100.16	100.28	100.01	99.92

1. Basalt, hessose-auvergnose, (II) III.5.4.4′. Fuji yama.
2. Pyroxene-andesite, andose-hessose, II.5.(3) 4.4. Aso, central crater.
3. Pyroxene-andesite, bandaite, bandose, II.4.4.4. Nazu, lava 1881.
4. Pyroxene-andesite, tonalose-bandose, II.4′.(3) 4.4. Zao San.
5. Pyroxene-andesite, tonalose-bandose, II.4.(3) 4.4. Shimasaki, Kumamoto
6. Pyroxene-andesite, tonalose-bandose, II.4.(3) 4.4. Asama yama, erupt. 1909.
7. Pyroxene-andesite, tonalose-bandose, II.4.(3) 4.′4. Ohachi, Takachiho,
8. Pyroxene-andesite, tonalose-bandose, II.4.(3) 4.4. Azuma yama, lava 1893.
9. Pyroxene-andesite, tonalose-bandose, ′II.4.(3) 4.4′. Komagatake, Hokkaido.

TABLE OF CHEMICAL ANALYSES AND NORMS OF JAPANESE LAVAS

9	10	11	12	13	14	15	16	17
59.76	58.63	61.89	64.98	65.89	70.43	70.72	73.64	76.64
17.69	17.32	17.89	15.89	16.02	15.55	14.79	14.35	12.08
5.13	4.53	3.73	1.89	1.97	1.24	0.75	0.40	0.73
2.16	3.12	1.66	4.22	2.16	1.19	1.35	0.99	0.66
2.65	2.98	2.09	1.83	2.20	0.78	0.94	0.21	0.24
6.97	7.40	6.48	3.22	3.93	3.53	1.50	0.63	1.28
3.15	3.50	3.39	3.31	3.67	4.08	4.19	4.33	3.54
0.93	1.20	1.19	1.87	2.67	1.41	4.25	3.79	3.11
0.09	0.01	0.18	0.04	0.00	0.42	0.02	0.04	0.00
0.24	0.12	0.27	0.62	0.31	0.71	0.57	0.47	0.99
0.56	0.54	0.50	0.75	0.51	0.13	0.34	0.03	0.25
0.23	0.12	0.01	0.26	0.26	0.12	0.12	0.23	0.12
0.44	0.13	0.09	0.70	0.42	0.15	0.23	0.18	0.32
0.00	0.00	n. d.	0.02	0.00	0.00	0.00	0.00	0.01
0.02	0.01	0.02	0.01	0.01	0.02	0.01	0.02	0.01
0.10	0.06	0.14	0.12	0.06	0.05	0.06	0.18	0.19
0.03	0.06	0.03	0.05	0.05	0.06	0.05	0.03	0.05
0.03	0.03	0.09	0.07	0.02	0.06	0.02	0.14	0.06
0.00	0.00	0.02	0.00	0.00	0.00	0.02	0.00	0.00
0.05	0.06	0.04	0.10	0.06	0.04	0.04	0.16	0.06
0.04	0.03	0.00	0.00	0.00	0.04	0.06	0.00	0.00
100.27	99.85	99.71	99.95	100.21	100.01	100.03	99.82	100.34

Norms

19.86	14.82	20.94	27.18	22.32	32.46	24.60	32.34	41.10
5.56	7.23	7.23	11.12	16.12	8.34	25.58	22.24	18.35
26.72	29.34	28.82	27.77	30.92	34.58	35.63	36.68	29.87
31.41	28.08	29.75	13.90	17.51	16.68	6.67	1.39	5.56
			3.37	0.61	1.22	0.71	2.65	0.92
1.51	6.15	1.73						
5.90	6.06	4.40	10.67	7.74	3.09	4.12	2.22	1.39
6.73	6.50	4.18	2.78	3.02	1.86	1.16	0.70	0.93
0.48		0.80						
1.06	0.91	0.91	1.37	0.91	0.30	0.64		0.46
0.34	0.34		0.67	0.67	0.34	0.34	0.67	0.34
0.58	0.37	0.77	1.02	0.50	1.38	0.85	1.02	1.36
100.15	99.80	99.53	99.85	100.32	100.25	100.30	99.91	100.18

10. Pyroxene-andesite, bandose-tonalose, II.4′.3′.4. Tarumai, lava 1909.
11. Pyroxene-andesite, bandose-yellowstonose, I (II).4.(3) 4.4. Iwakisan.
12. Pyroxene-andesite, shastaite, dacose-tonalose, ′II.′4.′3.4. Sakurajima.
13. Hornblende-mica-andesite, tonalose-yellowstonose, I (II).4.′3.′4. Mae yama, Shimabara.
14. Spherulitic dacite,-yellowstonose, I.(3) 4.′3.4′. Kosaka.
15. Glassy dacite, lassenose-toscanose, I′.4.′2.3 (4). Aso, west base, above Shinyu Inn.
16. Rhyolitic obsidian, liparose-kallarudose, I.(3) 4.1.(3) 4. Hiwashima.
17. Spherulitic rhyolite, tehamose-alsbachose, I.3′.′2.(3) 4. Coast north of Kagoshima.

IS THERE A TEMPERATURE COEFFICIENT FOR THE DURATION OF LIFE?

By Jacques Loeb and J. H. Northrop

ROCKEFELLER INSTITUTE FOR MEDICAL RESEARCH, NEW YORK

Received by the Academy, July 5, 1916

In 1908 one of us raised the question whether there is a definite temperature coefficient for the duration of life.[1] This might be expected if the duration of life depended upon the presence of certain substances which were used up during life; or if the duration of life were limited by the cumulative injurious effects of certain products of metabolism. Thus, Metchnikoff[2] has mentioned the possibility that the duration of the life of the moth of the silk worm is limited by the retention of certain poisonous substances contained in the urine. The rapidity of consumption of the necessary substance in the case of the first or the velocity of the accumulation or action of injurious substances in the case of the second hypothesis should increase with the temperature according to a certain law.

It seems that there exists for each species a pretty definite duration of life in spite of the fact that injuries of various types may shorten the life of the individual. The annual plants, the sequoia of the sierras, the human, the insects, have their characteristic duration of life. On the other hand, it was shown by Leo Loeb that the cancer cell is immortal and he pointed out that this might be the case for all cells. Then the problem arises, what is the cause of the fact that each species has a limited duration of life the magnitude of which is characteristic for the species? If the answer to this question is given by one of the two hypotheses mentioned in the first paragraph of this paper, it may be expected that there should be found a temperature coefficient for the duration of life of the order of magnitude of that of chemical reactions. A search for such a temperature coefficient can only be attempted on a form with a naturally short duration of life. We have selected for this purpose the fruit fly *Drosophila*.

Newly hatched flies were put into large Erlenmeyer flasks kept in thermostats 34°, 31°, 28°, 24°, 14°, and 9°. Each flask contained on the average about 100 flies. The number of dead were counted each day and the surviving flies were put into fresh flasks every two days. Each determination of the duration of life was based upon at least two and often as many as twelve cultures of about 100 flies each. The values for the mean length of life of the flies in the separate cultures, at a given temperature, was averaged and this value taken as the average duration of life for that temperature.

Three series of experiments were made. In one the flies were provided only with water, in the second with a 1% cane sugar solution, and in the third with fermented banana. In the third series the results were less regular than in the first two series, probably on account of differences in the nature of microorganisms present in the food. We intend to repeat these experiments with sterile cultures of flies which we are now raising, and will report on these experiments in a later publication.

The following table gives the average duration of life for different temperatures.

TEMPERATURE	DURATION OF LIFE IN DAYS	
	With H_2O	With 1% cane sugar
°C.		
34	2.1	6.2
28	2.4	7.2
24	2.4	9.4
19	4.1	12.3
14	8.3	
9	11.9	

If we consider the figures for temperatues between 28° and 9° for the cultures in water and in sugar we find that there exists a temperature coefficient for the duration of life of about the order of magnitude of that of chemical reactions, namely, of about 2 for a difference of 10°C. We find also that the coefficient is greater for the lower range of temperatures. The same temperature coefficient was also found by previous authors for the time required for the development of the eggs of animals and incidentally also by us for the larval period of these flies; with this difference only, that in the latter case the coefficient is more regular.

These experiments therefore show that the duration of life in the cases examined has a temperature coefficient of that order of magnitude which is characteristic for life phenomena and for chemical reactions in general.

[1] Loeb, J., *Arch. ges Physiol.*, **124**, 411 (1908).

[2] Metchnikoff, E., *Ann. Inst. Pasteur*, 29, 477, (1915).

ON THE SUGGESTED MUTUAL REPULSION OF FRAUNHOFER LINES

By Charles E. St. John

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Read before the Academy, April 18, 1916. Received, July 7, 1916

The correct interpretation of the results given by the spectroscope is so fundamental to progress in present day astronomy that every suggestion offering new points of view and avenues of approach to the problems furnished by the powerful instruments in commission deserves consideration. The spectrographs now used upon the sun yield spectra of such a scale and dispersion that minute changes in spectrum lines can be studied that were beyond the reach of the older instruments.

Changes in the relative position of spectrum lines are the phenomena most often under consideration. All astrophysicists recognize two conditions that produce displacements of the Fraunhofer lines, motion in the line of sight and differences in pressure, both capable of precise determination. Two others have been suggested, the gravitational effect of Einstein[1] and the anomalous dispersion hypothesis of Julius.[2] The former influence would displace all lines to a calculable amount and would, if found, add no serious difficulties to the solar problem. The effects of anomalous dispersion, however, would introduce an indeterminate factor into the spectroscopic problem. It becomes then a matter of prime importance to determine whether the relative positions of the Fraunhofer lines are measurably affected by this cause.

An accepted deduction from the theory is a mutual influence between neighboring lines, a quasi repulsion increasing with the proximity of the adjacent lines. The violet and red components of a close pair must then be displaced to the violet and red, respectively, in comparison with the displacement of the isolated lines of the same class. Displacements between the solar and arc lines give a direct and definite means of testing the question.

Iron, from its abundance of lines and the searching investigations given to them, furnishes the most reliable data. The wave lengths of the iron lines belonging to groups *a*, *b* and *c4* of the Mount Wilson classification are independent of arc conditions.[3] These lines therefore are capable of giving results of the highest possible precision. The mean sun-arc displacement for 213 lines of these groups is +0.0039 A. Those who assign an important rôle to anomalous dispersion in the solar atmosphere assume that solar lines within 0.5 A from each other are subject to mutual repulsion. The mean displacement for 30 lines with companions at an average distance of 0.314 A to the red is +0.0042A; for 59 lines with companions at an average distance of 0.270 A to the violet it is +0.0040 A. The displacements for the three cases are equal within the limits of accidental error, and, within such limits, the equality shows the absence of mutual influence.

Since a detectable mutual repulsion between adjacent solar lines should increase their separation over that determined from terrestrial sources, another approach to the question is found through such a comparison. It is usually assumed that repulsion occurs between all closely adjacent solar lines. For 45 pairs, mean separation 0.29 A, the differences Δλ Sun − Δλ Arc are positive for 17, negative for 21 and zero for 7 lines, the mean differences being zero. It has recently been suggested[4] that repulsion is to be expected only if the line in the solar spectrum has a closely adjacent line due to another substance not present in the arc. The following data from recent determinations do not show the differences required by the theory, though the lines are due to different elements and originate in separate arcs.

λ *Rowland*		Δλ *Sun*	Δλ *Arc*	Δλ *Sun* − Δλ *Arc*
4058.915	Fe			
.081	Mn	.170	.173	−0.003
4226.904	Ca			
.606	Fe	.700	.710	−0.010
4315.138	Ti			
.262	Fe	.115	.115	0.000
4427.266	Ti			
.482	Fe	.215	.213	+0.002
4454.552	Fe			
.953	Ca	.399	.399	0.000
4489.911	Fe			
.253	Mn	.340	.340	0.000
5208.596	Cr			
.776	Fe	.169	.170	−0.001
5446.797	Ti			
.130	Fe	.332	.334	−0.002
Mean........		.305	.307	−0.0018

Recently Albrecht[5] found by comparing the Rowland and International wave-lengths of iron lines that the violet and red components of a solar pair showed apparent displacements to the violet and red of 0.007 and 0.005A, respectively. He interpreted these as effects of anomalous dispersion, an interpretation accepted by Julius, who says,[6] "Only if the Fraunhofer lines are *mainly* due to anomalous dispersion will they be able to show a mutual influence of the observed kind and magnitude." The absence of mutual influence would, within the limits of error, remove any direct evidence that anomalous dispersion contributes to the production of the Fraunhofer lines.

As the Mount Wilson data for sun-arc displacements and for the comparative separation between close pairs of lines in solarand terrestrial spectra do not show the effect of mutual influence, it seems necessary to assume either that the Mount Wilson data are affected by systematic errors just sufficient to annihilate the effect of anomalous dispersion, or that the Rowland wave-lengths for lines in close pairs are systematically in error; a slight over-separation of such pairs would introduce an effect of the sign indicated by the theory. The results of an investigation upon "The Accuracy of the Measured Separations of Close Solar Pairs; Systematic Errors in the Rowland Table for such Lines" are given in a Contribution from this Observatory.[7] For pairs consisting of lines of intensities 3 and 4, with mean separations of 0.274, 0.145 and 0.075 A, the Rowland separations exceed the Mount Wilson values by 0.003, 0.008 and 0.013 A, respectively. As errors of this sign and magnitude would account for the deviations found by Albrecht, an exhaustive examination of the cases included in Albrecht's list was undertaken; details of this investigation will appear in a Contribution from this Observatory.

The wave-lengths of the 104 lines used by Albrecht have been referred to those of neighboring free-standing lines. The measurements have been made by two observers upon spectrograms of a scale and dispersion that previous experience had shown were best adapted to each case. The errors found in the Rowland values are systematic and of a sign and magnitude corresponding to the Albrecht deviations. This correspondence is marked, large and small values of the one being associated with large and small values of the other. The coefficient of correlation between the two sets of observations is $+0.55 \pm 0.05$, indicating that the correlation is a practical certainty. No explanation of this correlation seems possible other than that the errors in the Rowland wave-lengths and the Albrecht deviations are two phases of the same phenomenon, that in fact the deivations observed by Albrecht are a measurement of the Rowland errors.

Summary and Conclusion.—1. The violet and red components of close pairs of solar lines show the same displacement as isolated lines when compared with the spectrum of the arc.

2. The mean separation of close pairs in the solar spectrum is the same as that determined from terrestrial sources whether the component lines are due to the same or different elements.

3. The Rowland wave-lengths for close pairs of solar lines are sys-

tematically in error; the violet and red components being assigned values, respectively too small and too large.

4. The systematic deviations for lines with violet and red companions found by comparing the Rowland and International wave-lengths go pari passu with and are referable to the erros in Rowland wave-lengths. The coefficient of correlation is +0.55 ± 0.05.

5. These systematic deviations, therefore, do not furnish evidence that the relative positions of the Fraunhofer lines are systematically displaced by mutual influence. On the other hand, the sun-arc displacements and the relative separation of the components of close pairs in solar and arc spectra indicate that, within the limits of error, evidence of mutual influence is absent from the solar spectrum, and, in so far as mutual influence is a necessary corollary of anomalous dispersion in the sun, evidence for it also is absent.

[1] A. Einstein, *Leipzig, Ann. Phys*, **35**, 898 (1911).

[2] W. H. Julius, *Astrophys. J.*, **40**, 1 (1914).

[3] *Trans. Internat. Union Co-op. Solar Res.*, **4**, 74. Charles E. St. John and Harold D. Babcock, 'A Study of the Pole Effect in the Iron Arc,' *Mt. Wilson Contr.*, 106; *Astrophys. J.*, **46**, (1915).

[4] Sir Joseph Larmor, *Observatory*, **497**, 103 (1916).

[5] Sebastian Albrecht, *Astrophys. J.*, **41**, 333 (1915).

[6] W. H. Julius, *Astrophys. J.*, **43**, 53 (1916).

[7] Charles E. St. John and L. W. Ware, *Mt. Wilson Contr.*, —, *Astrophys. J.*, — (1916).

AN ATTEMPT TO DETECT THE MUTUAL INFLUENCE OF NEIGHBORING LINES IN ELECTRIC FURNACE SPECTRA SHOWING ANOMALOUS DISPERSION

By Arthur S. King

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

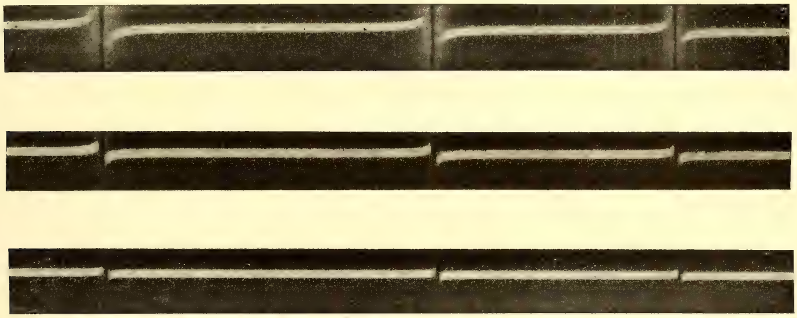
Received by the Academy, July 6, 1916

When light traverses a transparent medium, the variation of the index of refraction with the wave-length of the incident light may undergo decided changes if the medium shows selective absorption. If a beam of white light is passed through a mass of vapor which is itself emitting a spectrum, the sudden change of refractive index close to the absorption lines can be shown by suitable optical arrangements. One method of producing this effect is based on the principle of crossed prisms used by Kundt, and consists in making the absorbing medium form a prism whose refracting edge is at right angles to the slit of the analysing spectroscope. An absorption line showing anomalous dispersion will then cause the continuous spectrum to bend sharply in opposite directions on the two sides of the line. This method was employed by Wood in studying the anomalous dispersion of sodium vapor, the prism being formed by vapor enclosed in a horizontal steel tube heated from below, the upper portion being cooled.

According to a theory advanced by Julius, when anomalous dispersion is active, two vibrations of nearly the same period affect each other in such a way as to cause a mutual repulsion between the corresponding spectrum lines. It seemed possible to make laboratory tests which would set at least an upper limit to the amount of this effect. For this purpose, as well as to study the regular phenomena of anomalous dispersion, the writer has recently adapted the tube resistance furnace for the production of these effects. A beam of white light from a carbon arc outside the furnace chamber was passed through the furnace tube containing the metallic vapor to be studied. A strong density gradient in the vapor was produced by cutting away the upper portion of the tube and passing a water-cooled pipe above it. Examination under high dispersion with a plane-grating spectograph gave a means of studying the relative anomalous dispersion produced by different spectrum lines, and, by the use of mixed vapors in the furnace tube, to test the constancy of wave-lengths of lines when alone and when close to a line showing high anomalous dispersion given by another vapor.

A study of the iron spectrum over a considerable range of wave-length gave strong anomalous dispersion effects for those of the stronger lines which reverse easily in the arc and other sources. This feature appears to be general for the spectra which have been examined. It was found further that a certain temperature of the absorbing vapor is required to give the most efficient prism for a particular type of line. Thus the calcium line λ 4227 and the chromium lines $\lambda\lambda$ 4254, 4275, 4290 can all be made to show very strong anomalous dispersion, but the best temperature for the calcium prism is lower than that for chromium. This can be well shown by mixing the two vapors and causing each in turn to show anomalous dispersion. When the temperature is adjusted to give maximum anomalous dispersion for chromium, the effect usually disappears for λ 4227 of calcium, leaving only a wide absorption line. In several experiments, however, a high temperature condition has given a curvature of the spectrum adjacent to λ 4227 in the opposite direction to that prevailing for the chromium lines at the same time, and also the reverse of what the calcium line itself shows at a lower temperature. This indicates an inversion of the prism of vapor producing λ 4227, due probably to the cooler vapor above giving the line more strongly than the highly heated region below.

The density of the absorbing vapor is necessarily a function of the temperature when plenty of the material is present. At a given temperature, the amount of anomalous dispersion is proportional to the vapor density, which alters the width of the absorption line, so that for special purposes the effect can be regulated by changing the amount of vapor. The reproduction shows the anomalous dispersion for the chromium lines λ λ 4254, 4275, 4290, the temperature being approximately 2500°C. in each case, with varying vapor densities.



ANOMALOUS DISPERSION OF CHROMIUM LINES WITH VARYING VAPOR DENSITY.

The tests on the mutual influence of spectrum lines were carried out by mixing titanium and calcium vapors with that of chromium. By this means, a titanium line was obtained separated by 0.21 A from λ 4275 of chromium, while a calcium and a titanium line occur 0.36 A and 0.65 A respectively from λ 4290 of chromium. The titanium and calcium lines were thus well within the curved spectrum given by the strong anomalous dispersion of the chromium lines. The wave-lengths of the former, given as absorption lines under these conditions, were compared with their values when produced by the furnace as emission lines.

Twenty-seven spectograms showing varying degrees of anomalous dispersion were measured. The titanium line separated by only 0.21 A from the chromium line λ 4275 gave consistent measurements indicating an approach to the chromium line in the anomalous dispersion plates, the mean difference between the two sets of measures being 0.006 A. It is questionable, however, whether this displacement is genuine, as the spreading of the strong chromium line weakened the continuous spectrum on one side of the titanium absorption line, which would probably affect the micrometer settings. The calcium line separated by 0.36 A from λ 4290 of chromium showed an apparent approach amounting to 0.002 A, to which the same source of error may apply. The line of the set most favorable for accurate measurement was λ 4289.237 of titanium, 0.65 A from λ 4289.885 of chromium. Measurements of this line from three different standards gave no difference larger than 0.001 A and usually agreed exactly. It may be said, therefore, that the measurements have given no evidence whatever of a mutual repulsion between close lines when anomalous dispersion is active, such small apparent differences as were found being in the opposite direction.

The same lines were tested in the regular furnace spectrum under a dispersion of 1 mm. = 0.6 A to see whether the presence of the strong chromium lines affected the wave-length of the close line of titanium and calcium. The latter were photographed with and without the mixture of chromium vapor. The calcium and one titanium line gave differences of 0.002 A but these were opposite in direction and probably within the errors of measurement, while the other titanium line, closest of the three to a chromium line, agreed within 0.001 A when titanium was used alone and when mixed with chromium.

The material in this investigation has been limited by the scarcity of suitable pairs of lines, as in addition to being separated by a small interval, the lines must be given strongly in the furnace spectrum and one of them must show high anomalous dispersion. The lines tested have filled these requirements and the measurements have at least shown no tendency toward a repulsion between such close lines under these conditions.

SYNTHESIS OF THE BASE $C_3H_4ON_2$ DERIVED FROM METHYLAMINOMETHYL-3,4-DIHYDROXYPHENYLCARBINOL

By Chas. A. Rouiller

PHARMACOLOGICAL LABORATORY, THE JOHNS HOPKINS UNIVERSITY

Received by the Academy, July 3, 1916

Some thirteen years ago Abel[1] found that when methylamino-methyl-3,4-dihydroxyphenylcarbinol, 3,4-$(HO)_2C_6H_3CH(OH)CH_2NHCH_3$ (epinephrine, suprarenine, adrenaline, the substance known as the active principle of the medullary portion of the suprarenal capsules) is slowly added to nitric acid (density 1.2) an energetic reaction takes place and there are obtained as chief products oxalic acid and an unstable nitrogenous base in the form of a hygroscopic salt (probably the oxalate). With iodine trichloride this base gives a crystalline double compound which, although relatively stable, is also very hygroscopic, but with gold chloride is obtained a stable, beautifully crystalline chloroaurate. Abel analyzed this salt for carbon, hydrogen, nitrogen, gold and chlorine and the remarkably concordant results of all his determinations seem to establish beyond doubt that its composition is represented by the formula $C_3H_4ON_2.AuCl_3.HCl$. The free base itself could not be isolated and all the evidence tends to show that it decomposes as soon as it is set free from its salts; when any of the latter are treated with alkaline reagents (even such weakly basic ones as calcium carbonate or acetate) a characteristic, unpleasant, coniine-piperidine-like odor develops and vapors which turn moist litmus paper blue are evolved; among the products formed when the gold salt is boiled with sodium hydroxide solution were identified ammonia, methylamine and methylhydrazine.

Since the base must be formed by the action of the nitric acid on the side chain $CH_3NHCH_2CH(OH)$—of the epinephrine molecule, it seemed possible that it might also be abtained by treating methylaminoacetaldehyde, CH_3NHCH_2CHO, with nitric acid. Accordingly, methylaminoacetal, $CH_3NHCH_2CH(OC_2H_5)_2$, was prepared by heating under pressure chloroacetal, $ClCH_2CH(OC_2H_5)_2$, with an aqueous solution of one equivalent of sodium hydroxide saturated with methylamine, and was converted into the aldehyde by letting it stand overnight in concentrated hydrochloric acid; the solution was then concentrated *in vacuo* at 40° and finally at room temperature over sulphuric acid and solid sodium hydroxide. When the resulting syrup was treated with nitric acid there was obtained a substance agreeing in all respects with that described by Abel. It can also be prepared by treating the acetal directly with nitric acid, but in that case the reaction is very violent and even after it has apparently ceased sometimes starts again explosively and the whole product may be converted into a red tar. The synthetic base reduces ammoniacal silver nitrate and Fehling's solutions, evolves the characteristic coniine-piperidine odor with alkalies and gives with gold chloride a salt difficulty soluble in water and separating from acetone containing a little alcohol in beautiful yellow prisms containing the same percentages of gold and chlorine as Abel's salt.

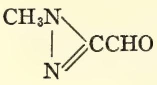
0.2484 g. salt decomposed with 10% aqueous Na_2O_2 gave 0.1156 g. metallic Au and the filtrate gave 0.3370 g. AgCl.

	Found	Calculated for $C_3H_4ON_2.AuCl_3.HCl$
Au	46.54	46.49
Cl	33.56	33.44

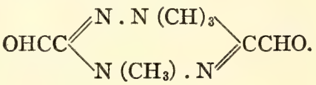
When the gold salt is treated with sodium hydroxide solution and the mixture is distilled to a small volume the distillate is found to reduce Fehling's solution strongly, and the residue in the distilling flask, after being filtered from the gold and acidified with acetic acid, gives with calcium chloride a crystalline precipitate soluble in hydrochloric acid and reprecipitated by ammonia.

The hydrochloric acid solution of the base, obtained by decomposing the water solution of the gold salt with hydrogen sulphide, gives with phenylhydrazine and potassium acetate at about 40° a reddish brown, partly crystalline, partly resinous solid, easily soluble in organic solvents (except ligroin), which, when dissolved in a little alcohol and poured into much boiling water, separates on cooling in fine felted needles melting at 159° when heated slowly and at 162–3° when the capillary is introduced into a bath previously heated to 150°.

With the data at his disposal Abel in 1903 suggested that the base might well be a highly unstable cyclic compound related to the pyrazolone series. In 1906 Curtius began publishing the results of his interesting investigations on the series of compounds obtained by the action of alkalies on diazoacetic ester,[2] and our new base in so many respects (namely, in its decomposition into methylhydrazine, methylamine, ammonia and oxalic acid) so closely resembles his 'pseudodiazoacetic acid' derivatives that it may not appear too hazardous to suggest that it contains the isodiazomethane or N-1,4-dihydro-1,2,4,5-tetrazine grouping and that its constitution may be represented by the formula



or



This suggestion is made with all reserve and it is hoped that certain experiments now being carried out will soon make it possible to decide whether or not it is tenable.

[This work was made possible by a Grant to Prof. John J. Abel from the Rockefeller Institute for Medical Research.]

---

[1] Abel, J. J., *Amer. J. Physiol. Proc.*, 8, 31 (1903); *Ber. D. chem. Ges.*, **36**, 1846 (1903); **37**, 368 (1904); Abel and Taveau, R. deM., *J. Biol. Chem* , **1**, 13 (1905).

[2] For a summary of the work of Curtius on these compounds, see *Ber. D. chem. Ges.*, **41**, 3161 (1908); also *Ibid.*, **42**, 3284 (1909).

EXTINGUISHED AND RESURGENT CORAL REEFS

By W. M. Davis

DEPARTMENT OF GEOLOGY AND GEOGRAPHY. HARVARD UNIVERSITY

Received by the Academy, June 22, 1916

In the diagrams by which Darwin originally illustrated his theory of upgrowing reefs on subsiding foundations, the reefs were drawn as growing upward and inward, in such a way that the diameter of their ring diminished, but he did not explain the cause of the diminution. Dana used similar diagrams and insufficiently explained the diminution of diameter by saying:—"a barrier, as subsidence goes on, gradually contracts its area, owing to the fact that the sea bears a great part of the material inward over the reef" (Corals and Coral Islands, 1872, 263). Certain later diagrams, especially one by Lendenfeld (Westermann's *Monatshefte*, 1896, 499–519), represents a reef as growing upwards and outwards, and hence of increasing diameter. Daly adopts essentially this view and adds:— "When one remembers that most of the detritus abraded from the main reef goes to form talus on the outer submarine slope; and, secondly, that the growth of new coral is much faster on that side, we cannot fail to expect a centrifugal tendency for the encircling reef, as the island sinks" (Glacial-Control Theory of Coral Reef, Proc. Amer. Acad., li, 1915, 247). Both increase and decrease of reef diameter may take place, for both outward and inward upgrowths are possible, but in atolls that have grown upward during long continued subsidence in a deep ocean, diminution of diameter with inward growth is the more probable for the following reasons.

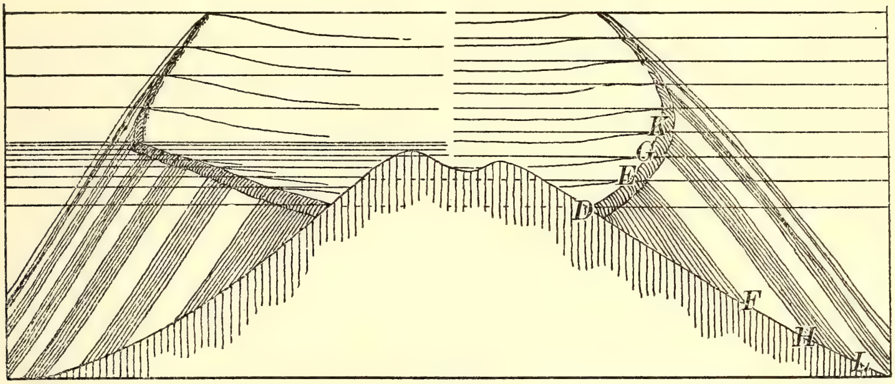


FIG. 1.

Let a sea-level fringing reef be formed around a young volcanic cone at *D*, figure 1, and let the horizontal lines on the right side of the figure represent successive levels of the sea with respect to a uniformly subsiding cone. Let the subsidence be so slow that the reef at first grows upward and outward, *DE*, on its own talus, which lengthens as subsidence progresses. The talus material consists chiefly of fragments broken by waves from the corals and other organisms which grow on the outer face of the reefs. In a given period of time, a growing reef face of given perimeter and 20 fathoms deep cannot produce more than a certain maximum volume of new growth; and as some of the new growth stands firm to build the reef upward, while some is broken off and washed inward to form the flat reef and the lagoon shoals, only the remainder can be sacrificed for the talus. Let this remainder be represented in section by *DEF*. If equal remainders are applied to talus building in equal time intervals, they may be represented by the shaded and unshaded quadrilaterals, *EFHG, GHLK*, etc., in which the breadth diminishes as the length increases.

So long as the reef growth is inclined outward, its increase in perimeter will, by providing a larger volume of coral growth, aid in supplying the demand for more talus material due to increase of depth; it is indeed conceivable that, if a reef foundation subsides very slowly and of the talus slants down to a level ocean floor of moderate depth, increase of perimeter may more than compensate for increase of talus length in the deepening water. In this case, outgrowth will continue at a constant or a lessening instead of at a steepening angle, as on the left side of figure 1. Especially favorable conditions for outward growth are provided when two islands stand so near each other that the submerged saddle between them is of moderate depth; for then the talus slopes on the line connecting the two island centers will be of decreasing length, if the rise of the talus intersection is faster than the subsidence of the islands. In such a case, reef cusps would tend to grow toward each other, and on uniting, the two concave reef faces would diminish their concavity by growing outward faster than the convex reef faces elsewhere, because the talus beneath a concave reef face is concentrated and thickened on converging lines of slope, while beneath a convex reef face it is distributed and thinned on diverging lines of slope. The double-looped barrier reef that encloses Makongai and Wakaya in central Fiji appears to be an example of this kind: on the other hand, four oval atolls north of the Exploring Isles in eastern Fiji, separated from each other by less distance than that which separates Makongai and Wakaya, show little tendency to develop approaching cusps; hence subsidence there may have been relatively rapid; and this is made probable by the occurrence of several 'drowned atolls' not far to the northeast.

But when the reef talus is built on the slope of a volcanic cone that rises from a deep ocean, the increase of coral growth on an enlarging reef perimeter may not fully compensate for increase of talus material demanded, especially if subsidence be relatively rapid; then the reef growth must be steeper in the second subsidence interval, *EG*, than in the first, *DE;* steeper in the third than in the second, and so on, in order that the thinner talus increments shall support it: otherwise the reef would have to grow outward in an overhanging, unsupported cornice. It is true that when upward growth becomes steep, the share of it that stands firm to build the reef upwards is smaller than before; thus the part that may be sacrificed for talus building is larger; but this will not make up for the increased demand due to increased talus length. A time must therefore come when upgrowth passes the vertical and thereafter inclines inward, as above *K:* then a decrease of perimeter sets in and the total volume of coral growth diminishes, while the demand for talus material still increases. Under these conditions, the angle of inward upgrowth must soon come to be but little steeper than the angle of talus slope. An atoll reef which has been reduced to a diameter of a mile when this condition is reached, will be reduced to a point when subsidence has progressed about 2000 feet farther. If subsidence then still continues the reef will be extinguished. Dana reached essentially the same conclusion, but without analysis; he briefly asserts that subsidence will gradually reduce the size of an atoll 'until it finally becomes so small that the lagoon is obliterated;' then, if "subsidence continues its progress . . . . it finally sinks the coral island, which, therefore, disappears from the ocean" (Corals and Coral Islands, 1872, 322, 323). Thus a difference may be drawn between atolls that are 'extinguished' by the reduction of their diameter to zero during a uniform subsidence (or during an intermittent subsidence at a constant average rate), and atolls that are 'drowned,' as Moresby phrased it, by a too rapid subsidence, whatever their diameter may be.

It is evident that many variable factors would enter the equation by which the ultimate extinction of a reef might be expressed; form of ocean floor, "rate of subsidence and length of intervening stationary periods" as Darwin phrased it, rate of coral growth, and strength of wave attack are the more important factors. The change in angle of upgrowth due to a change from slow to rapid subsidence is shown on the left side of figure 1. Further, an extinguished reef would be brought to light if subsidence were reversed into upheaval; such a reef might then increase in size if upheaval paused or halted; and thus increased, it might if subsidence were renewed again grow up for a while before being extinguished for a second time. Extinguished reefs thus brought to light may be called 'resurgent.'

Many smaller reefs in Fiji appear to be resurgent, as thus defined. A good example is Frost reef, *F*, figure 2, a third way between Mango, *M*, 7 miles to the east, and Vatu Vará, *V*, 13 miles on the west. Mango appears to be a denuded caldera ring 3 miles in diameter, which has been submerged 450 feet or more and rimmed at that level with a fringing reef, then uplifted 450 feet, and again fringed at present sea level; its highest summits now rise 650 feet above the sea. Vatu Vará is a former atoll, presumably formed during subsidence, but now standing 1030 feet above sea level and nearly 2 miles in diameter at its base; it is the highest elevated reef in Fiji. According to Andrews its steep slopes are contoured above and below mid-height by two corniced "water-lines;" hence its uplift must have been recent and rapid: like Mango, it is now fringed by a sea-level reef. Frost reef is a mile in diameter: it has no lagoon. Soundings of 103, 115, 134, 146, and 133 fathoms are charted at less than half a mile away; 200 fathoms and no bottom are recorded at three points toward Mango. If Frost reef be diminished by removing a fringe of somewhat less than the average sea-level fringe-width now surrounding Mango or Vatu Vará, it will be reduced to a mere point; and as such it was probably extinguished while Mango and Vatu Vará were subsiding; it must eventually have been submerged under more than 100 fathoms of water, when the highest reefs of Mango and Vatu Vará were at sea level, *ss;* but it was revealed as a resurgent reef when the following uplift brought sea level to *EE.*

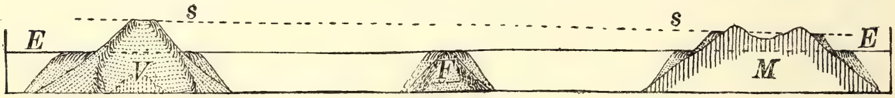


FIG. 2.

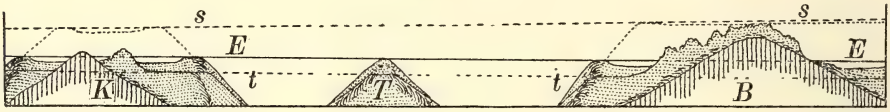


FIG. 3.

Midway in the eight-mile passage between the great barrier reef of the Exploring Isles, *B,* and Kimbombo reef, *K,* on the north is Trigger rock, *T,* figure 3, a minute reef in four fathoms of water, with the 100-fathom circle hardly half a mile in diameter, and soundings of 110, 121, 154 and 136 fathoms close outside. This rock must have been extinguished during the subsidence in which the now-elevated limestones of the Exploring Isles and Kimbombo were accumulating, and then submerged to a depth of 100 fathoms or more when the highest limestones were formed at the level *ss;* but a later upheaval brought it to light again as a resurgent reef, with sea level at *tt;* then it may have been recolonized for a time while the neighboring elevated limestones were suffering dissection; now, since the dissected limestones have again subsided, so that sea level is *EE,* the reef is almost extinguished for a second time. It is interesting to note that the upheaval of the Exploring Isles, by which the previously extinguished Trigger reef became resurgent, was of a significantly earlier date than the resurgence of Frost reef between Mango and Vatu Vará; for the limestones of these two islands are little dissected, while those of the Exploring Isles have been eroded to mere remnants of their former volume. It is the present almost-extinction of Trigger rock in a renewed subsidence of its region that is contemporaneous with the resurgence of Frost reef. Darwin's theory of intermittent subsidence is the only one of many coral-reef theories, which can account for the facts here adduced.

THE ORIGIN OF CERTAIN FIJI ATOLLS

By W. M. Davis

DEPARTMENT OF GEOLOGY AND GEOGRAPHY, HARVARD UNIVERSITY

Received by the Academy, June 22, 1916

The unconformable contact of many fringing reefs on the eroded submarine slopes of oceanic islands, and the embayment of such islands inside of barrier reefs are inconsistent with certain modern theories of coral reefs; and the recognition of these significant features has in recent years led several observers back to Darwin's earlier theory of upgrowth on intermittently subsiding foundations. My own experience two years ago, while on a Shaler Memorial voyage across the Pacific concerning which a brief report has been published in these PROCEEDINGS (**1**, 1915, 146–152), added evidence of reef formation during two periods of subsidence; the first subsidence being shown by the occurrence of elevated reef limestones resting unconformably on eroded volcanic foundations, as seen in Vanua Mbalavu and Avea, two of the reef-encircled cluster of the nine Exploring Isles in the eastern part of the Fiji group, the second subsidence being shown by the embayment of these now-dissected limestones, around which a new barrier reef has grown up, as stated more fully in the *American Journal of Science* for September, 1915.

Continued attention to this problem has lately enabled me to perceive that the evidence of two periods of subsidence and reef growth found in the Exploring Isles may be extended to several neighboring atolls, the area concerned being well shown in Plate 19 of Agassiz' "Islands and Coral Reefs of Fiji" (*Bull. Mus. Comp. Zool.*, xxxiii, 1899): thus singularly enough a reconciliation is permitted between Agassiz' theory of the formation of atolls on uplifted and worn-down limestone islands and Darwin's theory of the formation of atolls by upgrowth on subsiding islands. But it is not intended to imply that the view here announced applies to all atolls of the Fiji group, much less to all atolls elsewhere, most of which have probably been formed by upgrowth during intermittent subsidence without important interruption by uplift.

The essence of the case is as follows: In the Exploring Isles of the Fiji group, a portion of the largest volcanic island, Vanua Mbalavu, of well denuded form, 13 miles long and 930 feet high, is shown in the apex of sector M, figure 1; it is in part unconformably covered up to heights of 500 or 600 feet with heavy reef and lagoon limestones, deeply dissected and well embayed. Avea, a smaller member of the same cluster, two miles long, capped with limestones resting unconformably on denuded volcanic hills, rises 600 feet from the same broad lagoon, and is enclosed with the eight other Isles of the cluster by an irregular barrier-reef ring, 23 miles in longest diameter, part of which is shown in the same sector. All but one of these islands consist partly or wholly of elevated limestone: they therefore presumably represent, as Agassiz points out, "fragmentary remains of the land which must have once occupied the area of the lagoon." Outside of the barrier reef, twelve almost-atolls, atolls, and isolated reef patches rise from deep water, not more than 5 or 10 miles distant: four of these are represented in sector M: an almost-atoll in which the small central rocks are partly volcanic, partly limestone; a smaller almost-atoll, with limestone rocks alone rising from its lagoon; a true atoll; and a small reef on the verge of extinction. The islands and reefs are crowded together in the diagram to save space: the circular pattern which the barrier reef of the Explor-

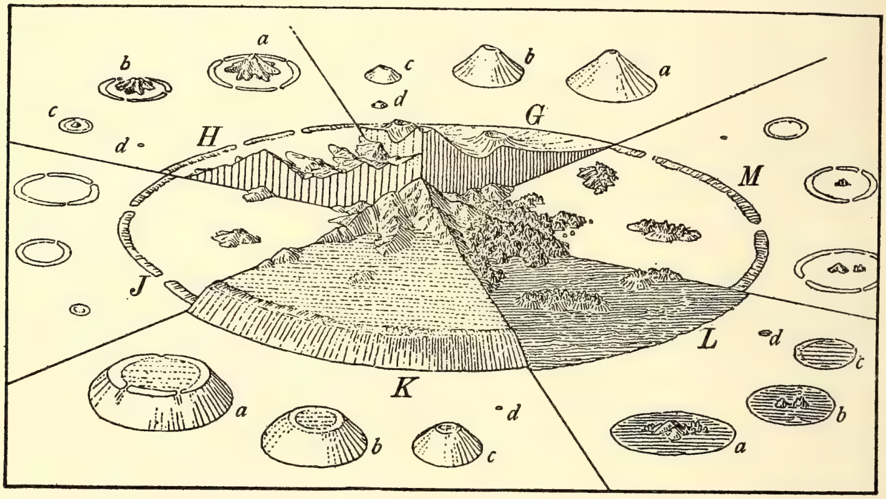


FIG. 1.

ing Isles here seems to have results from the repetition of one sector in six successive positions: the reef is really of irregular pattern and the islands are not centrally placed within it. A consideration of all the pertinent facts shows that, if the strong changes of level demanded by Vanua Mbalavu and Avea have extended to the exterior atolls, as is eminently probable, then the true atoll and the almost-atolls have been formed, during a recent subsidence of a few hundred feet, by upgrowth on the worn-down remnants of uplifted atolls, which had likewise been formed by upgrowth, presumably on a volcanic foundation, during an earlier time of greater subsidence.

The changes of level demanded by Vanua Mbalavu and Avea may be inferred from the structure of Avea, two miles long and 600 feet high, as shown in part in figure 2. It should be announced that this figure has been drawn from a rough outline made from a passing steamer; confidence is nevertheless felt in its essential correctness, for I fully agree with Gardiner and Agassiz that volcanic slopes and limestone cliffs in the Fijis are easily recognized at a distance. The underlying volcanic rocks here, as in Vanua Mbalavu, have the forms of rounded hills, subdued by deep erosion before they were covered with limestones. The eroded slopes dip under water, as at *x*; hence the island formerly stood at least *x* feet higher than now. The subdued hills and slopes are partly covered by limestones, now rising to a height of 600 feet; the limestones have suffered much erosion, which has greatly diminished their area, and has somewhat reduced their height, as at *y*. Hence the island after being eroded, must have sunk $x + 600 + y$ feet while receiving its unconformable limestone cover. The limestones are now dissected and their shore line is so well embayed as to necessitate a recent submergence of more than 100 feet; and it must have been during this submergence that the present great barrier reef has grown up. It may be noted in passing that the postglacial rise of ocean level may have had to do with the recent submergence, but for reasons elsewhere stated I believe that true subsidence has also been concerned in it.

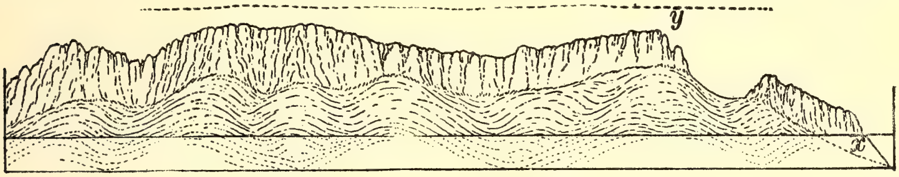


FIG. 2.

The six sectors, G to M, of figure 1 graphically represent the successive changes thus inferred. Sector G shows part of a group of confluent volcanoes, the area of which is about the same as that of the present Exploring Isles lagoon; several smaller cones rise near by; all together they resembled the present Lipari islands of the Mediterranean. After suffering prolonged dissection and partial submergence, the resulting embayed islands encircled by an upgrowing barrier reef are represented in sector H; this stage corresponds to that of Kandavu and its neighbors in southwestern Fiji. Down-sinking of the volcanic islands and upgrowth of the barrier reef continued until the total subsidence measured $x + 600 + y$ feet, when but few volcanic hills survived, as in sector J; the small Gambier islands in a large lagoon southeast of the Paumotus, or the small islands of Budd reef in northeastern Fiji represent this stage. An uplift of $600 + y + > 100$ feet then occurred, as in sector K: the resulting limestone plateau is typified by the uplifted atolls of the Loyalty group. The compound mass thus exposed to erosion was reduced over most of its limestone area to low relief surmounted here and there by residual hills, as in sector L; the hills of volcanic rock have smooth soil-covered slopes, those of limestone have steep cliffs and ragged crags. A recent submergence of 100 feet or more introduced the present conditions, as in sector M, where the lagoon floor has been smoothed by renewed deposition.

The exterior volcanic islands, a, b, c, d, of sector G, must have suffered essentially the same series of changes as the larger central volcanic islands. The atoll built on the largest cone, a, may have shown no volcanic knob in the stage of sector J, but one is afterwards laid bare in sector L, and its summit remains visible, along with a limestone knob, in the largest reefring of sector M. Island b having a less initial height in sector G, it now shows only limestone knobs, sector M. Island c, beginning as a small volcanic cone, sector G, was deeply covered with limestone in sector J, and reduced to a low surface without high limestone knobs in sector L; and this is reasonable enough, for its area is not so large as any one of several uninterrupted lagoon areas within the adjacent great barrier reef. Its present reef is the result of upgrowth during the subsidence which transformed sector L into sector M.

Seven of the twelve outlying reefs near the Exploring Isles are true atolls, like the true atoll of sector M. The foregoing discussion gives, I believe, a nearer approach to a demonstration of the origin of these sea-level atolls by upgrowth during sub-recent subsidence than has been provided for any other sea-level atolls, except Funafuti which has been penetrated by a deep boring. The smallest reef in sector M represents mere ledges of coral rock on the Admiralty chart: the 100-fathom line around each of three such reef rocks near the Exploring Isles is less than a mile in diameter, and the rocks are mere points; hence these minute reefs, beginning on small volcanic cones in the stage of sector G, must have been extinguished in sector J, resurgent in sector K, a little enlarged by outward growth in sector L, and almost extinguished again in sector M.

The upheaval and the sub-recent subsidence mentioned in the foregoing paragraph were not uniform, as has thus far been implied. The recent subsidence is believed to have increased to the east or northeast; first, as Agassiz pointed out, because the floor of the great lagoon of the Exploring Isles increases in depth from 20 fathoms or less on its western side to 80 or 100 fathoms on its eastern side; second, because several 'drowned atolls' or submerged banks lie to the northeast; indeed the northeastern-most of the seven outlying atolls is mostly submerged; third, because Mango, 10 miles to the southwest, has elevated reefs moderately dissected at an altitude of 500 feet, and 30 miles to the west, Yathata and Vatu Vará are uplifted, undissected atolls at altitudes of 840 and 1030 feet. The intermediate island of Kanathea, nearer to the Exploring Isles barrier reef than Mango is, also seemed to me to bear small uplifted reefs at a height of about 600 feet, but I was too far from this island to make sure of it.

The upheaval that preceded the subrecent subsidence must also have been unequal and greater to the east than to the west, because while the Exploring Isles were thus uplifted long enough ago to have been afterwards well dissected, Yathata and Vatu Vará were at that time presumably sinking and growing, preparatory to being uplifted recently as above stated. These unequal changes of altitude cannot be explained by changes in the level of the ocean, which are everywhere alike; they can be explained only by unequal subsidence and upheaval of the islands concerned.

INTERFEROMETER METHODS BASED ON THE CLEAVAGE OF A DIFFRACTED RAY

By C. Barus

DEPARTMENT OF PHYSICS, BROWN UNIVERSITY

Received by the Academy, June 20, 1916

The prismatic method of cleaving the incident beam of white light is available for the superposition of non-reversed spectra, under conditions where the paths of the component rays may have any length whatever.

It is thus an essential extension of the same method as used for reversed spectra, heretofore, and also of the methods in which the paths are essentially small.

In figure 1, *P* is the first prism cleaving the white beam, *L*, diffracted by the slit of the collimator. *M* and *N* are the opaque mirrors, the former on a micrometer. For greater ease in adjustment, the second prism *P'* is here right angled, though this is otherwise inconvenient, since the angle, $\delta = 90° - \varphi$, is too large. The rays reflected from *P'* impinge normally on the reflecting grating *G* (grating space, $D = 200 \times 10^{-6}$) and are observed by a telescope at *T*. *P*, *P'*, *M* and *N* are all provided with the usual three adjustment screws. *P'* must be capable of being raised and lowered and moved fore and aft. The field is brilliantly illuminated. When the path difference is sufficiently small the fringes appear and cover the whole length of superposed spectrum, strongly. They are displaced with rotation if *M* is moved normally to itself.

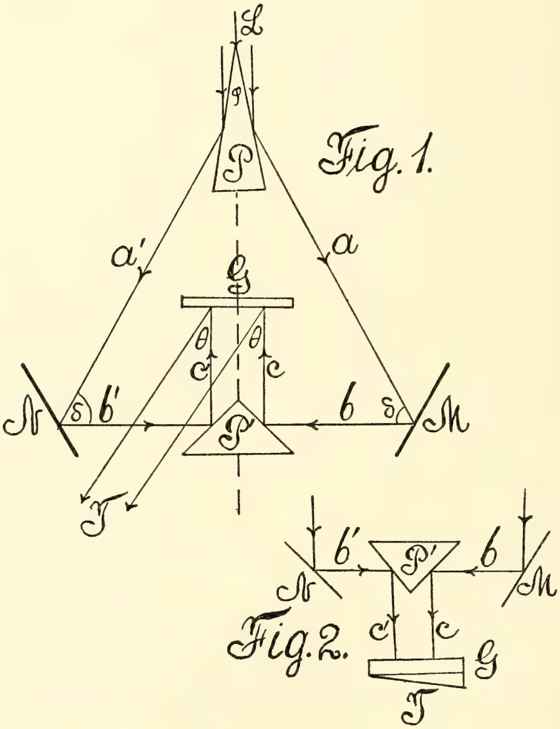


Fig. 1.

Fig. 2.

As first obtained the fringes were too close packed for accurate measurement; but the following example of the displacement, *e*, of the mirror

M, for successions of 40 fringes replacing each other at the sodium lines, show the order of results: $10^{-3}e = 1.55, 1.40, 1.60, 1.55$ cm., so that per fringe

$$\delta e = 39 \times 10^{-6} \text{ cm.}$$

The computed value would be (φ, the prism angle)

$\delta e = \lambda/2\cos(\delta/2) = 58.93\times10^{-6}/2\times0.81 = 36.4\times10^{-6}$ cm., assuming $\delta = 90°-\varphi$. The difference is due both to the small fringes which are difficult to count and to the assumed value of δ. The range of measurement is small (if *M* only moves), not exceeding 1.6 millimeters for a moderately strong telescope. Usually but one half of this displacement is available as the fringes increase in size (usually with rotation) from fine vertical hair lines to a nearly horizontal maximum and then abruptly vanish. But one half of the complete cycle is thus available.

If we regard the component beams, *a*, *b*, *c*, and *a′*, *b′*, *c′* as being of the width of the pencil diffracted by the slit of the collimator, it is clear that the maximum size of fringes will occur when *c* and *c′* are as near together as possible: furthermore, that as *M* moves toward *P′*, *c* continually approaches *c′*, until *b* drops off (as it were) from the right angled edge of the prism *P′*. To get the best conditions, i.e. the largest fringes, *c* must therefore also be moved up to the edge of *P* and very sharp angled prisms be used at both *P* and *P′*. The largest fringes (lines about 10 times the D_1D_2 distance) obtained with the right angled prism were often not very strong, though otherwise satisfactory. Much of the light of both spectra does not therefore interfere, being different in origin.

Results very similar to the present were described long ago[1] and found with two identical half gratings, coplanar and parallel as to rulings, etc., when one grating was displaced normal to its plane relative to the other. The edges of the two gratings must be close together, but even then the fringes remain small and the available paths also. Strong large fringes, but with small paths, were obtained by the later method[2] of two identical transmitting gratings, superposed.

If the prism *P′* is right angled, it may be rotated as in figure 2, so that the rays *c* and *c′* pass off towards the rear. They are then observed through an Ives' prism grating *G* and a telescope at *T*. This method admits of much easier adjustment. With the component beams, *a*, *b*, *a′*, *b′*, coplanar, horizontal and of about equal length in the absence of the prism *P′*, the latter is now inserted with its edge vertical (rotation) and the white slit images in *T* (without *G*) superposed, horizontally and vertically. *G* is then added and the micrometer at *M* or *N* manipulated till the fringes appears. As above, they are largest when c and c' are as nearly as possible coincident and vanish as horizontal fringes at the maximum; for the effective parts of c and c' are component halves of the same diffracted beam from the slit.

It is interesting to observe, since interference[3] also occurs when one of the superposed spectra is inverted on a line parallel to its length, that such diffraction is demonstrable in case of homogenous light, even when the slit is absent.

A fuller report of this work has been presented to the Carnegie Institution of Washington, D. C.

---

[1] *Phil. Mag.*, **22**, 118–129 (1911); *Carnegie Inst. Publ.*, No. 149, Chap. VI.

[2] *Physic. Rev.*, July, 1916; *Science*, **42**, 841 (1915).

[3] *Amer. J. Sci.*, **40**, §4, 491 (1915).

ON THE INHERITANCE OF CERTAIN GLUME CHARACTERS IN THE CROSS *AVENA FATUA* × *A. SATIVA* VAR. KHERSON

By Frank M. Surface

BIOLOGICAL LABORATORY, MAINE AGRICULTURAL EXPERIMENT STATION

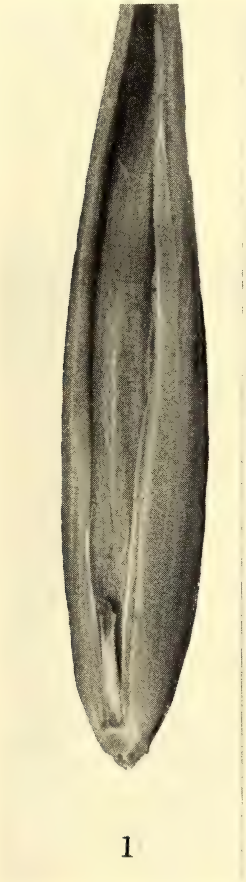
Received by the Academy, June 30, 1916

The investigation reported in the present paper[1] deals with the inheritance of certain characters of the flowering glumes in a cross between a wild oat *Avena fatua* and a cultivated variety (*Avena sativa*). The cultivated oat used is a selection from the Kherson variety. Both parent strains have been grown as pure lines for five years and are known to breed true.

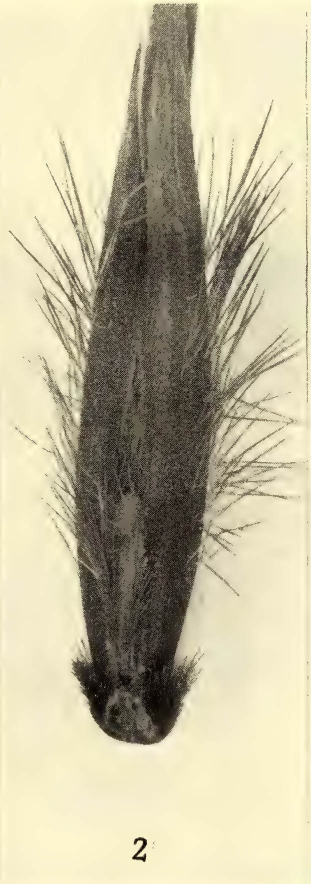
The parent varieties used in this cross possess the following contrasting glume characters.

CHARACTER	AVENA FATUA	AVENA SATIVA VAR. KHERSON
Grain color	Dark brown or black	Yellow
Base of grain	Wild type	Cultivated type
Shattering	Shatters	Does not shatter
Awns	Heavy, twisted and geniculate awns on both upper and lower grains of a spikelet	None or an occasional awn on the lower grain. None on the upper
Pubescence Base of grain	Thick pubescence on lateral and dorsal sides of callus on both grains	None or occasionally 1 or 2 hairs at the sides of the base of the lower grain
Back of grain	Heavy pubescence on the back of both grains	None
Pedicel	Heavy pubescence on both grains	None

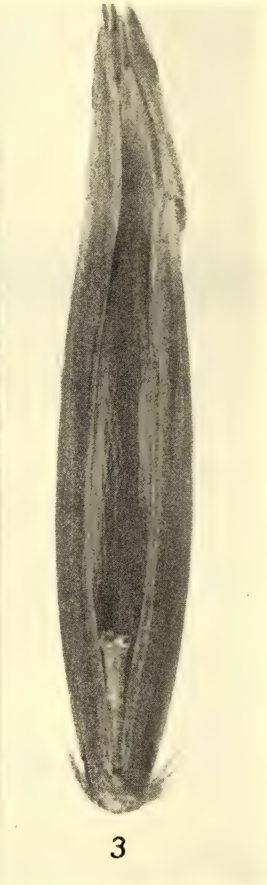
As shown in figure 2, the base of the grain or callus on the wild oat is expanded into a sucker-like ring. This large cleavage plane permits the grain to separate from the outer glumes very easily when mature. The cultivated grain (fig. 1) possesses a narrow contracted base and the grain does not shatter under ordinary conditions.



1



2



3

FIG. 1. VENTRAL VIEW OF THE LOWER GRAIN FROM A SPIKELET OF KHERSON OATS SHOWING THE CULTIVATED BASE OF THE GRAIN AND THE ABSENCE OF PUBESCENCE. × 8.

FIG. 2. VENTRAL SURFACE OF THE LOWER GRAIN FROM A SPIKELET OF WILD OATS SHOWING THE WILD BASE, THE PUBESCENCE ON THE PEDICEL AND THE SIDES OF THE GRAIN. × 8.

FIG. 3. VENTRAL VIEW OF THE LOWER GRAIN FROM AN F_1 PLANT SHOWING THE INTERMEDIATE BASE, PUBESCENCE AT SIDES OF THE BASE AND ABSENCE OF PUBESENCE ON THE PEDICEL. × 8.

In the F_1 generation the grain is brown in color but ordinarily not quite so dark as the pure wild. The base of the lower[2] grain in each spikelet is intermediate between the wild and cultivated condition (fig. 3).

The grain does not shatter. The base of the upper grain on each spikelet is like the upper grain of the cultivated variety. The F_1 grain is pubescent on the back of the lower grain of a spikelet but unlike the wild the upper grain of each spikelet is entirely free from pubescence. There is a tuft of hair at the sides of the base of the lower grain but none on the base of the upper grain. Medium strong to weak awns occur on the lower grain of most spikelets of the F_1 plants, but there are never any awns on the upper grain.

The F_2 generation consisted of 465 plants. A small F_3 generation was also grown. This consisted of short rows from 70 different F_2 plants.

The F_2 generation shows that the wild parent carries a factor for gray glume color as well as for black. It is also probable that the wild carries a gene for yellow color. The cultivated parent carries the factor for yellow color. These three colors segregate independently of each other, giving 12 black; 3 grey; 1 yellow. The numbers obtained were 347 black; 88 grey, 30 yellow, which is a very close agreement with expectation.

With regard to the base of the lower grain three types may be recognized: (1) the cultivated base (fig. 1); (2) the intermediate base like that found on the F_1 grain (fig. 3); and (3) the wild base (fig. 2). These three types appear in a 1: 2: 1 ratio on the F_2 generation. The observed numbers are 117 cultivated: 236 intermediate: 112 wild. The difference between the cultivated and wild base appears to be due to a single pair of genes. The heterozygous condition gives rise to the intermediate base.

The gene for the base of the lower grain segregates independently of the several color genes. For each of the three grain colors the plants with cultivated, intermediate and wild bases approach the 1: 2: 1 ratio.

There are, however, a number of characters which in the present cross are always associated with a particular type of base. In all there are seven pairs of characters which show this absolute correlation. The condition of each of these characters for the three types of bases on the lower grain is given in Table 1.

The present data are not sufficiently extensive to determine whether these several phenotypic characters are caused by the action of a single pair of genes or whether they are due to several pairs of genes very closely linked together. Nilsson-Ehle (1911) regards this group of characters as due to the presence or absence of a single inhibiting factor. In certain other crosses not yet completely analyzed but involving this same strain of *A. fatua* some of these correlations have been broken.

TABLE 1

CHARACTERS CORRELATED WITH THE THREE TYPES OF BASES ON THE LOWER GRAIN

CULTIVATED BASE	INTERMEDIATE BASE	WILD BASE
Cultivated base on the upper grain	Cultivated base on the upper grain	Wild base on the upper grain.
Absence or almost complete absence of awns on the lower grain	Medium awns on the lower grain of some spikelets	Very heavy awns on the lower grain of every spikelet.
Total absence of awns on the upper grain	Total absence of awns on the upper grain	Very heavy awn on the upper grain of every spikelet
Absence of pubescence on the pedicel on the lower grain.	Absence of pubescence on the pedicel on the lower grain	Heavy pubescence on the pedicel on the lower grains.
Absence of pubescence on the pedicel on the upper grain	Absence of pubescence on the pedicel on the upper grain	Heavy pubescence on the pedicel on the upper grain
None or sometimes very slight pubescence at the sides of the base of the lower grain	Medium heavy tuft of hair at the side of the base of the lower grain	Heavy pubescence on all sides of the base of the lower grain
No pubescence on the base of the upper grain	No pubescence on the base of the upper grain	Heavy pubescence on all sides of the base of the upper grain

This and other facts have led the writer to believe that some of these characters at least are controlled by separate genes.

Characters Showing Partial Linkage.—Two characters have been discovered in this cross which apparently show partial linkage with other genes. These characters are (1) pubescence on the back of the lower grain and (2) pubescence on the back of the upper grain. These two characters are inherited independently of each other but the phenotypic appearance of the latter character is dependent upon the presence of the former gene in the same zygote.

In the F_2 generation the plants which have pubescence on the back of the lower grain segregate from those lacking this character in a 3 to 1 ratio. The observed numbers are 347 pubescent to 118 smooth. This character further segregates independently of the type of base on the lower grain.

On the other hand there is a very close relation between the presence and absence of black color and the presence and absence of pubescence on the back of the lower grain. The observed numbers are shown in table 2.

There is an almost absolute correlation between the presence of pubescence on the lower grain and the black color. However, two black plants were found which lack this pubescence entirely. The grain from these two plants has been carefully examined and no evidence of

TABLE 2

RELATION OF THE PUBESCENCE ON THE BACK OF THE LOWER GRAIN TO THE COLOR OF THE GRAIN

	BLACK		NON-BLACK	
	PUBESCENT	SMOOTH	PUBESCENT	SMOOTH
Observed Number................	345	2	0	118

pubescence could be found. No plants with pubescent non-black grains have been found in this cross. However, in certain other crosses in which this same wild was used, a few non-black pubescent plants have been found.

While the present data are hardly sufficient to prove the point, it seems very probable that we have here a case of partial linkage. Although a straight F_2 generation is not the best kind of data with which to study linkage, certain points can nevertheless be made out.

As the observed figures stand they represent a coupling series in which the gametic ratio (in Bateson's sense) is approximately 240 : 1. If the present supposition is correct we might reasonably expect that some non-black pubescent individuals should appear in the 465 plants. Their absence is probably due to a chance fluctuation. If we may assume that one such individual is to be expected in 465, this will make a total of three crossover plants and the gametic ratio would be approximately 150: 1. This means that instead of appearing in equal numbers the different classes of gametes will be formed in the ratio 150 black, pubescent: 1 black, non-pubescent: 1 non-black, pubescent: 150 non-black, non-pubescent.

In regard to the gene for pubescence on the back of the upper grains it will be remembered that the absence of pubescence in this case is dominant over its presence. In the F_2 generation there are 378 plants without pubescence (smooth) to 87 which are pubescent on the upper grain. The ratio here is not 3 to 1, but 4 to 1. Likewise the relation between this gene and the gene for pubescence on the back of the lower grain gives a 9: 3: 4 ratio. No individuals occur which are smooth on the lower grain and pubescent on the upper. Apparently the gene for pubescence on the lower grain acts as a basic pubescence factor in a manner quite similar to the color factor (C) in mice, rabbits, sweet peas and many other organisms. In the absence of the factor for pubescence on the lower grain the factor for pubescence on the upper grain remains inactive.

The fact that this basic pubescence factor is linked with the black color factor disturbs the phenotypic ratio of the non-black plants when the character pubescence on the back of the upper grain, is correlated with other characters.

Table 3 shows the relation between this character and the type of base on the lower grain. In this table the cultivated and intermediate types of base are grouped together under the term 'cultivated.'

It is seen at once that there is distinct evidence of linkage between these two genes. Since the ratio in the non-black plants is disturbed by interdependence of two genes, it is necessary, in considering the question of linkage, to use the data from black plants only as given in the second line of the table.

Calculating the probable gametic ratio necessary to produce such an F_2 generation (using the black plants only) it is found that this ratio is approximately 65 to 1.

To sum up the main features of the present hypothesis, it is found that the gene for pubescence on the back of the upper grain segregates independently of the gene for pubescence on the back of the lower grain. However, the former gene is unable to produce a pubescence unless the factor for pubescence on the lower grain is present in the same zygote. The gene for pubescence on the back of the lower grain shows partial linkage with the gene for black color. The gametic ratio is apparently about 150 to 1. The gene for pubescence on the back of the upper grain is partially linked with the gene for the wild base. The gametic ratio in this case is approximately 65 to 1. It is very probable that these gametic ratios will be changed somewhat when larger numbers of individuals are available. It is believed that the present assumption represents the essential facts.

TABLE 3

RELATION BETWEEN THE PUBESCENCE ON THE BACK OF THE UPPER GRAIN AND THE CHARACTER OF THE BASE

	CULTIVATED		WILD	
	SMOOTH	PUBESCENT	SMOOTH	PUBESCENT
All plants	352	2	27	85
Black plants only	257	2	3	85

The brilliant work of Morgan and his collaborators upon linkage and its relation to the chromosome theory of inheritance makes it exceedingly attractive to point out the possible relation of these genes to the chromosomes. In the first place it has been shown by Nilsson-Ehle (1909) and partly by the present work that the three pairs of color genes segregate independently of each other. It may, therefore, be supposed that they lie in separate chromosomes. Further the gene for the character of the base of the grain segregates independently of the color genes. It, therefore, probably lies in a fourth chromosome. The seven characters given in table 1 are very closely if not absolutely linked with the character of the base. If these seven characters may be supposed to be due to separate pairs of genes these must lie in this fourth chromosome.

Still a ninth pair of genes is located in this fourth chromosome, viz., that for smoothness (lack of pubescence) on the back of the upper grain. This gene is apparently located at a slight distance from the group discussed above since it shows about 1.5% of crossovers with the members of that group.

The gene for pubescence on the back of the lower grain is linked with the gene for black color and is, therefore, to be regarded as located in the same chromosome. The per cent of crossovers between these two genes is probably less than 0.7%.

---

[1] This is an abstract of paper No. 95 from the Biological Laboratory of the Maine Agricultural Experiment Station. The complete paper is now in press in *Genetics*.

[2] A spikelet of common oats usually bears two grains. The larger of these is called the "lower" grain and the smaller one the "upper" grain. The upper grain is articulated with the lower by means of a short pedicel.

A COMPARISON OF THE RATES OF REGENERATION FROM OLD AND FROM NEW TISSUE

By Charles Zeleny

ZOÖLOGICAL LABORATORY, UNIVERSITY OF ILLINOIS

Received by the Academy, July 15, 1916

In determining the factors of regeneration one of the questions that arises is the extent to which rate of growth of the new organ is controlled by the character of the cells at the cut surface. Is regeneration wholly a matter determined by the characteristics of the local cells or is the process under more central control? If the former is true, change in the condition of the cells near the cut surface should modify the rate of regeneration, if the latter, such change does not necessarily mean change in rate.

The present study consists of a comparison of the rate from newly regenerated tissues with that from old tissues. When a portion of the tail of a frog tadpole is removed by a transverse cut there is near the cut surface a considerable degree of reorganization of the cells which are to give rise to the new organ. If, before the completion of regeneration, a second removal is made the regeneration will be from new cells if the second cut is distal to the level of the first and from old cells if it is proximal to that level. In order to insure equality of level the first cut was made at *A* in the figure for a new tissue basis and at *B* for an old tissue basis, the second regeneration level, in each case coming between *A* and *B*. If rate control is a function merely of the readiness of the cells the second regeneration from the new tissue level should be more rapid than that from the old tissue level.

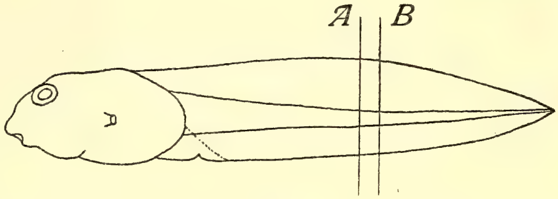


FIG. 1

Three groups of experiments were made on tadpoles of the green frog, Rana clamitans. In each it is clear that there is no striking difference between the rates from the two kinds of cells.

In experiment I the average regenerated length in ten individuals at the end of six days is 2.16 mm. from old tissue and 2.15 mm. from new tissue. The removed lengths are not exactly alike in the two cases and it is necessary to make a correction for the difference. It is shown elsewhere that within the limits of the present experiment rate of regeneration is directly proportional to length removed. The specific length regenerated or length per unit of removed length is therefore a constant within these limits. It is therefore fairer to use the specific than the absolute lengths. The specific lengths regenerated in the present experiment at six days are 0.196 for old and 0.204 for new tissue.

In the same experiment at eight days the average lengths are 3.19 mm. from old tissue and 3.12 from new and the specific lengths are respectively 0.303 and 0.310.

These values are given in table 1.

TABLE 1

COMPARISONS OF REGENERATIONS FROM OLD AND NEW TISSUE IN THE TAIL OF THE TADPOLE OF RANA CLAMITANS. SERIES 3628–3675

	TOTAL LENGTH	TAIL LENGTH	REMOVED LENGTH	REGENERATICN TIME SIX DAYS		REGENERATION TIME EIGHT DAYS	
				Regenerated length	Specific length regenerated	Regenerated length	Specific length regenerated
	mm.	*mm.*	*mm.*	*mm.*		*mm.*	
Average from old tissue.	32.9	21.5	11.3	2.16	0.196	3.19	0.303
Average from new tissue.	34.0	22.2	10.7	2.15	0.204	3.12	0.310
Old—ahead............				0.01		0.07	
New—ahead...........					0.008		0.007
Old—times ahead......				4	3½	3	3
New—times ahead.....				6	6½	3	3

In experiment II similar data were obtained for four, six, eight, ten, twelve and a half, eighteen and fifty-six days of regeneration. The removed lengths are quite different in the different individuals so that specific lengths alone are valid. Giving the old tissue level first in each case and using only removals of over 4 mm. these values for the seven times are respectively 0.043 and 0.045, 0.135 and 0.143, 0.216 and 0.224, 0.292 and 0.293, 0.331 and 0.337, 0.352 and 0.348, and 0.345 and 0.346. They are given in more detail in table 2.

TABLE 2

COMPARISON OF REGENERATIONS FROM OLD AND NEW TISSUE IN THE TAIL OF THE TADPOLE OF RANA CLAMITANS. SERIES 3676–3765

DAYS OF REGENERATION	OLD TISSUE SPECIFIC LENGTH OF REGENERATION	NEW TISSUE SPECIFIC LENGTH OF REGENERATION	OLD AHEAD	NEW AHEAD
4	0.043	0.045		0.002
6	0.135	0.143		0.008
8	0.216	0.224		0 008
10	0.292	0.293		0.001
12½	0.331	0.337		0.006
18	0.352	0.348	0.004	
56	0.345	0.346		0.001
Average....				0.003

Similar results were obtained when the removed lengths were under 4 mm.

In experiment III completed lengths from old and new tissue are compared in each of the third, fourth and fifth regenerations. It is necessary to point out that in the tadpole tail the completed regeneration is always less in length than the removed tail. The comparison here therefore concerns the degree of replacement of the lost part and not its rate. The average regenerated lengths giving the old tissue levels first are 7.9 mm. and 7.9 mm. for the third, 5.3 and 5.5 for the fourth and 6.6 and 5.9 for the fifth successive regeneration.

The data as a whole show clearly that there is no essential difference between rate of regeneration from new cells and from old cells. There is only a very slight advantage in favor of the new cells while a striking one would be expected if rate were largely determined by local cell characteristics. Rate of regeneration seems therefore to be under central control. This conclusion is in agreement with the results of experiments on other factors.

The full data will be published in the *University of Illinois Biological Monographs*.

THE EFFECT OF SUCCESSIVE REMOVAL UPON THE RATE OF REGENERATION

By Charles Zeleny

ZOÖLOGICAL LABORATORY, UNIVERSITY OF ILLINOIS

Received by the Academy, July 15, 1916

One of the most interesting facts in connection with regeneration is the ability to replace a part after repeated removal. Former studies by the writer show that as a rule the rate of regeneration following a removal is no greater than that following second and later removals if the effect of age is eliminated. Where a difference exists it is in favor of the later regeneration.

The matter is of very great interest in connection with general problems of development and particularly in connection with the question as to the existence or non-existence of a necessary limit to the amount of living substance that a single individual may produce during its life cycle. Does the production of a group of tissues use up a part of a certain store of developmental energy possessed by the individual or is this store inexhaustible or perchance even increased by exercise of the function? These questions warrant more extended study especially in view of the additional analysis that has been made of other factors controlling the rate of regeneration. The new data support the conclusion previously reached and make possible a further analysis of the character of the difference between successive regenerations.

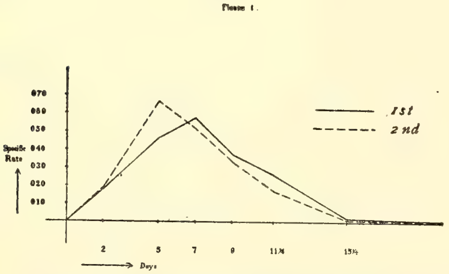
In making a comparison of successive regenerations a method must be devised for eliminating the effect of age. If the rate of a second regeneration is compared with that of a first regeneration in the same individual any difference that is made out may be due not to the pure effect of successive injury but to the effect of difference in age.

The method pursued in the present experiments consists in the initial removal of a portion of the tail in one-half of a set of Amphibian larvae of equal age. When regeneration has proceeded for several days there is a second removal of the part accompanied by a first removal in the half of the set that had not previously been operated upon. The second regeneration that ensues in one-half of the set may be compared directly with the first regeneration in the other half. There is no difference in age.

In experiment I approximately one-half of the tail was removed in tadpoles of the green frog, Rana clamitans. At six days the average first regeneration length is 2.01 mm. and the average second regeneration length 2.18 mm. The first exceeds the second in five sets and the second exceeds the first in six. The corresponding specific lengths or lengths regenerated per unit of removed length are 0.194 and 0.205. The first regeneration exceeds the second in two sets, the second exceeds the first in five and one is tied. The second regeneration has the advantage in all the comparisons.

At eight days the average first regeneration length is 3.06 mm. and the second 3.42 mm. The first exceeds the second in three sets and the second exceeds the first in seven. The corresponding average specific lengths are 0.298 and 0.323. The first regeneration exceeds the second in four sets and the second exceeds the first in six. The second regeneration has the advantage in all the comparisons.

This advantage of the second regeneration over the first in experiment I holds true of second regenerations from both old and new tissue levels.



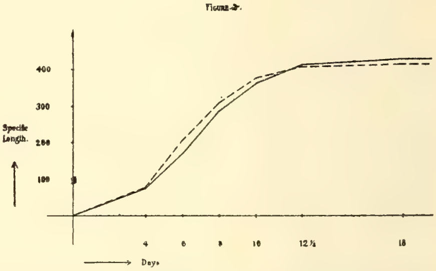


FIG. 1. CHANGE IN SPECIFIC RATE OF REGENERATION DURING THE REGENERATION PERIOD FOR BOTH FIRST AND SECOND REGENERATIONS. TADPOLE TAIL OF RANA CLAMITANS. UNBROKEN LINE = FIRST REGENERATION. BROKEN LINE = SECOND REGENERATION.

FIG. 2. SPECIFIC REGENERATED LENGTHS DURING THE REGENERATION PERIOD FOR BOTH FIRST AND SECOND REGENERATIONS. TADPOLE TAIL OF RANA CLAMITANS. UNBROKEN LINE = FIRST REGENERATION. BROKEN LINE = SECOND REGENERATION.

In experiment II, also with Rana clamitans, operations were made at the $\frac{1}{10}$, $\frac{1}{3}$, $\frac{1}{2}$ and $\frac{2}{3}$ levels in a sufficient number of individuals to yield valid data. Regeneration measurements were made at each of these levels 4, 6, 8, 10, 12½, 18 and 56 days after the operations. The second regeneration at these levels is ahead of the first until about the twelfth day after which the first regeneration catches up. The maximum rate for both regenerations is reached before this time, and earlier for the second than for the first regeneration. The relation between the rates is given in figure 1 and between the total regenerated lengths in figure 2.

In experiment III two-thirds of the tail was removed in larvae of the salamander, Amblystoma punctatum. A comparison of the first, second and third regenerations was made at 2, 4, 6, 8, 10 and 14 days. At two days the first, second and third regenerations average respectively 0.22, 0.25 and 0.26 mm. The corresponding values at four days are 0.66, 0.75 and 1.00; at six days 1.36, 1.40 and 1.36; at eight days 2.18, 2.68 and 2.68; at ten days 3.55, 3.82 and 4.20; at fourteen days 5.34, 6.12 and 6.08. The advantage is in favor of the second and third regenerations as opposed to the first and of the third as opposed to the second. Individual comparisons at each of the different times show similar results.

The evidence as a whole shows clearly that when the age factor is eliminated there is no decrease in rate of regeneration with successive removal. On the contrary the second regeneration is more rapid than the first up to the period of maximum rate. The second regeneration however passes its maximum sooner than does the first and after the tenth day the latter therefore catches up in total amount regenerated. There is no striking difference between the second and the third regenerations but in each comparison the third has a slight advantage.

When successive regenerations within single individuals are compared there is a progressive decrease in rate and it is obvious that this is due to age.

The possibility has suggested itself that the second regeneration starts out at a more rapid rate than the first because the cells at the cut surface were undergoing regenerative changes at the time of the new operation and can therefore start the process much faster than can the old cells at the first surface of regeneration. Following a first removal there is a considerable degree of reorganization of the cells at the cut surface, accompanied by active migration. During this period which in Rana clamitans lasts two or three days there is little or no mitotic cell division. Then follows a division period which reaches its maximum at seven to ten days. Its decline is associated with the oncoming of tissue differentiation.

A special study has been made of the relative rates of second regeneration from old cells following a cut inside of the first removal level and from new cells following a cut outside of the first level. This comparison shows only a very slight difference in favor of the new cells and this is largely confined to the early stages, the period of cell migration.

The period of increase in rate is the period of active cell multiplication and the decline in rate is associated with cell differentiation. The second regeneration therefore reaches the period of differentiation slightly in advance of the first regeneration.

Apart from the slowing due to age there is no indication of a limitation of the amount of new material that may be produced by regeneration. The actual limitation comes not from the using up of regenerative or developmental energy or of 'determiners' by repeated regeneration but from changes in the non-regenerating part associated with age. In another place there is a discussion of the possibility that there may be an effect upon the rate of developmental processes in the organism as a whole due to continued regeneration of a part. This is studied particularly in connection with the effect of regeneration upon rate of metamorphosis in Amphibia.

Regeneration studies in general and those on successive regeneration in particular make it improbable that there is a definite number of cell regenerations between the fertilized egg and the end product, the differentiated cell. The possibility that certain cells may remain in an early cell generation can not be wholly excluded as an explanation of at least part of first regeneration phenomena. Under suitable stimulation such cells may be postulated to take up development where it has left off. The definite descriptions of de-differentiations of cells as well as other facts of regeneration argue against this conclusion. The view that there can be no such definite number of cell generations is strengthened by the facts of successive regeneration. It does not seem probable that embryonic cells of an early cell generation can be held in reserve through repeated regenerations.

The explanation of regeneration by the theory of duplicate sets of determiners meets difficulties in undiminished successive regenerations. The greater the number of repeated regenerations the greater the difficulties of explanation on this basis. Of course the difficulty does not hold for the hypothesis that every cell or nearly every cell contains a full set of determiners.

The earlier appearance of the maximum rate in the second than in the first regeneration may be due to the more rapid progress of the cells in the early cell migration period alone or it may be due to the acceleration of the whole developmental cycle.

The full data will be published in the *University of Illinois Biological Monographs.*

THE GEOLOGIC RÔLE OF PHOSPHORUS

By Eliot Blackwelder

DEPARTMENT OF GEOLOGY, UNIVERSITY OF WISCONSIN

Received by the Academy, July 19, 1916

Phosphorus appears in nature in many forms and in many situations. Its numerous transformations, however, follow an orderly sequence. In a broad way, the changes form a cycle but since the path of change divides here and there, the element really progresses through a series of subordinate cycles all having a common beginning and capable of being brought eventually to a common end.

Omitting the rare meteorites, we find the primary source of phosphorus in magma (molten lava). Almost all magmatic rocks, whether they solidify in the interior of the earth or are poured out upon its surface, contain this element. According to Clarke,[1] the average igneous rock of the world contains 0.29% of P_2O_5. The proportion is somewhat greater in the basic rocks, such as gabbro and the peridotites; and in the peripheral portions of certain magnetite ore bodies, which are believed to be merely ultrabasic segregations from a less basic magma, the quantity of P_2O_5 may exceed 10%. In these rocks the phosphorus almost invariably appears in the form of the mineral apatite, a compound of tricalcium phosphate with calcium fluoride or chloride.

The hot solutions and vapors emitted by subterranean bodies of magma, especially during the progress of crystallization, permeate the adjacent rocks, and, through the crystallization of their constituents upon the walls of fissures, are believed to form the veins known as 'pegmatites.' In rare instances pegmatites contain noteworthy quantities of apatite—generally in large crystals. Some of the Norwegian apatite deposits are probably of this origin.

There is another type of phosphatic veins which is even rarer than the pegmatitic variety, although locally in central Spain there are important deposits of this kind. These are the fibrous veins consisting not of apatite, but of the minerals staffelite or dahllite—hydrous calcium carbo-phosphates[2] containing about 38% of P_2O_5. These veins traverse quartzite, slate and limestone, but have a suggestive association with intrusions of granite. While their origin is not known, it may be suggested that they will eventually prove to have crystallized from magmatic solutions at a considerable distance from the parent intrusion and at a comparatively low temperature. Certain even rarer primary deposits containing phosphorus minerals may be neglected in this brief sketch.

At and near the surface of the earth, and under favorable climatic conditions, rocks of all kinds are subject to chemical decomposition. One of the chief agents of this process is ground water containing carbonic acid and other solvent substances. In this weak solution apatite and the hydrous phosphates such as dahllite dissolve somewhat readily, as compared with most other common minerals, although much less rapidly than calcite ($CaCO_3$).

The dissolved portion circulates through the rocks and is disposed of in several ways. Much of it is absorbed by plants and thus vicariously by animals, but returns again to the soil after the death of the organism, if not before. A very large part either immediately or eventually joins the streams and finds its way to the ocean, suffering on the way certain minor losses which may be passed over here.

Of the vast quantities of dissolved mineral matter delivered to the ocean annually by streams, it is estimated that nearly $\frac{1}{2}$ of 1% (0.45%) consists of phosphoric acid. Using the best available figures for the amount of water reaching the ocean, we may calculate that if the phosphatic material in the form of tricalcium phosphate were loaded into standard railroad cars it would fill a train stretching continuously from New York to Seattle and would be from 7 to 12 times as great as the world's total production of phosphate rock in 1911. Nevertheless so great is the volume of the oceans, and so vast the area of their floors, that if all of this material were converted into phosphatic minerals and deposited uniformly over the bottom of the sea, it would serve to form annually a layer less than $\frac{1}{5}$ mm. thick. Of the phosphorus poured into the sea, so large a proportion is utilized by living things that the net working balance dissolved in oceanic water at present averages less than 0.005%, expressed as P_2O_5, or about $\frac{1}{6}$% of the salts dissolved. This seemingly represents the most diffuse state reached by phosphorus in the course of its complex migrations. Its subsequent transformations, now to be described, generally tend toward ever greater concentration almost until the cycle is closed upon itself.

Soluble phosphates are absorbed by the myriads of oceanic plants. In some measure they become chemically linked in organic compounds but for the most part they probably remain as salts or ions. Living diatoms and other algae contain in both of these states $\frac{1}{10}$ to $\frac{2}{10}$% of P_2O_5. These organic substances containing phosphorus are devoured by an all but infinite variety of aquatic animals, each of which is in its turn liable to a similar fate. This endless process of devouring is recognized even in the ancient Chinese proverb to the effect that "the big fish eat the little fish, and the little fish eat the shrimp, and the shrimps eat the mud." In the individual animal, the phosphorus forms a constituent of its tissues, cells, liquids, and in some groups also its teeth, bones, shells and other hard parts.

There are at least two ways in which the phosphorus may escape from this oceanic cycle. The organisms of the sea may be eaten by land animals, chief of which are the birds, or the phosphorus may be fixed among the solid deposits on the sea bottom and eventually buried beneath the accumulating sediments. The second of these processes will be considered first.

At somewhat rare intervals, marine animals are killed in enormous numbers by sudden changes in temperature, by volcanic eruptions, submarine earthquakes, and other catastrophes. The carcasses of such animals generally float; but some may come to rest upon the sea bottom. In either event the omnipresent forces of decay rapidly convert the carcasses to soluble form and thus return their constituents to the oceanic solution before they are deeply buried. It is conceivable that, under the most favorable circumstances, a layer of bones and teeth might be left on the sea floor; but at best the reality of fixation in this manner is doubtful.

As an incident to the normal life of vast numbers of organisms both on the sea bottom and in its upper waters, shells and pellets of solid excrement are incessantly falling to the sea bottom. Dredging operations have shown that in many places the latter material forms an appreciable part of the bottom sediments[3] and in a few places the sediment has been found to consist almost entirely of such pellets, especially those of holothurians, marine worms, and echinoids. Under normal conditions, this material is devoured by the various scavengers of the sea bottom, and any portion that may be left is fermented by bacteria. Insofar as this action prevails, phosphorus cannot well become a solid part of the sediment deposited on the sea floor.

Under conditions probably anaërobic but not yet well understood, a quite different process may operate. The organic refuse fermented by bacteria in the absence of free oxygen yields such compounds as hydrocarbons, carbon dioxide, hydrogen sulphide and ammonia. Under favorable chemical conditions the phosphoric acid also liberated will react with various substances, particularly lime salts. With the latter it produces the mineral collophanite, a hydrous calcium carbo-phosphate.[4] Where calcareous shells lie on the sea bottom, they become phosphatized. Even organic matter such as excretory pellets, pieces of wood, and other non-calcareous materials, are altered in this way. Bones which initially contained about 58% calcium phosphate are still further phosphatized until that percentage rises to 85 or more. In addition, the collophanite forms little round grains resembling the oolitic grains common in certain limestones. It is also deposited as a cement between the particles of all kinds, thus producing hard nodules or even continuous solid beds of phosphate rock. At the same time, probably by the influence of carbonic acid evolved during the decay of the organic matter, lime carbonate is dissolved. Many facts indicate that in these situations only the phosphatic mineral is stable, even though lime carbonate shells may at the time be falling from the surface to the bottom of the sea. In such a phosphatic deposit the content of phosphorus pentoxide may range from less than one to at least 36%. Sediments of this type, subsequently lifted above the sea, have become the rare but locally voluminous source of our most important commercial phosphates, or have served as a necessary antecedent for the later formation of such deposits.

Reference has already been made to the fact that through the agency of birds and other animals of the land, phosphorus may escape from the cycle of changes in the ocean. Upon islands where they are out of reach of predaceous mammals, sea-birds congregate in extraordinary numbers, and the amount of excrement annually deposited by them on the surface of these islands is large.

In moist regions bacterial fermentation decomposes this material and the soluble resultants are removed by rain water so rapidly that little residue is left. On those islands, however, which are situated in the dry trade-wind belts, this process is almost inoperative, and hence the guano accumulates from year to year. The dry guano—before subsequent alterations have taken place—contains as much as 28% P_2O_5, plus organic matter. During moist seasons, transient though they may be, the guano is subject to fermentation by microörganisms such as the bacteria, with the result that the nitrogenous matter is largely converted into ammonia, and is lost. The occasional rains dissolve out some of the soluble phosphates and most of the nitrates, leaving a residue of solid 'stone guano' containing as much as 39% of P_2O_5, largely in the form of hydrous acid and basic calcium phosphates.

The solutions derived from the guano above sink downward through the underlying rocks, and produce characteristic alterations in them. If the subjacent terrane be limestone, it is slowly converted into a mass of calcium phosphate, in which the minerals collophanite and stafellite (?) are the characteristic forms. Observations on the islands in the Indian Ocean indicate that coralline limestone has been changed in recent times at the rate of 2 or 3 feet in twenty years.[5] In the laboratory, a coral skeleton became 60% phosphatized in only two months.[6] On volcanic cones, even such refractory igneous rocks as andesite have been converted superficially into a mass of aluminum and iron phosphates.

These processes of alteration near the surface of the earth affect not only the guano deposits, but also the marine sediments which contain phosphatic minerals. Some of the Cretaceous chalk deposits of Belgium, which it appears originally contained from 1 to 4% of P_2O_5, have been converted by the differential leaching of lime-carbonate by carbonated waters into much thinner beds, in which the proportion of P_2O_5 has arisen to more than 30%. As in the case of guano, more or less of the lime phosphate is dissolved along with the lime carbonate, and sinking down through subjacent limestones, converts them into secondary beds of phosphate rock. Our important Florida deposits (the hard rock variety) are believed by Sellards[7] to have originated in this manner. Other examples of the same type are probably to be found in the limestone plateau of southern France, some of the phosphates of central Tennessee, and many other regions.

Phosphatic deposits of any of the types already described may be buried to great depths and there be subjected not only to great vertical pressures but to the even more intense horizontal strains that accompany mountain folding. In harmony with the well established principle that most rocks tend to recrystallize under these conditions, the phosphatic rocks are probably reorganized. The hydrous minerals characteristic of the surface must become dehydrated, and there is also a tendency for carbonic acid to be driven off. As a result, beds which originally consisted of the hydrous carbo-phosphates—collophanite, dahllite and others—are believed to pass over into pure apatite, in which the proportion of P_2O_5 may rise to nearly 43%—the maximum attainable. Although none of the known apatitic deposits have thus far been proved to have originated in this way, it is more than probable that illustrations of this type exist. It has been suggested that the rich apatitic beds associated with the Grenville marbles and gneisses near Ottawa, Canada, are really the highly anamorphosed representatives of phosphatic sediments once deposited on the surface of the land or sea bottom.

There is no obvious reason why deeply buried layers of phosphate rock invaded by fluid lava from below should not be dissolved by the latter, diffused through its mass, and eventually crystallize as apatite disseminated through the resulting igneous rock. Insofar as this occurs it closes the cycle, for the phosphorus is thus brought back to its original condition, albeit after untold ages of migration and transformation in the surficial portion of the earth.

[1] Clarke, F. W., Data of Geochemistry, *Bull. U. S. Geol. Survey*, No. 616, p. 27 (1916).

[2] According to A. Lacroix, *Mineralogie de la France*, v. 4, pp. 555 et seq.

[3] Murray, Sir. J., and Philippi, E., *Wissensch. Ergeb. der Deutschen Tiefsee Exped.*, Bd. X, Lf. 4, (1905) (Carl Chun, editor).

[4] Lacroix, A., *Mineralogie de la France*, v. 4.

[5] Willis, J. L., *Ottawa Naturalist*, 6, 18 (1892).

[6] Collet, L. W., *Proc. R. Soc. Edinburgh* 25: 882.

[7] Sellards, E. H., *Florida Geol. Survey*, *5th Ann. Rept.*, pp. 37–66 (1913).

DOMINANTLY FLUVIATILE ORIGIN UNDER SEASONAL RAINFALL OF THE OLD RED SANDSTONE

By Joseph Barrell

DEPARTMENT OF GEOLOGY, YALE UNIVERSITY

Received by the Academy, July 17, 1916

The Devonian system is represented in the British Isles by a series of rocks many thousands of feet in thickness, none of which, except in the south of England, hold marine fossils. Red sandstones constitute the predominant exposures, the colors ranging from light red to deep chocolate-brown, but in places are also found sandstones and shales of green, gray, or yellow colors. Besides the sandstone, the system includes much shale and conglomerate, the latter in places consisting of large-sized and subangular débris. Much volcanic material is intermixed in certain localities. This system of rocks, so distinct from the marine formations, has been familiarly known since the days of Hugh Miller as the Old Red Sandstone.

The sediments are mostly barren of organic remains, but at certain horizons many fossils have been found,—plant fragments, estheria, scorpions, myriopods, eurypterids, ostracoderms, and fishes. Several distinct faunas are found, representing successive epochs. These fossil faunas contain the oldest known ganoid fishes and the oldest well preserved representatives of those primitive and spinose sharks,—the acanthodeans. Ostracoderms are well represented also. In contrast to this abundant preservation of chordates in certain horizons of the Old Red Sandstone, the rocks of earlier geologic periods show only rare and usually fragmentary remains belonging to this phylum.

Our own ancestral line is regarded as having ascended through elasmobranchs and crossopterygian ganoids. These groups evidently flourished at the time of the Devonian period within the environment under which the sediments of the Old Red Sandstone were laid down. If, then, we can determine the conditions of origin of those sediments we shall restore by so doing the life surroundings of the primitive ancestral vertebrates; life surroundings which were related to their passage from the realm of the waters to that of the land and air.

Godwin-Austen, noting the distinctness of the Old Red Sandstone formations from the typical marine Devonian, was led to a view in 1855, previously maintained by Dr. John Fleming, that the sediments were laid down in great freshwater lakes or inland seas. This interpretation rapidly supplanted the older view advocated by Hugh Miller, that the Old Red Sandstone was of marine origin. This lacustrine interpretation prevails in British opinion to the present day, though it is true that Macnair and Reid[1] reverted in 1896 to the hypothesis of marine origin, and Goodchild in 1904,[2] pressing in the other direction, urged "that the whole of this vast formation was accumulated under continental conditions, partly in large inland lakes, partly as torrential deposits, partly as old desert sands, and partly as the result of extensive volcanic action." In 1908 Walther published a volume which consisted of a general review of historical geology. In the section touching on the Old Red Sandstone, he held that the lakes of the British geologists were in reality desert basins which "possessed a hot desert climate whose dry periods were broken only seldom by the downpour of thunder storms."[3]

In other places Kayser and Walther have expressed the view that the Old Red Sandstones were deposited in lagoons not far from the sea, the water being thought of as occasionally replenished by the sea and the deposition as taking place in lagoons and as dunes on their margins.

Thus we see that a great variety of conclusions, amounting in fact to flat contradictions, have been expressed and are still entertained by geologists in regard to the nature of the habitat of the early vertebrate faunas whose remains are found in the formations of the Old Red Sandstone. Yet this is to the geologist and biologist a most important problem, especially as it may be shown that the conditions of the environment held important causal relations to the rise of amphibians.

The differences in conclusions are not due to differences of opinion in regard to the actual facts of stratigraphy, but to radical differences in the interpretation of those facts. At the present stage of investigation it is of the first importance therefore to determine the criteria for interpretation and their degree of validity. It is that subject especially to which the writer has given attention in previous investigations. The present conclusions rest therefore upon the following papers.

In November, 1906, the writer, introduced by Professor Davis, gave a paper before the National Academy of Sciences entitled 'Continental Sedimentation with Applications to Geological Climates and Geography.' The articles on this general subject were published *in extenso* in the *Journal of Geology* in 1906 and 1908.[4] In 1913 and 1914 articles were published in the *American Journal of Science* on the formations in the eastern United States which correspond to the Old Red Sandstone system of the British Isles.[5] In the present article, therefore, the establishment of the criteria on which the conclusions rest may be omitted and the resulting interpretation directly given.[6]

This interpretation leads to the conclusion that the deposits which make up the Old Red Sandstone, although they undoubtedly contain lacustrine beds and other beds laid down in shifting, shallow, and variable bodies of water, are dominantly fluviatile in origin; laid down over river flood plains by streams in times of flood, exposed to air in times of drought. They record in this way the existence of an alternation of seasons of rainfall and drought—a climate with an arid season, but not an arid climate. This type of climate is best defined as semi-arid, and is existent over broad areas at present, especially in much of the torrid zone. Such a climate is to be sharply distinguished in thought, on the one hand, from that of a typically humid character, such as now prevails in northwestern Europe and northeastern North America; and, on the other hand, from truly desert climates, such as prevail in central Asia or northern Africa. The conclusion as to dominantly fulviatile origin,—similar to the mode of origin of the sediments now accumulating within the Great Valley of California, or on the Plains of Mesopotamia, is also to be sharply distinguished from the conception of great and permanent lakes, as well as from the opposite conception of torrential and eolian conditions in desert basins. Thus the present interpretation is distinctly different, both in regard to climatic and physiographic conditions of origin, from either the prevailing British or the German conceptions, but approaches nearest to that given by Goodchild.

The character of this interpretation may be called American, for in its main lines the writer does not stand alone. W. M. Davis, Hatcher, and others have shown the general importance of flood plain deposition. Especially may be mentioned the work of Grabau who has independently reached similar conclusions in regard to the importance in certain Paleozoic formations of fluviatile deposition. He has studied the Old Red Sandstone in the field and, though he has not published on that subject, has publicly stated that his conclusions are the same as those here expressed. More recently Schuchert has reviewed the evidence and expresses similar conclusions, though the part played by rivers as distinct from lakes is not so sharply differentiated as in the papers of Garbau and the writer.[7]

In closing we may draw a picture of the geography of Great Britain in Lower Devonian times. The region was then a part of a continent which extended an unknown distance northward and westward. Beyond the northwestern side of Great Britain extended a mountain system. The region of Great Britain was, in this newer interpretation, made up of a sequence of river basins separated by minor ranges of mountains, the whole marginal to the greater mountain system. Sediment was brought into these basins by rivers from the bordering uplands and from the more distant regions to the northwest. The excess beyond what was laid down by the rivers in time of flood to maintain their grade across the sinking basins was carried through to the shallow sea which lay on the surface of the continent to the southwest. The relations were somewhat similar to those which now prevail between the ranges of the North American Cordillera and the Tertiary basins which lay between them and especially on the west between the Sierra Cascade chains and the Coast Ranges. The Great Valley of California may therefore in the present epoch, both in physiography and in climate be cited as a striking illustration of the nature of the Old Red Sandstone basins.

[1] Macnair and Reid, *Geol. Mag. Decade* IV, 3, 106–116, 217–221 (1896).

[2] Goodchild, J. G., The Older Deutozoic Rocks of North Britain, *Geol. Mag. Decade* V, 1, 591–602 (1904).

[3] Walther, J., *Geschichte der Erde und des Lebens*, 259 (1908).

[4] Barrell, J., Relative Geological Importance of Continental, Littoral, and Marine Sedimentation, *J. Geol.*, **14**, 316–356, 430–457, 524–568 (1906); Relations Between Climate and Terrestrial Deposits, *Ibid.*, **16**, 159–190, 255–295, 363–384 (1908).

[5] Barrell, J., The Upper Devonian Delta of the Appalachian Geosyncline, *Amer. J. Sci.*, **36**, 429–472 (1913), **37**, 87–109, 225–253 (1914).

[6] This paper was given in brief form at the meeting of the American Society of Vertebrate Paleontology at New Haven, Conn., on December 26, 1907 [Loomis, F. B., Report of the Secretary, The American Society of Vertebrate Paleontology, *Science*, **27**, 254 (1908)], and more fully at the meeting of the Geological Society of America at Washington, December 28, 1915. It will be published in full in the *Bulletin of the Geological Society of America* in 1916. The present paper is a digest and its chief importance is because of its bearing on the environment of early vertebrates. In that way it is introductory to a paper on the Influence of Silurian-Devonian Climates on the Rise of Air-breathing Vertebrates which will follow in these PROCEEDINGS.

[7] Pirsson-Schuchert, *Text Book of Geology*, Part II, *Historical Geology*, by C. Schuchert, 714–721 (1915).

THE INFLUENCE OF SILURIAN-DEVONIAN CLIMATES ON THE RISE OF AIR-BREATHING VERTEBRATES

By Joseph Barrell

DEPARTMENT OF GEOLOGY, YALE UNIVERSITY

Received by the Academy, July 17, 1916

The problems of organic evolution have many aspects and ramify into many fields of science. The subject was at first embraced chiefly in the domain of the old time naturalist—zoologist or botanist. But the problems of variation and heredity have passed into the hands of the experimental evolutionist; and there are other problems whose answers are found in the geologic record—but these are of two rather opposite aspects. On the one hand, the paleontologist specializes particularly on the organic features, the successions and relations of fossil faunas or floras. On the other hand, it is the work of physical and historical geology to restore the ancient environments. The relations of the environments to the biotas is a field wherein physical geology and paleontology meet, to give a better understanding of the underlying physical surroundings associated with organic response and progress. It is from the standpoint of physical and historical geology, rather than from that of paleontology that the present study is made.

In this subject are two major problems—first as to the environment in which fishes developed; second, the changes in the environment and the associated organic responses which brought forth amphibians from fishes. It is the solution of the second problem which is here especially sought,[1] but it involves a statement of the evidence in regard to the first.

The question in regard to the origin of fishes is as to whether they developed first in the sea and later invaded the land waters, as has been generally assumed, or whether their expansion was in the opposite direction—from rivers to embayed waters and thence finally to the sea. Chamberlin appears to have been the first to have seriously suggested that the latter may have been the real direction of their emigration.[2] He pointed out that the fishes appeared as fossils only in the embayed waters until they had attained such dominance that they spread into the truly oceanic realm. Their body form, furthermore, Chamberlin notes, is peculiarly adapted to the stemming of currents and suggests an initial adaptation to rivers.

The evidence for this hypothesis of the continental origin of fishes has been examined by the writer, by taking up the earliest faunas and studying the mode of origin of the associated sediments. The results strongly support Chamberlin's position. The exclusively marine habitats of the lowest remaining chordates, constitute really negative evidence on the subject, for these forms are far removed from fishes, and they leave no fossil record. The original habitats may have been far wider and the progressive forms may have lived a freer life than is shown by these retrogressive and more or less aberrant relics. The positive evidence given by the early fish fossils is more definite and conclusive. A review of this is as follows:

The earliest known ostracoderm remains were found by Walcott in 1891 near Canyon City, Colorado, in a horizon belonging to the base of the Middle Ordovician, in sandstones *which rest upon the Pre-Cambrian.*[3] These sandstones are marginal marine deposits, holding species of Lingula, Orthoceras, and Beyrichia. With these are found abundantly at several levels waterworn scales and plates of ostracoderm fishes. The sandstones are succeeded by marine limestones of Trenton age, but in the limestones, although marine fossils are abundant, the fish remains are absent. If we seek to interpret the origin of these sandstones and their fish remains, it is to be noted that the thickness, 86 feet, is large to have been formed by wave action on a nearly base leveled land. But rivers bring most of the sediment to the sea which is laid down as sandstone, and it is probable that most of this sand was of river derivation. The waterworn character of the fish remains indicates a degree of transportation which the other fossils do not show, and suggests driftage into the region of burial. The absence of all fish remains in both Ordovician and Silurian deposits of solely marine nature strengthens the view that these rare and fragmentary finds were floated into the marginal waters from the rivers.

In mid Silurian red shales of Pennsylvania and in the Upper Silurian Ludlow rocks of Great Britain have been found the next clear appearance of fish remains, mostly of ostracoderms, in associations which seem to indicate at this geologic stage a life of these bottom dwellers not exclusively in the rivers but also probably in protected and brackish embayments of the sea. More clearly, however, they did not yet live in the open sea.

The first true fish fauna, as represented by sharks, in better preservation and considerable variety, is found in the Lower Old Red Sandstones of the Caledonian basin. These are lowest Devonian in age. But the sediments are of a different nature from the marine or brackish water deposits of the Ordovician and Silurian which held the earlier fossils. Viewed in the light of the modern knowledge of the nature of alluvial deposits, the sediments of the Old Red Sandstone are interpreted as having accumulated as river deposits on flood plains or in shallow lakes of an interior basin, lakes subject to marked shrinkage in area during the dry season.

The ganoid fishes appear for the first time and in force in the Middle Old Red Sandstone formations of the Orcadian basin of northeastern Scotland, twenty species belonging to seven genera being known from that basin. Dipnoans also appear at the same time. The enveloping deposits are regarded as Lower Devonian in age, though younger than the deposits of the Caledonian basin.

In contrast to this land water record of the lowest Devonian, the sharks are not found in declaredly marine rocks until at least the Middle

Devonian, and they did not dominate the sea in such exclusive fashion as they dominated the continental deposits of the Lower Devonian until the opening of the Mississippian (Lower Carboniferous) period. In contrast also to the abundant ganoid fauna in the continental waters, only a single species of ganoid is found in the marine Ulsterian (Middle Devonian) rocks of the interior of North America, and they have but scanty representation in the seas until the Mesozoic era. This record appears to show conclusively that the center of piscine evolution was within the land waters, and that the ganoids represented a closer adjustment to that environment than did the sharks.

This brings us to the second problem, the relation of environment to the origin of the amphibians. The nature of the sediments shows that through Silurian and Devonian times the climates, although subject to variation, tended to be warm and semi-arid, that is, marked by a pronounced alternation of wet and dry seasons. In the middle Silurian in fact a high degree of aridity existed in certain regions. The initial development of ganoids, distinguished in their organization from sharks by their capacity to supplement the use of water in respiration by the direct use of air in an air bladder, probably goes back to this Silurian epoch of aridity; since in the Middle Old Red Sandstone stage they are already well differentiated and expand in numerous species through the fresh waters. We must look to the previous period therefore for their origin. During the Upper Devonian the climate became more markedly semi-arid than in the Lower Devonian. Then the traces of sharks disappear completely from the fresh waters, but dipnoans and ganoids continue to exist. This migration of the sharks in habitat is logically to be correlated with the fact that elasmobranchs have no air bladder, not even in rudimentary or vestigial form, whereas the other two orders are so provided. It is notable that the few living species of dipnoans, or lung fishes, are restricted at the present time to tropical regions whose flowing waters are restricted to a wet season, and show such marked adaptations for surviving through seasons of drought that in those environments they possess a permanent advantage over other fishes.

The fossils of fishes in the Old Red Sandstone are apt to be crowded into occasional layers and are associated with bitumen, indicating in those layers an unusual amount of organic matter buried without being exposed to atmospheric oxygen. Judged by the environment, they appear to have died in shoals owing to crowding into the foul waters of pools shrinking in the dry seasons. The seasons of drought formed the critical factor in their environment. No traces of amphibians are found, though by the law of anticipation they must have been in existence in Upper Old Red Sandstone times; since they are typically developed at the opening of the following period. The amphibians seem therefore to have originated in more limited regions where fishes could not survive and the waters were not sufficiently permanent for the preservation of their remains.

The preceding arguments have been drawn from the geologic record. Let us now turn to another record,—that embalmed in the nature of living vertebrates, especially lung fishes and amphibians; for they preserve practically unaltered a stage of respiratory and circulatory evolution which had been attained in the Devonian. We may find by examining their organic nature whether the development of lungs was a spontaneous, internally directed organic advance, giving increased activity and efficiency, or whether at first it was a mere makeshift response to the pressure of adverse external conditions.

The air-bladder of ganoids and higher fishes is related to the intestinal region, not the pharyngeal region, as is shown both by its direct connection with the oesophagus in ganoids and in the embryos of other fishes. The blood system shows the same, the dipnoans, ganoids and amphibians having the air-bladder or lungs supplied with blood from a *branch of the fourth efferent gill artery.* In each round of the circulation only a portion of the blood is passed through the primitive lungs. The efficiency of this is so limited that the higher fishes living in well-aerated waters, though descendents of ganoids, have turned a primitive breathing organ to other uses and have reverted wholly to the use of gills. The function of the air-bladder was, then, to tide over periods of partial asphyxiation in foul waters, and seems to have been initially developed from a habit of swallowing air, gasping and finding this mode of relief. If forced to rely wholly upon this, it sufficed merely to keep the fishes alive until a new season of floods should recur, expanding the sphere of action and the supply of food simultaneously with the renewed gill-respiration and increased activity of the inhabitants of the river and playa waters. The use of gills, furthermore, could not become completely eliminated until such great changes had progressed in the heart and circulation inherited from elasmobranchs as are found in modern lung fishes and amphibians. It was this initial inefficiency of air-breathing which seems the strongest proof that it was an adaptation forced by the compulsion of nature, not the expression of a hypothetical innate tendency for advancement in organization. Of course, the organisms must possess at the same time an instability of germ plasm, permitting variations or mutations to arise. These would under the law of probabilities be more often degenerative than ascensive, but the strenuous environmental conditions cut off unsparingly all but the favored few which showed increased efficiency in meeting the critical factor in the environment. The rising death rate owing to recurrences of seasons of marked aridity was the ultimate cause back of the rise of air-breathing vertebrates. Natural selection, by the survival of the fittest, is shown thus, by matching the advance in life to the necessities of environment, to be the overarching cause which led the vertebrates to the possession of the land, their future theater of high evolution. It is doubtful in view of this conclusion if vertebrates could have attained their present dominance of the land without the driving effect of climatic adversities such as are found in the Devonian.

The biologist who studies variations, mutations, Mendelian factors, hybridization; the paleontologist who studies orthogenetic variations, the budding and expansion of phyla, are looking at the expressions of internal forces. These, however, do not constitute a complete picture of the cause and effects of organic evolution any more than the observer of a running antelope could fully explain its flight as the result of the coördination of nervous and muscular actions on a skeletal framework. He must also take into account the ultimate cause—the carnivore behind, and also the carnivores which pursued its ancestors through millions of years. The ultimate controls of evolutions are found in a study of the geologic record, though the possibilities of evolution must be latent within the organism. Natural selection, although discredited as a cause determining specific variations, appears nevertheless to be a major factor in evolution, the driving cause in association with changes in environment, which has forced the great advances in organic progress.

[1] This paper was given orally by the author before the American Society of Vertebrate Paleontology, December 26, 1907 [Loomis, F. B., *Secretary,* The American Society of Vertebrate Paleontology, *Science,* 27, 254–256 (1908)], and also before the Geological Society of America at Washington, December 28, 1915, and the complete paper will be published in the *Bulletin* of that Society in 1916. Some discussion of the subject has also been given by Professor Schuchert in his recent text on *Historical Geology* (Part II of Pirsson-Schuchert, *Text Book of Geology.*

[2] Chamberlin, T. C., On the Habitat of the Early Vertebrates, *J. Geol.*, 8, 400–412 (1900).

[3] Walcott, C. D., Preliminary Notes on the Discovery of a Vertebrate Fauna in Ordovician Strata, *Bull. Geol. Soc. Amer.*, 3, 153–172 (1892).

DENSITY OF RADIO-LEAD FROM PURE NORWEGIAN CLEVEITE

By T. W. Richards and C. Wadsworth, 3d

WOLCOTT GIBBS MEMORIAL LABORATORY, HARVARD UNIVERSITY

Received by the Academy, August 4, 1916

Through the kindness of Prof. Ellen Gleditsch of the University of Kristiania we have been so fortunate as to receive a specimen of lead sulphide from carefully selected Norwegian cleveite. According to Dr. Gleditsch, "The Norwegian uraninites are very old and very unaltered. They are found in well developed crystals and occur in connection with the pegmatite dykes in southeastern Norway." The sample in question occurred in cubic crystals near Langesund.

As Hönigschmid has already pointed out[1] the properties of radio-lead (this name being used provisionally to designate lead produced by radioactive transformations) obtained from pure minerals of this sort are far more interesting and significant than those of lead obtained from ordinary uranium ores, which doubtless contain some admixture of ordinary lead. Hönigschmidt has shown that the lead from pure cleveite has an atomic weight as low as 206.06, and our own experience with the sample referred to above essentially confirms this result, as will be shown in another communication. So far as we know, however, the density of lead of this kind has not yet been determined, and accordingly the present paper recounts such a determination, which forms an interesting sequel to the recently published results on the density of Australian radio-lead.[2]

The purification of the sulphide, which doubtless contained traces of sulphides of other metals was carried out as follows: The specimen was dissolved in nitric acid and crystallized three times with centrifuging as nitrate,—a process which Baxter's experience has shown to be an excellent one for the purification of lead from other metals.[3] From this purified nitrate the chloride was precipitated by pure hydrochloric acid, and this salt was crystallized three times. The final crystals, after draining on the centrifuge, were stored in a vacuum desiccator over caustic soda. The chloride thus prepared was used for the determination of the atomic weight, the density being determined in the material saved from the filtrates from that determination. These filtrates contained excess of silver, therefore enough hydrochloric acid was added to bring the concentration of the dissolved chloride ion to 0.01 normal, because at this concentration silver chloride is most nearly insoluble.[4] When the precipitated silver chloride had settled and had been removed by filtration through a Gooch-Munroe crucible with platinum mat, the resulting solution was concentrated and crystallized once more as nitrate. The pure crystals were electrolyzed, and the pure radio-lead treated exactly as in the case of the other samples previously described.[5]

As the amount of substance at hand was very small, the work could not be done quite so accurately as before. The density determinations were made in the same pycnometer as before, by the second method described on page 223 of our previous publication, the volume of the pycnometer having been redetermined because its tip had been broken in the meantime. Four identical determinations gave 5.7200 as the weight of water in the pycnometer at 19.94°, weighed in air. Therefore, the volume of the pycnometer was 5.7361 cm.

TABLE 1

DENSITY OF LEAD FROM CLEVEITE

OBSERVED WEIGHT	WEIGHT IN VAC. (W)	OBSERVED WEIGHT WATER NOT DISPLACED	CORRESPONDING VOLUME	VOLUME OF PYCNOMETER	VOLUME (V) OF WATER DISPLACED	DENSITY (W/V)
4.4252	4.4250	5.3287	5.3437	5.7361	0.3924	11.277
4.4252	4.4250	5.3286	5.3436	5.7361	0.3925	11.274
4.4252	4.4250	5.3285	5.3435	5.7361	0.3926	11.271
4.4252	4.4250	5.3285	5.3435	5.7361	0.3926	11.271
					Average	11.273

The density of this sample, presumably a nearly pure isotope, is thus 11.273, distinctly less than 11.289, the density of the Australian radio-lead, and still less than the density 11.337 found for ordinary lead. The decrease is almost exactly proportional to the decrease of the atomic weight in these samples; for the atomic weight of the Australian lead was about 206.35, and that of this sample 206.085. Thus the atomic volume of the isotope, 18.281 (equal to 206.08/11.273), is almost identical with that of pure lead, as indicated by our previous experiments. Thus 18.281 is essentially equal, within the limit of error of experiment, to the value 18.277, found for ordinary lead, and to the value 18.279, found for Australian radio-lead. It is interesting to note that Australian radio-lead would be essentially duplicated as to these properties by a mixture consisting of three parts of pure isotope to one of ordinary lead.

---

[1] Hönigschmid, *Sitzungsber. Wiener Akad.*, 123, II a, 20 (1914).

[2] Richards and Wadsworth, *J. Amer. Chem. Soc.*, 38. 221 (1916).

[3] Baxter and Grover, *J. Amer. Chem. Soc.*, 37, 5 (1915).

[4] Forbes, *J. Amer. Chem. Soc.*, 33, 1937 (1911).

[5] Richards and Wadsworth, *loc. cit.*, p. 224.

NATIONAL RESEARCH COUNCIL

PRELIMINARY REPORT OF THE ORGANIZING COMMITTEE TO THE PRESIDENT OF THE ACADEMY

By George E. Hale, Chairman

On April 19, 1916, at the closing session of the annual meeting, the Academy voted unanimously to offer its services to the President of the United States in the interest of national preparedness. The Council of the Academy was authorized to execute the work in the event of the President's acceptance.

On April 26 the President of the Academy, accompanied by Messrs. Conklin, Hale, Walcott, and Woodward, was received at the White House by the President of the United States. In presenting the resolution adopted at the annual meeting, it was suggested that the Academy might advantageously organize the scientific resources of educational and research institutions in the interest of national security and welfare. The President accepted this offer, and requested the Academy to proceed at once to carry it into effect.

Immediately following this visit, the President of the Academy, in harmony with resolutions adopted by the Council on April 19, appointed the following Organizing Committee: Messrs. Edwin G. Conklin, Simon Flexner, Robert A. Millikan, Arthur A. Noyes, and George E. Hale (*Chairman*).

At a meeting of the Council of the Academy, held in New York on June 19, the Organizing Committee presented the following statement of work accomplished up to that date.

Much time was devoted during the first five weeks to the organization of committees to meet immediate needs, including those on Nitric Acid Supply (A. A. Noyes, Chairman), in co-operation with the American Chemical Society; Preventive Medicine (Simon Flexner, Chairman), in co-operation with the Committee of Physicians and Surgeons, and Synthetic Organic Chemistry (M. T. Bogert, Chairman), in co-operation with the American Chemical Society. Special attention was also given to arrangements for co-operation with the scientific Bureaus of the Government, the Committee of Physicians and Surgeons, the Naval Consulting Board, the national societies devoted to branches of science in which committees were immediately needed, the national engineering societies, the larger research foundations, certain universities and schools of technology, and the leading investigators in many fields of research, both on the industrial and the educational side. The hearty encouragement received from all of these men and institutions leaves no doubt that, as soon as a general request for co-operation is sent out, it will meet with universal acceptance.

During this preliminary period a more comprehensive plan of organization was developed, and finally embodied in the form indicated below. It was recognized from the outset that the activities of the committee should not be confined to the promotion of researches bearing directly upon military problems, but that true preparedness would best result from the encouragement of every form of investigation, whether for military and industrial application, or for the advancement of knowledge without regard to its immediate practical bearing. The scheme of organization must be broad enough to secure the co-operation of all important agencies in accomplishing this result.

After considering a variety of plans the Organizing Committee presented to the Council of the Academy the following recommendations:

"That there be formed a National Research Council, whose purpose shall be to bring into co-operation existing governmental, educational, industrial, and other research organizations with the object of encouraging the investigation of natural phenomena, the increased use of scientific research in the development of American industries, the employment of scientific methods in strengthening the national defense, and such other applications of science as will promote the national security and welfare.

"That the Council be composed of leading American investigators and engineers, representing the Army, Navy, Smithsonian Institution, and various scientific Bureaus of the Government; educational institutions and research endowments; and the research divisions of industrial and manufacturing establishments.

"That, in order to secure a thoroughly representative body, the members of the Council be chosen in consultation with the presidents of the American Association for the Advancement of Science, the American Philosophical Society, the American Academy of Arts and Sciences, the American Association of University Professors, and the Association of American Universities; that representatives of industrial research be selected with the advice of the Presidents of the Society of Civil Engineers, the American Institute of Mining Engineers, the American Society of Mechanical Engineers, the American Society of Electrical Engineers, and the American Chemical Society, and that members of the Cabinet be asked to name the representatives of the various Departments of the Government.

"That Research Committees of two classes be appointed, as follows: (*a*) Central committees, representing various departments of science, comprised of leading authorities in each field, selected in consultation with the president of the corresponding national society. (*b*) Local committees in universities, colleges and other co-operating institutions engaged in scientific research."

The Organizing Committee also recommended the following plan of procedure, subject to such modifications as the National Research Council may deem desirable.

"(1) The preparation of a national inventory of equipment for research, of the men engaged in it, and of the lines of investigation pursued in co-operating Government Bureaus, educational institutions, research foundations, and industrial research laboratories; this inventory to be prepared in harmony with any general plan adopted by the proposed Government Council of National Defense.

"(2) The preparation of reports by special committees, suggesting important research problems and favorable opportunities for research in various departments of science.

"(3) The promotion of co-operation in research, with the object of securing increased efficiency; but with careful avoidance of any hampering control or interference with individual freedom and initiative.

"(4) Co-operation with educational institutions, by supporting their efforts to secure larger funds and more favorable conditions for the pursuit of research and for the training of students in the methods and spirit of investigation.

"(5) Co-operation with research foundations and other agencies desiring to secure a more effective use of funds available for investigation.

"(6) The encouragement in co-operating laboratories of researches designed to strengthen the national defense and to render the United States independent of foreign sources of supply liable to be affected by war."

The Council of the Academy voted to accept the proposals of the Organizing Committee, and instructed it to proceed with the formation of the National Research Council in accordance with the plan recommended by the committee.

In consultation with the presidents of the various societies already mentioned, most of the members of the Council have now been chosen.

The endorsement of the President of the United States and the authority to secure the appointment of government representatives, is conveyed in the following letter to the President of the Academy:

> WASHINGTON, D. C., July 24, 1916.
>
> DR. WILLIAM H. WELCH,
> *President of the National Academy of Sciences, Baltimore, Maryland.*
> MY DEAR DR. WELCH:
>
> I want to tell you with what gratification I have received the preliminary report of the National Research Council, which was formed at my request under the National Academy of Sciences. The outline of work there set forth and the evidences of remarkable progress towards the accomplishment of the object of the Council are indeed gratifying. May I not take this occasion to say that the Departments ot the Government are ready to co-operate in every way that may be required, and that the heads of the Departments most immediately concerned are now, at my request, actively engaged in considering the best methods of co-operation?
>
> Representatives of Government Bureaus will be appointed as members of the Research Council as the Council desires.
>
> Cordially and sincerely yours,
> [Signed] WOODROW WILSON.

Under this authority, the appointment of representatives of the Army, Navy and various scientific Bureaus of the Government will now be arranged with the members of the Cabinet. It is expected that the first meeting of the Council will be held in September.

It has already been stated that cordial desire to co-operate has been encountered on every hand. Special reference may now be made to certain striking cases. The first of these illustrates how the Council, taking advantage of the increased appreciation of the value of science and the spirit of national service which have resulted from the war, may obtain the co-operation of educational institutions and assist them in adding to their endowments for scientific research. Throop College of Technology, in Pasadena, California, is a small institution of high standards which gives special attention to research. President Scherer, hearing of the plans of the Research Council, offered the assistance and co-operation of the recently endowed Research Laboratory of Chemistry and secured at once an additional endowment of one hundred thousand dollars for scientific research. Under somewhat similar circumstances, a gift of $500,000 has been made to the endowment of the Massachusetts Institute of Technology, with the expectation that much of the income will be used for research at that Institution.

Another illustration of friendly co-operation, of special importance because it assures the support of the National Engineering Societies, is afforded by the following resolution of the Engineering Foundation of New York, adopted at the annual meeting of the Foundation, on June 21, 1916:

> "WHEREAS, the National Academy of Sciences of the United States of America has taken the initiative in bringing into co-operation existing governmental, educational, industrial, and other research organizations with the object of encouraging the investigation of natural phenomena, the application of scientific principles in American industries, the employment of science in the national defense, and such other objects as will promote the national welfare, and
>
> "WHEREAS, these objects are among the objects for which The Engineering Foundation was created,
>
> "*Now, Therefore, be it Resolved,* that The Engineering Foundation hereby registers its approval of the co-ordination and federation of the research agencies of the country undertaken by the National Academy of Sciences and expresses its willingness to join with and assist the National Academy in accomplishing the above federation."

The Foundation also offered to devote its entire income for the coming year (including a special gift of $5000 for this purpose from its founder, Mr. Ambrose Swasey) toward the expenses of organization, and to provide a New York office for the Council in the Engineers Building.

The Presidents of the American Philosophical Society, of the American Association of University Professors, and of Yale University have already expressed their intention of proposing the adoption of similar resolutions by the institutions which they represent and of recommending the appointment of committees to co-operate with the National Research Council; and it is expected that other societies and educational institutions will take similar action.

Respectfully submitted by the Organizing Committee,

GEORGE E. HALE (Chairman),
EDWIN G. CONKLIN,
SIMON FLEXNER,
ROBERT A. MILLIKAN,
ARTHUR A. NOYES.

CONTENTS

Page

PHYSICS.—THE ABSORPTION COEFFICIENTS OF SOFT X RAYS . . *By C. D. Miller* 441

PATHOLOGY.—FURTHER EVIDENCE AS TO THE RELATION BETWEEN CROWN GALL AND CANCER . . . . *By Erwin F. Smith* 444

ZOOLOGY.—LOCOMOTION OF SEA-ANEMONES . . . . *By G. H. Parker* 449

ZOOLOGY.—THE BEHAVIOR OF SEA-ANEMONES . . . . *By G. H. Parker* 450

PETROLOGY.—A CONTRIBUTION TO THE PETROGRAPHY OF JAPAN . . . . *By J. P. Iddings and E. W. Morley* 452

PHYSIOLOGY.—IS THERE A TEMPERATURE COEFFICIENT FOR THE DURATION OF LIFE? *By Jacques Loeb and J. H. Northrop* 456

ASTRONOMY.—ON THE SUGGESTED MUTUAL REPULSION OF FRAUNHOFER LINES . . *By Charles E. St. John* 458

PHYSICS.—AN ATTEMPT TO DETECT THE MUTUAL INFLUENCE OF NEIGHBORING LINES IN ELECTRIC FURNACE SPECTRA SHOWING ANOMALOUS DISPERSION . . *By Arthur S. King* 461

CHEMISTRY.—SYNTHESIS OF THE BASE $C_3H_4ON_2$ DERIVED FROM METHYL-AMINO-METHYL-3,4-DIHYDROXYPHENYLCARBINOL . . . . *By Chas. A. Rouiller* 464

GEOLOGY.—EXTINGUISHED AND RESURGENT CORAL REEFS . . *By W. M. Davis* 466

GEOLOGY.—THE ORIGIN OF CERTAIN FIJI ATOLLS . . . . . *By W. M. Davis* 471

PHYSICS.—INTERFEROMETER METHODS BASED ON THE CLEAVAGE OF A DIFFRACTED RAY . . . . *By C. Barus* 475

GENETICS.—ON THE INHERITANCE OF CERTAIN GLUME CHARACTERS IN THE CROSS *Avena Fatua* × *A. Sativa* VAR. KHERSON . . . . . . *By Frank M. Surface* 478

ZOOLOGY.—A COMPARISON OF THE RATES OF REGENERATION FROM OLD AND FROM NEW TISSUE . . . . *By Charles Zeleny* 484

ZOOLOGY.—THE EFFECT OF SUCCESSIVE REMOVAL UPON THE RATE OF REGENERATION . . . . *By Charles Zeleny* 487

GEOLOGY.—THE GEOLOGIC RÔLE OF PHOSPHORUS . . . . *By Eliot Blackwelder* 490

GEOLOGY.—DOMINANTLY FLUVIATILE ORIGIN UNDER SEASONAL RAINFALL OF THE OLD RED SANDSTONE . . . . *By Joseph Barrell* 496

GEOLOGY.—THE INFLUENCE OF SILURIAN-DEVONIAN CLIMATES ON THE RISE OF AIR-BREATHING VERTEBRATES . . . . *By Joseph Barrell* 499

CHEMISTRY.—DENSITY OF RADIO-LEAD FROM PURE NORWEGIAN CLEVEITE . . *By T. W. Richards and C. Wadsworth, 3d* 505

NATIONAL RESEARCH COUNCIL.—PRELIMINARY REPORT OF THE ORGANIZING COMMITTEE TO THE PRESIDENT OF THE ACADEMY. *By George E. Hale, Chairman* 507

VOLUME 2 SEPTEMBER, 1916 NUMBER 9

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES
OF THE
UNITED STATES OF AMERICA

EDITORIAL BOARD

Arthur A. Noyes, *Chairman* Edwin B. Wilson, *Managing Editor*
Arthur L. Day, *Home Secretary* George E. Hale, *Foreign Secretary*

J. J. Abel
W. B. Cannon
J. McK. Cattell
J. M. Clarke
E. G. Conklin
J. M. Coulter
C. B. Davenport
Simon Flexner
E. B. Frost
R. G. Harrison
W. H. Holmes
J. P. Iddings
R. A. Millikan
E. H. Moore
Alexander Smith

Publication Office: Williams & Wilkins Company, Baltimore
Editorial Office: Massachusetts Institute of Technology, Boston
Home Office of the Academy: Washington, D. C.

Entered as second-class matter at the Post Office at Baltimore, Md., under the Act of August 24. 1912

INFORMATION TO CONTRIBUTORS

The Proceedings is the official organ of the Academy for the publication of brief accounts of important current researches of members of the Academy and of other American investigators, and for reports on the meetings and other activities of the Academy. Publication in the Proceedings will supplement, not replace, that in journals devoted to the special branches of science. The Proceedings will aim especially to secure prompt publication of original announcements of discoveries and wide circulation of the results of American research among investigators in other countries and in all branches of science.

Articles should be brief, not to exceed 2500 words or 6 printed pages. Technical details of the work and long tables of data should be reserved for publication in special journals. But authors should be precise in making clear the new results and should give some record of the methods and data upon which they are based. The viewpoint should be comprehensive in giving the relation of the paper to previous publications of the author or of others and in exhibiting where practicable, the significance of the work for other branches of science.

Manuscripts should be prepared with a current number of the Proceedings as a model in matters of form, and should be typewritten in duplicate with double spacing, the author retaining one copy. Illustrations should be confined to text-figures of simple character, though more elaborate illustrations may be allowed in special instances to authors willing to pay for their preparation and insertion. Particular attention should be given to arranging tabular matter in a simple and concise manner.

References to persons should include initials; citations of literature, numbered consecutively, should be placed at the end of the article; and short footnotes should be avoided. Indicate 'subject' for 'Contents.'

Papers by members of the Academy may be sent to Edwin Bidwell Wilson, Managing Editor, Mass. Institute of Technology, Boston, Mass. Papers by non-members should be submitted through some member.

Proof will be sent to authors only on their request, in order to avoid delay in publication. Authors are therefore requested to make final revisions on the typewritten manuscripts. The editors cannot undertake to do more than correct obvious minor errors.

Reprints should be ordered at the time of submission of manuscript. They will be furnished to authors at cost, approximately as follows:

Reprints of - -	2 pp.	4 pp.	6 pp.	8 pp.	Covers extra
Charge for first 100 copies	$1.00	$1.30	$2.20	$2.20	$2.00
Charge for additional 100s	.25	.45	.85	.85	.50

INFORMATION TO SUBSCRIBERS

Subscriptions at the rate of $5.00 per annum should be made payable to the National Academy of Sciences, and sent to Williams & Wilkins Company, Baltimore, or Arthur L. Day, Home Secretary, National Academy of Sciences, Smithsonian Institution, Washington, D. C. Single numbers, $0.50.

Copyright, 1916, by the National Academy of Sciences

JUN 24 1921
National Museum

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES

Volume 2 SEPTEMBER 15, 1916 Number 9

THE MECHANISM OF DIFFUSION OF ELECTROLYTES THROUGH ANIMAL MEMBRANES

By Jacques Loeb

ROCKEFELLER INSTITUTE FOR MEDICAL RESEARCH, NEW YORK

Received by the Academy, August 7, 1916

1. In a former paper Loeb and Cattell[1] have shown that embryos of the marine fish *Fundulus* poisoned in a KCl solution until all the hearts stop beating, cannot recover their heart beat when put into H_2O or into a solution of cane sugar, although they will live in such a medium indefinitely; but that they recover in less than a day when put into a salt solution. Eggs which after being poisoned with KCl were kept in H_2O for twelve days without recovering, recovered when put into the solution of an electrolyte.

Conversely it could be shown that when eggs of *Fundulus* are kept for twenty-four hours or longer in distilled water they are poisoned by a solution of potassium salts much more slowly than when the eggs are put into KCl solutions of the same concentration directly from sea water. The washing in distilled water frees the surface layer of the egg membrane from salts and this retards the diffusion of KCl into the egg.

These facts suggested to me the idea that it was impossible for KCl to diffuse through a membrane free from salts and that for such a diffusion two conditions are required: first, a sufficiently high osmotic pressure of the KCl solution, and second, a certain modification of the membrane by an electrolyte, which we will call the 'salt effect.' When the concentration of the KCl solution was sufficiently high, *e.g.*, 3/8m or m/2, the KCl itself might be able to supply this 'salt effect' upon the membranes of washed eggs; but when the solution of KCl was m/8 or below, the solution might not be sufficiently concentrated to supply the 'salt effect' required to allow the diffusion of KCl. This idea was found to be correct.

An m/2 KCl solution caused cessation of the heartbeat of unwashed eggs in less than two hours, and of the washed eggs in about nine hours. An m/8 KCl solution required six days to cause the cessation of the heartbeat in 70% of the eggs previously washed for twenty-four hours in distilled water, while the same solution had the same effect upon unwashed eggs in less than five hours. This difference was due, according to the theory mentioned above, to the presence of the salts of the sea water in the superficial layers of the membrane of the egg. On this assumption it was to be expected that by adding enough sea water to an m/8 KCl solution to bring this solution to a total salt concentration of 3/8m or m/2 the washed eggs should be poisoned much more rapidly. This was found to be the case. While 70% of 'washed' eggs were poisoned by m/8 KCl in H_2O in about six days, the same effect was usually produced in less than one day when the m/8 KCl solution was prepared in m/4 sea water. The same effect was produced by the addition of any salt of Li, Na, or NH_4, and others, to the m/8 KCl solution instead of sea water. Since it might be argued that the addition of a single salt like NaCl might act by merely injuring the membrane and thus increasing its permeability, emphasis should be laid on the fact that these experiments succeeded as well when the 'salt effect' was supplied by balanced solutions like sea water or NaCl + $CaCl_2$ as by non-balanced solutions.

When any of these salts were added to the m/8 KCl solution in a slightly higher concentration than the moderate concentration required to produce the 'salt effect' upon the membrane, the opposite result was noticed: the diffusion of KCl into the egg was again retarded or rendered impossible (antagonistic salt action). Thus the hearts of *Fundulus* embryos were still practically all beating in m/8 KCl made up in m/1 (NaCl + $CaCl_2$ + $MgCl_2$) after eighteen days (when the experiment was discontinued), while they had all stopped beating in less than three days and more than 50% in less than one day in m/8 KCl made up in m/8 (NaCl + $CaCl_2$ + $MgCl_2$). This disposes of the possibility that the general 'salt effect' consists in an increase of the 'permeability' of the membrane through injury by the salt.

The most important fact is that only salts act in this way, namely to accelerate the diffusion of KCl through the membrane when added in moderate concentrations and to retard it again when added in higher concentration. Non-electrolytes, like sugars, glycerin, ethyl or methyl alcohol, and urea, do not show this behavior.

2. The difference in the behavior of electrolytes and non-electrolytes, and the influence of the chemical nature of the salt in this general 'salt

effect' upon the membrane can best be studied in experiments on the recovery of eggs previously poisoned with KCl. Eggs were put directly from sea water into m/2 KCl for twelve hours. These eggs must have contained an excess of KCl, since in this solution all the hearts of unwashed eggs stop beating in less than two hours. The heartbeat was resumed when enough KCl had left the eggs after they had been transferred into the proper solutions. The resumption of the heartbeat became, therefore, an indicator for the rapidity of diffusion of the KCl from the heart into the membrane or the surrounding solution. Since the concentration of KCl was approximately equal in each egg at the beginning of the experiment, the differences in time required for recovery were due to the influence of the outside solution upon the rapidity of the diffusion of KCl from the egg into the membrane. Table I shows the percentage of hearts which recovered from the effects of KCl in various salts of Na, NH_4, and Mg in twenty-four hours. It is obvious that beginning with a limiting lower concentration all the hearts were beating again at that time in all the solutions, although most of the solutions are so poisonous that they would have killed the embryo in a short time had they been able to diffuse in more than traces into the egg. They acted by bringing about a certain modification of the membrane rendering possible the diffusion of the potassium salts.

TABLE 1

	PERCENTAGE OF HEARTS RECOVERING FROM POTASSIUM STANDSTILL IN TWENTY-FOURS IN													
	m/1	m/2	m/4	m/8	m/16	m/32	m/64	m/128	m/256	m/512	m/1024	m/2048	m/4096	H_2O
Sea water		100	100	91	60	50	26	15	45	15	10	25	35	5
$NaCl + CaCl_2$	100	95	100	90	55	55	25	25	5	5				10
NaCl	100	100	100	100	100	90	40	25	15	10				5
NaBr	100	100	100	100	90	75	66	50	35	30	40			5
$NaNO_3$				100	100	70	35	15	10	15	10	0	10	5
Na_2acetate		100	100	100	100	100	85	35	15	0	5	5	0	5
Na_2SO_4		100	100	100	100	90	93	90	81	38	14	5		
$NaHCO_3$				100	100	100	100	75	35	10	5			
Na_2CO_3									35	30	20	20	5	5
Na_2HPO_4					95	100	90	55	50	25	10	14	5	0
Na_3 citrate									95	60	80	30	20	
NH_4Cl		38	72	89	83	77	41	16	15	15				11
$(NH_4)_2SO_4$			45	75	70	75	65	65	50	31	15	10	30	10
$(NH_4)_3$ citrate									80	68	55	25	14	
$MgCl_2$			100	85	95	75	55	25	25	25	15	10	0	5
$MgSO_4$		100	95	70	95	90	80	45	70	30	10	10	15	

The limiting lower concentration to bring about the recovery of 70% or more eggs in twenty-four hours depends in a definite way upon the nature of the anion of all these salts, being for Cl: SO_4: citrate in the ratio of approximately 1:4:16, as stated in a previous publication.[1]

In time recovery took also place in the solutions below the limiting concentration. In the recovery experiments no antagonistic salt action in noticeable, probably on account of the fact that the decisive boundary is the inner surface of the membrane which receives only traces of the salt from the outside solution, enough to accelerate the diffusion of KCl but not enough to inhibit this process.

Table II shows the behavior of the eggs in twenty-four hours when put into solutions of non-electrolytes. The characteristic influence of increasing concentrations of the solution upon recovery which is so striking in Table I is entirely lacking in this case, m/2 solutions of non-electrolytes acting no better than weaker ones. What little recovery takes place in the solutions of non-electrolytes is 'accidental' and where the figures for recovery are 20% or more, it is probably due to the secondary formation of some acid or alkali through microorganisms or in the embryo. The main fact is that the number of hearts which recovered did *not* increase when the eggs were left for a number of days in the solution of non-electrolytes. When the eggs were transferred from the solution of non-electrolytes to a solution of the proper electrolyte the 'salt effect' and hence the recovery took place.

TABLE II

	PERCENTAGE OF HEARTS WHICH RECOVERED FROM KCl POISONING IN											
	m/1	m/2	m/4	m/8	m/16	m/32	m/64	m/128	m/256	m/512	m/1024	H_2O
Glucose	0	15	10	5	0	0	0	5	0	0	0	15
Cane sugar	14	10	14	20	14	15	15	37	21	32	10	5
Glycerin		20	20	15	11	11	0	15	5	15		10
Methyl alcohol	20	45	25	15	20	10	10	10	20	10	15	10
Ethyl alcohol		5	25	25	35	0	15	5	5	15		10
Urea	5	40	15	10	5	30	10	25	10	20	15	10

Table III illustrates the effect of the cation suggesting an influence of the periodic law. Thus recovery occurs neither in RbCl and CsCl nor in the salts of the alkali earth metals occupying the corresponding position in the next group of the periodic series; namely, Sr and Ba. These cations act by preventing the diffusion of the KCl from the egg for if we transfer the eggs from such a solution to one with an efficient cation, like Na, they will recover. In experiments on the diffusion of KCl into the egg these salts acted somewhat differently, Mg and especially Ca inhibiting the diffusion.

TABLE III

INFLUENCE OF CATION UPON THE RECOVERY

	PERCENTAGE OF EMBRYOS POISONED BY KCl RECOVERED AFTER TWENTY-FOUR HOURS IN													
	m/1	m/2	m/4	m/8	m/16	m/32	m/64	m/128	m/250	m/512	m/1024	m/2058	m/4096	H_2O
LiCl		30	45	80	60	80	60	30	5	0	10			5
NaCl	90	100	100	100	100	90	40	25	15	10				0
RbCl		0	0	0	5	5	0	1	0	2	0			0
CsCl		5	5	0	0	5	5	0	0	0	0			
$MgCl_2$			100	85	95	75	55	25	25	25	15	10		5
$CaCl_2$				100	100	75	45	20	20					
$SrCl_2$				30	15	30	15	20	0	5	25	0	0	10
$BaCl_2$			5	0	5	10	15	25	10	20	35	10	20	0
NH_4Cl		38	72	89	83	77	41	16	15	15				11
$N(C_2H_5)_4Cl$					100	78	60	23	20	14	5	14	12	10

3. If we wish to formulate a theory of the general 'salt effect' upon the membrane required to make the diffusion of potassium salts possible, we must remember that the conditions for the diffusion of acid into the egg resemble those described for KCl.[2] We know that acids form salts with proteins, which dissociate rather completely into protein ion and the anion of the acid used; while ordinarily the protein molecule (being usually a very weak base and acid) does not dissociate electrolytically to any extent. We can understand the phenomena described in this paper if we assume that the ionization of the proteins of the membrane is the prerequisite for the diffusion of KCl as well as of acid through the membrane. When this ionization of proteins of the membrane is to be brought about by neutral salts, as in the case of the diffusion of KCl, a much higher concentration of the salts is required than when the ionization is caused by acid or base, and the limiting concentration varies with the constitution of the salts. This ionizing effect need not be produced entirely by KCl but, for the 'salt effect,' the KCl can be partly replaced by other neutral salts. When more than a moderate amount of salt is added the ionization of the proteins of the membrane is diminished instead of increased, and the diffusion of KCl may again become as impossible as if a non-electrolyte had been added. A fuller

discussion of the theory will be given in the complete paper. Those cases in which the possibility of diffusion through a membrane was diminished by the addition of too much salt or by a salt with bivalent cation have thus far been singled out for discussion under the heading of antagonistic salt action, but the proof furnished in this paper that the absence of electrolytes acts in a similar but more complete way shows that the case of antagonistic salt action is only a part of the more general problem of the role of electrolytes (or of the ionization of the protein molecules of the membrane) in the diffusion of salts through animal membranes.

Summary and Conclusions.—Our experiments show that for the diffussion of certain electrolytes (potassium salts and acid) through animal membranes besides the osmotic pressure of the electrolyte a second effect is required which we call the 'salt effect' upon the membrane. This consists probably in an ionization of the protein molecules of the membrane. KCl and acid cannot diffuse through a membrane free from salts. The condition necessary for the diffusion of acid and KCl is produced when a trace of acid or a moderate amount of salt is added whereby the protein moleules of the membrane become ionized ('salt effect'). The addition of more salt (whereby the dissociation of the protein molecules of the membrane is diminished and probably other changes are brought about) annihilates this condition again. The latter fact is the special case of antagonistic salt action. The influence of the nature of the salt upon the limiting concentration for the 'salt effect' is stated in the paper.

The nature of the forces by which the ionization of the protein molecules of the membrane may bring about the diffusion of acids and potassium salts (and possibly of electrolytes in general) will be discussed in the main paper.

---

[1] Loeb, J., these PROCEEDINGS, **1**, 473 (1915); Loeb, J., and Cattell, McK., *J. Biol. Chem.*, **23**, 41 (1915); Loeb, J., and Wasteneys, H., *Biochem. Zs.*, **31**, 450 (1911); **32**, 155.

[2] Loeb, J., *J. Biol. Chem.*, **23**, 139 (1915); *Science*, **34**, 653 (1911); *Biochem. Zs.*, **37**, 127 (1912); Loeb, J., and Wasteneys, H., *Biochem. Zs.*, **33**, 489 (1911); **39**, 167 (1912).

THE ROTATION AND RADIAL VELOCITY OF THE SPIRAL NEBULA N. G. C. 4594

By Francis G. Pease

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, July 18, 1916

The question of the rotation of nebulae has received a considerable amount of attention within recent years. The extreme faintness, however, of these objects, particularly of the spiral nebulae, makes spectroscopic investigation most difficult and laborious; and for only a very few of them are direct photographs available of such quality and separated by such intervals of time as to provide a reasonable hope of detecting changes of position by direct measurement.

In the case of the planetary nebulae Hartmann[1] in 1902 believed that he had found spectroscopic evidence of rotation in the large nebula in Draco and in Struve No. 6. His values, however, are not in accordance with the measures of Campbell and Moore, who recently have published definitive values for several of these nebulae.[2]

In his report for 1913 Wolf[3] stated that the lines in the spectrum of Messier 81, N. G. C. 3031 Ursae Majoris appeared doubly bent near the nucleus, indicating a rotation of 100 km. per sec. near the nucleus, the east side approaching. In *Lowell Observatory Bulletin*, No. 62, Slipher announced the rotation of the nebula which is the subject of this communication, and a few months later gave the numerical data referred to below. Later Wolf[4] mentions attempts to detect rotation in the Ring and Dumb-bell nebulae, but his apparatus was not well adapted to the work.

Van Maanen,[5] from measures of direct photographs, obtained evidence of rotation in Messier 101, together with a possible outward component for the individual knots, which seem to move along the arms of the spiral.

The great lengths of exposure necessary for spectrum work on faint nebulae has suggested the following modification of the slit end of the focal plane spectrograph so that several points in a nebula may be observed simultaneously. In this arrangement the metal jaws of the slit are replaced by a silvered glass plate. This plate and a direct photograph of the nebula taken at the same focus as that at which the spectrograph is to be used are mounted on a comparator, and for each bright spot in the nebula a slit (fig. 1) is cut in the silver film with a dividing attachment, the cut projecting beyond the nebulosity far enough to allow the addition of a comparison spectrum. Spots are chosen so that their spectra do not overlap. For the comparison spectrum another silvered glass plate is made in which for each continuous cut in the slit-plate there is an interrupted cut (fig. 2), the central portion covering the nebula while the comparison exposure is made on a spark between iron electrodes. This second plate is mounted on a metal frame which swings it almost into contact with the slit-plate when the spark is used. When not in use it is turned out of the way. A dense piece of opal glass between the spark and the slit serves to diffuse the light thoroughly. No projecting lens is employed. It is of course necessary in this arrangement to place the photographic plate at right angles to the axis of the camera.

At the Seventeenth Meeting of the American Astronomical Society, Slipher[6] gave for the nebula N. G. C. 4594 a radial velocity of + 1100 km. per sec., and stated that it has a rotation analogous to that of the planets. He assigns a value of 100 km. per sec. to the linear velocity of rotation at a distance of 20 seconds of arc from the nucleus.

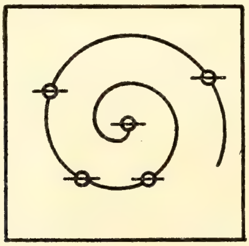


FIG. 1.

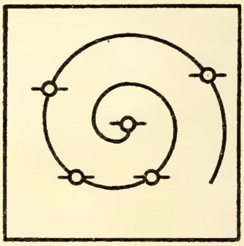


FIG. 2.

A direct photograph (fig. 3) of this nebula shows it to be a spiral seen almost edge on, with the convolutions so nearly concentric that it is not possible to state whether it is of the right or left-handed type. The nebula is crossed by a dark streak which appears to absorb or obstruct the light. This streak seems to lie at the periphery, being possibly an outer ring of cooler material, or perhaps the unilluminated edge of the thin disk of nebulous matter surrounding the brilliant nucleus. With possible differences of temperature in mind, a photograph was taken on a red sensitive plate through a color screen transmitting the region λ 5650–7600, to test for changes of contrast in visual and photographic light. No certain difference was found between this photograph and one taken upon a Seed '23' plate.

A spectrum exposure of 80 hours was made at intervals during March, April, and May, the slit being placed along the major axis of the spindle across the nucleus. Since the nebula is long and narrow a single straight line was in this instance cut in the slit-plate, while the spark plate was made in such a way that the nebular spectrum was traversed longitudinally by comparison spectra at intervals of 1 mm. The photographic plate accordingly was used in its regular position, and thus a large part of the spectrum was in focus.

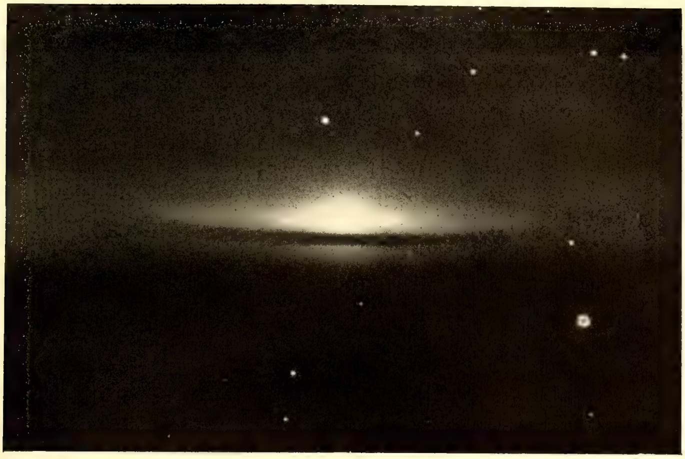


FIG. 3.—N. G. C. 4594 EXPOSURE = 2 HOURS. 1 MM = 6.7

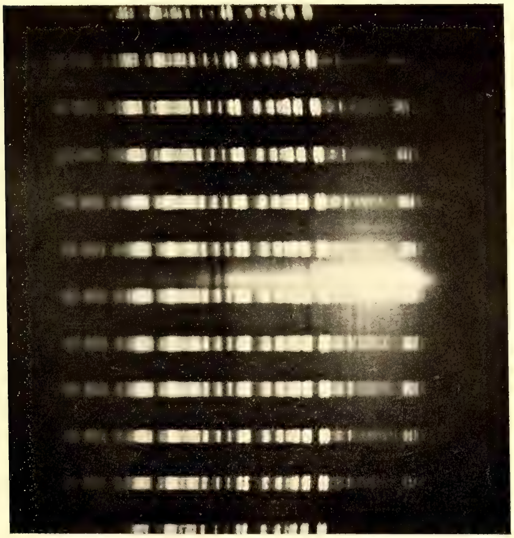


FIG 4.—SPECTRUM OF N. G. C. 4594. EXPOSURE 80 HOURS.

The resulting spectrum (fig. 4) represents approximately the central half of the nebula, and it has been found possible to obtain measures in five spaces ($2\frac{1}{4}$ minutes of arc) on either side of the nucleus. The type is about F5 and seems to be alike in all parts of this nebula. The linear scale of the spectrum is 4.3 mm. from λ 3930 to λ 4950 and most of the lines measured lie in the more refrangible portion of this region. A valuable check has been furnished by a photograph of the spectrum of the sky and spark taken in the same way as the nebular plate. The results of the measures by Mr. Adams and Miss Burwell are given in Table I. The correction for the earth's motion amounts to −1 km. for the mean epoch of the photograph and has been neglected.

		Adams		Burwell		Mean Velocity
		No. lines	Mean *km.*	No. lines	Mean *km.*	*km.*
West Side	5th space	2	+ 830	4	+ 775	+ 800
	4th space	4	960	6	905	930
	3rd space	4	980	6	930	955
	2d space	6	970	6	970	970
	1st space	6	1155	8	1105	1130
Nucleus		5	1210	7	1190	1200
Adjoining Nucleus		6	1250	7	1275	1260
East Side	1st space	7	1275	7	1275	1275
	2d space	7	1330	9	1350	1340
	3d space	5	1390	5	1395	1390
	4th space	4	+1380	4	1600	1490
	5th space			2	+1630	+1630

These velocities are shown in figure 5, the ordinates being kilometers per second and the abscissae seconds of arc. The velocity curve

$$y = -2.78x + 1180$$

was determined by a least squares solution. From this equation we find:

1. The radial velocity of the nebula is + 1180 km., a value in good agreement with that of + 1100 km. found by Slipher.
2. The linear velocity of rotation at a point 2 minutes of arc from the nucleus is over 330 km.
3. Within the limits of accuracy of the measures the change of rotational velocity is linear, although there may be some variation in individual parts of the nebula.

The third result may be interpreted either as indicating that the nebula is rotating as a solid body, or that the material is moving in accordance with a law which will give a linear velocity curve. The latter seems the more probable explanation, since the high velocity is opposed to our conceptions of stability in the case of a rotating solid body. In any event the results seem to be inconsistent with a system involving planetary motion about a central nucleus, since this would require an increase of linear velocity toward the center of the nebula.

If we assume that the rotational component of the internal motion in this nebula is of the same order as that found by van Maanen for Messier 101, we can at once derive a value for the parallax. Van

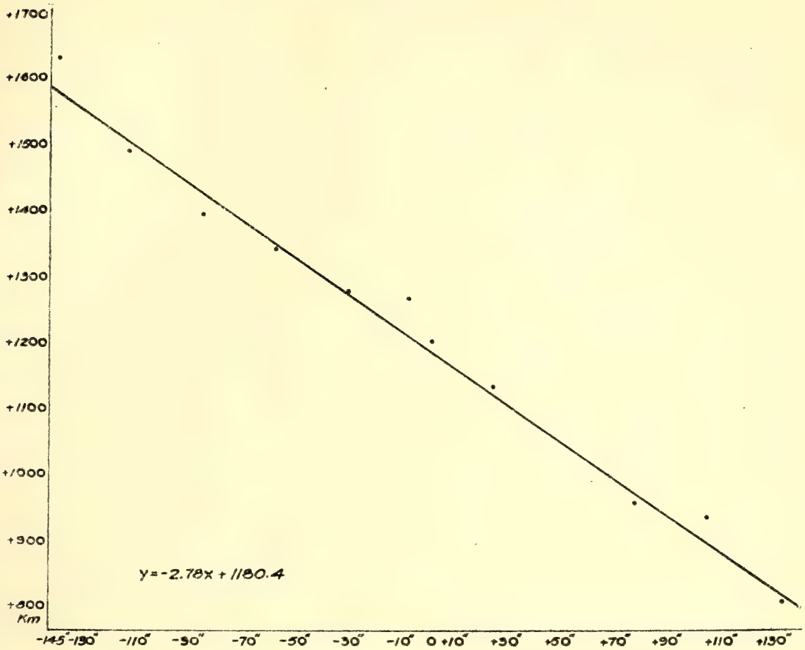


FIG. 5.

Maanen finds for Messier 101 an annual rotational component of $0''.022$ for a distance of 5′ from the center. Combining this with the rotational velocity of 330 km. per sec. for a distance of 2′ gives $0''.00013$ for the parallax of N. G. C. 4594. The result is of the same general order as that found by Curtis for spiral nebulae as a class from a discussion of their proper motion.[7]

---

[1] *Astroph. J.*, **15**, 287 (1902).

[2] *Lick Obs. Bull.*, 278 (1916); *Pub. Astr. Soc. Pac.*, **28**, 119–20 (1916).

[3] *Leipzig, VierteljSchr. astr. Ges.*, **49**, 162 (1914).

[4] *Ibid.*, **50**, 97 (1915).

[5] Van Maanen, these PROCEEDINGS, **2**, 386, July 1916.

[6] *Pop. Astr.*, **23**, 21 (1915).

[7] *Pub. Astr. Soc. Pac.*, **27**, 217 (1915).

A SIMPLE METHOD FOR DETERMINING THE COLORS OF THE STARS

By Frederick H. Seares

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, July 24, 1916

Aside from visual estimates, such as those by Müller and Kempf in the *Potsdam Durchmusterung*, three methods of determining the colors of stars have been more or less widely applied. The earliest of these involves only the use of the spectroscope; the determination of a star's spectral type at once fixes its color, at least within narrow limits. But usually spectroscopic observations are concerned with other physical factors and we ordinarily think of them as only incidentally determinations of color.

The second method depends upon a comparison of photographic and visual or photovisual[1] magnitudes. A red star, owing to a deficiency of light of shorter wave-lengths, makes little impression on the ordinary photographic plate. It will therefore be faint photographically even though visually bright. The light of a white star, on the other hand, affects strongly both the eye and the plate, and its photographic and visual brightness are essentially the same. The difference in the photographic and visual magnitudes is therefore frequently used as a measure of a star's color. This difference is commonly known as the color-index.

The third method determines the color of stars by finding the effective or predominating wave-lengths in the light which they emit.[2] The observations are made by means of a coarse objective grating attached to the end of the telescope. The central diffraction image of each star in the field will have adjoining it on either side tiny images of the first-order spectra. For a red star the blue ends of the spectral images will be weak; the distance separating the points of maximum photographic density is accordingly greater than for a bluer object. From the observed value of this distance the wave-length for the point of maximum density can be calculated. The result is the effective wave-length.

Since spectral type is determined mainly by the number, intensity, and position of the Fraunhofer lines, its relation to color, which depends upon the distribution of intensity in the continuous spectrum background, is not necessarily constant. Thus stars of the same type show differences in color whose origin may be traced to differences in their intrinsic luminosity or absolute magnitude; and any selective absorption or scattering of light during its passage through space must also introduce an element into the star's color which escapes a determination based upon spectral classification.[3] The first method, therefore, does not necessarily give exact values of the color. Moreover, it cannot be used for very faint stars.

The second method is free from the objections attached to the first, but it is not always easy of application. Both the photographic and photovisual magnitudes must be known on the absolute scale. Aside from the difficulties arising from systematic errors in the slope of the scales, there are others, connected with the determination of their zero points, which at present are likely to prove troublesome. With care satisfactory results can be obtained, but the operation is tedious at best, and the uncertainties are greatest for just those objects whose colors we should like most to know—the faint stars.

The third method, although spectroscopic in character, gives real values of the color, and not merely the spectral type. It is also free from the zero-point and scale errors which complicate precise determinations of the color-index; and although subject to its own peculiar errors, these do not seem to be very serious. The chief objection is that the method is wasteful of light, the limiting magnitude for the determination of effective wave-lengths being about three magnitudes less than that for direct photography.[2] If a large telescope is to be used, the size and weight of the objective grating are also objectionable and make it difficult to pass quickly from the determination of effective wave-lengths to other kinds of observational work.

The following method seems to offer some advantages, and has also the merit of simplicity and convenience. The limiting magnitude to which it can be applied should be about the same as that for which photovisual magnitudes can be determined. It consists simply in determining the ratio of exposure-times necessary to produce photographic and photovisual, or more briefly blue and yellow, images of the same size. For convenience, and also as a matter of precision, the images should be on the same plate. An isochromatic plate exposed behind a yellow filter registers the yellow image as usual. The same plate used without filter gives the blue image. This will include to some extent the effect of the longer wave-lengths, but owing to the relatively small sensitiveness of the isochromatic plate to the yellow and orange rays, the shorter wave-lengths will still be of predominating influence.

In the simplest case there will be for each star one yellow image and a series of three or four blue images, with the exposure times for the latter increasing in geometrical progression—as a constant ratio, 2 seems to be the most convenient. The diameters of the blue images, or their scale readings,[4] are plotted against the logarithm of the exposure, and from the nearly linear curve thus derived can be read the exposure time for a blue image of the same size as the yellow image. The ratio of the interpolated exposure to that which produced the yellow image—the exposure ratio—compared with similar ratios for stars of known color, gives at once the color-index or color-class.[3]

Various modifications necessary to meet special conditions immediately suggest themselves; but whatever the details, consideration should be given to the following points: First, if there are differences of gradation between the blue and yellow images, the exposure-ratios of bright and faint stars of the same color will be different. In other words, the ratios will depend upon the size of the images. The matter is easily investigated by observing repeatedly the same stars with different apertures and exposure times; the variation of exposure-ratio with the size of image can thus be determined. In practice it will be advantageous, wherever possible, to adjust exposure and aperture so that the resulting yellow image is of a standard size. Preliminary tests indicate that the systematic gradation difference for the blue and yellow images is small. Small deviations from the standard image are therefore of no consequence, and larger differences are readily taken into account in the reductions.

Second, atmospheric extinction reduces the intensity of blue light more than that of yellow and therefore modifies the exposure-ratio. Although the effect enters differentially, its influence is appreciable, except for small zenith distances. Tables of corrections can be derived by determining the exposure-ratio for the same star at different zenith distances.

Third, care should be taken to bring the plates into equilibrium with the atmospheric conditions before beginning the exposures; otherwise the absorption of moisture during the exposure may introduce variations of sensitiveness and gradation which are capable of influencing seriously the results.

No extensive test of the method has yet been made, but it has been tried sufficiently to demonstrate its practicability. About 40 plates, mainly of individual bright stars, were exposed on three successive nights, two or more plates being obtained on each of 10 different stars. Two yellow and five blue exposures were made on each plate; further three exposures through a filter transmitting to the violet of λ 4000 were also added in each case in order to test the precision of results derived from different regions of the spectrum. To avoid gradation errors, the apertures and exposure times were adjusted so that the images differ little from a standard size.

For the plate and filter used the exposure ratio of blue to yellow varies from about $\frac{1}{8}$ to $\frac{1}{3}$ for spectral types A to M. That of blue to ultra-violet would be a much smaller quantity but for the fact that the large exposure factor of the ultra-violet filter was offset by a change in the aperture—the exposures for blue and yellow always being made through a wire-gauze screen absorbing about 3 magnitudes.

PLATE	DATE	HOUR ANGLE	EXPOSURE-RATIOS	
			Blue/Yellow	Blue/Violet
P 3138	1916, June 25	25°W	0.168	0.496
3150	26	1E	0.156	0.542
3163	27	27E	0.178	0.558
3167	27	6E	0.180	0.595
3171	27	15W	0.158	0.600
3173	27	37W	0.164	0.533
3177	27	58W	0.172	0.547
Mean			0.168	0.553
Average deviation from mean			±0.007	±0.027
Change in ratio for 0.1 spectrum interval			0.004	0.014
Spectrum interval corresponding to average deviation			±0.180	±0.190

From the accordance of results for the same star, derived mainly upon different nights, it is found that the average deviation in the exposure-ratio from a single plate corresponds to about 0.2 of a spectrum interval, or 0.08 mag. in the color-index. The greater redness of stars of high luminosity, spectral type remaining the same, is easily seen in the results from even a single plate. The precision seems to be the same whether the yellow or the violet is used for comparison with the blue. As none of the results have yet been corrected for atmospheric extinction, the average deviation includes the effect of this disturbing factor. For illustration the results for seven plates on one of the stars, 72 *w* Herculis, are given in the table.

[1] Photovisual magnitudes express the visual brightness of a star, but are determined photographically. The procedure is the same as for the determination of photographic magnitudes, except that an isochromatic plate combined with a yellow filter is used instead of the ordinary photographic plate.

[2] See for example Hertzsprung, *Mt. Wilson Contr.*, No. 100; *Astroph. J.*, **42**, 92 (1915).

[3] These PROCEEDINGS, **1**, 481 (1915).

[4] *Mt. Wilson Contr.*, No. 80, 19; *Astroph. J.*, 39, 325 (1914).

STUDIES OF MAGNITUDES IN STAR CLUSTERS, III. THE COLORS OF THE BRIGHTER STARS IN FOUR GLOBULAR SYSTEMS

By Harlow Shapley

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, August 3, 1916

In a former communication dealing with the probable sequence of spectral types in stellar development[1] it was shown that the brightest stars in the great Hercules cluster are of the redder spectral types, while among the fainter magnitudes blue stars were conspicuously predominant. Since all the stars of the cluster are at sensibly the same distance from the earth, the apparent difference in brightness between red and blue stars represents an actual characteristic of the brighter objects of the system. This result, however, is not in harmony with the relations heretofore found between color and intrinsic brightness in the less condensed bright clusters and in other selected groups of stars, and it is of importance to seek verification of the phenomenon in additional globular clusters. If it is proved that in these remote, gigantic stellar systems the most luminous objects are chiefly of the redder, low-temperature spectral types, and consequently of very great dimensions relative to the thousands of other stars each cluster contains, the result may have a significant bearing on theories of the evolution of stars. For instance, a satisfactory hypothesis of the evolutionary sequence of spectral and color classes would need to account explicitly for the ancestral relationships of these extremely voluminous second-type (red) stars and the comparatively small bodies of the bluer types.

Measures of the magnitudes and color indices are now available for stars in three other globular clusters. For none of them is the work completed in all its detail, but the preliminary values, taken in conjunction with final results for the Hercules cluster, are sufficiently definite to justify a report regarding the present problem. The observational material and the final discussion will appear in extended form in the *Contributions from the Mount Wilson Solar Observatory*.

The names of the clusters and their positions for the epoch 1900.0 are given in Table I.

TABLE I

DESIGNATION			RIGHT ASCENSION	DECLINATION	GALACTIC LATITUDE	GALACTIC LONGITUDE
Messier	3…….	N.G.C.5272	$13^h\ 38^m$	+28°53′	8°	+77°
	5…….	5904	15 13	+ 2 27	333	+45
	13…..	6205	16 38	+36 39	26	+40
	15……	7078	21 25	+11 44	33	−29

A number of stars that undergo periodic light variations have been found in these clusters. Such objects were avoided in collecting the results of which a summary appears in Table II. The inclusion of the variables would scarcely change the averages, however, for the plates were taken in pairs and only extremely rapid light changes could enter as systematic errors in the colors. Stars near the center were also omitted from the present treatment in order to avoid possible errors similar to those suspected for the dense central part of Messier 13.[2]

The tabulation of the material needs no general explanation. The photographs now available for Messier 3 do not give a large interval of photovisual magnitude, and consequently, in order to test for the presence of blue stars in this cluster, recourse was had to the short period variable stars. The last average, then, depends entirely on the colors deduced for a number of these variables[3] and, representing a group that is probably peculiar for blueness, is entitled to diminished weight.

The data in Table II show that in these four clusters the average color index is decidedly larger for the brighter stars. Not only is this true in the averages but a detailed examination of the magnitudes involved shows that with few exceptions relatively high luminosity is always accompanied by greater redness. (The exceptions may indicate real anomalies among the members of the cluster, or may be due to the superposed non-cluster stars.) Preliminary consideration of the colors and magnitudes in three additional globular clusters—Messier 4, 9 and 14—shows that the same relation holds for them as that deduced numerically in Table II.

The condition clearly indicated by the above results is that the total light emission of the bluer stars (small color indices), for which the surface temperature is presumably in excess of 10,000 degrees Centigrade, is less than that of many of the redder stars (large color indices),

TABLE II

AVERAGE MAGNITUDES AND COLORS IN FOUR GLOBULAR CLUSTERS

PHOTOVISUAL MAGNITUDE	AVERAGE COLOR-INDEX	NUMBER OF STARS	PHOTOVISUAL MAGNITUDE	AVERAGE COLOR-INDEX	NUMBER OF STARS
Messier 3			*Messier 13*		
12.71	+1.34	10	12.28	+1.28	13
13.34	+1.16	10	13.00	+0.92	23
13.91	+0.88	10	13.50	+0.96	23
14.24	+0.88	9	14.10	+0.73	76
14.51	+0.71	10	14.70	+0.65	110
14.78	+0.73	11	15.30	+0.39	222
(15.20	+0.05	14)	15.70	+0.34	27
			16.45	+0.32	28
Messier 5			*Messier 15*		
12.46	+1.52	6	13.04	+1.35	6
12.93	+1.22	6	13.52	+1.17	7
13.24	+1.20	6	14.19	+0.83	7
13.66	+1.02	6	14.87	+0.86	7
14.42	+0.59	6	15.18	+0.68	7
14.72	+0.44	7	15.54	+0.42	7
14.83	+0.29	7	15.65	+0.32	6

with surface temperatures but half as much. But the light emission per unit area is much less for red stars than for blue. The obvious conclusion is, therefore, that in all these clusters, and probably in all globular clusters, the volumes of the bright red stars are very great in comparison with the stars that are fainter and relatively blue.

---

[1] These PROCEEDINGS, **2,** 12 (1916).

[2] *Mt. Wilson Contr.*, No. 116, Sections IV and VIII (1916).

[3] *Publ. Astr. Soc. Pac.*, No. 163, April 1916.

THE EFFECT OF AN ELECTRIC FIELD ON THE LINES OF LITHIUM AND CALCIUM

By Janet T. Howell

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, August 10, 1916

During the last three years a great deal of work has been done on the electric decomposition of spectral lines. Stark[1] examined the effect in canal rays, subjected to a strong auxiliary field, and Lo Surdo[2] photographed the light immediately in front of the cathode, where the luminosity of the negative glow and the sudden fall of potential fulfilled the required conditions. Hydrogen and helium have been examined by both methods and Stark has investigated the transverse effects for lithium, mercury, and a number of other elements. So far, only H, He, and Li have shown large effects, and the results found for H and He by the two methods differ considerably.

In spite of the great number of data accumulated by Stark, Lo Surdo, and their co-workers since the discovery of the electric effect, the work in this important field is only begun and offers great opportunities for further work. The apparatus used by Stark is very difficult to construct and needs the constant services of a skilled glass blower. Moreover, Stark has already examined most of the more promising elements. The method of Lo Surdo is very simple and has been applied, so far, only to hydrogen and helium. A survey of a number of elements was therefore made with the Lo Surdo form of apparatus, under low dispersion, and in the course of the work some new and interesting results were obtained with calcium and lithium.

A full description of the apparatus used will be published in the *Astrophysical Journal*, but it was essentially of the Lo Surdo form. The tube had an internal diameter of 6 mm. and a length of 20 cm. The discharge from an induction coil was used, rectified by a valve tube. The spectrum was photographed with a three prism, quartz and ultra-violet glass, spectroscope which gave a dispersion of 1 mm. = 18 A. U. at Hγ, and 1 mm. = 12 A. U. at the H and K lines.

Lithium and calcium were examined, both for the longitudinal and transverse effects, and the results are given below in tables I and II. With both elements the spectra were obtained by covering the cathode with a thin layer of the chloride, which gave the characteristic spectrum of the metal under the bombardment of the anode rays. The tubes were filled with hydrogen, and the field strength was determined for every plate by measuring the separation of the outer components of Hγ and comparing it with Stark's results. The field strength was about 25,000 volts per cm. close to the cathode. In the following tables the numbers marked + indicate components to the red, − to the violet.

TABLE I

LITHIUM.

Transverse effect for 20,000 volts per centimeter

λ IN A	COMPONENTS POLAR PARALLEL	INT.	COMPONENT POLAR PERPENDICULAR	INT.	REMARKS
4602.37	+1.00	8	+0.48	8	
	−2.48	6	−2.00	6	
4132.93	+2.26	2	+1.78	2	
	−0.18	5	−0.18	5	Unpolarized
	−3.10	1	−2.24	1	
Longitudinal effect for 20,000 volts per centimeter					
4602.37	+0.57	8	+0.34	6	
	−2.01	6	−1.53	3	
4132.93	+1.16	1	+0.77	1	
	−0.26	5	−0.26	5	Unpolarized
	−1.99	0	−1.50	0	

TABLE II

CALCIUM H AND K

Transverse effect for 20,000 volts per centimeter

λ IN A	COMPONENTS POLAR PARALLEL	INT.	COMPONENTS POLAR PERPENDICULAR	INT.	REMARKS
3968.63	+0.22	6	+0.16	6	Unpolarized
	−0.86	2	−0.74	2	
3933.83	+0.22	9	+0.22	9	Unpolarized
	−0.92	3	−0.74	3	
Longitudinal effect for 20,000 volts per centimeter					
3968.63	+1.27	3	+1.23	3	Unpolarized
	+0.01	8	−0.02	8	Unpolarized
	−1.17	0	−1.11	0	Unpolarized
3933.83	+1.42	4	+1.38	4	Unpolarized
	+0.06	9	−0.02	9	Unpolarized
	−1.30	1	−1.26	1	Unpolarized

It is interesting to find that the lithium lines λ4602 and λ4132 show polarized components in the longitudinal effect. The longitudinal effects in hydrogen and helium, the only ones investigated up to this time, had given unpolarized components.

Previously, only the diffuse series of elements had shown large electric effects which makes the calcium results most unexpected. H and K belong to a principle pair series and the lines of the diffuse series at λλ4457, 4435, and 4425 show no effect at all, under low dispersion.

A full account of this investigation will be published shortly in the *Astrophysical Journal.*

[1] J. Stark, *Ann. Physik,* 43, 965 (1914); J. Stark and G. Wendt, *Ibid.,* **43**, 983 (1914) J. Stark and H. Kirschbaum, *Ibid.,* 43, 991 and 1017 (1914); J. Stark, *Ibid,* **48**, 193 (1915).

[2] A. Lo Surdo, *Roma, Rend. Acc. Linci,* **23**, 1st. sem., 82, 143, 252, 326 (1914).

A PROOF OF WHITE'S PORISM

By A. B. Coble

DEPARTMENT OF MATHEMATICS, JOHNS HOPKINS UNIVERSITY

Received by the Academy, August 8, 1916

The interesting theorem of Professor White[1] to the effect that *if two cubic curves in space admit a configuration* Δ_7—i.e., *seven points of the one and seven planes of the other such that each of the points is on three of the planes and each of the planes is on three of the points—then they admit* ∞^1 *such configurations* furnishes perhaps the only important generalization of the Poncelet polygons.[2] Analytically expressed the theorem states that if for a (3, 3) form $F(\lambda,\mu)$ there exists a set of seven parameters λ and seven parameters μ such that $F = 0$ for each λ together with three μ's and for each μ together with three λ's, then there exists ∞^1 such sets Δ_7.

The published proof of this theorem fails owing to an error of enumeration.[3] This error, originally overlooked by Professor White and myself, was noted subsequently by him. That however the theorem itself is true can be shown as follows.

Let $G(\lambda_1, \lambda_2) = 0$ be the condition that distinct values λ_1, λ_2 determine in $F(\lambda, \mu) = 0$ the same value of μ. Then G is a symmetrical (6, 6) form. If $F(\lambda, \mu)$ has a Δ_7, the seven λ's constitute an involutorial set of G, i.e., a set such that any two of the λ's satisfy $G = 0$. Conversely if G has an involutorial set, then $F(\lambda, \mu)$ has a Δ_7. For if λ_1 and any one of $\lambda_2, \ldots, \lambda_7$ satisfy $G = 0$ then, since λ_1 can determine in $F = 0$ at most three μ's, λ_1 must be associated with three pairs of the remaining λ's in such a way that each pair determines with λ_1 a common value μ.

Thus at most seven μ's can be determined by the seven λ's. If however G has one involutorial set, it has ∞^1 such sets.[4] If therefore F admits one Δ_7, it must admit ∞^1 Δ_7's.

---

[1] A variable system of sevens on two twisted cubic curves, these PROCEEDINGS, **2**, 337 (1916).

[2] In the sense that other analogous porisms can be based on the properties of an algebraic (2, 2) form.

[3] The number of poristic forms of the special type which factor into three bilinear forms was estimated at ∞^3 instead of ∞^5.

[4] Coble, Symmetric binary forms and involutions, *Amer. J. Math.*, **31**, 189.

A CONTRIBUTION TO THE PETROGRAPHY OF THE PHILIPPINE ISLANDS

By J. P. Iddings and E. W. Morley

BRINKLOW, MARYLAND, AND WEST HARTFORD, CONN.

Received by the Academy, August 10, 1916

A very large part of the volcanic rocks of the Philippine Islands occur as tuff-breccias with flows and dikes of massive lava, and a very great area of country is covered with vegetation, so that it will be many years before anything like a thorough knowledge of the igneous rocks of the region will be acquired. Fragmentary contributions to the petrography of the islands are the most that may be expected for some time. Already it is known that the principal volcanic rocks are andesites with much fewer basalts and dacites, and almost no rhyolites. A short visit to Luzon in 1910 enabled one of us to study the collections in the Mining Department of the Bureau of Science [Iddings, J. P., *Philippine J. Sci.*, 5, 155, 1910,] and to collect specimens from some of the more accessible localities on the island. From these specimens the accompanying chemical analyses have been made by Dr. Morley.

The first three analyses are from basaltic lavas having andesitic characters. The rocks are porphyritic with andesitic habit to the groundmass and a variable amount of modal olivine, which is present in the rocks from Antipolo and Taal, but does not appear in the specimen from Mayon which was analyzed, though it occurs in other lava from the same volcano. The normative plagioclase in each case is labradorite.

The basalt from Antipolo, anal. 1, is massive lava from near the waterfall. It is magnophyric and seriate porphyritic with phenocrysts of labradorite and smaller ones of olivine. The groundmass consists of prismoid and anhedral feldspar, anhedral pyroxene and euhedral magnetite. The norm contains a small amount of quartz, no olivine, and plagioclase which is approximately $Ab_2 An_3$.

The basalt from the south base of Taal volcano, anal. 2, is massive lava on the shore of the lake which surrounds the volcano, and is overlaid by tuffs from more recent eruptions. It has small phenocrysts of plagioclase, augite and olivine, and has more mafic minerals than the rock from Antipolo, but the feldspar is less calcic. The rock is transitional to andose.

The lava from Mayon volcano, anal. 3, is labradorite-andesite, with augite and hypersthene and without noticeable olivine. The norm contains 5% of quartz, but the rock is hessose, closely similar to the basalts from Taal and Antipolo. Other lavas from Mayon contain olivine.

The rock from Benguet Road, anal. 4, is holocrystalline hornblende-andesite or andesite-porphyry, slightly altered. The phenocrysts of plagioclase are zonal with alternating shells which differ in composition, more or less calcic. The hornblende is brownish green, in part altered to chlorite, epidote and calcite. The rock is intruded in andesitic breccias and lavas. The hornblende-andesite from Olongapo, anal. 5, is seriate magnophyric with phenocrysts of zonal plagioclase with alternating zones differing in composition as in the rock from Benguet Road, the outermost zone being strongly calcic instead of strongly sodic as is commonly the case. The hornblende is brown with black border, and opaque paramorphs of hornblende are abundant.

The dacite from Corregidor Island in Manila Bay, anal. 6, is a dense white porphyry forming a large massive body on the sea coast near the landing. It is dopatic and seriate mediophyric, with phenocrysts of glassy plagioclase, quartz and biotite. The groundmass is holocrystalline and seriate microporphyritic, consisting of minute phenocrysts of prismoid plagioclase and abundant microscopic euhedral bipyramidal quartzes in a microcrystalline matrix which besides quartz and alkalic feldspar must contain some kaolin, judging from the chemical analysis and norm, for there is 5% of normative corundum, a large amount of normative quartz and 2.5% of water. The calculated kaolin is 13% which is probably more than is actually present, as some of the normative corundum and water belongs to the modal biotite. The rock is alsbachose, and is the most siliceous volcanic rock from the Philippines so far analyzed. It is interesting to note that the volcanic rocks in these islands are like many in Japan and in western America.

CHEMICAL ANALYSES OF ROCKS FROM LUZON, P. I.

	1	2	3	4	5	6
SiO_2	50.54	52.33	53.06	58.54	60.13	72.68
Al_2O_3	21.63	17.18	19.68	16.79	17.51	15.77
Fe_2O_5	3.64	3.51	3.24	1.89	3.25	0.65
FeO	4.22	5.73	4.94	3.33	0.91	0.21
MgO	3.06	5.30	3.64	3.27	3.35	0.41
CaO	10.47	10.71	9.41	7.25	6.54	1.66
Na_2O	2.88	3.17	3.24	3.76	3.78	3.26
K_2O	1.57	0.94	0.94	1.62	0.77	2.19
H_2O-	0.19	0.00	0.01	0.04	1.45	1.18
H_2O+	0.72	0.38	0.06	1.24	0.90	1.38
TiO_2	0.34	0.31	0.77	0.70	0.44	0.09
P_2O_5	0.24	0.22	0.33	0.66	0.68	0.22
CO_2	0.04	0.02	0.00	0.69	0.02	0.00
ZrO_2	0.00	0.00	0.00	0.01	0.00	0.00
Cl	0.08	0.08	0.08	0.04	0.09	0.02
F	0.02	0.05	0.07	0.05	0.02	0.01
S	0.01	0.03	0.03	0.04	0.02	0.05
Cr_2O_3	0.01	0.01	0.00	0.01	0.00	0.00
MnO	0.17	0.24	0.59	0.13	0.27	0.16
BaO	0.05	0.07	0.06	0.04	0.03	0.02
SrO	0.04	0.03	0.02	0.01	0.01	0.03
	99.92	100.31	100.17	100.31	100.17	99.99
Norms						
q	0.90	0.96	4.98	11.94	18.00	41.46
or	9.45	5.56	5.56	9.45	5.00	12.79
ab	24.63	27.25	27.25	31.96	31.96	27.77
an	41.14	29.75	36.42	24.19	27.52	7.51
c					0.31	5.30
di	8.34	18.27	6.80	5.81		
hy	8.00	11.82	11.85	8.97	8.40	1.00
mt	5.34	5.10	4.64	2.78	2.78	0.93
il	0.61	0.61	1.52	1.37	0.76	0.15
hm					1.44	
ap	0.34	0.34	0.67	1.68	1.68	0.34
etc.	1.16	0.67	0.33	2.17	2.54	2.69
	99.91	100.33	100.02	100.32	100.39	99.94

1. Basalt, hessose, II.5.4.4. Antipolo.
2. Basalt, andose-hessose, II (III).5.(3)4.4′. Taal volcano.
3. Labradorite-andesite, hessose, II,′5.′4.4′. Mayon volcano.
4. Hornblende-andesite, tonalose, II.4′.3.4. Benguet road.
5. Hornblende-andesite, placerose-tonalose, (I) II.4.3′.4(5), Olongapo.
6. Dacite, alsbachose, I.3.2.4. Corregidor Island.

SALT ANTAGONISM IN GELATINE

By W. O. Fenn

LABORATORY OF PLANT PHYSIOLOGY, HARVARD UNIVERSITY

Received by the Academy, July 28, 1916

In studying the effects of electrolytes on living organisms Loeb[1] has distinguished two groups of salt antagonisms. To the first belongs the antagonism between NaCl and $CaCl_2$. Here the antagonism is between a monovalent salt and a salt with a strong (bivalent or trivalent) cation. The antagonistic effect increases with the valence of the cation. To the second group belongs the inhibition of the toxic action of KCl on the eggs of *Fundulus* by means of sodium salts, the sulphate and citrate being the most effective. In this second group the antagonism is between a monovalent salt and a salt with a strong (bivalent or trivalent) anion, the effect increasing with the valence of the anion.

A recent explanation of the first kind of antagonism has been presented by Clowes,[2] who assumes that NaCl has a more strongly adsorbed anion and is therefore able to antagonize more strongly adsorbed cations (like Ca). His general conclusion is that electrolytes with a more strongly adsorbed anion are able to antagonize electrolytes with a more strongly adsorbed cation. No explanation is given of antagonism between monovalent salts and salts with strong anions (like Na_2SO_4).

From experiments on gelatine the writer has formulated a hypothesis by which both groups of antagonism may be explained. According to this hypothesis (in many respects similar to that of Clowes) electrolytes with a more strongly adsorbed cation should antagonize electrolytes with a more strongly adsorbed anion, *and in the case of salts like NaCl, the effect of the Na ion should predominate in the presence of strong anions and the effect of the Cl ion should predominate in the presence of strong cations.*

In agreement with this hypothesis it is found possible to duplicate experimentally both groups of antagonism, as defined by Loeb, i.e., NaCl is found to antagonize both anions and cations. This is not only in agreement with biological results, but it also correlates the well known fact that NaCl tends to maintain the neutrality of protein sols to which either acid or alkali have been added,[3] and to decrease the viscosity of both alike.[4]

The results of the gelatine experiments in so far as they concern antagonism are represented more completely in figure 1 in diagrammatic form. Only a few typical electrolytes are included in the diagram, but the general principles which they exemplify are applicable to all cases thus far studied.

According to this scheme, electrolytes are found to fall into three groups.

Group I includes salts with monovalent cations and monovalent anions, the effect of the former predominating in the presence of Group II, the effect of the latter in the presence of Group III.

Group II includes alkalies and salts with bi- and trivalent anions, the effect of the anion predominating.

Group III includes acids, and salts with bi- and tri-valent cations, the effect of the cations predominating.

Members of any one group are additive to each other but antagonistic to members of either of the other two groups. In general the antagonism between Groups II and III is greater than between Groups I and II or between Groups I and III. The valence of the ion (except H and OH), while not the only factor, seems of chief importance. Acids and alkalies are in general more effective than neutral salts.

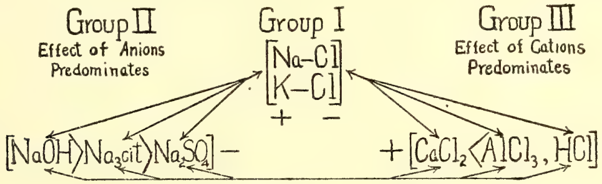


FIG. 1

Aditive combinations are bracketted together. Antagonistic combinations are connected by arrows. In general, the length of the arrows indicates the amount of antagonism expected.

An exception to this rule is found in the case of $MgCl_2$ which is found to antagonize both NaCl and $CaCl_2$. This exception, however, is in agreement with biological results.

As a criterion of antagonism, the precipitation of gelatine by 95% alcohol was used. It is found that *less alcohol is required for the precipitation of the gelatine in the presence of two antagonistic salts than when either is present alone.*

Let us for convenience designate the antagonistic salts as A and B. We find that as more and more of salt A is added to gelatine, more and more alcohol is required to produce an opaque precipitate, until finally a maximum is reached. This maximum, according to Pauli's theory,[5] corresponds to the greatest ionization of the protein and the greatest viscosity. At this point the hydration of the protein is greatest and offers greatest resistance to the dehydration effect of the alcohol, by which precipitation is ultimately accomplished.

If now salt A be gradually replaced by an antagonistic salt B, less and less alcohol is required for precipitation until a minimum is reached. This minimum is at the isoelectric point, corresponding to greatest aggregation. *At this point, if the concentration of the salts be sufficiently high the gelatine is precipitated without alcohol.* If all of salt A be replaced by salt B, the amount of alcohol needed for precipitation again rises to the point characteristic of that salt in the concentration used. Thus an antagonism curve can be plotted, such as that shown in figure 2 for $CaCl_2$ and sodium citrate and in figure 3 for NaCl and $CaCl_2$. The probable additive effect in each figure is represented by the dotted base line. Such lines at least have resulted from similarly arranged experiments with non-antagonistic salts such as NaCl vs. KSCN and $CaCl_2$ vs. $MnCl_2$.[6]

It is frequently more convenient in determining antagonism to keep the concentration of one salt constant and gradually increase that of the other. If the two salts are *additive* to each other, the alcohol needed for precipitation is thereby *increased.* Thus when $CaCl_2$ is added to gelatine containing HCl, or when Na_2SO_4 is added to gelatine containing NaOH, the amount of alcohol needed for precipitation is increased.[7] If two *antagonistic* salts are used, on the other hand, there is a *decrease* in the amount of alcohol needed for precipitation until a minimum is reached. This is again an isoelectric point and here the gelatine may be precipitated without alcohol if the salt concentration be sufficiently high. If still more of the salt be added, there is an increase of the amount of alcohol needed for precipitation and a decrease of precipitability by salts in the absence of alcohol. Such results are observed when NaCl, for example, is added to gelatine containing HCl, NaOH, $AlCl_3$, $CaCl_2$, Na_2SO_4 or Na_3-citrate.

The molecular proportions of salts at the minimum, i.e. at the isoelectric point or point of greatest precipitability, appears to be characteristic of the particular salts used. Thus for NaCl and $CaCl_2$, it is about 100 to 20; for NaCl and $AlCl_3$, about 100 to 1.5.

Mixtures of NaCl and $AlCl_3$ in the above proportions[8] gives maximum precipitation in gelatine, milk, and olive oil emulsions.[9] Gelatine containing NaCl 0.5M shows maximum precipitation in presence of $AlCl_3$0.0078M and $Ce(NO)_3$0.00625M. The molecular proportions at these two points are 100 to 1.5 and 100 to 1.2. Mines[10] has observed a maximum precipitation of blood corpuscles in NaCl plus $CeCl_3$, the molecular proportions being between 100 to 3.3 and 100 to 0.03.

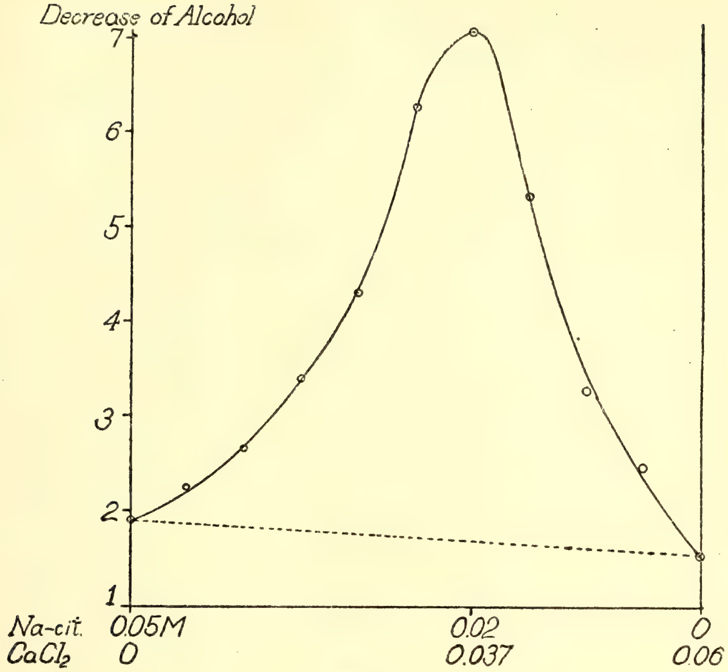


FIG. 2

Antagonism between $CaCl_2$ and Na-citrate in gelatine. Decrease of alcohol required for precipitation is plotted as ordinates (as 12 minus the number of cc. of 95% alcohol added to 5 cc. of the gelatine-salt mixtures in order to produce an opaque precipitate). As abscissae are plotted the concentrations of NaCl and $CaCl_2$ simultaneously present in the gelatine before titrating. The dotted line represents the additive effect. A pure grade of commerical gelatine was used in 3% solution. Each point represents the mean of two titrations. Each titration is accurate to 0.2 cc. The gelatine was always allowed to stand at least 15 hours in an incubator at 26°C. in order to reach equilibrium.

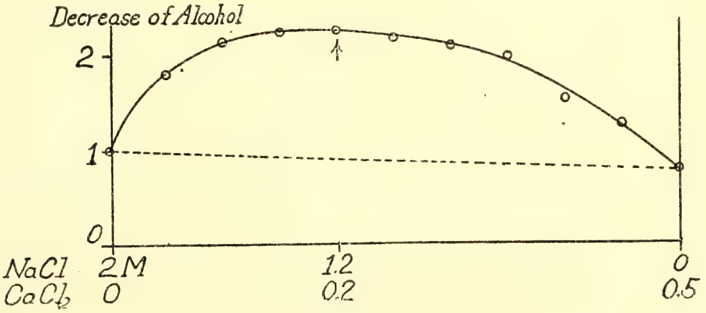


FIG. 3

Antagonism between NaCl and $CaCl_2$ in gelatine (for explanation see figure 2). At the maximum the molecular proportion of NaCl to $CaCl_2$ is 100 to 17.

In pure NaCl there was no precipitation and the corpuscles wandered to the anode. In excess of cerium there was no precipitation and the corpuscles wandered to the cathode. These results become intelligible if the point of maximum precipitation is an isoelectric point due to the predominating effect of the Cl ion (of NaCl) in the presence of the trivalent cerium cation.

From these experiments it may be reasoned that antagonistic salt solutions produce a precipitate or a state of aggregation of an isoelectric nature in protoplasm. Physicochemically, therefore, a physiologically balanced solution would be merely a mixture producing high precipitability. By varying the amount of the precipitate changes in the permeability of the protoplasm might be brought about.

In connection with the hypothesis developed here attention may be called to the suggestion of Loeb[11] that antagonism between salts and acids furnishes an excellent parallel to the results of Procter on gelatin and indicates that the protoplasmic substances which are responsible for antagonism are protein in nature.

Summary.—1. The experiments on gelatine support the hypothesis that anions antagonize cations in their effects upon organisms.

2. The hypothesis here developed resembles that of Clowes except that it requires that NaCl should antagonize any electrolyte which has either a strong anion or a strong cation.

3. The point of maximum antagonism is an isoelectric point at which the amount of alcohol needed for precipitation is at a minimum, and the aggregation or amount of precipitation is at a maximum.

---

[1] Loeb, These PROCEEDINGS, **1**, 473 (1915).

[2] Clowes, *J. Phys. Chem.*, **20**, 407 (1916) and *Science*, **43**, 750.

[3] Chick and Martin, *Koll.-Chem. Beihefte*, **5**, 49 (1915).

[4] Pauli, *Biochem. Zs.*, **24**, 239 (1910).

[5] Pauli, *Kolloid Zs.*, **12**, 222 (1913); **7**, 241 (1910).

[4] Pauli u. Wagner, *Biochem. Zs.*, **27**, 296 (1910).

[6] For "additive effect" see Osterhout, *Bot. Gaz.*, **58**, 178 (1914).

[7] This is true at least in low concentrations. In higher concentrations, the results are complicated by other factors and have not been fully determined.

[8] These proportions would be expected to vary with the medium used and the conditions of the experiment. Clowes (l.c.) finds that the critical point for NaCl and $CaCl_2$ in oil-water emulsions varies between 500 to 1 and 10 to 1.

[9] Clowes has shown that in critical mixtures of NaCl and $CaCl_2$, the phases of a water-oil emulsion separate out.

[10] Mines, G. R., *Koll.-Chem. Beihefte* **3**, 191 (1912).

[11] *Science*, **34**, 653 (1911); *Biochem. Zs.* **33**, 489 (1911) and **47**, 127 (1912).

SIMILARITY IN THE BEHAVIOR OF PROTOPLASM AND GELATINE

W. O. Fenn

LABORATORY OF PLANT PHYSIOLOGY, HARVARD UNIVERSITY

Received by the Academy, July 28, 1916

In the course of experiments on salt antagonism in gelatin, it occurred to the writer that it would be possible to find an analogy in gelatine for the processes which occur in protoplasm when it is acted upon by antagonistic salts. These processes may be illustrated by figure 1, which shows changes in the electrical resistance of living protoplasm produced by NaCl and $CaCl_2$. If these changes are due to the diffusion of salts into the protoplasm (and the resulting increase of their concentration) it is possible that similar changes would occur in gelatine if it were divided up into small portions corresponding in size to the masses of protoplasm which form the individual living cells. The increase in the concentration of the salts as they diffused into the gelatin would produce changes in the precipitability (and other properties) of the gelatine. The manner in which the precipitability varies with the concentration of salts has been explained in the previous article.[1]

The experiments already reported[1] indicate that the favorable action of a physiologically balanced solution is due to the fact that such a solution produces a precipitate[2] (or some condition accompanying precipitation) in the protoplasm, the maximum precipitate being found at the isoelectric point. By varying the amount of this precipitate, variations in electrical resistance might be brought about.

If, however, the normal condition of protoplasm is isoelectric, the addition of *any* electrolyte would change this balance and decrease the precipitate, thus increasing permeability. It is well known, however, from Osterhout's experiments on the electrical resistance of *Laminaria*,[3] that salts like $CaCl_2$ cause a decrease of permeability at first, but this is later followed by an increase, while NaCl causes only an increase. To make the analogy between gelatine and protoplasm more complete, therefore, we should assume that the normal condition of protoplasm is somewhat on the anion side of the isoelectric point, i.e., protoplasm must bear a negative charge. The addition of $CaCl_2$ would neutralize this charge, increase the amount of precipitate or aggregation in the protoplasm and thereby decrease the permeability. In excess of Ca, however, the precipitate disappears and the permeability increases.

Figure 1 shows a set of Osterhout's electrical resistance curves for *Laminaria*, in which resistance is plotted against time. In his experiments resistance was used as a measure of permeability. From what has been said, it is to be expected that the *Laminaria* curves can be duplicated in alkaline gelatine which bears, of course, a negative charge.[4] This is done in the following way.

To a series of test tubes containing gelatine made alkaline (by adding enough NaOH to make the concentration 0.005M) is added NaCl, $CaCl_2$, or mixtures of the two, in increasing concentrations. Each tube is then titrated with 95% alcohol until an opaque precipitate is produced. Figure 2 shows the results of such an experiment. Inspection of the curves shows that low concentrations of $CaCl_2$ cause a decrease in the amount of alcohol needed for precipitation while higher concentrations cause an increase. NaCl causes only an increase. Mixtures of NaCl and $CaCl_2$ take intermediate positions. A comparison of these gelatine curves with the *Laminaria* curves in figure 1 shows a striking resemblance between the two.

In spite of the difference in the methods of obtaining the curves in protoplasm and in gelatine, the two are comparable on the assumption, that the effect of time in the *Laminaria* experiments is to increase the concentrations of the salts in the cells of the tissue. We might then suppose that variations in the concentration of the salts produced corresponding variations in the amount of a precipitate (e.g., at the plasma membrane) which offered a resistance to the passage of ions and so increased electrical resistance. This assumption would be rendered more probable if evidence were obtainable that the electrical conductivity of gelatine varies directly with the amount of alcohol required to produce precipitation. Some evidence on this point is already available.

Samec[5] working on a starch gel, measured the electrical conductivity and the alcohol precipitability simultaneously in lots of starch which had been heated for different lengths of time, and found that the two ran parallel to each other. Pauli[6] and his pupils state that the conductivity is a measure, other things being equal, of the number of electrically neutral particles present. Conductivity would therefore be least at an isoelectric point where, also, least alcohol is needed for precipitation.

There are of course quantitative discrepancies in this analogy, but one could hardly expect gelatine to be as sensitive to electrolytes as protoplasm. Qualitatively, however, the resemblance between the two curves is remarkably close, even in the following details.

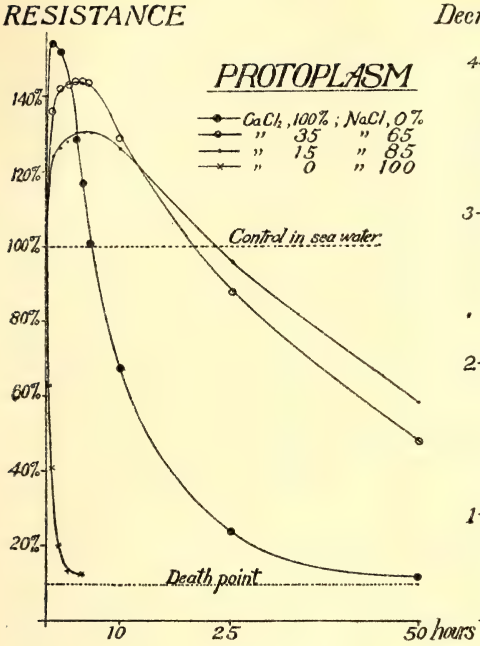


FIG. 1.

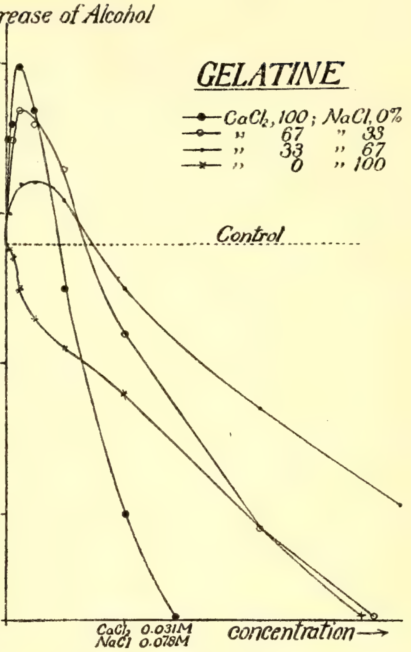


FIG. 2.

FIG. 1

Curves from Osterhout's experiments showing the changes in the electrical conductivity of protoplasm when discs of *Laminaria* are transfered from sea water to NaCl 0.52M or to $CaCl_2$ 0.27 8M or to mixtures of the two (all having the same conductivity as sea water.) Ordinates represent per cent of resistance (the resistance of the discs in sea water being taken as 100%). When the resistance has fallen to 10% of the control, the death point is reached. Here the protoplasm has the same conductivity as sea water.

FIG. 2

Curves showing how the effects of NaCl and $CaCl_2$ on the electrical resistance of protoplasm may be imitated by the effects of NaCl and $CaCl_2$ on the alcohol precipitability of gelatine (containing NaOH 0.005M). The ordinates represent the decrease (8 minus the number of cc.) of 95% alcohol required to produce an opaque precipitate, the abscissae salt concentrations. For comparison with *Laminaria* it is assumed that the salts diffuse into the gelatin (as into the protoplasm of *Laminaria*) $CaCl_2$ diffusing 80% as fast as NaCl. The molecular concentration of the NaCl solution is twice as great as that of the $CaCl_2$ solution (which is approximately the case in the *Laminaria* experiments). At the end of a given time, when the concentration of $CaCl_2$ in the gelatin is 0.031M, the concentration of NaCl will be $0.031 \times 2 \times (100 \div 80) = 0.078$M. This point is indicated in the figure. The concentrations in mixtures at this point can easily be calculated (e.g., in NaCl 50% + $CaCl_2$ 50% they would be half of those just mentioned).

It is therefore evident that we may regard the abscissae as representing time as well as concentration and that from this point of view the figure becomes comparable with figure 4, showing the changes in alcohol precipitability and in resistance which would be expected to occur with time if the salts were diffusing into the gelatin.

The last point on each curve is obtained by interpolation between the preceding point and a later one not shown in the diagram.

1. The maximum is higher in pure $CaCl_2$ than in mixtures of $CaCl_2$ and NaCl.

2. The maximum is reached slightly sooner (or at a slightly lower concentration) in pure $CaCl_2$ than in mixtures of $CaCl_2$ with NaCl.

3. There is a slight lag at the start of the NaCl curve in both *Laminaria* and gelatine. In the case of gelatine, this lag appears only if more NaOH is put into the gelatine at the start than in the experiment recorded in figure 5. With sufficient NaOH, the lag may be magnified into a distinct rise. This is in agreement with the fact, that NaCl will antagonize both anions and cations. The lag in the *Laminaria* experiments is seen in the fact that the velocity constant of the loss of resistance in NaCl is lowest at the start.[7]

The NaCl curve falls more slowly in figure 2 than in figure 1 but this may be paralleled by the results with *Rhodymenia* and other plants where the NaCl curve falls slowly.

4. The final increase in conductivity (or of alcohol) in the mixtures of NaCl and $CaCl_2$ is less rapid than in either of them taken separately. In general, the greater the percent of $CaCl_2$ in the mixture, the less rapid is the increase.

In the *Laminaria* experiments the curves approach a death point where the conductivity of the protoplasm is the same as that of sea water. No further increase of conductivity is possible. Of course this behavior would not be expected in gelatine.

In a recent article in *Science*, Spaeth[8] has presented a colloid-chemical theory of the 'vital equilibrium' of protoplasm as an explanation of Osterhout's *Laminaria* experiments. Spaeth's theory differs from that outlined above in the following fundamental points.

1. According to Spaeth the 'vital equilibrium' is between liquefying agents like NaCl and precipitating agents like $CaCl_2$. Reasoning from the gelatine analogy the equilibrium is simply between positive and negative ions.

2. Spaeth supposes that $CaCl_2$ and similar electrolytes increase the viscosity of the protoplasm by an *irreversible coagulation,* after which the viscosity becomes that of the dispersion medium, due to the complete separation of the two phases. According to the gelatine experiments, on the other hand, low concentrations of Ca cause a perfectly *reversible* precipitate, or increase of aggregation, of an isoelectric nature, which disappears gradually with excess of $CaCl_2$. Irreversible changes resulting in death appear only in much higher concentrations of $CaCl_2$.

It should be noted that the viscosity of a colloid (a conception which Spaeth uses extensively in his theory) is not a simple property of a system but is dependent upon many different factors which may sometimes be opposing.[9] Thus Pauli and his pupils find minimum viscosity at an isoelectric point in protein sols due to minimum ionization of the protein. In the writer's gelatine experiments, however, the gelation or gelation viscosity of the gel was distinctly at a maximum at the isoelectric point due to maximum aggregation. It appears necessary to distinguish between these two kinds of viscosity.

Summary.—A close analogy to Osterhout's experiments on the electrical resistance of *Laminaria* is found in gelatine (plus NaOH), if we assume that the effect of time in the *Laminaria* experiments is to increase the concentrations of the salts in the cells of the tissue.

---

[1] These PROCEEDINGS, **2**, 534 (1916).

[2] For the sake of brevity the word precipitate is used throughout to denote not only an actual precipitate, but any accompanying conditions which vary with the amount of precipitate or the degree of precipitability.

[3] Cf. Osterhout, *Science*, **41**, 255 (1915) for summary of results.

[4] Pauli has concluded for other reasons that protoplasm reacts much like protein soils containing alkali. *Biochem. Zs.*, **24**, 239 (1910).

[5] Samec, *Koll.-Chem. Beihefte*, **5**, 141 (1913).

[6] Pauli, loc. cit.

[7] Osterhout, *Science*, **39**, 544 (1914).

[8] Spaeth, *Science*, **43**, 502 (1916).

[9] Ostwald, *Kolloid. Zs.*, **12**, 213 (1913).

ON CERTAIN ASYMPTOTIC EXPRESSIONS IN THE THEORY OF LINEAR DIFFERENTIAL EQUATIONS

By W. E. Milne

DEPARTMENT OF MATHEMATICS, BOWDOIN COLLEGE

Received by the Academy, July 6, 1916

The nature of the solutions of a certain linear differential equation containing a complex parameter has been investigated by Prof. G. D. Birkhoff,[1] who discovered the asymptotic character of the solutions when the parameter is large in absolute value. These results he employed in the study of expansion problems connected with the particular differential equation

$$\frac{d^n u}{dx^n} + P_2(x)\frac{d^{n-2}u}{dx^{n-2}} + \quad . \; . \; . \quad + P_n(x)\,u + \rho^n u = 0, \tag{1}$$

together with n linearly independent linear boundary conditions

$$W_1(u) = 0, W_2(u) = 0, \quad . \; . \; . \; . \quad, W_n(u) = 0. \tag{2}$$

It is the aim of this paper to present asymptotic formulas for n linearly independent solutions $y_1, y_2, \; . \; . \; . \; , y_n$ of equation (1) which are in some respects more precise than those obtained by Birkhoff, and also to present similar asymptotic formulas for the n functions $\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_n$, related to the n y's by the n identities

$$\sum_{i=1}^{n} y_i^{(k)} \bar{y}_i = \begin{cases} 0, & k = 0, \quad . \quad . \quad . \quad , n-2, \\ 1, & k = n-1. \end{cases} \tag{3}$$

These functions $\bar{y}$ play an important rôle in the theory of linear differential equations. As is well known they form a system of linearly independent solutions of the equation adjoint to equation (1), while the expression

$$\sum_{i=1}^{n} y_i(x)\, \bar{y}_i(t)$$

is of fundamental importance in Lagrange's method of solving the non-homogeneous equation of which (1) is the reduced equation, as well as in the formation of the Green's function of the system (1) and (2). Asymptotic forms for the $\bar{y}$'s were also used by Birkhoff in his paper on expansion problems.[2]

I was led to make refinements in the forms of the y's and the $\bar{y}$'s in connection with a paper treating the degree of convergence of the expansion associated with the differential system (1) and (2).

Birkhoff divided the plane of the complex parameter ρ into $2n$ equal sectors,

$$S_k : k\pi/n \leqq \arg \rho \leqq (k+1)\pi/n, \qquad k = 0, 1, \quad . \quad . \quad . \quad , 2n-1,$$

and then numbered the n n-th roots of -1, $w_1, w_2, \ldots, w_n$, in such a manner that when ρ is on any given sector S_k, the inequalities

$$R(\rho w_1) \leqq R(\rho w_2) \leqq \quad . \quad . \quad . \quad \leqq R(\rho w_n)$$

are satisfied, where $R(\rho w_i)$ denotes the real part of ρw_i. He then proved that if the coefficients $P_s(x)$ in (1) have continuous derivatives of all orders in the closed interval $a \leqq x \leqq b$, there exist for ρ in any given S_k n independent solutions of (1),

$$\begin{aligned} y_i &= u_i(x, \rho) + e^{\rho w_i (x-c)} E_{i0}/\rho^{m+1}, \\ y_i^{(k)} &= u_i^{(k)}(x, \rho) + e^{\rho w_i (x-c)} E_{ik}/\rho^{m+1-k}, \\ i = 1, 2, \quad . \quad . \quad . \quad &, n; \; k = 1, 2, \quad . \quad . \quad . \quad , n-1, \end{aligned} \tag{4}$$

in which

$$u_i(x, \rho) = e^{\rho w_i (x-c)} \left[1 + \frac{u_{i1}(x)}{\rho} + \quad . \quad . \quad . \quad + \frac{u_{i\,m+1}(x)}{\rho^{m+1}} \right],$$

where m is any positive integer or zero.[3] The E_{ij} are functions of x and ρ which are bounded for x in (a, b) and for ρ in S_k and large in absolute value. The y_i are analytic in ρ in S_k, and the $u_{ij}(x)$ have derivatives of all orders with respect to x in (a, b).

The modification here proposed is this: *If the coefficients $P_s(x)$ have continuous derivatives of order $(m + n - s)$ in (a, b), m being any positive integer or zero, then there exist n solutions of (1) of the form (4), analytic in ρ in the sector S_k, and the functions $u_i(x,\rho)$ are of the form*

$$u_i(x, \rho) = e^{\rho w_i (x-c)}[1 + \varphi_1(x)/\rho w_i + \dots + \varphi_m(x)/(\rho w_i)^m],$$

where the functions $\varphi_j(x)$ have continuous derivatives of order $(m + n - j)$, and are independent of i.

The improvement in precision over Birkhoff's formulas consists primarily in putting the $u_i(x,\rho)$ in the form indicated, where the $\varphi_j(x)$ are independent of i; the details concerning the number of derivatives of the P's and the φ's are of secondary importance. A similar remark applies to the statement concerning the $\bar{y}$'s, which is as follows:

The n functions $\bar{y}_i$, determined by the n equations (3), have when $|\rho|$ is large the asymptotic form

$$\bar{y}_i = \frac{e^{-\rho w_i (x-c)}}{n(\rho w_i)^{n-1}}[v_i(x, \rho) + \bar{E}_i/\rho^{m+1}], \quad i = 1, 2, \dots, n,$$

where

$$v_i(x, \rho) = 1 + \psi_1(x)/\rho w_i + \dots + \psi_m(x)/(\rho w_i)^m,$$

in which the functions $\psi_j(x)$ are independent of i and have continuous $(m + n - j)$-th derivatives in (a, b).

The proof of the asymptotic formulas for the y's is simply an adaptation of the proof given by Birkhoff. In the case of the $\bar{y}$'s, we verify the formulas by substituting the values of the $\bar{y}_i$ given above into equations (3) and showing that the ψ's and E's with the desired properties can be chosen to satisfy them.

---

[1] *Trans. Amer. Math. Soc.*, 9, 219–231, 373–395 (1908).

[2] Loc. cit., p. 391, formula (56).

[3] For the formulas here quoted see loc. cit., pp. 381–2, formulas (21) and (23). They are quoted in (4) with some slight changes

ON NEWTON'S METHOD OF APPROXIMATION

By Henry B. Fine

DEPARTMENT OF MATHEMATICS, PRINCETON UNIVERSITY

Received by the Academy, August 8, 1916

In the first of the following theorems[1] a condition is given under which Newton's method of approximation for computing a real root of an equation $f(x) = 0$ and the extension of this method used in computing a real solution of a system of equations $f_i(x_1, x_2, \ldots, x_n) = 0$, $(i = 1, 2, \ldots, n)$, will with certainty lead to such a root or solution. The condition relates to the absolute value of $f(x)$ or of the functions $f_i(x_1, x_2, \ldots, x_n)$ corresponding to the initial values of x and of the variables $x_1, x_2, \ldots, x_n$ respectively. No assumption is made as to the existence of a solution. On the contrary it is proved that under the condition to which reference has been made one and but one solution exists in a certain designated neighborhood of the initial x or $(x_1, x_2, \ldots, x_n)$. The second theorem is the extension of the first (for $n = 1$) to the case of a complex root of an analytic equation $f(z) = 0$.

Theorem 1. *Let $f_i(x_1, x_2, \ldots, x_n)$, $(i = 1, 2, \ldots, n)$, be a system of real functions of the real variables $x_1, x_2, \ldots, x_n$ which have continuous first and second derivatives in the region R, $(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)})$ a set of values of $x_1, x_2, \ldots, x_n$ belonging to this region, and $\xi_1, \xi_2, \ldots, \xi_n$ the set of numbers determined by the equations.*

$$f_i\,(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}) + \sum_{k=1}^{n} \frac{\partial f_i}{\partial x_k^{(0)}}\,\xi_k = 0 \qquad (i = 1, 2, \ldots, n)$$

and let S denote the interval, circle, sphere, or hypersphere whose center is $(x_1^{(0)} + \xi_1, \ldots, x_n^{(0)} + \xi_n)$ and whose radius is $\rho = \left(\sum_{k=1}^{n} \xi_k^2\right)^{\frac{1}{2}}$, S_ρ being supposed to belong to R.

Suppose also that in S the functional determinant F of the functions f_i does not vanish, $\mu(<\infty)$ is the upper bound of the absolute values of the fractions whose denominators are F and whose numerators are the several first minors of F, and $\nu(<\infty)$ is the upper bound of the absolute values of the second derivatives of the functions f_i.

Then, if

$$\left[\sum_{i=1}^{n} \left\{ f_i\,(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}) \right\}^2\right]^{\frac{1}{2}} < \frac{1}{n^{7/2}\,\mu^2\,\nu} \qquad (a)$$

the equations $f_i = 0$ *have one and but one solution in* S, *and an approximate value of this solution, as close as may be desired, will be obtained by determinations first of* $\xi_1^{(j)}, \xi_2^{(j)}, \ldots, \xi_n^{(j)}$ *and then of* $x_1^{(j+1)}, x_2^{(j+1)}, \ldots, x_n^{(j+1)}$, *for* $j = 0, 1, 2 \ldots$ *successively, by the equations*

$$\begin{aligned} f_i\,(x_1^{(j)}, x_2^{(j)}, \ldots, x_n^{(j)}) + \sum_{k=1}^{n} \frac{\partial f_i}{\partial x_k^{(j)}}\, \xi_k^{(j)} &= 0, \qquad (i = 1, 2, \ldots, n) \\ x_k^{(j+1)} = x_k^{(j)} + \xi_k^{(j)}&, \qquad (k = 1, 2, \ldots, n), \end{aligned} \qquad (b)$$

(*where* $\xi_k^{(0)} = \xi_k$), *the solution being* $\lim_{j=\infty} (x_1^{(j)}, x_2^{(j)}, \ldots, x_n^{(j)})$.

Moreover the equations (*b*) *will yield a similar sequence of approximations to this solution if instead of* $(x_1^{(0)}, x_2^{(0)}, \ldots x_n^{(0)})$ *any other point in* S *be chosen as the point of departure.*

For the case of a single equation $f(x) = 0$ *the condition* (*a*) *reduces to*

$$|\,f(x_0)\,| < \frac{\lambda^2}{\nu}, \qquad (a')$$

where λ (>0) *is the lower bound in the interval* S *of the values of* $|\,f'(x)\,|$.

For let

$$f_i^{(j)} = f_i\,(x_1^{(j)}, x_2^{(j)}, \ldots, x_n^{(j)}),\ \varphi_j = \left\{ \sum_{i=1}^{n} [f_i^{(j)}]^2 \right\}^{\frac{1}{2}},\ \rho_j = \left\{ \sum_{k=1}^{n} [\xi_k^{(j)}]^2 \right\}^{\frac{1}{2}}.$$

Then, if both $(x_1^{(j)}, x_2^{(j)}, \ldots, x_n^{(j)})$ and $(x_1^{(j+1)}, x_2^{(j+1)}, \ldots, x_n^{(j+1)})$ are in S,

$$f_i^{(j+1)} = f_i^{(j)} + \sum_{k=1}^{n} \frac{\partial f_i}{\partial x_k^{(j)}}\, \xi_k^{(j)} + \tfrac{1}{2} \sum_{k,l=1}^{n} \frac{\partial^2 f_i}{\partial x_k^{(j_i)}\, \partial x_l^{(j_i)}}\, \xi_k^{(j)}\, \xi_l^{(j)}, \quad (i = 1, 2, \ldots, n), \quad (1)$$

where each $x_k^{(j_i)}$ lies between $x_k^{(j)}$ and $x_k^{(j+1)}$.

Hence if the numbers $\xi_k^{(j)}$ be so taken as to satisfy the equations

$$f_i^{(j)} + \sum_{k=1}^{n} \frac{\partial f_i}{\partial x_k^{(j)}}\, \xi_k^{(j)} = 0, \qquad (i = 1, 2, \ldots, n), \qquad (2)$$

we shall have

$$f_i^{(j+1)} = \tfrac{1}{2} \sum_{k,l=1}^{n} \frac{\partial^2 f_i}{\partial x_k^{(j_i)}\, \partial x_l^{(j_i)}}\, \xi_k^{(j)}\, \xi_l^{(j)}, \qquad (i = 1, 2, \ldots, n), \qquad (3)$$

Solving the equations (2), we find

$$\xi_k^{(j)} = -\frac{\sum_{i=1}^{n} A_{ik}^{(j)} f_i^{(j)}}{F_j}, \qquad (k = 1, 2, \ldots, n), \qquad (4)$$

where F_j is the value of F for $(x_1, x_2, \ldots, x_n) = (x_1^{(j)}, x_2^{(j)}, \ldots x_n^{(j)})$, and $A_{ik}^{(j)}$ the cofactor of the element in the ith row and the kth column of F_j. But

$$\left| \sum_{i=1}^{n} A_{ik}^{(j)} f_i^{(j)} \right| \leqq \left\{ \sum_{i=1}^{n} [A_{ik}^{(j)}]^2 \right\}^{\frac{1}{2}} \left\{ \sum_{i=1}^{n} [f_i^{(j)}]^2 \right\}^{\frac{1}{2}}$$

Hence

$$\rho_j^2 \leqq \frac{\overset{n}{\underset{i,k=1}{[A_{ik}^{(j)}]^2}}}{F_j^2} \sum_{i=1}^{n} [f_i^{(j)}]^2 \leqq n^2 \mu^2 \phi_j^2$$

and therefore

$$\rho_j \leqq n\, \mu\, \phi_j. \tag{5}$$

Again, the expressions under the summation sign in the equations (3) are of the form

$$\sum_{k,l=1}^{n} a_{kl}\, \xi_k\, \xi_l$$

and

$$\left| \sum_{k,l=1}^{n} a_{kl} \xi_k \xi_l \right| = \left| \sum_{k=1}^{n} \left(\sum_{l=1}^{n} a_{kl}\, \xi_l \right) \xi_k \right| \leqq \left\{ \sum_{k=1}^{n} \left(\sum_{l=1}^{n} a_{kl}\, \xi_l \right)^2 \right\}^{\frac{1}{2}} \left\{ \sum_{k=1}^{n} \xi_k^2 \right\}^{\frac{1}{2}}$$

$$\leqq \left(\sum_{k,l=1}^{n} a_{kl}^2 \right)^{\frac{1}{2}} \sum_{k=1}^{n} \xi_k^2 \leqq na \sum_{k=1}^{n} \xi_k^2, \tag{6}$$

where a denotes the greatest of the numbers $|a_{ik}|$.

Hence

$$\varphi_{j+1}^2 \leqq n \left(\frac{n\nu}{2} \right)^2 \left\{ \sum_{k=1}^{n} [\xi_k^{(j)}]^2 \right\}^{\frac{1}{2}} \leqq \frac{n^3 \nu^2}{2^2} \rho_j^4$$

and therefore

$$\varphi_{j+1} \leqq \frac{n^{3/2} \nu}{2} \rho_j^2. \tag{7}$$

From (5) and (7) it follows that

$$\varphi_{j+1} \leqq k_1 \phi_j^2, \text{ where } k_1 = \frac{n^{7/2} \mu^2 \nu}{2}, \tag{8}$$

$$\rho_{j+1} \leqq k_2\, \rho_j^2, \text{ where } k_2 = \frac{k_1}{n\mu} = \frac{n^{5/2} \mu \nu}{2} \tag{9}$$

Hence $k_1 \varphi_{j+1} \leqq (k_1 \varphi_j)^2$ and therefore, by setting $j = 0, 1, 2, \ldots$ successively,

$$k_1 \varphi_j \leqq (k_1 \phi_0)^{2^j}. \tag{10}$$

Similarly

$$k_2\,\rho_j \leqq (k_2\,\rho)^{2^j}, \tag{11}$$

Let r denote any positive number less than 1, and suppose that $k_1\,\varphi_0 < r$. Then since, by (5), $\rho \leqq n\,\mu\,\phi_0$, we also have $k_2\,\rho < r$, and therefore, by (8) and (9),

$$k_2\,\varphi_j < r^{2^j}, \qquad k_2\,\rho_j\, r^{2^j}. \tag{12}$$

Again $\rho_{j+1} \leqq k_2 \rho_j\,\rho_j$ and therefore

$$\rho_{j+1} < r^{2^j}\,\rho_j. \tag{13}$$

Also, $k_2\,\rho_j \leqq k_2\,\rho\,(k_2\,\rho)^{2^j-1}$ and therefore

$$\rho_j < r^{2^j-1}\,\rho. \tag{14}$$

Therefore, since $r < 1$, the sum $\rho_1 + \rho_2 \ . \ . \ . \ \rho_j$ increases with j to a limit which is less than $\rho r\,/\,(1 - r^2)$.

But $\rho r\,/\,(1 - r^2) < \rho$ when $r \leqq 1/2$.

Therefore when $r = 1/2$, that is when the condition $k_1\,\varphi_0 < r$ becomes

$$\varphi_0 < \frac{1}{2k_1} = \frac{1}{n^{7/2}\mu^2\nu}, \tag{15}$$

the point whose coordinates are $x_k^{(j)} = (x_k^{(0)} + \xi_k^{(0)}) + \xi_k^{(1)} + \ . \ . \ . + \xi_k^{(j-1)}$, $(k = 1, 2, \ . \ . \ . \ , n)$, will remain in S as j increases and will approach a definite limiting position $(c_1, c_2, \ . \ . \ . \ , c_n)$ in S as $j = \infty$. Moreover by (12), $\lim_{j=\infty} \varphi_j = 0$, and therefore, since the functions f_i are continuous, $f_i\,(c_1, c_2, \ . \ . \ . \ c_n) = 0$, $(i = 1, 2, \ . \ . \ . \ n)$. Therefore the equations $f_i = 0$ have the solution $(c_1, c_2, \ . \ . \ . \ c_n)$ in S.

Observe that it follows from (5) that when (15) is satisfied,

$$\rho < \frac{1}{2k_2} = \frac{1}{n^{5/2}\mu\,\nu} \tag{16}$$

The equations $f_i = 0$ have no other solution than $(c_1, c_2, \ . \ . \ . \ c_n)$ in S. For suppose that $(c_1 + h_1, \ . \ . \ . \ , c_n + h_n)$ represents another such solution. Then by developing as in (1) and applying (6)

$$0 = \sum_{k=1}^{n} \frac{\partial f_i}{\partial c_k} h_k + \frac{\theta_i}{2} n\nu \sum_{k=1}^{n} h_k^2, \quad (i = 1, 2, \ . \ . \ . \ , n), \tag{17}$$

where $|\,\theta_i\,| \leqq 1$.

Solving these equations for the numbers h_k in terms of $\sum_{k=1}^{n} h_k^2$, we obtain, by the method used in deriving (5) from (2),

$$\left[\sum_{k=1}^{n} h_k^2\right]^{\frac{1}{2}} \leqq \frac{n^{5/2}\mu\nu}{2} \sum_{k=1}^{n} h_k^2,$$

and therefore, if $\mathop{}_{k=1}^{n} h_k^2 \neq 0$,

$$\left[\sum_{k=1}^{n} h_k^2\right]^{\frac{1}{2}} \geqq \frac{2}{n^{5/2}\mu\nu} > 2\rho. \tag{18}$$

Hence $(c_1 + h_1, \ldots, c_n + h_n)$ lies outside of S. Therefore the existence of a second solution in S is impossible.

The equations (b) will also yield a sequence of approximations to $(c_1, c_2, \ldots, c_n)$ similar to the sequence $(x_1^{(j)}, x_2^{(j)}, \ldots x_n^{(j)})$ if instead of $(x_1^{(0)}, x_2^{(0)}, \ldots x_n^{(0)})$ any other point in S be taken as the point of departure. For let $(a_1^{(0)}, a_2^{(0)}, \ldots, a_n^{(0)})$ be any point in S, and $\xi_k^{(j)}$, $a_k^{(j+1)}$, $d_k^{(j)}$, $(k = 1, 2, \ldots, n)$, the numbers determined by the equations

$$f_i(a_1^{(j)}, a_2^{(j)}, \ldots, a_k^{(j)}) + \sum_{k=1}^{n} \frac{\partial f_i}{\partial a^{(j)}} \xi_k^{(j)} = 0, \quad (i = 1, 2, \ldots, n),$$

$$a_k^{(j+1)} = a_k^{(j)} + \xi_k^{(j)}, \quad d_k^{(j)} = c_k - a_k^{(j)}, \qquad (k = 1, 2, \ldots, n),$$

for $j = 0, 1, 2, \ldots$ successively.

Then since

$$c_k = a_k^{(j)} + d_k^{(j)} = a_k^{(j)} + (\xi_k^{(j)} + d_k^{(j+1)}), \quad (k = 1, 2, \ldots, n),$$

we shall have, on developing as in (1) and taking into account the equations which determine the numbers $\xi_k^{(j)}$,

$$0 = \sum_{k=1}^{n} \frac{\partial f_i}{\partial a_k^{(j)}} d_k^{(j+1)} + \tfrac{1}{2} \sum_{k,\,l=1}^{n} \frac{\partial^2 f_i}{\partial x_k^{(ji)} \partial x_l^{(ji)}} d_k^{(j)} d_l^{(j)}, \ (i = 1, 2, \ldots, n), \tag{19}$$

where each $x_k^{(ji)}$ lies between $a_k^{(j)}$ and c_k, $(k = 1, 2, \ldots n)$.

From (19) it follows, as in the proof of (5), that

$$\delta_{j+1} \leqq k_2\, \delta_j^2, \tag{20}$$

where $\delta_j = \left\{\sum_{k=1}^{n} [d_k^{(j)}]^2\right\}^{\frac{1}{2}}$ and, as above, $k_2 = n^{5/2}\mu\nu/2$.

From (20) in turn it follows, as in (10), that

$$k_2\, \delta_j \leqq (k_2\, \delta_0)^{2^j}; \tag{21}$$

therefore, if $k_2\, \delta_0 < 1$, that $\lim_{j=\infty} \delta_j = 0$ and $\lim_{j=\infty} (a_1^{(j)}, a_2^{(j)}, \ldots, a_n^{(j)}) = (c_1, c_2, \ldots, c_n)$.

But δ_0 is the distance of the point $(a_1^{(0)}, a_2^{(0)}, \ldots, a_n^{(0)})$ from the point $(c_1, c_2, \ldots, c_n)$, and $k_2\delta_0 < 1$ if $\delta_0 < 1/k_2$; therefore, by (16), if

$\delta_0 < 2\rho$. And the condition $\delta_0 < 2\rho$ is satisfied if $(a_1^{(0)}, a_2^{(0)}, \ldots, a_n^{(0)})$ be taken anywhere in S.

Again since $k_2\,\delta_0 = r < 1$ we have, as in (12), $\delta_{j+1} \leqq (k_2\,\delta_j)\,\delta_j \leqq r^{2^j}\,\delta_j < \delta_j$. Hence each approximation in the sequence $(a_1^{(j)}, a_2^{(j)}, \ldots, a_n^{(j)})$, $(j = 0, 1, 2, \ldots)$, is closer than the one which precedes it.

Finally, by (21), $\delta_j \leqq r^{2^j}/k_2$. Let j' be any value of j such that $r^{2^{j'}}/k_2 < 0.1$ and $r^{2^{j'}} < 0.1$ and set $j = j' + l$ $(l = 0, 1, 2, \ldots)$. We then have

$$\delta_{j'+l} \leqq \frac{r^{2^{j'+l}}}{k_2} = \frac{r^{2^{j'}}}{k_2}\left(r^{2^{j'}}\right)^{2^l-1} < (0.1)^{2^l}.$$

Hence $\left\{\sum_{k=1}^{n} [a_k^{j'+l}]^2\right\}^{\frac{1}{2}}$ will coincide with $\left\{\sum_{k=1}^{n} c_k^2\right\}^{\frac{1}{2}}$ and therefore $a_k^{j'+l}$ with c_k $(k = 1, 2, \ldots, n)$, to at least the 2^lth decimal figure.

Theorem 2. *Let* $f(z)$ *be any function of the complex variable* z *which is analytic at* $z = z_0$, *and* h *the number determined by the equation*

$$f(z_0) + f'(z_0)\,h = 0.$$

Let S *be the circular region whose center is* $z_0 + h$ *and whose radius is* $\rho = |h|$, *and suppose that, in* S, $f(z)$ *is everywhere analytic,* $\lambda (> 0)$ *is the lower bound of the values of* $|f'(z)|$ *and* $\nu (< \infty)$ *is the upper bound of the values of* $|f''(z)|$. *Then, if*

$$|f(z_0)| < \frac{\lambda^2}{\sqrt{2}\,\nu}, \qquad (a'')$$

the equation $f(z) = 0$ *has one and but one root in* S, *and this root will be approximated to uninterruptedly by successive determinations of* h_j *and* z_{j+1} *by the formulas*

$$f(z_j) + f'(z_j)\,h_j = 0, \quad z_{j+1} = z_j + h_j, \quad (j = 0, 1, 2, \ldots), \qquad (b')$$

(where $h_0 = h$*), the root being* $\lim_{j=\infty} z_j$.

For let

$$z_j = x + iy, \quad f(z_j) = \varphi(x, y) + i\,f(x, y), \quad h_j = \xi + i\eta.$$

Then if z_j and z_{j+1} are in S, we shall have

$$f(z_j + h_j) = f(z_j) + f'(z_j)\,h_j + \tfrac{1}{2}\left(\frac{\partial^2\varphi}{\partial x'^2}\xi^2 + 2\frac{\partial^2\varphi}{\partial x'\partial y'}\xi\eta + \frac{\partial^2\varphi}{\partial y'^2}\eta^2\right)$$
$$+ \frac{i}{2}\left(\frac{\partial^2\psi}{\partial x''^2}\xi^2 + 2\frac{\partial^2\psi}{\partial x''\partial y''}\xi\eta + \frac{\partial^2\psi}{\partial y''^2}\eta^2\right) \qquad (22)$$

where x', x'' lie between x and $x + \xi$; y', y'' between y and $y + \eta$. But

$$\left|\frac{\partial^2\varphi}{\partial x'^2}\xi^2+2\frac{\partial^2\varphi}{\partial x'\partial y'}\xi\eta+\frac{\partial^2\varphi}{\partial y'^2}\eta^2\right|=\left|\frac{\partial^2\varphi}{\partial x'^2}(\xi^2-\eta^2)-2\frac{\partial^2\psi}{\partial x'^2}\xi\eta\right|$$

$$\leqq\left\{\left[\frac{\partial^2\varphi}{\partial x'^2}\right]^2+\left[\frac{\partial^2\psi}{\partial x'^2}\right]^2\right\}^{\frac{1}{2}}\left\{(\xi^2-\eta^2)^2+4\xi^2\eta^2\right\}^{\frac{1}{2}}\leqq\nu\,|h_j|^2.$$

Similarly

$$\left|\frac{\partial^2\psi}{\partial x''^2}\xi^2+2\frac{\partial^2\psi}{\partial x''\partial y''}\xi\eta+\frac{\partial^2\psi}{\partial y''^2}\eta^2\right|\leqq\nu\,|h_j|^2.$$

Therefore

$$f(z_j+h_j)=f(z_j)+f'(z_j)\,h_j+\frac{\theta_1+i\theta_2}{2}\nu\,|h_j|^2,\quad -1<\theta_1,\theta_2<1,\quad (23)$$

Hence if h_j be so taken as to satisfy the equation $f(z_j)+f'(z_j)\,h_j=0$ and if, as in the proof of Theorem 1, we set $|h_j|=\rho_j$, $|f(z_j)|=\phi_j$, we shall have

$$\rho_j\leqq\frac{1}{\lambda}\varphi_j,\qquad \varphi_{j+1}\leqq\frac{\sqrt{2}\,\nu}{2}\rho_j^2.$$

But these are the same as the inequalities (5) and (7) in the proof of Theorem 1 for the case $n=1$ except that ν is here replaced by $\sqrt{2}\nu$. It therefore at once follows from that proof, and in particular from the inequalities (15) and (18), that if $\varphi_0<\lambda^2/\sqrt{2}\nu$ and therefore $\rho=\rho_0<\lambda/\sqrt{2}\nu$, the equation $f(z)=0$ has one and but one root is S; also that the equations (b)′ will yield a sequence of approximations to the root if instead of z_0 any other point in S be taken as the point of departure.[2]

[1] This paper was read before the American Mathematical Society, April 29, 1916.

[2] The distinctive feature of the method used in this paper is the determination of a number C such that if $\varphi_0<C$, then $\lim_{j=\infty}\varphi_j=0$. In this respect it differs from other discussions of Newton's method which I have been able to find. Of such discussions the following should be mentioned. Cauchy (Oeuvres [Ser. 2, Vol. 4], p. 573) obtained $\rho</\lambda 2\nu$ as the condition for the existence in S of a real root of a real equation obtainable by Newton's method, and $\rho<\lambda/4\nu$ as the corresponding condition for a complex root. Quite recently G. Faber (*J. Math., Berlin*, **138**) has proved, for an analytic equation $f(z)=0$, that if $|f(z)f''(z)/[f'(z)]^2|<\alpha<1$ in the circle whose center is z_0 and radius $\rho/(1-\alpha)$, a root of $f(z)=0$, obtainable by Newton's method, exists in this circle. I have been unable to discover any previous proofs of the existence of a solution for the case $n>1$. But Runge (*Encyc. Math. Wiss.*, **1**, p. 446) and E. Blutel (*Paris, Acad. Sci., C. R.* **151**, 1109) have given proofs that if a solution exists at a certain point C, there must also exist a region R about C from any point of which a steady approach to C will be made by Newton's method Blutel's method of proof (an extension of one employed by Cauchy) being that which I have used in the proof of the corresponding part of Theorem 1.

CONTENTS

Page

PHYSIOLOGY.—THE MECHANISM OF DIFFUSION OF ELECTROLYTES THROUGH ANIMAL MEMBRANES . . . . *By Jacques Loeb* 511

ASTRONOMY.—THE ROTATION AND RADIAL VELOCITY OF THE SPIRAL NEBULA N. G. C. 4594 . . . . *By Francis G. Pease* 517

ASTRONOMY.—A SIMPLE METHOD FOR DETERMINING THE COLORS OF THE STARS . . *By Frederick H. Seares* 521

ASTRONOMY.—STUDIES OF MAGNITUDES IN STAR CLUSTERS. III. THE COLORS OF THE BRIGHTER STARS IN FOUR GLOBULAR SYSTEMS . . . *By Harlow Shapley* 525

PHYSICS.—THE EFFECT OF AN ELECTRIC FIELD ON THE LINES OF LITHIUM AND CALCIUM . . . . *By Janet T. Howell* 528

MATHEMATICS.—A PROOF OF WHITE'S PORISM . . . . *By A. B. Coble* 530

PETROLOGY.—A CONTRIBUTION TO THE PETROGRAPHY OF THE PHILIPPINE ISLANDS . *By J. P. Iddings and E. W. Morley* 531

PHYSIOLOGY.—SALT ANTAGONISM IN GELATINE . . . . *By W. O. Fenn* 534

PHYSIOLOGY.—SIMILARITY IN THE BEHAVIOR OF PROTOPLASM AND GELATIN . . . *By W. O. Fenn* 539

MATHEMATICS.—ON CERTAIN ASYMPTOTIC EXPRESSIONS IN THE THEORY OF LINEAR DIFFERENTIAL EQUATIONS . . . . *By W. E. Milne* 543

MATHEMATICS.—ON NEWTON'S METHOD OF APPROXIMATION . *By Henry B. Fine* 546

VOLUME 2 OCTOBER, 1916 NUMBER 10

PROCEEDINGS

OF THE

NATIONAL ACADEMY OF SCIENCES

OF THE

UNITED STATES OF AMERICA

EDITORIAL BOARD

Arthur A. Noyes, *Chairman*
Arthur L. Day, *Home Secretary*
Edwin B. Wilson, *Managing Editor*
George E. Hale, *Foreign Secretary*

J. J. Abel
W. B. Cannon
J. McK. Cattell
J. M. Clarke
E. G. Conklin
J. M. Coulter
C. B. Davenport
Simon Flexner
E. B. Frost
R. G. Harrison
W. H. Holmes
J. P. Iddings
R. A. Millikan
E. H. Moore
Alexander Smith

Publication Office: Williams & Wilkins Company, Baltimore
Editorial Office: Massachusetts Institute of Technology, Cambridge
Home Office of the Academy: Washington, D. C.

Entered as second-class matter at the Post Office at Baltimore, Md., under the Act of August 24, 1912

INFORMATION TO CONTRIBUTORS

THE PROCEEDINGS is the official organ of the Academy for the publication of brief accounts of important current researches of members of the Academy and of other American investigators, and for reports on the meetings and other activities of the Academy. Publication in the Proceedings will supplement, not replace, that in journals devoted to the special branches of science. The Proceedings will aim especially to secure prompt publication of original announcements of discoveries and wide circulation of the results of American research among investigators in other countries and in all branches of science.

ARTICLES should be brief, not to exceed 2500 words or 6 printed pages. Technical details of the work and long tables of data should be reserved for publication in special journals. But authors should be precise in making clear the new results and should give some record of the methods and data upon which they are based. The viewpoint should be comprehensive in giving the relation of the paper to previous publications of the author or of others and in exhibiting where practicable, the significance of the work for other branches of science.

MANUSCRIPTS should be prepared with a current number of the Proceedings as a model in matters of form, and should be typewritten in duplicate with double spacing, the author retaining one copy. Illustrations should be confined to text-figures of simple character, though more elaborate illustrations may be allowed in special instances to authors willing to pay for their preparation and insertion. Particular attention should be given to arranging tabular matter in a simple and concise manner.

REFERENCES to persons should include initials; citations of literature, numbered consecutively, should be placed at the end of the article; and short footnotes should be avoided. Indicate 'subject' for 'Contents.'

PAPERS by members of the Academy may be sent to Edwin Bidwell Wilson, Managing Editor, Mass. Institute of Technology, Cambridge, Mass. Papers by non-members should be submitted through some member.

PROOF will be sent to authors only on their request, in order to avoid delay in publication. Authors are therefore requested to make final revisions on the typewritten manuscripts. The editors cannot undertake to do more than correct obvious minor errors.

REPRINTS should be ordered at the time of submission of manuscript. They will be furnished to authors at cost, approximately as follows:

Reprints of - -	2 pp.	4 pp.	6 pp.	8 pp.	Covers extra
Charge for first 100 copies	$1.00	$1.30	$2.20	$2.20	$2.00
Charge for additional 100s	.25	.45	.85	.85	.50

INFORMATION TO SUBSCRIBERS

SUBSCRIPTIONS at the rate of $5.00 per annum should be made payable to the National Academy of Sciences, and sent to Williams & Wilkins Company, Baltimore, or Arthur L. Day, Home Secretary, National Academy of Sciences, Smithsonian Institution, Washington, D. C. Single numbers, $0.50.

Copyright, 1916, by the National Academy of Sciences

Smithsonian Institution
JUN 24 1921
National Museum

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES

Volume 2 OCTOBER 15, 1916 Number 10

PRELIMINARY RESULTS ON THE COLOR OF NEBULAE

By Frederick H. Seares

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, August 4, 1916

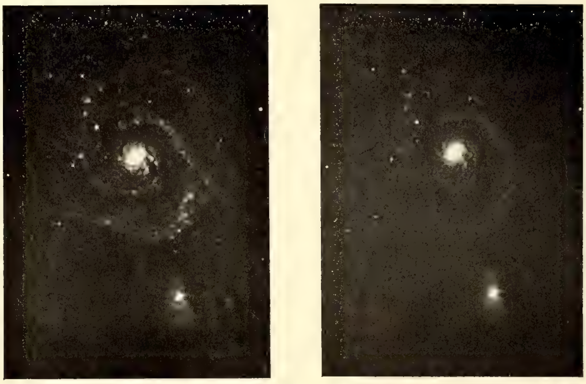
Important differences in the spectra of gaseous nebulae, even those of the same general class, have been known for many years. Although the characteristic nebular lines λ 5007 and λ 4959 seem to maintain a constant ratio of brightness, their behavior varies greatly with respect to $H\beta$, λ 4686, λ 3727, and other lines. Thus $H\beta$ is usually fainter than the chief nebular line λ 5007, but there is a wide range in the relative intensities, and intensity reversals may even occur. Again, lines usually present are as yet unobserved in certain objects, and in some we have apparently only the monochromatic radiation λ 3727.[1]

This diversity is reminiscent of the differences in the spectra of stars. These are universally accepted as evidence of evolutionary change, and if nebulae have any part in the scheme of development, it is likely that their spectral peculiarities will find a similar interpretation. In this event spectral correlations must exist, more or less clearly defined according to the development of the nebulae compared. The important investigations of Wright have already revealed relationships presumably of this kind.

Further, experience in other fields suggests the probability of spectrum differences within the same nebula. No mixture of luminous vapors is homogeneous in its radiation, for the pressure, temperature, electrical conditions, etc., are never constant throughout. Thus the spectrum changes from point to point in the flame and the electric arc; the radiation from the attenuated gases of a vacuum tube varies with the region examined; and, on a larger scale, the spectrum of the corona, the chromosphere, and the reversing layer—or, even better, spectroheliograms for hydrogen and calcium—reveal the selective distribution of vapors within the solar atmosphere.

Lack of homogeneity has already been observed in numerous nebulae; but probably the phenomenon is more nearly universal than has hitherto been supposed, and one naturally inquires whether internal differences may not provide useful criteria of development supplementing those based on spectral variations from nebula to nebula. In fact Wright has already called attention, in the case of planetary nebulae, to the gradually increasing concentration of λ 4686 within the nucleus as we approach the Wolf-Rayet stars in his graded series of spectra.[2] He has made this circumstance a basis for classification, and it seems likely that such criteria will be even more used in the future.

Various methods for investigating the constitution of nebulae are available. Measures with a spectral photometer should reveal differences in the distribution of the gases; but few objects are bright enough for an application of the method, which at best is tedious and uncertain. Monochromatic photographs for each spectrum line can be obtained with a slitless spectrograph or an objective prism; but if the nebula is large, or the dispersion insufficient, the images overlap. The method is useful for planetary nebulae, which are small and relatively bright. For these the ordinary slit spectrograph gives valuable results, the distribution of luminosity being indicated by the length of the lines. That localized in the nucleus produces short lines, while longer lines originate in gases more widely distributed. The method has been used effectively by Campbell, Wright, and Wolf.

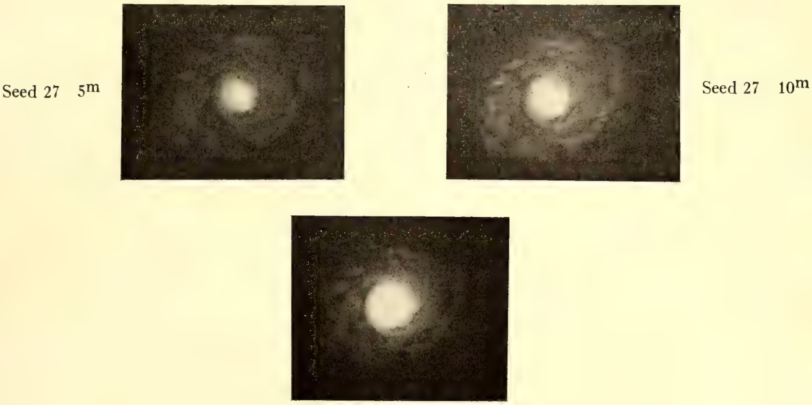


Seed 27 10^m Iso 1^h

FIG. 1. MESSIER 51.

For a general survey photographs on ordinary and photovisual[3] plates should yield useful information. This method was first used by

Keeler[4] to confirm Campbell's results for the nebula in Orion. It shows admirably, but without differentiation, the distribution of whatever luminosity lies between the limiting wave-lengths determined by the filter and the color curve of the plate. Some refinement of analysis can be effected by using filters of limited transmissibility. Thus Hartmann[5] showed that the great photographic activity noted by Keeler in certain regions of the Orion nebula is due to λ 3727, whereas the wider spectral region embraced in Keeler's photographs left the identity of the active radiation in doubt. Again, a red-sensitive plate and a filter transmitting nothing to the violet of λ 5600 enabled Hale[6] to demonstrate the reality of the reddish fringe observed by Barnard, and later by

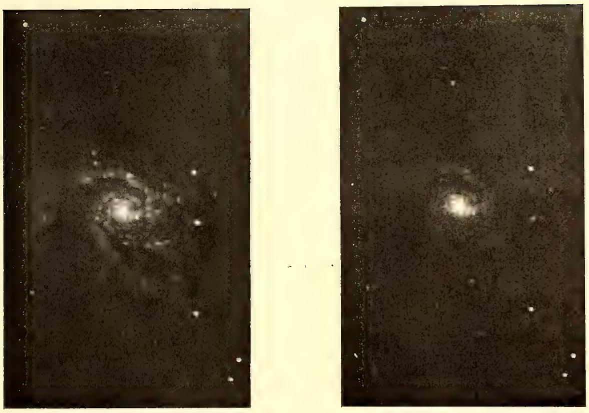


Seed 27 5^{m}

Seed 27 10^{m}

Iso 1^{h}

FIG. 2. MESSIER 94.



Seed 27 20^{m}

Iso 2^{h}

FIG. 3. MESSIER 99.

Keeler, along the southern boundary of the Huyghenian Region, and to show that the color was probably due to the $H\alpha$ line of hydrogen. Spectrum photographs by Adams subsequently placed the matter beyond doubt.[6]

The usefulness of monochromatic photographs seems thus to have been demonstrated, but it does not appear that they have been employed except for the Orion Nebula. A wider application has therefore seemed desirable, particularly for objects too faint for detailed spectroscopic observation. Among these one naturally turns to the spiral nebulae because of the unexpected result uniformly found for the central region, namely, an absorption spectrum of intermediate or late type. Even admitting the presence of superimposed bright lines, which has in some cases been claimed, the result is still remarkable for objects whose finer details are suggestive of some of the gaseous nebulae.

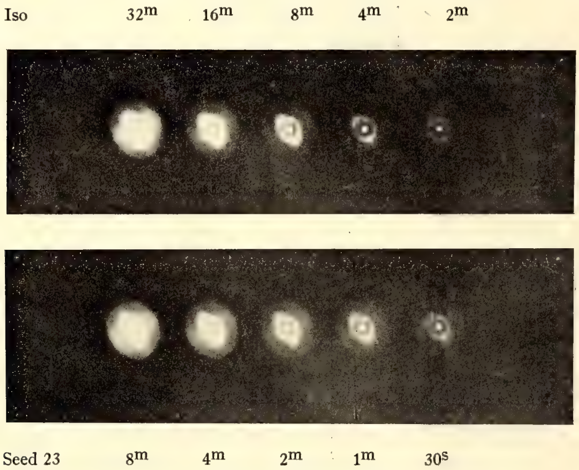


FIG. 4. N. G. G. 3242.

The illustrations show the distribution of the blue and yellow light in three of the well-known spirals, Messier 51, 94, and 99. The photographs were made with the 60-inch reflector on Seed's '27' and Cramer's 'Instantaneous Isochromatic' plates, the latter exposed behind a yellow filter. The exposure time for yellow light was six times that for blue, in order that the images for the bluer stars might be comparable on the two photographs.

Few if any of the yellow images are smaller than the corresponding blue images, at least for objects which are certainly stars; but this is by no means the case for the condensations and nebulosity comprising the branches of the spirals. For these the relative weakness of the yellow light is a striking feature for each of the objects mentioned. Quantitative measures of intensity have not yet been made, but there is every reason to believe that the nebular condensations will reveal negative color indices of large amount. The knots of nebulosity are certainly bluer than the bluest of the neighboring stars, and one is reminded of the great photographic activity of the central star in the Ring Nebula in Lyra.

Whatever the spectral character of these outlying regions may prove eventually to be, it must differ from that of the central nucleus, for the three central nuclei and the secondary nucleus at the end of one of the branches of M 51 are all much stronger in yellow light than in blue. Here the color seems to be in accordance with the typical absorption spectrum found in all similar objects thus far observed.

It is still too early for any general conclusion, but preliminary photographs of other spirals suggest similar results; and it seems not unlikely that the phenomena described are typical of this class of objects.

In contrast to the spirals it is of interest to note the results for the bright planetary N.G.C. 3242, which is also illustrated. In this instance no important differences are revealed by the blue and yellow exposures, at least none which cannot be accounted for by possible differences in gradation on the two kinds of plates.

[1] Newcomb-Engelmann, *Populäre Astronomie,* Fünfte Auflage, p. 672, Leipzig, 1914.
[2] These Proceedings, **1**, 590 (1915).
[3] An isochromatic plate exposed behind a yellow filter.
[4] *Astroph. J.*, **9**, 133 (1899).
[5] *Ibid.*, **21**, 389 (1905).
[6] An unpublished result obtained in 1909.

THE ACTION OF ALKALI IN THE PRODUCTION OF LIPOLYTICALLY ACTIVE PROTEIN

By K. George Falk
HARRIMAN RESEARCH LABORATORY, ROOSEVELT HOSPITAL, NEW YORK

Received by the Academy, August 22, 1916

Introduction.—A summary of an extended experimental study of the lipolytic or ester-hydrolyzing enzymes was presented in these Proceedings last year.[1] The changes in the lipases themselves under various conditions were the main objects of the investigation at that time, as a preliminary to the possible elucidation of the chemical structure of the active groupings. The investigation of the factors which control the loss or destruction of this enzymatic hydrolyzing activity appeared to offer the most promising field for further study; a preliminary report on the results of this study is here presented.

The materials used were *Esterase preparation,* a clear, dialyzed and filtered water-extract of oil-free and husk-free castor beans 0.5 g. to 60 cc.; and *lipase preparation,* a mixture from the 1.5 normal NaCl solution extract of water-extracted castor beans, 1.0 g. to 100 cc., dialyzed until salt-free. The former, a clear colorless solution, was more suitable for following the changes under different conditions than the latter, which was a suspension of globulin in water.

The activity tests were carried out with 1 cc. ethyl butyrate or 0.5 cc. glyceryl triacetate at 38° for 24 or 48 hours, and the results given as the number of cubic centimeters of 0.1 normal alkali required for neutralyzing the acid produced with phenolphthalein as indicator, with suitable corrections for blanks.

Inactivation of the enzymes by acid.—The hydrogen-ion concentrations were determined by color comparison with the standard solutions and indicators recommended by A. A. Noyes.[2] The esterase preparation, $H^+ = 10^{-7.0}$, brought to $H^+ = 10^{-3.5}$ with acetic acid for 24 hours and then back to $H^+ = 10^{-7.0}$, lost its activity completely. At $H^+ = 10^{-4.5}$ under the same conditions for the same length of time, one-third of the activity was lost, while at $H^+ = 10^{-6.0}$ about one-eighth was lost. The difficulty of color comparison with the lipase preparation made difficult an exact determination of the H^+ ion concentration at which inactivation took place. A larger H^+ ion concentration than with the esterase preparation appeared to be necessary to produce a corresponding effect.

Inactivation by alkali.—The esterase preparation lost one-eighth of its activity after being kept at $H^+ = 10^{-8.0}$ for 24 hours, and became practically inactive at $H^+ = 10^{-10.5}$ and $10^{-11.0}$. The lipase preparation dissolved to form a cloudy solution at $H^+ = 10^{-11.0}$. Kept at this for 18 hours and then brought back to $H^+ = 10^{-7.0}$, one-half of the activity was lost. At about $H^+ = 10^{-12}$, one-third of the activity remained, and at about $H^+ = 2 \times 10^{-12}$, one-sixth remained.

Inactivation by alcohols and by acetone.—It was shown previously that dilute solutions of methyl alcohol, ethyl alcohol, and acetone inhibited the action of both preparations. Solid preparations made by precipitation and washing with alcohol were always inactive. Solid esterase preparations, precipitated and washed with acetone, were active in a number of cases; but the activity was much smaller than that of the corresponding solutions from which they were prepared. Similar solid lipase preparations were always inactive.

Inactivation by salts.—The action of salts on these enzymes has already been described. Some produced marked inactivation, others less, while some accelerated the hydrolytic actions.

Inactivation by heat.—Like all enzymes, the esterase and lipase are both inactivated by heating their aqueous solutions or suspensions for a few minutes at 100°. The original oil-free and husk-free castor beans, on being heated dry at 100–110° lost 50–80% of their lipolytic activity; the same loss of weight in a vacuum desiccator over phosphorus pentoxide was not accompanied by loss in activity. Drying first, and then heating (the latter causing only 0.1–0.2% greater loss in weight) produced 50–80% loss in activity.

Nature of the chemical changes involved in the inactivations.—The summary of the different ways in which the esterase and lipase preparations may be inactivated makes it appear at first sight as if different reactions occurred in the inactivations. If, however, a definite chemical group is responsible for a definite enzyme action, it might perhaps be more reasonable to assume that inactivation followed a definite reaction. The preparations were essentially protein in character. There is no evidence that a dehydration, or loss of the elements of water, causes the inactivation. Some of the reactions indicate that a possible hydrolysis may be a cause of inactivation. With proteins, hydrolysis is generally taken to occur with the $-$ CO–NH $-$ group, which goes over into the $-$ COOH NH_2 $-$ groups. Experiments with all the inactivations showed in no case an increase in the formal titration as would be expected in such a reaction, and therefore makes the assumption of such a hydrolysis improbable. Coagulation of the material accompanied some of the inactivations. This physical change alone does not appear satisfactory as an explanation, some change in chemical structure unquestionably accompanying or producing the physical phenomenon. Furthermore, the lipase material in suspension in water showed the same activity as in 1.5 normal sodium chloride solution when tested immediately.

The explanations of the chemical changes accompanying inactivation so far suggested are not satisfactory. The reagents used are simple. It is difficult to conceive of a very deep-seated chemical reaction taking place under so many different conditions, none of a complex nature. To the writer the only chemical change which appears probable under these conditions is that involving a simple rearrangement within the molecule, such as a tautomeric change involving the change in position of a hydrogen atom. In considering the structure of proteins, it is evident that such a rearrangement is possible in the peptide linking.

The hypothesis to be suggested is that the active grouping of the esterase and lipase preparations is of the enol structure $-C(OH)=N-$, the specific actions being dependent in part upon the groups combined with the C and N, and that inactivation consists primarily in a rearrangement to the keto group $-CO-NH-$. Such structures have been proposed at different times as indicating the difference between proteins in living matter, and proteins not in living matter.

Since strong alkali, as a rule, favors the formation of the enol structure in such tautomeric changes, a way is open to test the hypothesis.

Activation of proteins by alkali.—One gram of inactive solid lipase-preparation, washed and dried by means of alcohol, was allowed to stand 24 hours with 25 cc. 10% NaOH solution (and toluene). The brown suspension or mixture was diluted with 100 cc. water and dialyzed against running water for 24 hours to remove the greater part of the alkali. The volume increased to 410 cc. The mixture was brought to a very faint pink color toward phenolphthalein with hydrochloric acid, and the hydrolytic action of 50 cc. portions (corresponding to 0.12 g. original material) was tested for 48 hours. An action (corrected for blanks) of 0.24 cc. was found with ethyl butyrate, and 0.73 cc. with glyceryl triacetate.

Many experiments were also carried out with casein (Kahlbaum's preparation "nach Hammarsten"), of which the following may be cited.[1] Two grams of casein were mixed with 25 cc. 1 normal NaOH solution (and toluene). After 24 hours at room temperature, a grayish-brown and dialyzed 48 hours, the volume increasing to 280 cc.; the H^+ ion concentration was then brought to $10^{-7.0}$, and the hydrolytic actions of 40 cc. portions, corresponding to 0.3 g. of casein each, were tested for 48 hours. The actions found were 0.08 cc. with ethyl butyrate and 0.48 cc. with glyceryl triacetate. A large number of similar experiments were carried out in which the alkali was removed either by dialysis alone or by direct neutralization with acid. There was marked action on glyceryl triacetate, but only very slight action on ethyl butyrate.

The strength of the alkali, between 0.1 normal and 3 normal, used in the preliminary treatment, appeared to have small influence on the activity produced; but great influence was exerted by the H^+ion concentration of the solutions in the activity tests. This last effect may be shown by a series of results with 1 normal NaOH solution in which the alkali was neutralized directly to different points. Twenty-four hours' action on glyceryl triacetate gave the following results:

H^+ ion concentrations	10^{-4}	10^{-6}	10^{-8}	10^{-10}
Activities	0.15	0.10	1.17	1.47

Two similar experiments gave the following results:

H^+ ion concentrations	10^{-7}	10^{-8}
Activities	0.91	1.68

The action was therefore greater in slightly alkaline solution.

[1] These PROCEEDINGS, **1,** 136 (1915).

[2] *J. Amer. Chem. Soc.* (1911).

THE EXCRETION OF ACIDS BY ROOTS

By A. R. Haas

LABORATORY OF PLANT PHYSIOLOGY, HARVARD UNIVERSITY

Received by the Academy, August 28, 1916

Whether roots excrete acid, other than carbonic, has long been a matter of controversy. The problem is important not only because acids dissolve plant food from the soil, but also because it involves the fundamental questions of the reaction of protoplasm and of the mechanism of secretion.

The problem has been greatly complicated by the failure of many vestigators to distinguish between the effects of dead and of living ls.

Becquerel early pointed out that all seedling roots when laid on moist neutral litmus paper possess the property of giving it a lasting red color, which he believed was due to excretion of acetic acid. Boussingault thought that the acid might be lactic, while other investigators left the nature of the acid undetermined. The alkali salts of formic acid have been reported by Czapek[1] in the culture solution of *Lepidium* and *Hordeum* seedlings. The formic acid was considered as coming not from the root hairs but from the sloughing off of root cap cells and their secondary decomposition. The drops occurring on root hairs in a moist atmosphere were found to give no acid reaction. The solvent[2] action of plant roots has been considered to be due not alone to the respired CO_2 but also to organic acids because phosphorite (which can be taken up to some degree by plant roots), requires a strong acid to dissolve it. Kunze[3] believes that there is no free mineral acid in the root excretion of higher plants, but that the acid reaction is due to organic acids and that the quantity of detectable acids is very small, lying below the limit of sensitivity of litmus. Czapek[4] concluded that roots gave off K, Ca, Mg, HCl, H_2SO_4 and H_3PO_4 to water. Stocklasa and Ernest[5] have been unable to find K or H_3PO_4 in root excretions. They found that CO_2 is the only gaseous product in root excretions, with a possibility of H. No free mineral acids were considered to be present in root excretions, and organic acid excretion was considered to occur only when the oxygen percentage was low. Breazeale and LeClerc[6] state that the roots of wheat seedlings in distilled water excrete acids other than CO_2.

In some of the investigations mentioned above no control of distilled water without seedlings was run, and the amount of alkali used to titrate a boiled sample of distilled water, in which seedlings had grown, appears to be approximately the amount required to cause coloration of the indicator in such a volume of neutral solution. Seeds were in some cases in contact with the solution and might have given off acid from dead cells. In some cases it appears certain that dead cells of the roots affected the results. The use of carbon black by some investigators introduces an unknown factor which may alter the results.

The experiments of the writer were made upon early sweet corn by soaking the seed for several hours and then germinating it on moist filter paper in porcelain pans covered by inverted pans. When the roots were about one to one and one-half inches long, the seedlings were placed in the water cultures. Glass tumblers were steamed and thoroughly cleansed, and the outside covered with paper to prevent the growth of algae. Dental napkins, in which one rather large and numerous small holes were cut, were used to cover the tumblers after the distilled water was run in.[7] The cloth covers were drawn tight by the use of string and never were allowed to come into contact with the solution. The larger hole was used to run in more water as evaporation proceeded while in the small holes the seedling roots were placed. Covers of various kinds were used because the water evaporated too rapidly from the tumbler through the cloth. The same kind of cover, with a thin coat of high grade paraffine, prevented much evaporation since the large corn seed covered the hole through which the root had been thrust. Also no tying of the cover was necessary; simply folding down the overhanging edges of the cover about the sides of the tumbler sufficed. Great care was taken to use only roots which had no dead cells (except those of the root cap).

Experiment 1. Twenty-five seedlings were grown in each tumbler in 190 cc. of distilled water. Controls were set up similar in every respect (except that they contained no seedlings) and having the holes in the covers partially covered so as to obtain about equal amounts of evaporation in all the tumblers. A series of phosphate buffer solutions, of known hydrogen ion concentration, and checked by the use of the gas chain, were made up. Phenolsulphonephthalein (12 mgm. per 100 cc.) was used as indicator (3 drops to 10 cc. of solution). The indicator was added to a series of tubes containing the various buffer solutions whose PH^+ value was known.[8] After the plants had grown for 8 days, 10 cc. of the water from each tumbler was run into a tube of the same diameter as those containing the buffer solutions and after adding 3 drops of the indicator, the CO_2 was expelled by a stream of hydrogen (washed free from impurities). The hydrogen was allowed to run through each tube for one-half to three-quarters of an hour to be sure of expelling all the CO_2.

TABLE 1

Results after CO_2 *was expelled*

Water from tumblers containing roots			Water from controls without plants		
1.	P_H+	7.38	1.	P_H+	7.38
2.	"	7.38	2.	"	7.38
3.	"	7.38	3.	"	7.38

Experiment 2. Seedlings with vigorous roots and with apparently no dead cells, were grown for 5 days in culture. The distilled water was different from that in Experiment 1.

TABLE 2

Results after CO_2 *was expelled*

Water from tumblers containing roots	Water from controls without plants
The water from 7 tumblers each P_H+ 7.16	The water from 6 controls each P_H+ 7.16

A similar experiment in glass tumblers was run for 19 days. During that time the cultures received no attention whatsoever. Analysis showed no increase of acidity, even though the roots were full of brown spots.

The use of glass tumblers might be objectionable because of the possible dissolving of the glass, resulting in the possible neutralization of any excreted acid, but in that event the controls ought to indicate an increased alkalinity.

The experiment was then repeated, using large quartz dishes instead of glass tumblers. The covers were made of large sheets of filter paper impregnated and coated on both sides with a thin layer of high grade paraffine, melting point 60–62°C. Holes of the required size were cut out of the cover with a sharp cork borer. The cover was laid on the dish after the distilled water had been added, and care was taken not to let the cover touch the water at any time. The two quartz dishes each contained 650 cc., and two smaller quartz dishes each contained 50 cc. of pure distilled water. One of the large quartz dishes had 65 corn seedlings growing in it while the other had 77 seedlings. The seeds never came in contact with the water. The seedlings were grown in the culture for 7 days. The water in the dishes was made up daily to the same volume as at the start and the evaporation was kept approximately the same in each dish by covering or uncovering the extra holes in the covers. At the end of 7 days the corn roots were about 6 inches long and apparently free from brown spots. The water was then tested as in the other experiments, the CO_2 being expelled from the samples by running hydrogen (free from impurities) through them for 45 minutes each.

TABLE 3

Results after CO_2 *was expelled*

Quartz dishes containing seedlings	Quartz dishes containing only water
65 seedlings: Water P_H+ 8.04	No seedlings: Water P_H+ 7.88
77 seedlings: Water P_H+ 7.88	No seedlings: Water P_H+ 7.88

The two covers bearing the seedlings were then lifted from the top of the quartz dishes and by means of a sharp razor blade (that had been thoroughly cleansed) the roots were cut off a short distance below the seed and were allowed to fall into the dishes in which they had been growing. The tops and the attached seeds were discarded. The paraffin covers were put on the dishes and the holes in the covers were closed by laying several layers of filter paper over them. After two weeks, numerous brown spots appeared on the roots, and many of the roots became soft and gelatinous toward the tip. After 4 weeks from the time that the experiment was first begun, the water in which the roots were decomposing appeared brownish, but possessed no disagreeable odor. Analysis of the water was made as described before.

TABLE 4

Results after CO_2 *was expelled*

Quartz dishes without roots: control	Quartz dishes containing cut-off roots
Control quartz dish......Water P_H^+ 7.88	Quartz dish with decaying roots..............Water P_H+ 8.04
Control quartz dish......Water P_H^+ 7.88	Quartz dish with decaying roots..............Water P_H+ 8.04
	Two new samples of water were taken to verify the results:
	Quartz dish with decaying roots..............Water P_H+ 8.04
	Quartz dish with decaying roots..............Water P_H+ 8.04

After the experiment was ended, it was found that the mass of roots was so soft as to be easily compressible by the fingers into a small ball. The solutions were tested with litmus paper prior to taking out the roots from the dishes. Experience with litmus paper has brough out the fact that it may require some time before the paper indicates the reaction of a solution which is close to the neutral point. The water, that contained the decaying roots, gave no evidence of being acid to blue litmus paper even when the water contained CO_2.

The corn seedlings had been used because of the convenience in making cultures, and because of the opportunity to choose only healthy seedlings having no brown spots on the roots. Whenever any brown spots appeared on the roots before the time for the first analysis the culture was discarded.

Having found that no acid, other than CO_2 was excreted from the roots of corn, experiments were made upon wheat seedlings. Porcelain pans (with no spots on the inside of the pan) were used. The pans were about twelve inches in diameter at the top and were quite shallow. The same volume of water was run into each pan. Bronze screening was dipped a few moments into nitric acid and then washed a long time in running water. If any acid had remained adhering to the wire, we should expect to find acid in certain of the pans at the end of the experiment. The screening was dried and then dipped into hot high grade paraffine until the screening was coated. The paraffined screens were placed as covers upon the pans of water but were not allowed to come in contact with the water.

Wheat seeds were soaked over a half day in water and were sown in a uniform layer over four of the screens, the other two screened pans of water being used as controls. In two of the four seeded screens, the screens were purposely pushed down until they touched the water and in these two pans the seeds also were partially in contact with the water. Pans were inverted over the seed until they had germinated and their roots had entered the distilled water. The seedlings were then exposed to the light and the control pans were always treated in the same manner.

The cultures were allowed to grow for four weeks without attention of any kind. The plants had grown about 6–8 inches in height in about a week after germination and then ceased to grow. At the end of the experiment the roots in no case were coiled up but grew straight downward. The roots of the seedlings had softened and were well decayed after four weeks. Analysis of the water was made as in the other experiments.

TABLE 5

Results after CO_2 *was expelled*

Control (screen not touching the distilled water)	Water P_H+ 7.6
Control (screen not touching the distilled water)	Water P_H+ 7.6
Pan with seedlings (screen or seed not touching the distilled water)	Water P_H+ 8.0
Pan with seedlings (screen or seed not touching the distilled water)	Water P_H+ 8.0
Pan with seedlings (screen and seed touching the distilled water)	Water P_H+ 8.3
Pan with seedlings (screen and seed touching the distilled water)	Water P_H+ 8.2

The experiments with corn seedlings indicate that no acid other than CO_2 were excreted by the roots. The data for corn seedlings shows an exceedingly small increase in the alkalinity of the distilled water when the roots were permitted to decompose in the water. The increase in alkalinity of the one culture, of 65 seedlings of corn in quartz at the end of 7 days, probably indicates the presence of some dead cells that were not apparent. Distilled water, into which only the roots of wheat seedlings extended, showed a very slight increase in alkalinity when the roots had decayed, but when the screen, germinated and ungerminated seeds, and roots were in the water, the increase in alkalinity was slightly greater.

---

[1] Czapek, F., *Biochemie der Pflanzen*, **2**, 872, 1905.

[2] T. Pfeiffer u. E. Blanck, *Landw. Versuchstat, Berlin*, **77**, 217 (1912).

[3] Kunze, G., Jahrb. wiss. Bot., **42**, 357 (1906).

[4] Czapek, F., Zur Lehre von den Wurzelausscheidungen, *Ibid.*, **29**, 321.

[5] Stocklasa, J., and Ernest, A., *Ibid.*, **46**, 73 (1908).

[6] Breazeale, J. F., and LeClerc, J. A., *Bull. U. S. Dept. Agric. Div. Chem.*, No. 149 (1912). See also Meurer, R., *Jahrb. wiss. Bot.*, **46**, 503 (1909), and Johnson, H. V., *Amer. J. Bot.*, **2**, 250 (1915).

[7] The water was distilled from a glass still (which had been used for some months) without the use of cork or rubber. The first and last parts of the distillate were discarded.

[8] This method was adopted because the indicator must be accurately calibrated by buffer solutions in order to obtain results of any value. Phenolsulphonephthalein is very useful for such investigations.

SPECTROGRAPHIC OBSERVATIONS OF RELATIVE MOTIONS IN THE PLANETARY NEBULAE

By **W. W. Campbell and J. H. Moore**

LICK OBSERVATORY, UNIVERSITY OF CALIFORNIA

Received by the Academy, September 6, 1916

In these PROCEEDINGS, **2**, 129 (1916), we described certain rotation phenomena of the planetary nebula N. G. C. 7009 (R. A. = 20 h. 58 m.), as observed by means of the Mills spectrograph attached to the 36-inch refracting telescope. In the past nine months, as opportunity offered, we have extended this line of investigation to others of the brighter planetary nebulae. We have employed 3-prism dispersion and cameras either 16 or 32 inches in focal length. The linear dispersions at H_β are respectively 20 and 10 angstroms to the millimeter. Up to the present time thirty-three planetary nebulae have been examined for evidences of rotational effects. These effects manifest themselves by Dopper-Fizeau displacements of the spectral lines with reference to the normal directions of these lines as defined by the lines of the comparison spectra of hydrogen and helium impressed upon the same photographic plates. The general results of this work are briefly summarized below. It is convenient to speak at first of the results as relative motions in the nebulae rather than as rotations, the former being the more general term.

1. Of the 33 planetaries observed, 16 give definite evidence of relative motions, and for 5 others we suspect that the nebular lines are slightly inclined in some cases and distorted in others. For 12 of the 33 observed nebulae no indications of relative motion have been found.

2. Attention should be called to the apparent effect of the size of the nebular images upon the results. For the so-called 'stellar' nebulae, or those whose images are essentially round and less than 5 seconds of arc in diameter, the chances of observing line inclinations or distortions are reduced by two causes: first, our experience with nebulae elliptical in outline has been that the relative motions are most pronounced along the major axes of figure, and in the stellar nebulae the positions of the major axes are unknown; and secondly, the spectral lines from the stellar nebulae are very short. Of the stellar objects only one has certainly shown rotational effects. Seven of the 12 objects mentioned in the preceding paragraph as giving no evidence of relative motion belong to the stellar class.

3. Of the nebulae whose diameters are between 5 and 20 seconds of arc, 7 give certain evidence of relative motion, 3 are suspected and 5 seem to give no evidence.

4. All of the 8 nebulae whose diameters exceed 20 seconds of arc give strong evidence of relative motion.

5. With one or two exceptions, the images of the nebulae in which relative motion has been observed are approximately elliptic in form. The greatest observed effects occur in general with the elliptical forms of greatest eccentricity.

6. The 5 nebulae with diameters between 5 and 20 seconds of arc which show no relative motions are sensibly round, which suggest that the axes of rotation may lie very nearly in the line of sight; in which cases the spectrographic test would not be applicable.

7. About half of the nebulae in which relative motion has been detected give spectral lines of forms and degrees of inclinations which are satisfactorily interpreted as due to fairly rapid rotations of the central parts of the nebulae about axes approximately perpendicular to the longer axes of figure, and slower to rotations of the outer parts of the nebulae. These objects belong in general to the class of smaller planetaries whose diameters are of the order of 10 seconds of arc.

8. Five nebulae, most of which are ring nebulae, give lines which suggest the presence of other form or forms of relative internal motion in addition to those interpretable as rotational.

9. The spectra of 3 of the larger ring nebulae, among them the well-known ring nebula in Lyra, while definitely showing internal motions of considerable magnitude, are apparently not interpretable on a simple rotation hypothesis. If rotation effects exist, they appear to be modified or concealed by some other type of motion whose nature we have not yet determined.

10. The central sections of the lines in the spectra of the ring nebula in Lyra and a few other nebulae, corresponding to the central areas of the nebular images, are of bowed form, convex to the red, and are broadened; the central section of the lines in the spectrum of N. G. C. 7662 (R. A. 23 h. 21 m.) is doubled, with one bowed form convex to the red and the other convex to the violet; and the central section of the lines in the spectra of several nebulae are broadened toward the red. These phenomena, if interpreted as Doppler-Fizeau effects, suggest motion of nebular materials toward the nebular nuclei at the centers of the objects, but the high values of the corresponding velocities make the acceptance of such an idea difficult. However, the hypothesis should at least be given careful consideration.

11. The observations indicate that in the nebulae which are more and more condensed as the center is approached we seem to be dealing with simple cases of rotating bodies whose outer strata rotate more slowly both linearly and angularly than the strata nearer the center. In other nebulae, of the ring form with central nuclei, rotation effects seem to be combined with motions of other nature.

12. Measures of the rotational velocity of a nebula enable us to draw some interesting conclusions concerning its probable mass. Assuming that a particle in a nebula at a given angular distance from the center is moving in obedience to the gravitational force of the central mass, then, if the central mass is assumed to be equal to the mass of our Sun, the maximum distances of 9 nebulae whose relative motions seem to be interpretable as rotations lie between 250 light years and 5

light years. These are improbably small distances. If we assume that these nebulae are at distances of 1000 light years, which we have reason to believe is of the order of more probable distance, the minimum values of their masses, in terms of the Sun's mass, lie between 4 and 210.

13. As explained in the former paper, it is difficult to avoid the conclusion that the so-called ring nebulae are in reality not ring forms in space, but ellipsoidal shells. If these forms are rotating about the minor axis of figure, which also seems highly probable, it is difficult to account for their apparent equilibrium under rotational and gravitational forces: it would seem that the nebular materials in the polar regions—the regions of the extremities of the rotation axes—should be depressed toward the central nuclei. Are other forces, possibly including radiation pressure, involved?

14. Inasmuch as the observer will not in general be situated in the planes of the equators of rotation of the nebulae, the observed rotational velocities are smaller than their true values. On this account the actual masses of the observed nebulae should be greater than the minimum values assigned above. The effect of radiation pressures would likewise make the deduced masses too small. The indications are that the nebulae under consideration are capable of developing into systems much more massive than is our solar system.

NEW DETERMINATIONS OF PERMEABILITY

By S. C. Brooks

LABORATORY OF PLANT PHYSIOLOGY, HARVARD UNIVERSITY

Received by the Academy, August 31, 1916

The purpose of these experiments was to investigate permeability by new and independent methods, in order to test as far as possible the conflicting views now held by different investigators. For a number of reasons the investigation was confined to electrolytes.

1. A new method of determining permeability was devised, which is independent of other methods. This consists in direct measurements of the rate of diffusion of dissolved salts through a diaphragm of tissue from the thallus of *Laminaria Agahrdii* (formerly identified as *L. saccharina*). Discs of tissue (*E*, fig. 1) were placed between two short lengths of glass tube (*A* and *B*), the ground ends of which were covered with a suitable grease (*F*), and applied to the tissue in such a manner as to make the joint water-tight. One of these cells was closed at the end away from the diaphragm by a rubber tube and pinchcock (*C* and *D*). The 'lower cell' thus formed was filled with sea water or with a salt solution having

the same conductivity, and the open 'upper cell' with a measured amount of a solution of the same salt as that in the lower cell but of one-half the concentration. The rate of increase in the conductivity of the solution in the upper cell served as a measure of the rate of passage of salts through the tissues, and hence of the permeability of the tissue to that salt. The errors due to diffusion of salts from the tissue, to individual differences in the age and thickness of the fronds, and to variations in the area of tissue through which salt could pass, were eliminated by suitable controls.

The effect of the intercellular material was determined by experiments in which dead tissue took the place of living. The outcome of these experiments was independent of the method of killing. Sodium and calcium chlorides, lanthanum nitrate and the salts of sea water passed through the diaphragm of dead tissue at a very considerable rate which did not differ materially in the case of the different salts; the rate of change of conductivity of the solution in the upper cells was between 2.1% and 2.2% per hour during successive four and one-half hour periods of diffusion of the different salts through the tissue. There was therefor no selective permeability of the intercellular substance to the ions used, and no alteration of its permeability caused by these salts.

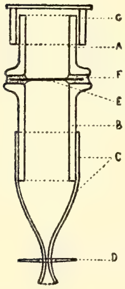


FIG. 1.

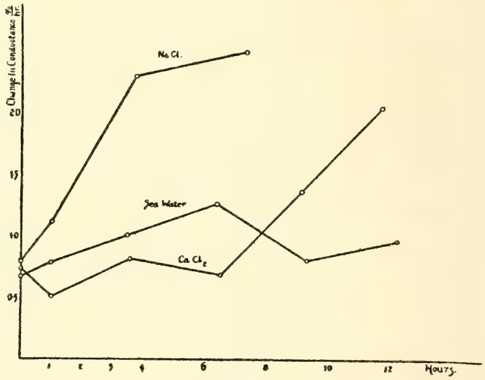


FIG. 2.

FIG. 1. APPARATUS FOR THE DETERMINATION OF PERMEABILITY BY DIFFUSION THROUGH A DIAPHRAGM OF TISSUE.

FIG. 2. PROGRESSIVE CHANGES IN THE PERMEABILITY OF *LAMINARIA* TISSUE CAUSED BY SOLUTIONS OF DIFFERENT SALTS.

Table 1, based on experiments with living tissue, shows that the rate of diffusion of different salts through the protoplasm was considerable, and that it was greatly increased by sodium chloride in pure solution; that it remained nearly normal in sea water (the increase in the rate of diffusion of the salts of sea water in the later periods of an experiment was apparently due to injury of the protoplasm by unfavorable conditions necessarily imposed by the experiment), and that it was decreased considerably by calcium chloride and still more by lanthanum titrate.

Figure 2 shows that the decrease in permeability due to calcium chloride, is temporary. It is succeeded by an increase of permeability, which in the case of calcium chloride (as well as in the case of sodium chloride, which produces an immediate increase), culminates in a rate of diffusion which is the same as that through killed tissue.

These experiments show that the protoplasm of *Laminaria* is permeable to inorganic salts, and that characteristic alterations in its permeability are produced by various salts. Sodium chloride causes an increase in permeability culminating in death. Calcium chloride causes a temporary decrease in permeability, followed by an increase, culminating in death. Certain preliminary experiments indicate that the great decrease in permeability produced by lanthanum nitrate is followed by an increase, presumably culminating in death.

TABLE 1

PERMEABILITY OF LIVING *Laminaria*

EXPERIMENT NUMBER	UPPER SOLUTION	FIRST PERIOD			SECOND PERIOD				RATIO 2D PERIOD / 1ST PERIOD
		Lower solution	Duration h. m.	Change per cent per hour	Upper solution	Lower solution	Duration h. m.	Change per cent per hour	
18	½ sea water	sea water	2.05	0.73	½ sea water	sea water	2.00	0.78	1.07
19	½ sea water	sea water	2.00	0.79	NaCl 0.26 M	NaCl 0.52 M	2.00	1.11	1.41
22	½ sea water	sea water	2.04	0.73	$CaCl_2$ 0.14 M	$CaCl_2$ 0.28 M	2.02	0.51	0.70
17a	½ sea water	sea water	1.35	0.73	$La_2(NO_3)_6$ 0.05 M	$La_2(NO_3)_6$ 0.10 M	1.35	0.33	0.45

It was possible to demonstrate by suitable modifications of the diffusion methods, described above, that the cell walls of the epidermis from the inner surface of the bulb scales of the onion are exceedingly impermeable to sodium, calcium and aluminium chlorides, sodium hydroxide, eosin and Bordeaux red, and slightly permeable to hydrochloric acid.

2. The observations of many investigators are best explained on the assumption that when cells in equilibrium with the liquid normally fill-

ing the intercellular spaces come into contact with a dilute solution which replaces the intercellular sap, a disturbance of equilibrium occurs, involving loss of solutes from the cell. Such an 'exosmosis' would delay the recovery of a plasmolysed cell.

The comparative rate of exomosis of electrolytes was studied in strips of the peduncles of the common dandelion (*Taraxacum officinale Weber*) and was determined in the following manner: comparable lots of tissue were placed for a period of twenty minutes in isotonic solutions of sodium, calcium and aluminium chlorides, and in an isotonic balanced solution containing salts in the following molecular proportions: NaCl 68.4%, $CaCl_2$ 19.8%, $MgCl_2$ 6.7%, $MgSO_4$ 3.3% and KCl 1.9%, and in distilled water. After removal from these solutions the lots of tissue were rinsed and placed in measured amounts of distilled water, whose conductivity was determined at suitable intervals. It had previously been determined that outward diffusion of the salts which had entered the tissue from these solutions was practically complete in thirty minutes. After thirty minutes the rate of change of conductivity of the distilled water bathing the tissue was therefor a measure of the rate of exosmosis of electrolytes normally present in the cell.

The rate of exosmosis from tissue previously exposed to distilled water or to the balanced solution was less than that from tissue treated with sodium chloride, and greater than that from tissue treated with calcium chloride. The tissue treated with cerium chloride showed a slow exosmosis which after a time became quite rapid, ultimately exceeding that from any other lot of tissue. This effect was probably due to the toxic effect of the cerium, which, like lanthanum, causes an increase of permeability following the initial decrease. The data are graphically presented in figures 3 and 4.

It is therefore apparent that exosmosis from living cells is influenced by salts such as are frequently used in plasmolytic experiments. In *Taraxacum*, as in *Laminaria*, sodium increases permeability, while calcium cerium and lanthanum cause an initial decrease. A balanced solution may be prepared, of a constitution such that it will cause no appreciable alteration in the permeability of the protoplasm of a given plant.

3. The conclusions derived from the experiments on exosmosis were were confirmed and amplified by experiments in which changes in turgidity were used to measure the rate of penetration of sodium potassium, calcium, magnesium, aluminium and cerium chlorides, potassium nitrate, saccharose, and the balanced solution heretofore described.

Strips of peduncle were fastened at one end in a horizontal position in a suitable dish, and in such a manner that the free end moved horizontally as the curvature of the strip changed in response to changes in its turgidity. The movement of the free end was then observed by means of a microscope. The strips were first allowed to come into osmotic equilibrium with a solution originally very nearly isotonic with the cells. A slight increase in concentration was then made, and the time required by the strip to return to its original position after the loss of curvature thus caused, was noted. Immediately upon the recovery of a strip, the concentration of the solution bathing it was again increased, and the 'recovery time' again noted. In this manner a series of 'recovery times' of the same strip was obtained. In the absence of exosmosis the molecular increase in concentration, divided by the 'recovery time' affords an empirical measure of the rate of penetration of the plasmolysing substance.

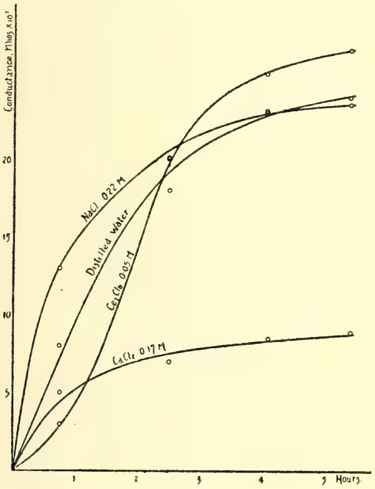


FIG. 3.

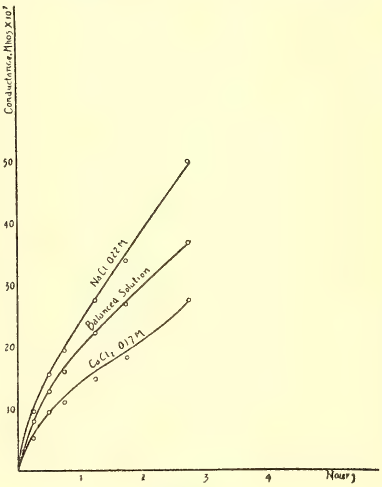


FIG. 4.

FIG. 3. THE EFFECT OF PREVIOUS TREATMENT OF PEDUNCLES OF *TARAXACUM* WITH SOLUTIONS OF DIFFERENT SALTS ON THE CONDUCTIVITY OF DISTILLED WATER IN CONTACT WITH THE TREATED TISSUE.

FIG. 4. THE EFFECT OF PREVIOUS TREATMENT OF PEDUNCLES OF *TARAXACUM* WITH PURE AND BALANCED SALT SOLUTIONS, ON THE CONDUCTIVITY OF DISTILLED WATER IN CONTACT WITH THE TREATED TISSUE.

The data graphically presented in figures 5 and 6, show that the rate of recovery was increased by the salts of the monovalent kations, and by saccharose, while in the salts of bi- and trivalent kations it was less than that in the balanced solution. The first group exhibits also a characteristic secondary decrease in the rate of recovery which may be due to exosmosis from the cells.

It will be seen that these observations are in accord with those made by other methods; and in addition it shows the advantage of a balanced solution over saccharose (which is often used on the assumption that it does not penetrate nor alter the permeability of protoplasm) in experiments in which it is desirable to maintain the normal permeability of living cells.

Summary.—Determinations of permeability have been made by a new method, which is independent of other methods, as well as by improved forms of existing methods.

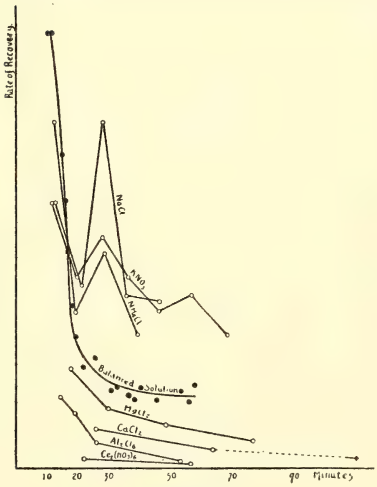


FIG. 5.

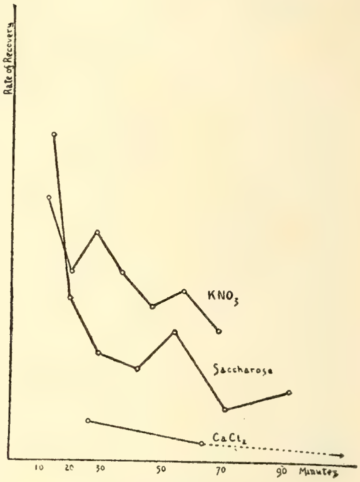


FIG. 6.

FIG. 5. THE RATE OF PENETRATION OF SALTS IN PURE AND BALANCED SOLUTION INTO LIVING CELLS OF *TARAXACUM*.

FIG. 6. THE RATE OF PENETRATION OF SACCHAROSE AND OF SALTS IN PURE SOLUTION INTO LIVING CELLS OF *TARAXACUM*.

The results agree in showing:

(1) That living protoplasm is normally permeable to the salts studied.

(2) That salts in pure solution may alter permeability, some (like sodium chloride) causing an increase of permeability while others (like calcium, lanthanum and cerium chlorides) cause a decrease, followed by an increase, of permeability.

(3) In a properly balanced solution the permeability remains normal.

(4) Cell walls may be semipermeable to an extent which renders them important in such experiments.

POINT SETS AND CREMONA GROUPS. PART III

By Arthur A. Coble

DEPARTMENT OF MATHEMATICS, JOHNS HOPKINS UNIVERSITY

Received by the Academy, August 23, 1916

In Part I[1] of this series projectively distinct sets P_n^k of n points in S_k were mapped upon points of a space $\Sigma_{k(n-k-2)}$ and a certain Cremona group $G_{n!}$ in Σ was obtained by permutation of the points of the set. In Part II[2] the G_n appeared as merely a subgroup of a more important group $G_{n,\,k}$ in $\Sigma_{\,k(n-k-2)}$ which also is defined by P_n^k. In particular the G_6, in Σ_4 attached to P_6^2 is a subgroup of the $G_{6,\,2}$ in Σ_4 which has the order 51840 and is isomorphic with the group of the lines on a cubic surface.

The purpose of this Part III is to utilize the $G_{6,\,2}$ in the problem of determining the lines of a cubic surface C^3. It appears that there is a one-to-one correspondence between the invariants of C^3 and the invariant spreads of $G_{6,\,2}$ in Σ_4. The lines of C^3 can be rationally expressed in terms of a solution of the form problem of $G_{6,\,2}$ by means of a typical representation of C^3 in the hexahedral form with the aid of the linear covariants of C^3. In order to solve the form problem of $G_{6,\,2}$ the simplest linear system of irrational invariants of C^3 is employed. This system is of dimension 9 and the members appear in Σ_4 as quintic spreads. Under the invariant subgroup $\Gamma_{6,\,2}$ of $G_{6,\,2}$ of index two this linear system separates into two skew linear systems each of dimension 4 with the important property that the members of the two systems are permuted under the operations of $G_{6,\,2}$ precisely as the points and S_3's of a linear space S_4 are permuted under the elements of a correlation group in S_4 whose collineation subgroup is the Burkhardt group G_{25920} in S_4. The form problem of $G_{6,\,2}$ can then be solved in terms of a solution of the form problem of G_{25920} by using the point invariants of G_{25920} and in addition five invariants of G_{25920} linear in the S_3 coördinates and of degrees 1, 7, 9, 13, 15 in the point coördinates.

The method for solving the form problem of G_{25920} is suggested by the properties of the normal hyperelliptic surface M_2^{18} of grade 3 in S_8 obtained parametrically by using 9 linearly independent theta functions of the third order and zero characteristic. The M_2^{18} admits a collineation group $G_{2.81}$ which contains 81 involutions. If I is one of these involutions the fixed S_3 and fixed S_4 of I meet M_2^{18} in 6 and 10 points respectively. The M_2^{18} is projected from the fixed S_4 upon the fixed S_3 into a doubly covered Weddle surface and from the fixed S_3 upon the fixed S_4 into a doubly covered 2-way N_2^6 which has a node at α. If N_2^6 be projected from α it becomes a Kummer surface. There is a family of ∞^3 surfaces M_2^{18} with the same $G_{2\cdot 81}$. By projection we obtain a family of N_2^6's whose node α runs over a quartic spread J_4—the simplest invariant of G_{25920}. The 10 points in the S_4 of I run over the Hessian J_{10} of J_4. The spread J_4 is its own Steinerian and the polar cubic of a point α on J_4 as to J_4 is a Segre cubic spread with nodes at the 10 points on J_{10}, and of course a simple point at α. The point α on its polar cubic determines a binary sextic—the fundamental sextic of the hyperelliptic functions. In this way the solution of the form problem of G_{25920} in terms of hyperelliptic modular functions becomes apparent at once *in the special case* when $J_4 = 0$. This restriction is removed later by a conventional method. The conclusions above all are drawn from the existence of a set of 9 quadrics whose complete intersection is the normal spread M_2^{18} and whose coefficients are the modular forms α.

The above determination of the lines of C^3 differs from that of Klein[3] in that no equation of degree 27 or other resolvent equation is employed. All the processes are effected within the domain of the invariants and linear covariants of C^3. Klein also uses as fundamental form problem that of the Maschke collineation group in S_3 rather than the Burkhardt form problem. This implies the isolation of a root of the underlying binary sextic. The accessory irrationalities required are thereby somewhat simpler.

---

[1] These PROCEEDINGS, **1**, 245 (1915); *Trans. Amer. Math. Soc.*, **16**, 155 (1915). This series of investigations has been pursued under the auspices of the Carnegie Institution of Washington, D. C.

[2] These PROCEEDINGS, **2**, 244 (1916); *Trans. Amer. Math. Soc.*, **17**, 345 (1916).

[3] That an equation of degree 27 for the lines of a cubic surface could be solved by hyperelliptic modular functions was first pointed out by Klein, *J. Math.*, *Paris*, Ser. 4, **4**, 169 (1888). His suggestions were elaborated by Witting, *Math. Ann.*, *Leipzig*, **29**, 167 (1887); by Maschke, *Ibid.*, **33**, 317 (1889); and by Burkhardt, *Ibid.*, **35**, 198 (1890), **38**, 161 (1891), **41**, 313 (1893).

THE INTERFERENCES OF SPECTRA BOTH REVERSED AND INVERTED

By Carl Barus

DEPARTMENT OF PHYSICS, BROWN UNIVERSITY

Received by the Academy, September 6, 1916

This is an interesting combination of the two methods of investigation hitherto given (Carnegie Publications, No. 249, 1916, §4) and not very difficult to produce. Retaining the adjustment for inverted spectra the white light impinging on the grating is previously dispersed, preferably by an Ives direct vision grating (with auxiliary prism). The rulings of both gratings (the Ives grating being between the collimator, at some distance, and the first grating of the interferometer) are to be parallel. If the grating constants D_1 are different ($D = 167 \times 10^{-6}$ cm. film, and $D = 352 \times 10^{-6}$ cm. ruled grating, were employed) the spectra in the telescope are naturally of different lengths: for the dispersion of the Ives grating is increased on one side and decreased on the other side, by the grating of the interferometer. Moreover this decrease from the larger dispersion of the first grating is beyond zero (achromatism) into negative values. Hence the corresponding duplicate spectrum in the telescope is a small spectrum with a large spectrum reversed relatively to it, while the inversion remains intact. In the experiment made, the larger doublet, D'_1 D'_2, was somewhat more than twice as broad as the smaller D_1 D_2.

It is now merely necessary to place any longitudinal axis (line of symmetry) of the two spectra seen in the telescope in contact; or it is but necessary that the spectra are longitudinally parallel and overlap. The phenomenon then appears at the intersection of the lines of longitudinal and of transverse symmetry. It is thus proportionately nearer the smaller D_1 D_2 and farther from the larger D' D' doublets, but always between them. If the D_1 D_2 lie within the D'_1 D'_2 lines, the fringes lie within the D_1 D_2 pair.

The phenomenon which should be observed with a powerful telescope, usually consists of three or fewer small elongated dots, lying within an elliptic locus, the locus usually having a transverse axis (parallel to the Fraunhofer lines) about two or three times as long as the longitudinal axis (parallel to lengths of spectra). Usually the width was D_1 D_2 and the length larger than D'_1 D'_2, but this ratio may be changed by screening off the wavefront. The fringes were not more than one-half of D_1 D_2 apart.

The interesting result is here again met, incidentally, that spectra from the same white source, though of different lengths, are nevertheless quite capable of producing strong interferences on overlapping.

In further experiments with the long collimator and very bright spectra, a variety of other forms was obtained. In most patterns the elliptic outline, sometimes circular, is always evident from the enhanced brightness of the bright fringes of the spot. As any adjustment of overlapping spectra suffices, the D lines may be quite out of the field, or the spectra may be slightly separated with the interference spot in the gap.

The experiment was also made of crossing the spectra at some other angle than 0° or 180°. For instance, the rulings of the Ives grating were placed at right angles to those of the interferometer grating, as in Newton's method of crossed prisms. Seen in the telescope (adjusted for inverted spectra) the two spectra now made an elbow with each other, while the D_1 D_2 lines were still parallel and could be put in coincidence. On using a long (meter) collimator, strong interferences were obtained in the line of symmetry of the elbow and normal to the D lines. They have the same characteristics as the preceding and persist during a displacement of mirror of about 0.3 cm.

SEX INTERGRADES IN A SPECIES OF CRUSTACEA

By Arthur M. Banta

STATION FOR EXPERIMENTAL EVOLUTION, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, August 30, 1916

In unisexual forms we are accustomed to think of the term male as signifying the possession by the animal indicated not only of a sperm-producing organ and the associated accessory male reproductive structures but also of certain definite secondary sex characters, definite characters of form and structure and, further, of conspicuous physiological and psychological characteristics. The term female implies definite and marked characters contrasting with those of a male on all these points. In other words maleness and femaleness are generally taken to indicate definite and precise opposed and alternative states, only one of which may obtain in a single individual.

The occurrence of known deviations from the conditions of maleness and femaleness are in the main confined to a comparatively few isolated and sporadic cases of hermaphroditism and gynandromorphism. Gynandromorphs are really sex mosaics inasmuch as a definite portion of the body, frequently one-half, possesses *in toto* the definite characters of one sex and the remainder of the body is distinctively of the other sex. However two workers have published concerning stocks in which considerable numbers of individuals are intermediate in their sexuality—intermediate not as sexual mosaics but quantitatively and as a whole different from either the normal male or the normal female. Riddle[1] has demonstrated intermediate sex forms in his pigeons and Goldschmidt has obtained such forms in the offspring of certain crosses in the gypsy moth.

In a species of crustacean the writer has obtained intermediate sex forms (for which the term *sex intergrade* seems appropriate) constituting a large percentage of the population. The significance of this material may better be considered after some acquaintance with it.

About four years ago two lines of a species of small phyllopod crustacean, *Simocephalus vetulus,* were taken from outdoor ponds, and their propagation continued in the laboratory. Throughout the experiment young have been isolated in individual bottles when not more than a day old. They reproduce parthenogenetically beginning when seven to twelve days of age, the first brood ordinarily containing from 8 to 20 young.

In addition to numerous other lines of this and other species of Cladocera, six separate strains of one line, '740,' have been propagated. For 130 generations these six strains (like all the other strains of Simocephalus to date) produced nothing but parthenogenetic females. Several lines of each of four other species of Cladocera have been propagated for long periods of time in the same manner as Simocephalus and except for a few males in one of the many lines they have produced only females.

But last October in the 131st generation of one of the strains of line 740 there suddenly appeared a large percentage of males together with normal females and a large number of sex intergrades—males with one or more female secondary sex characters, females with one to several male characters and some hermaphrodites with various combinations of male and female secondary sex characters.

At this point we may consider the criteria of sex in this species. The character of the sex gland,—whether a testis or an ovary—is readily determined by microscopic observation of the living animal. In addition, eight secondary sex characters distinguish the female from the male sex: (1) *body size,* the females rapidly outstrip the males in growth and are considerably larger at sexual maturity; (2) *size and position of eye,* the eye in the female is smaller than in the male and does not crowd the margin of the head as in the male; (3) *outline of the head,* the antero-ventral margin of the head is less angulated in the female than in the male; (4) *absence of the nuchal protuberance,* there is a slight dorsal protuberance immediately anterior to the cervical suture, typically in the male only; (5) and (6) *character of the first (rudimentary) antennae,* in the female the basal portion of the antenna is not enlarged and bears a single lateral spine (or stylet), whereas in the male the swollen condition of the basal portion of the antenna and the presence of two lateral stylets are distinctive. Since the two antennae in the same individual frequently are unlike, the character of each antenna counts as a separate diagnostic character;

(7) and (8) *outline and armature of the lateral post-abdomen margins,* in the female these margins are decidedly concave and bear posteriorly no teeth or only rudimentary 'teeth.' In the male the margin is slightly or not at all concave and bears larger teeth which also extend posteriorly much farther than in the female. Since the two post-abdominal margins often differ in character the nature of each margin constitutes a diagnostic sex character.

That these eight characters are really diagnostic secondary sex characters for the female is indicated by the fact that they occur in all individuals in the lines of Simocephalus which are reproducing parthenogenetically in the usual fashion, i.e., producing nothing but females. Again in a derived strain, in which for a time there appeared both females and males, but no sex intergrades, each female and male possessed all the secondary sex characters indicated above for its sex.

The sex intergrades are of almost all possible sorts from females with a single male secondary character to females with all eight of the secondary characters male, hermaphrodites with many combinations of secondary sex characters, and, finally males with one to several (never as many as eight, however) female characters.

The highly male-like female intergrades produce few young. Not infrequently they are entirely sterile. A female intergrade with as many as six strong male secondary characters rarely produces young. Many female intergrades with fewer than six male secondary sex characters are likewise sterile. In most of the sterile individuals eggs are produced but either they fail to develop after passing into the brood pouch or, more generally, the ovarian development is not completed and the eggs disintegrate in the ovary. To cite illustrative cases: two female intergrades in the 132d generation each had five male characters and though eggs began to form, the ovarian development was not completed and no eggs ever appeared in the brood pouch. A sibling of these had only the antennae male in character yet produced only a single brood. This case however is rather unusual as such individuals are usually quite prolific. In the 133d generation a female intergrade with four male secondary characters produced three broods consisting in all of only 15 individuals. On the other hand some individuals with several secondary male characters prove to be very prolific. A sibling of the last-mentioned individual had five secondary male characters and in addition one side of its post-abdomen was somewhat male in character yet it had a high prolificacy. In general in addition to being more prolific one may say that female intergrades with few or less distinctly male characters produce a smaller percentage of males and sex intergrades than those having a larger number or more definitely male characters.

Males that bear one or more female secondary sex characters in nearly every case have incompletely developed reproductive systems—one or both testes, though filled with sperm, being shorter than normal or lacking all or part of the sperm duct. In most cases the entire sperm duct is lacking. Some males, normal in their secondary sex characters, have similar incomplete reproductive systems.

By propagating from female intergrades we are able to continue the production of mixed broods—females, males and sex intergrades. This has now continued for 16 generations with no apparent change in the ratio of the various sex forms and with no apparent tendency of the stock to lose vigor or become less prolific.

From the sex intergrade stock we have derived in its third generation (the 133d generation under laboratory conditions) a normal strain producing nothing but typical females.

The origin of this all-female-producing strain is of interest. An almost sterile hermaphroditic intergrade produced only two young. One of these offspring was a female intergrade. This individual produced 202 young, 2 of which were normal females, 16 were female intergrades, 4 were hermaphrodites, 12 were male intergrades and 67 were normal males. In addition 3 individuals were not examined while 10 females (including female intergrades) and 88 males (including male intergrades) were undescribed. Probably only two individuals (1% of the whole), from the entire number of offspring from this individual were normal females. The only sibling of this female intergrade was a normal female which produced nothing but typical females, and from this individual came a derived normal strain which has now descended for 23 generations in the usual fashion without reversion to the production of sex intergrades. For two generations, however (the 13th and 14th from its origin from the sex intergrade strain), this derived strain did produce a considerable percentage of males. These males were all normal however and their sisters were all normal females, there being no return to the production of sex intergrades as in the parent stock. Following these two generations, in which males were produced, this derived strain has produced nothing but normal females in the nine generations which have occurred to date (August 10). From the 11th generation of the sex intergrade strain a second normal strain producing only typical females has been derived. More recently two additional normal all-female-producing strains have appeared.

As suggested above the various mothers in the sex intergrade strain differ markedly in the proportion of normal sex forms and sex intergrades while they produce. For example, for a given mother the percentage of males—normal males and definitely male intergrades of all grades—varies from 8 or 10% to 100%, the more usual figures being 45 to 85%. The percentages from the same mother from brood to brood vary of course but the sex array and its proportion in the first brood provide a basis for an estimate of what is to be expected in future broods.

The main interest in this material would seem to lie in the remarkable array of sex forms, the stock in general consisting of perhaps 40% normal males and about 8% normal females, the remainder being intergrades with *almost* every combination of male and female secondary and primary sex characters. More than half the individuals are neither wholly male nor wholly female but possess definite morphological sex characters of both sexes. The writer has attempted to classify these individuals on the basis of sex characters into more than twenty classes or grades. A grade at one end of the scale contains normal females; at the other end of the scale there is a class of normal males. Between these extreme grades are others for the many types of sex intergrades. Even this large number of grades is scarcely sufficient; many individuals occur which are intermediate between these grades and can with difficulty be assigned to any of them.

Sex here reveals itself not as a fixed and definite state but as a purely relative thing. No arbitrary classification into males and females is justifiable or possible; not only because of the confusing admixture of male and female secondary sex characters but also because the same individual, even the same sex gland, may develop eggs and sperm at the same time or sperm at one time and eggs at another time.

It seems probable that in this species sex depends upon a number of elements (I believe them to be largely or exclusively environmental factors) which influence the general physiological whole. In the stock which produces the sex intergrades the more usual association and the culmination of these influences has become interrupted. There is a consequent abnormal physiological balance resulting in haphazard combinations of sex-characters and a shifting of the balance one way or the other. Sometimes the result is a balance so unstable that in certain cases it seems to be shifted within a few days in the same individual or within portions of that individual as indicated by changes in the sex products.

Of secondary interest is the method of origin of the sex intergrade strain. The usual sex balance has become disturbed. The cause or method of this disturbance is not definitely known. But whatever the cause, the disturbance influenced not only the individual but also its germ plasm and the disturbed balance is evident throughout succeeding generations. The origin of this disturbance of the sexual balance may be referred to as a mutation. While there is little evidence concerning its cause, there seems abundant evidence concerning its permanent character so far as this strain is concerned.

The derivation from this sex intergrade strain of several strains which produce only *normal* (so far as may be judged by their morphological characters at any rate) females, and, on occasion, *normal* males, is a phenomenon similar to that of the sudden appearance of the sex intergrade strain and might with equal propriety be called a return mutation.

I am inclined to believe from evidence from many Cladocera, and from other forms reproducing parthenogenetically during most of the time and by means of sexual reproduction at irregular and uncertain intervals, that environmental factors in all such forms wield the determining influence. The evidence at hand in the present case, however, is not very conclusive and must be reserved for the larger paper.

[1] The following references may be cited:

Goldschmidt, R., Erblichkeitsstudien an Schmetterlingen. I, *Zs. ind. Abs.—Vererbungslehre,* **7**, 1-62 (1912); and a preliminary report on further experiments in inheritance and determination of sex, these PROCEEDINGS, **2**, 53-58.

Riddle, Oscar, Statement run in the *Carnegie Inst. Washington Year Book*, **12**, 322 (1913); and Sex control and known correlations in pigeons, *Amer. Nat.*, **50**, 385-410 (1916).

SOME PROBLEMS OF DIOPHANTINE APPROXIMATION: A REMARKABLE TRIGONOMETRICAL SERIES

By G. H. Hardy and J. E. Littlewood

TRINITY COLLEGE, CAMBRIDGE, ENGLAND

Received by the Academy, August 7, 1916

1. The title of this note is perhaps not very appropriate: we retain it because the contents of the note form a natural sequel to those of three papers which we have published under same title elsewhere,[1] and in particular those of our second paper in the *Acta Mathematica.* We there discussed in detail the series

$$\sum e^{\alpha\pi i n^2 + 2\theta\pi i n}, \qquad (1.1)$$

and other similar series associated with the elliptic Theta-functions, and used our results to elucidate a variety of difficult points in the theory of Taylor's series and trigonometrical series. We have since discovered that even simpler and more elegant illustrations may be derived from the series

$$\sum e^{\alpha\pi i n \log n + 2\theta\pi i n}. \tag{1.2}$$

This series behaves, for different values of the parameters α and θ, far more regularly than does the series (1.1). To put the matter roughly, the behaviour of the series *does not, in its most essential features, depend upon the arithmetic nature of* α.

2. Our fundamental formula is

$$\sum_{0}^{\infty} a^{\rho n} e^{-ya^n} = \frac{1}{\log a} \sum_{-\infty}^{\infty} \Gamma\left(\rho + \frac{2\pi i n}{\log a}\right) y^{-\rho - \frac{2\pi i n}{\log a}} - \sum_{0}^{\infty} \frac{(-y)^n}{n!\,(a^{\rho+n}-1)}. \tag{2.1}$$

Here $a > 1$, ρ is real, and $\mathrm{R}\,(y) > 0$. The formula becomes illusory when ρ is zero or a negative integer, but the alterations required are of a trivial character. The formula is easily proved by means of Cauchy's Theorem: similar formulae were proved by one of us in a paper published in 1907.[2]

We now write $y = \sigma + it$, where $t > 0$, suppose that $\sigma \to 0$, and approximate to the series of Gamma-functions by means of Stirling's Theorem. We thus obtain

$$\frac{Ht^{-\rho}}{\log a} e^{-\frac{1}{4}\pi i} f(z) = F(\sigma) + \phi(\sigma), \tag{2.2}$$

where

$$f(z) = \sum_{1}^{\infty} n^{\rho-\frac{1}{2}} e^{\alpha i n \log n} z^n; \tag{2.21}$$

$$\alpha = \frac{2\pi}{\log a},\ H = \frac{(2\pi)^{\rho}}{(\log a)^{\rho+\frac{1}{2}}},\ z = re^{i\theta},\ r = e^{-\alpha\sigma/t},\ \theta = \alpha \log\left(\frac{\alpha}{et}\right), \tag{2.22}$$

so that $r \to 1$ when $\to \sigma$ 0;

$$F(\sigma) = \sum_{0}^{\infty} a^{\rho n} e^{-(\sigma+it)\,a^n}; \tag{2.23}$$

and $\phi(\sigma)$ is of one or other of the forms

$$A + o\,(1),\ O\left(\log \frac{1}{\rho}\right),\ O\,(\sigma^{-\rho+\frac{1}{2}}),$$

according as $\rho < \frac{1}{2}$, $\rho = \frac{1}{2}$, or $\rho > \frac{1}{2}$.

3. It is known[3] that, if $\rho > 0$,

$$F(\sigma) = O\,(\sigma^{-\rho}),\ F(\sigma) = \Omega\,(\sigma^{-\rho}), \tag{3.1}$$

when $\sigma \rightarrow 0$, the second of these formulae meaning[4] that $F(\sigma)$ is *not* of the form $o(\sigma^{-\rho})$, and the two together that

$$0 < h = \overline{\lim}\, \sigma^{\rho} F(\sigma) < \infty. \tag{3.2}$$

These relations all hold uniformly in t. It follows that, if $\rho > 0$ and $r = |z| \rightarrow 1$, *the function* $f(z)$ *is exactly of the order* $(1 - r)^{-\rho}$, *and this uniformly in* θ. Incidentally, of course, it follows that every point of the unit circle is a singular point: but this is known already.[5]

The series furnishes an example in which the orders in the unit circle of the functions $f(z) = \Sigma\, a_n z^n$ and $g(z) = \Sigma\, |a_n|\, z^n$ differ by exactly $\frac{1}{2}$, the maximum possible.[6]

When $\rho = 0$, $f(z)$ is bounded, but does not tend to a limit when z approaches any point of the unit circle along a radius vector. We know of no other example of a function possessing this property. When $\rho < 0$, $f(z)$ is continuous for $|z| \leq 1$.

4. Let

$$s_n = \sum_{1}^{n} k^{\rho - \frac{1}{2}}\, e^{\alpha i k \log k + 2\theta\pi i k}; \tag{4.1}$$

and suppose first that $\rho > 0$. Then it is easy to deduce from the results of §3 that s_n is of the form $\Omega(n^{\rho})$ when $n \rightarrow \infty$. The corresponding 'O' result lies a little deeper: all that can be proved in this manner is[7] that $s_n = O(n^{\rho} \log n)$. But a direct investigation, modelled on that of the early part of our second paper in the *Acta Mathematica*, shows that the factor $\log n$ may be omitted. It should be observed that an essential step in our argument depends on an important lemma due to Landau,[8] according to which

$$\left| \int_{1}^{X} x^{\gamma}\, e^{ix \log(\eta x)}\, dx \right| < 23\, X^{\gamma + \frac{1}{2}} \tag{4.2}$$

for $X \geq 1$; $\gamma \geq 0$, $\eta > 0$. We thus find that s_n *is, for every positive value of* α, *exactly of the order* n^{ρ}, *and this uniformly in* θ. *The series*

$$\sum n^{\rho - \frac{1}{2}}\, e^{\alpha i n \log n + 2\theta\pi i n} \tag{4.3}$$

is never convergent, or summable by any of Cesàro's means.

When $\rho = 0$, s_n is bounded, but the series is never convergent or summable. When $\rho < 0$ it is convergent; and uniformly in θ.

5. For further applications it is necessary to consider the real and imaginary parts of our function and series separately, and this is most easily effected by introducing some restriction as to the value of α. Suppose that a is an integer, not of the form $4k + 1$. Thus we may take $a = 2$, $\alpha = 2\pi / \log 2$. Then the results of §§3–4 hold for the real and

imaginary parts of the function or the series. In particular *the series*

$$\sum n^{\rho-\frac{1}{2}} \cos\,(\alpha n \log n + 2\theta\pi n) \quad (\rho \geq 0) \tag{5.1}$$

is never convergent or summable for any value of θ, *and is accordingly not a Fourier's series.* We thus obtain a solution of what, in our former paper, we call Fatou's[9] problem which combines all the advantages of those given previously by Lusin,[9] Steinhaus,[9] and ourselves.

We can also obtain in this manner exceedingly elegant examples of continuous non-differentiable functions. Thus *the function*

$$f\,(\theta) = \sum \frac{\sin\,(\alpha n \log n + 2\theta\pi n)}{n^{\beta}} \quad (1 < \beta \leq \tfrac{3}{2}) \tag{5.2}$$

does not possess a finite differential coefficient for any value of θ.

---

[1] G. H. Hardy and J. E. Littlewood, Some problems of Diophantine approximation · (i) *Proc. Fifth Int. Congress Math.*, Cambridge, **1**, 223–229 (1912); (ii) *Acta Math.*, **37**, 155–190 (1914); (iii) *Ibid.*, 193–238.

[2] G. H. Hardy, On certain oscillating series, *Quarterly J. Math.*, **38**, 269–288 (1907).

[3] G. H. Hardy, Weierstrass's non-differentiable function, *Trans. Amer. Math. Soc.*, **17**, 301–325, (1916).

[4] *l. c. supra* (1) (iii), p. 225.

[5] G. N. Watson, The singularities of functions defined by Taylor's series, *Quarterly J. Math.*, **42**, 41–53 (1911).

[6] G. H. Hardy: (i) A theorem concerning Taylor's series, *Ibid.*, **44**, 147–160 (1913); (ii) Note in addition to a theorem on Taylor's series, *Ibid.*, **45**, 77–84 (1914).

[7] Cf. E. Landau, Abschätzung der Koeffizientensumme einer Potenzreihe: (i) *Arch. Math. Physik*, ser. 3, **21**, 42–50 (1913); (ii) *Ibid.*, 250–255; (iii) *Ibid.*, **24**, 250–260 (1915).

[8] E. Landau, Uber die Anzahl der Gitterpunkte in gewissen Bereichen, *Göttinger Nachrichten*, 687–771 (p. 707), (1912).

[9] For references see p. 232 of our paper (1) (iii).

STERIC HINDRANCE AND THE EXISTENCE OF ODD MOLECULES (FREE RADICALS)

By Gilbert N. Lewis

CHEMICAL LABORATORY, UNIVERSITY OF CALIFORNIA

Received by the Academy, September 6, 1916

The discovery of certain unpredicted facts in organic chemistry has led to the employment of the elusive phrase 'steric hindrance,' a phrase, however, which seems too vague in its significance to connote a real scientific theory. If a steric influence upon a chemical reaction be defined as one which is due to the room occupied by a large atom or group of atoms, such a definition leaves an opportunity for that kind of confusion, which is too frequently found in chemical literature, between the factors that determine the speed of a reaction and those that determine the conditions of final equilibrium. In order to avoid such confusion it is necessary to distinguish between two possible types of steric influence:

1. A group of atoms, owing to its size and to its position in a molecule, may stand in the way so as to impede a reaction in which that molecule is involved. In other words, it may act merely to diminish the speed of such a reaction.

2. The space about a given atom may be so completely occupied by certain large groups of atoms, before the natural valence of the central atom is satisfied, that the entrance of another large group will be prevented; or if such a group occasionally enter, it will again be forced out through lack of room.

I think it will be admitted that these ideas, plausible as they are, should not be introduced into the already very complex body of chemical theory until phenomena are known which cannot be so well explained in other ways. In this paper I wish to discuss the so-called free radicals of organic chemistry, in the interpretation of which the term 'steric hindrance' has so often been employed, and to show that, in this case at least, another explanation can be offered which is more satisfactory in that it correlates a larger number of known phenomena.

After Gomberg, by treating triphenylmethyl chloride with metals, obtained a substance which he believed to contain triphenylmethyl, thus giving rise to one of the most interesting chapters of modern organic chemistry, it was at first commonly believed that by some other method a hexaphenylethane might be produced which would be stable and unreactive, and which would be formed from triphenylmethyl except for the retardation of this reaction by the first type of steric hindrance which I have just defined. We are now convinced that this is not the case. The later work of numerous investigators, which has been very completely reviewed by Gomberg[1] in his interesting summary, 'The Existence of Free Radicals,' has satisfactorily cleared an extremely intricate situation. A careful study of the work of these authors must, I believe, lead to the acceptance of the following perfectly definite conclusions concerning the class of hexa-arylethanes, of which hexaphenylethane is the simplest example.

I. The hexa-arylethanes, aside from the irreversible processes of intramolecular rearrangement and autoxidation to which they are subject and which we shall not further consider, exist in a state of tautomeric equilibrium between quinoid forms and the benzoid form which represents the simple substituted ethane. As in most similar cases the equilibrium is shifted in favor of the quinoid type in polar solvents, and in a very polar solvent such as SO_2 the molecules may dissociate into positive and negative ions of which the former (and probably the latter also) is largely in the quinoid form.

The recognition of the existence of the quinoid form is indispensable to an understanding of numerous reactions which characterize the hexa-arylethanes, but since we are here concerned with that part of the substance which remains in the benzoid form we may proceed at once to the next conclusion.

II. In nonpolar solvents such as benzene the substituted ethanes exist chiefly in the benzoid form and are to a greater or less extent dissociated into the triarylmethyls. The undissociated part is colorless. The product of dissociation, though colored, has also the benzoid form.

The fact of dissociation has been abundantly demonstrated. The mere fact that the equilibrium between colorless and colored forms changes in favor of the latter with increasing dilution[2] shows that the change from colorless to colored is attended by an increase in the number of molecules. Measurements of the degree of dissociation, whether through molecular weight determinations or by colorimetric methods, show that while the parent substance, hexaphenylethane, is at ordinary temperatures only to a few per cent dissociated, the dissociation is nearly complete in the case of some other aryl derivatives. Schmidlin[3] has shown that the process of dissociation is not instantaneous but requires a short though appreciable time, and hence he was able to show that it is the colored substance which reacts rapidly with oxygen or iodine, thus exhibiting the characteristics of a free radical. Indeed it might be possible under favorable circumstances to obtain the degree of dissociation in a new way by rapid titration of the colored substance against atmospheric oxygen.

Gomberg has hesitated to adopt the conclusion that the colored triarylmethyls retain the benzoid form, on the ground that there is no analogy to support the assumption that a colored substance could be formed by the mere dissociation of a colorless substance such as the substituted ethane. It is true there is no analogy in organic chemistry except among these very compounds of trivalent carbon, and the corresponding compounds of bivalent and quadrivalent nitrogen which have been investigated with great perspicuity by Wieland and his collaborators. If, however, we examine the whole field of chemistry, we see that triphenylmethyl belongs to a class of substances which are almost invariably colored and which are typified by NO_2 which, like triphenylmethyl, associates at low temperature or high concentration to form the colorless double molecule. They are the substances which possess what I have called, in a recent paper,[4] *odd molecules*.

I will review briefly the significance of this term. If we assume that every neutral atom possesses in its outer shell a number of electrons corresponding to the ordinal number of the element's group in the periodic system, for example, H, 1; C, 4; N, 5; O, 6; Cl, 7, then in almost all compounds there is an even number of such electrons. Those few substances which possess an odd or unpaired electron are said to have odd molecules. Thus sodium in the state of vapor has 1, monatomic iodine has 7, NO_2 has 17, ClO_2 has 19, $(C_6H_5)_3C$ has 91, $(C_6H_5)_2N$ has 63, and $(C_6H_5)_2NO$ has 69. Since an unpaired electron is always held in the molecule by weak constraints, it is usually capable of oscillating with so low a characteristic frequency as to absorb visible light. This absorption, moreover, involving only a single electron, might be expected to be of the simplest possible character.

In fact it has been shown by Wieland[5] that several substances of this type, with odd molecules, have very characteristic absorption spectra possessing ordinarily a single absorption band. Thus the spectrum not only of $(C_6H_5)_2NO$ but also of the triarylmethyls is like that of NO_2 and by no means resembles that of such quinoid substances as triphenylmethyl ion. There seems therefore every reason to believe that triphenylmethyl is colored not because it assumes the quinoid form but because it possesses an odd electron which, as in every other known substance of this class except NO, has a sufficiently low natural frequency to absorb visible light.

But whether or not this point be admitted, the fact that equilibrium is rapidly established between the ethanes and their corresponding methyls shows conclusively the non-existence of the first type of steric hindrance. Instead of a retardation due to the large groups we find a reaction velocity which, among organic compounds, is exceptionally high.

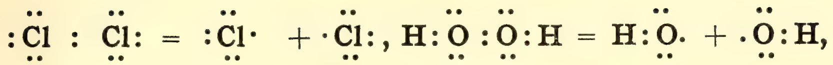
Therefore in the discussion of this type of reaction the term, steric hindrance, tacitly came to be employed in the second sense, implying a crowding out of large groups by other large groups, and this view seemed to be favored by the discovery by Schlenk that the degree of dissociation of the hexarylethanes ordinarily increases with increasing complexity of the substituted aryl groups.

In order to obtain a comprehensive view let us consider four substances which in the light of the periodic system must in certain respects be regarded as analogous, namely, Cl—Cl, HO—OH, $H_2N—NH_2$, $H_3C—CH_3$. The chlorine molecule acquiring rotational energy due to temperature has a certain tendency, through centrifugal force, to break at the bond of union into monatomic molecules. The molecule of bromine with its heavier atoms has, especially in view of modern quantum theory, a higher moment of momentum and is therefore dissociated to a higher degree, while iodine with its still heavier atoms is to a large degree dissociated at easily attainable temperatures. No one has suggested that this dissociation of iodine vapor is due to steric hindrance. The volume occupied by an atom is at best a most indefinite conception, but the atomic volume commonly attributed to iodine is not greater than that attributed to bromine or chlorine. It is doubtless the mass of the iodine atom which is chiefly responsible for its greater dissociation.

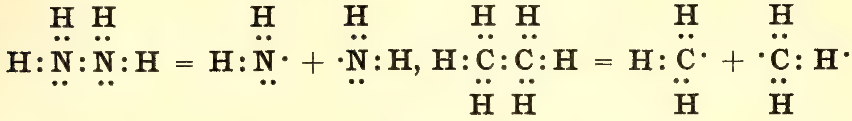
Since diatomic iodine dissociates to give nul-valent iodine we may expect to find a properly substituted hydrogen peroxide dissociate to give a compound of univalent oxygen, as the substituted hydrazines and ethanes have been found to give compounds of bivalent nitrogen and trivalent carbon. Both in hydrazine and in ethane it is the substitution of heavy groups which causes appreciable dissociation. Thus pentaphenylethane does not dissociate sufficiently to show the properties of a free radical below 180°, where it absorbs oxygen readily.[6] Hexaphenylethane is at room temperature dissociated to the extent of several per cent. The successive substitution of phenyl by biphenyl[7] increases the dissociation regularly. Now it is possible, but not obvious, that biphenyl occupies more space immediately about the methyl carbon than phenyl does. It certainly seems better to assume that by increasing the mass of the group and thus increasing the centrifugal force at the central bond, it produces a greater dissociation.

Another interesting illustration of the effect of the mass of the substituent upon dissociation is afforded by the interesting substance obtained by Wieland in which one oxygen of NO_2 is replaced by two phenyl groups. While NO_2 is largely associated even at ordinary temperatures $(C_6H_5)_2NO$ remains monomolecular even at the lowest temperature obtainable with solid CO_2 and ether.

The relation between nul-valent chlorine, univalent oxygen, bivalent nitrogen and trivalent carbon is more readily visualized if we express the equations of dissociation in the symbols of my previous paper:



$$:\overset{\cdot\cdot}{\underset{\cdot\cdot}{\mathbf{Cl}}}:\overset{\cdot\cdot}{\underset{\cdot\cdot}{\mathbf{Cl}}}: = :\overset{\cdot\cdot}{\underset{\cdot\cdot}{\mathbf{Cl}}}\cdot + \cdot\overset{\cdot\cdot}{\underset{\cdot\cdot}{\mathbf{Cl}}}:, \ \mathbf{H}:\overset{\cdot\cdot}{\underset{\cdot\cdot}{\mathbf{O}}}:\overset{\cdot\cdot}{\underset{\cdot\cdot}{\mathbf{O}}}:\mathbf{H} = \mathbf{H}:\overset{\cdot\cdot}{\underset{\cdot\cdot}{\mathbf{O}}}\cdot + \cdot\overset{\cdot\cdot}{\underset{\cdot\cdot}{\mathbf{O}}}:\mathbf{H},$$



$$\mathbf{H}:\overset{\mathbf{H}}{\overset{\cdot\cdot}{\underset{\cdot\cdot}{\mathbf{N}}}}:\overset{\mathbf{H}}{\overset{\cdot\cdot}{\underset{\cdot\cdot}{\mathbf{N}}}}:\mathbf{H} = \mathbf{H}:\overset{\mathbf{H}}{\overset{\cdot\cdot}{\underset{\cdot\cdot}{\mathbf{N}}}}\cdot + \cdot\overset{\mathbf{H}}{\overset{\cdot\cdot}{\underset{\cdot\cdot}{\mathbf{N}}}}:\mathbf{H}, \ \mathbf{H}:\overset{\mathbf{H}}{\overset{\cdot\cdot}{\underset{\underset{\mathbf{H}}{\cdot\cdot}}{\mathbf{C}}}}:\overset{\mathbf{H}}{\overset{\cdot\cdot}{\underset{\underset{\mathbf{H}}{\cdot\cdot}}{\mathbf{C}}}}:\mathbf{H} = \mathbf{H}:\overset{\mathbf{H}}{\overset{\cdot\cdot}{\underset{\underset{\mathbf{H}}{\cdot\cdot}}{\mathbf{C}}}}\cdot + \cdot\overset{\mathbf{H}}{\overset{\cdot\cdot}{\underset{\underset{\mathbf{H}}{\cdot\cdot}}{\mathbf{C}}}}:\mathbf{H}\cdot$$

The symbols in heavy type represent the atomic kernels, while the dots represent the electrons of the outer shells. It is the odd electrons that gives the free radicals color and high reactivity. It will be understood that the particular substances used for illustration are dissociated only to an extremely small degree; only upon substitution of heavier radicals will the dissociation be appreciable at ordinary temperatures.

In emphasizing the effect of mass and thus of centrifugal force in breaking a chemical bond we must not forget, however, that this is only one of the two main factors that determine the stability of a molecule. The other factor is the strength of the constraints which constitute the bond itself. (Electrochemical influences which are so frequently important do not concern us here since we are considering a bond symmetrically placed between two identical radicals.) I have pointed out, in the paper already cited, that the bond between two carbon atoms is weakened when either atom has a double bond or is attached directly to an atom with a double bond. Thus the great change produced by the substitution of phenyl for hydrogen or alkyl is due to both increasing mass and a weakened bond. If the phenyl groups in hexaphenylethane are replaced by the somewhat heavier benzyl groups the dissociation will nevertheless be diminished, since the benzyl will not aid in weakening the central bond. On the other hand there are radicals which cause a still greater weakening of the central bond than the phenyl group. Thus Gomberg has shown that the linking of two of the phenyl groups of triphenylmethyl by an atom of oxygen in the ortho positions greatly increases the percentage of free radical.

The odd molecules produced by the thermal dissociation of the substances here considered may combine not only with their like but with other free radicals to produce mixed types. Thus the reaction $(Ar)_2N-N(Ar)_2+(Ar)_3C-C(Ar)_3=2(Ar)_3C-N(Ar)_2$ is analogous to the reaction $(Ar)_3C-C(Ar)_3+I_2=2(Ar)_3C-I$. Wieland[8] has found a very interesting reversible reaction $(C_6H_5)_3C-C(C_6H_5)_3+(C_6H_5)_2N-N(C_6H_5)_2=2(C_6H_5)_3C-N(C_6H_5)_2$.

By some reaction of this type, if proper protection against intramolecular rearrangement be provided, we may predict that a di-aryl substituted hydrogen peroxide will be prepared, which in turn will dissociate into mono-aryl oxide, ArO, the odd molecule, or free radical, of univalent oxygen. Such a peroxide would probably be less dissociated than the similarly substituted hydrazine, just as the latter is less dissociated than the corresponding ethane.

[1] Gomberg, *J. Amer. Chem. Soc.*, **38**, 770 (1916).

[2] Piccard, *Liebig's Ann. Chem.*, **381**, 347 (1911).

[3] Schmidlin, *Ber. D. Chem. Ges.*, **41**, 2471 (1908).

[4] Lewis, *J. Amer. Chem. Soc.*, **38**, 770 (1916).

[5] Wieland and Offenbecher, *Ber. D. Chem. Ges.*, **47**, 2111 (1914); Meyer and Wieland, *Ibid.*, **44**, 2557 (1911).

[6] Chichibabin, *Ibid.*, **40**, 367 (1907).

[7] Schlenk, Weickel and Herzenstein, *Liebig's Ann. Chem.*, **372**, 1 (1910).

[8] Wieland, *Ibid.*, **381**, 200 (1911).

NEWTON'S METHOD IN GENERAL ANALYSIS

By Albert A. Bennett

DEPARTMENT OF MATHEMATICS, PRINCETON UNIVERSITY

Received by the Academy, August 8, 1916

The present paper is essentially an extension of the methods and results given by Dean H. B. Fine, On Newton's Method of Approximation,[1] and the results there obtained need not be explained here.

An illustration of some general notions.—As might be expected, Newton's Method is of very wide application and may be used, for example in the following three cases to find a real 'root' of the real functionelle $F[x(s)]$ where F has the property that in a certain domain, and for every $x(s)$ and $x(s) + h(s)$ in this domain, there exist functionelles F_1 and F_2 for which

$$F[x(s)+h(s)] \equiv F[x(s)] + \int_0^1 F_1[x(s), r]h(r)dr + \int_0^1 \int_0^1 \tfrac{1}{2} F_2[\xi(s), r_1, r_2]\, h(r_1)\, h(r_2)\, dr_1\, dr_2, \qquad \text{(A)}$$

(1) where $\xi(s)$ is such that $\max_s |\,\xi(s) - x(s)\,| \leqq \max_s |\,h(s)\,|$ and $\max_s$ $x(s) + h(s) - \xi(s)\,| \leqq \max_s |\,h(s)\,|$, if by '$\max_s$' is meant the maximum as s varies, and by a 'root' of the above equation is meant a function $x(s)$, such that $\max_s |\,F[x(s)]\,| = 0$; or (2) where $\xi(s)$ is such that $\sqrt{\int_0^1 [\xi(r) - x(r)]^2 dr} \leqq \sqrt{\int_0^1 h^2(r)\,dr}$ and $\sqrt{\int_0^1 [x(r) + h(r) - \xi(r)]^2 dr} \leqq \sqrt{\int_0^1 h^2(r)\,dr}$, and by a 'root' is meant a function $x(s)$, such that $\sqrt{\int_0^1 F[x(r)]^2 dr} = 0$; or (3), where $\xi(s)$ is such that $\int_0^1 |\,\xi(r) - x(r)\,|\,dr \leqq$

$\int_0^1 | h(r) | dr$, and $\int_0^1 | x(r) + h(r) - \xi(r) | dr \leqq \int_0^1 | h(r) | dr$ and by a 'root' is meant a function $x(s)$ such that $\int_0^1 | F[x(r)] dr = 0$.

The above equation (A) we can write symbolically in the form

$$F(x+h) = F(x) + F'(x)h + \tfrac{1}{2}F''(\xi)h^2. \qquad (A')$$

We must regard x, h, and ξ as functions whose independent variables range over the same interval, viz., from zero to one. Any linear transformation of x must be accompanied by the same transformation on h and ξ. We shall speak of x, h, and ξ as covariant and of equal weight, viz., *one*, on the first of the two ranges that we shall consider. On the other hand the symbol h^2 or $h(r_1)\ h(r_2)$ is covariant with h but of weight *two*. Now h and F are not covariant, but *divariant*. We shall say that F is defined on the second range. Clearly F, F' and $\frac{1}{2}F''$ are covariant and of weight one on this second range. Similarly F' and h are *contravariant*, for only if F' be subjected to the transformation contragredient to that on h, will the term $F'h$ be left invariant. Similarly $\frac{1}{2}F''$ must be subjected to the square of the contragredient transformation. The whole situation may be succinctly expressed by introducing the notion of *signature*, which denotes the range and weight and whether covariant or contravariant. We shall say that x, h, and ξ are of signature (1, 0) being defined on the first but not on the second range, and of weight one on the first, h^2 is of signature (2,0) being of weight two. F is of signature (0, 1), F' of signature $(-1, 1)$ being contravariant with functions of weight one on the first range, and covariant with functions of weight one on the second, $\frac{1}{2}F''$ is of signature $(-2, 1)$. A constant we may speak of as of signature (0, 0).

The three expressions (1) $\max_s | z(s) |$, (2) $\sqrt{\int_0^1 z^2(r)dr}$, (3) $\int_0^1 | z(r) | dr$ we may subsume under the notation $\| z \|$ for the ranges (1, 0) and (0, 1). If the integration be in the sense of Lebesgue, it is true that if $z(s) = 0$ identically in s, then $\| z \| = 0$ in each of the cases, but in the second and third cases it does not follow from $\| z \| = 0$ that $z(s) = 0$ identically in s.

We may speak of x, h, ξ, F, F', $\frac{1}{2}F''$ as vectors, this being an extension of the familiar notion of a real vector in three dimensions. The symbolic expression (A') brings in symbolic products of apparently different types. But if the signature be kept in mind, no ambiguity results, for in each case, the product of two vectors has for its signature, the matrical sum of the signatures of the factors. For example in $\frac{1}{2}F''(\xi)h^2$, we have $(-2, 1) + (1, 0) + (1, 0) = (0, 1)$ as desired. The norm $\| z \|$ of the vector z may also be readily defined for h^2 F' and $\frac{1}{2}F''$, so as to satisfy certain useful inequalities. We shall define in the three cases respectively; (1) $\| h^2 \| \equiv \max_{s_1 s_2} | h(s_1)\, h(s_2) |$, $\| F'[x(s), r] \| \equiv \max_s \int_0^1 | F[x(s), r] | \, dr$, $\| \tfrac{1}{2} F''[x(s), r_1, r_2] \| \equiv \max_s \int_0^1 \int_0^1 | \tfrac{1}{2} F''[x(s), r_1, r_2] | \, dr_1\, dr_2$; (2) $\| h^2 \| \equiv \int_0^1 \int_0^1 h^2(r_1)\, h^2(r_2)\, dr_1\, dr_2$, $\| F'[x(s), r] \| \equiv \sqrt{\int_0^1 \int_0^1 F'[x(s), r]^2\, ds\, dr}$, $\| \tfrac{1}{2} F''[x(s), r_1, r_2] \| \equiv \sqrt{\int_0^1 \int_0^1 \int_0^1 \{\tfrac{1}{2} F''[x(s), r_1, r_2]\}^2\, ds\, dr_1\, dr_2}$; (3) $\| h^2 \| \equiv \int_0^1 \int_0^1 | h(r_1)\; h(r_2) | \, dr_1\, dr_2$, $\| F'[x(s), r] \| \equiv \int_0^1 \{\max_r | F[x(s), r] | \}\, ds$, $| \tfrac{1}{2} F''[x(s), r_1, r_2] \| \equiv \int_0^1 \{\max_{r_1 r_2} | \tfrac{1}{2} F''[x(s), r_1, r_2] | \}\, ds$. With these definitions $\| z_1 + z_2 \| \leqq \| z_1 \| + \| z_2 \|$ and $\| z_1 z_2 \| \leqq \| z_1 \| \; \| z_2 \|$, where $z_1 z_2$ means the symbolic product. Furthermore we may define $\| z \|$ for z of range (0, 0) as identical with $| z |$.

The case of Newton's method for one variable and n variables has been discussed by Dean Fine in the paper referred to. These cases and the case of integration in which the ranges are continuous may be treated by the methods discussed in the present paper. But the conditions here used are of an abstract sort and may be used in much more extensive cases as will be clear to those familiar with the recent work of E. H. Moore, M. Fréchet, F. Riesz, V. Volterra, etc. The scalars, or vectors of signature (0, 0), which in all of the classical instances are ordinary real numbers, may be taken as Hensel p-adic numbers or elements in any *perfekte bewertete Körper* of Kürschák,[2] but more generally do not need to constitute a field as division is not essential. To avoid repetition the further discussion of this case will not be treated separately but may be regarded as included in the following sections.

Preliminary concepts.—Starting with an arbitrarily chosen range, which we shall refer to by the signature (1, 0), we shall suppose that we may construct ranges of the signatures (0, 0), (0, 1), (1, 1), (−1, 1), (−2, 1), (2, 0), respectively, where the range of signature (0, 0) contains but one element. An explicit definition of signature will not be required.

By the term *vector* or *function on a range*, will be meant a correspondence from the elements of a range to a set of *scalars* where each element determines one and only one scalar. A vector of signature (0, 0) is by definition a scalar, and conversely. The sum and the difference of two scalars will be required to exist uniquely, addition being associative. The sum and the difference of two vectors will be the usual vector or matrix sum and difference, respectively, and will be required to exist uniquely in the cases considered. The product of two vectors will be required to exist uniquely in the cases considered, and to be (1) associative, (2) completely distributive with respect to addition—so that addition is proved to be commutative, (3) continuous in each factor, continuity being defined as below, and (4) such that the signature of the range of the product is the matrix sum of the signatures of the ranges of the factors.

The notation $\| z \|$ will be used for the *norm* of z. We shall require only that (1) $\| z \|$, $= \| -z \|$, is a uniquely defined real non-negative number such that $z \equiv 0$ implies $\| z \| = 0$, but not necessarily conversely; (2) $\| z_1 + z_2 \| \leqq \| z_1 \| + \| z_2 \|$, $\| z_1 z_2 \| \leqq \| z_1 \| \ \| z_2 \|$, and in particular, if $\| z_1 \| = 0$, $\| z_1 + z_2 \| = \| z_2 \|$, the sum and the product being defined as above; (3) the signature of z being given and a positive number ϵ, being given, there exists a z, such that $0 < \| z \| < \epsilon$; and (4) if $z_1, z_2, \ldots, z_n, \ldots$ be a denumerable sequence of vectors on the same range, and such that $\| z_1 \| + \| z_2 \| + \ldots + \| z_n \| + \ldots$ converges to a finite limit, then there is at least one z on the same range such that given any positive number ϵ, there exists a number m such that $n > m$ implies that $\| z - \Sigma_1^n z_i \| < \epsilon$ and $\lim_{n=\infty} \| \Sigma_1^n z_i \| = \| (z) \|$. Such a z will be called a *limit* of $z_1 + z_2 + \ldots + z_n + \ldots$, and we notice that for any two such limits z and z', $\| z - z' \| = 0$. We shall define two vectors z and z' for which $\| z - z' \| = 0$ as equivalent. It is by virtue of this last-named property that the vectors of a given range form in some sense a closed set.

A vector b with the signature $(-1, 1)$ will be said to be *non-singular*, if and only if there exists a unique vector b^{-1}, with the signature $(1, -1)$ such that the equation $a = bx$ where a is of signature $(0, 1)$, and x is required to be of signature $(1, 0)$, always has one and only one solution given by $x = b^{-1}a$.

We shall extend the usual definition of continuity to the case where $g(x)$ and x are of any signature as follows:—The function $g(x)$ is *continuous* in x at x' if for every assigned positive constant ϵ, there exists a positive constant δ, such that $\| x - x' \| < \delta$ implies that $\| g(x) - g(x') \| < \epsilon$.

Description of Newton's method.—In describing Newton's Method, we presuppose that we are given initially a function $f(x)$, where x is of signature $(1, 0)$ and $f(x)$ of signature $(0, 1)$, and such that we may expand $f(x+h)$ as follows:—

$$f(x+h) = f(x) + f'(x)h + \tfrac{1}{2} f''(\xi)h^2, \tag{1}$$

where h and ξ are of signature $(1, 0)$, h^2 of signature $(2, 0)$, f' of signature $(-1, 1)$, f'' of signature $(-2, 1)$, the expansion being valid in a known domain. The ξ is supposed to be dependent on the x and h, but such that simultaneously $\| \xi - x \| \leqq \| h \|$ and $\| (x+h) - \xi \| \leqq \| h \|$. The functions f, f', and f'' are all supposed to be continuous in the domain considered. Equation (1) may also be written, for convenience, in the form $f(x+h) = a + bh + ch^2$.

Newton's Method consists in the following steps. Choose any x_0 in the domain and then select h_0 as a solution of $a_0+b_0h_0=0$. Put $x_1=x_0+h_0$, and repeat. Thus $a_i+b_ih_i=0$, and $x_{i+1}=x_i+h_i$, $i=0, 1, \ldots,$ where in general a_i, b_i, c_i mean $a(x_i)$, $b(x_i)$, $c(x_i)$ respectively. If $\lim_{i=\infty} x_i$ exists uniquely, $(=x')$, and is in the domain, then x' is a root of $f(x)=0$.

In order that Newton's Method may be applied, we note that the equations $a_i+b_ih_i=0$ must be solved uniquely for h_i, so that $b(x)$ must be non-singular in the domain. We shall write $\| a_i\| =\alpha_i$, $\| b_i^{-1} \| = 1/\beta_i$, $\| c_i \| =\gamma_i$. Taking the norm of both members of equation (1), and recalling that $a_i+b_i h_i=0$, we have $\| f(x_i+h_i) \| = \| c(\xi_i) h_i^2 \| \leqq \gamma_i \| h_i \|^2$. From $h_i = -b_i^{-1} a_i$, we have

$$\| h_i \| \leqq (1/\beta_i)\alpha_i, \tag{2}$$

Thus we may write

$$\alpha_{i+1} \leqq \gamma_i(1/\beta_i)^2\alpha_i^2 \text{ or } (\alpha_i\gamma_i/\beta_i^2)\alpha_i.$$

If the sequence of approximations x_i is to yield a root x' as a limit, it is necessary that the sequence of values of $\| f \|$, viz., $\alpha_{i+1}= \| f(x_{i+1}) \| = \| f(x_i+h_i) \|$ approach zero as a limit, and hence necessary although not sufficient that the limit of $(\alpha_i\gamma_i/\beta_i^2)$ shall not exceed unity. A refinement of these considerations suggests the theorems of the following section.

Justification of Newton's Method.—Theorem 1. *Let x_0 be given and let D be a positive number such that $f(x)$ is expansible in the form* (1) *for every x and $x+h$ (if either be denoted by u) in the domain $\| u-x_0 \| < D$. Let $f'(x)$ be non-singular in the domain, and B be a positive number such that $\| f^{-1}(x) \| < 1/B$ in the domain, and C a positive number such that $\| \frac{1}{2} f''(x) \| < C$ in the domain. Then Newton's Method yields a root x' in the domain, whenever $(CD/B) \geqq \varphi[C \| f(x_0) \| /B^2]$ where* $\varphi(\lambda) \equiv \lambda+\lambda^2+\lambda^4+\lambda^8+ \ldots +\lambda^{2^i}+ \ldots$.

An idea of the behavior of $\varphi(\lambda)$ may be had by noting that $\varphi(\lambda)$ has the unit circle in the complex plane for natural boundary, so that $\varphi(\lambda)$ is not defined for real positive λ's except when λ is not greater than one. Obviously $\varphi(\lambda)$ increases with positive λ's. Some of the numerical values of $\varphi(\lambda)$ are given by the following table.

λ	$\varphi(\lambda)$	λ	$\varphi(\lambda)$
0.	0.	0.566126	1.000000
0.1	0.110100	0.6	1.106678
0.2	0.241603	0.7	1.491062
0.3	0.398165	0.8	2.046305
0.356497	0.500000	0.9	3.017540
0.4	0.586255	1.	∞
0.5	0.816421	$\varphi[1/(k+1)] < 1/k, k > 0.$	

The proof of the above theorem is immediate. We shall use the previous notation. From (3), $\alpha_{i+1} \leqq (C/B^2)\alpha_i^2$, and from (2), $\| h_i \| \leqq (1/B)\alpha_i$. Hence $\| \Sigma_0^\infty h_i \| \leqq \Sigma_0^\infty \| h_i \| \leqq (1/B) [\alpha_0 + (C/B^2)\alpha_0^2 + (C/B^2)^3 \alpha_0^4 + \ldots + (C/B^2)^{2i-1} \alpha^{2^i} + \ldots] \leqq (B/C) \varphi[(C/B^2)\alpha_0]$. Thus if $\| \Sigma_0^\infty h_i \| = \| (x_0 + \Sigma_0^\infty h_i) - x_0 \| < D$, we have by hypothesis $x' = x_0 + \Sigma_0^\infty h_i$ as a vector in the domain, for which also $\lim_{i=\infty} \alpha_i = 0$.

Theorem 2. *If x' is a root of $f(x)$, and $\| f'(x')^{-1} \|$ exists and is equal to $1/\beta'$ and if $\| \frac{1}{2} f''(x) \| \leqq \beta'/D'$ in the region defined by $\| x - x' \| \leqq D'$ and if $f(x)$ is expansible in the form* (1) *in this region, then there is no other nonequivalent root of $f(x)$ in this region.*

Suppose if possible x'' is another nonequivalent root in the region. Then $f(x'') = f(x') + 'f(x') (x'' - x') + \frac{1}{2} f''(\xi') (x'' - x')^2$ and since x' and x'' are roots of $f(x)$, the equation $f'(x') (x'' - x') = f(x'') - f(x') - \frac{1}{2} f''(\xi') (x'' - x')^2$ reduces to $\| x'' - x' \| = \| f'(x')^{-1} \| \,.\, \| [-\frac{1}{2} f''(\xi')] (x'' - x')^2 \|$ or $\| x'' - x' \| \leqq (1/\beta') (\beta'/D') \| x'' - x' \|^2$, so that since $\| x'' - x' \| \mp 0$, $\| x'' - x' \| \geqq D'$, contrary to hypothesis.

The following theorem is a corollary of Theorems 1 and 2 where $D' = 2D$. The proof is trivial and will be omitted.

Theorem 3. *Let x_0 be given and $f(x)$ be expansible in the form* (1) *in the domain $\| \tilde{x} - x \| < B/(2C)$, these being defined as in Theorem 1. If $\| f(x_0) \| \leqq B^2/(3C)$, then Newton's Method yields a root x' in the domain and any root in the domain is equivalent to x'.*

Conclusion.—It may be noted that in these theorems no explicit use is made of h_0, x_1 and similar terms, derivable but not initially given. This feature and the statement of the general Theorems 1, and 2, as against Theorem 3, distinguish the form of these results from those of Dean H. B. Fine, already referred to. The steps in the proofs are in essential taken from the article by Dean Fine, where however the proofs hold only for a finite range, and are here extended to a general range, with a consequent notational simplification. The method here used of obtaining a general result by a mere reinterpretation of the case of one variable, offers several features of novelty and is suggested as, perhaps, of even more interest than the results obtained by its particular application to the present problem.

The present theory applies to nonlinear functionelles and integral equations with quadratic terms. A complete expansion in integral power series is not presupposed. In addition to interpretations of $\| z \|$ as $\sqrt{\Sigma\, z_{(n)}^2}$ and its generalizations, $\| z \|$ may be interpreted as $\max_n | z_{(n)} |$ or as $\Sigma | z_{(n)} |$ or in various other ways for the range (1, 0), with obvious extensions to the other cases. We may even with Riesz

(Equations Linéaires) introduce an arbitrary parameter p and write $\| z \| \equiv \sqrt[p]{\Sigma \left| z^{p}_{(n)} \right|}$ as an instance.

---

[1] H. B. Fine, these *Proceedings*, **2**, 546 (1916).

[2] J. Kürschak, *J. Math., Berlin*, **142**, 211–253 (1913).

THE COBALTAMMINES

By William D. Harkins, R. E. Hall and W. A. Roberts

KENT CHEMICAL LABORATORY, UNIVERSITY OF CHICAGO

Received by the Academy, August 22, 1916

The metallic elements whose properties of aggregation are such that they have low atomic volumes, or high 'cohesion,' form a series of complex ammonia compounds whose stability seems to increase with the cohesion, or decrease with the atomic volume of the metallic aggregate. These complex ammonia compounds, or ammines are of peculiar interest because, like the hydrates, their structure has not yet been explained in a satisfactory way from the standpoint of ordinary valence theories. Werner, however, has developed a special theory of valence which seems to fit very well the chemical behavior of these particular compounds, whether or not it is in agreement with the general behavior of other chemical substances.

Now perhaps the most important point to be established with respect to a series of salts such as these ammines, is the type of each salt with reference to its ionization in solution. When we turn to the work of Werner and of other investigators in this field, it is found that no work has been done which determines directly the type for these salts, although it might be considered that their molecular conductance as determined by Werner, together with their chemical action, makes it seem probable that they belong to the types according to which he has classified them.

The freezing-point method should give the most easy and certain method for the determination of the type of such salts as these, but it is just here that not only the work of other investigators, but even that of Werner[1] himself, does not agree with his theory. Since in this laboratory there is a double combination potentiometer system specially designed for us by Dr. W. P. White, to give with a fifty-couple copper-constantan thermocouple a reading to one twenty-thousandth of a degree, it seemed advisable to make for the first time a series of accurate freezing-point measurements upon the special series of cobaltammines which was most used by Werner in his work. The analyses were made by a Haber-Zeiss water interferometer, loaned to us by the Geophysical

Laboratory through the kindness of Drs. John Johnston and L. H. Adams. The only troublesome feature of the analyses was that caused by the color of these salts; and the difficulties encountered were such that it seems advisable in future work upon these salts to determine the concentration of the solutions by the conductivity method, since that method would render unnecessary the frequent gravimetric analyses which had to be resorted to, in order to make certain that the proper interferometric bands were being read.

In order to give a comparison of the results with those obtained formerly, the following table is inserted, which gives a few of our values of the equivalent freezing-point-lowerings ($\Delta t/N$) at rounded concentrations, upon a number of these salts, including the interesting isomeric salts dinitrotetrammine cobalt chloride (croceo) and dinitrotetrammine cobalt nitrate (flavo), which are shown by the freezing-point results to be of the di-ionic type. The values obtained by Werner and Herty and by Peterson are given within parentheses. The table also contains the corresponding values of the van't Hoff coefficient or mol-number i, and those of the percentage ionization (including those derived from the conductivity-ratio).

While the results of Werner and Herty were very severely criticised by Peterson,[2] it will be seen that they are of the right order of magnitude, while Peterson's own results are about 100% too large, a divergence difficult to explain. The table shows that, while for the croceo salt the values of Werner and Herty are only about 8% too low, for the hexammine they obtained values which were so high (see figure) as to indicate that they belong to other types of salts than is the case; but on the whole the deviations of their results are explained by the fact that they used a Beckmann thermometer, and by the fact that most of the cobaltammines are not very soluble, so that the measurements have to be made in very dilute solutions, which makes the percentage accuracy much less than it would be at such concentrations as are commonly used.

The salts investigated are given in the following list, together with the number of ions which we have found that each salt produces.

	Ions
Hexammine (luteo) cobalt chloride	4
Nitropentammine (xantho) cobalt chloride	3
Chloropentammine (purpureo) cobalt chloride	3
Dinitrotetrammine (flavo) cobalt chloride	2
Dinitrotetrammine (croceo) cobalt chloride	2
Trinitrotriammine cobalt	0
Potassium tetranitrodiammine cobaltate	2
Ammonium tetranitrodiammine cobaltate	2
Sodium cobaltic nitrate	4

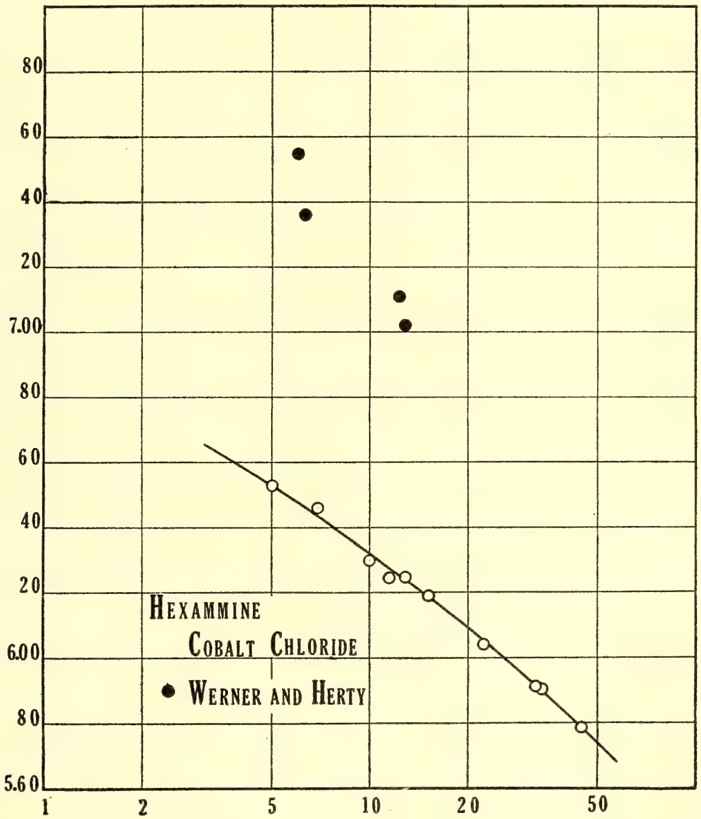
TABLE I

Concentration in milli-equivalents	Equiv. fr. pt. lowering $\Delta t/N$	Mol-number i		Percentage Fr.-Pt. Lower	Ionization from Conductivity
		for the cobaltammine	for potassium iodate		
		Flavo Cobalt Nitrate			
2	3.65	1.964	1.967	96.4	96.6
5	3.595	1.935	1.958	93.5	93.8
(5)	(7.00)*				
10	3.531	1.900	1.937	90.0	91.2
(10)	(6.60)*				
20	3.446	1.855	1.908	85.5	
(20)	(6.15)*				
		Croceo Cobalt Chloride			
2	3.66	1.97	1.967	97.0	95.4
(4.35)	(3.45)†				
(4.72)	(3.60)†				
5	3.622	1.949	1.958	94.9	93.2
10	3.590	1.932	1.937	93.2	90.7
20	3.556	1.914	1.908	91.4	
		Hexammine Cobalt Chloride			
5	6.53	3.51		83.7	
(6)†	(7.54)	(4.055)			
	(7.36)	(3.95)			
10	6.32	3.40		80.0	
(12)†	(7.11)				
	(7.02)				
20	6.09	3.22		74.0	
50	5.74	3.09		69.0	
		Chloro-pentammine Cobalt Chloride			
5	5.35	2.88		94.0	
10	5.18	2.785		89.0	
20	4.95	2.66		83.0	
		Nitro-pentammine Cobalt Chloride			
5	5.26	2.83		91.5	
10	5.13	2.76		88.0	
20	4.935	2.655		82.7	
		Potassium tetra-nitro-diammine			
2	3.675	1.975		97.5	
5	3.65	1.95		95.	
10	3.62	1.945		94.5	
20	3.57	1.92		92.	
50	3.505	1.885		88.5	
		Ammonium tetra-nitro-diammine			
2	3.66	1.95		95.	
5	3.62	1.945		94.5	
10	3.565	1.92		92	
20	3.49	1.88		88.	

* Peterson. † Werner and Herty.

These results are in accord with Werner's theory in so far as the number of ions is concerned.

When the apparent ionization values for these salts obtained by the conductivity and freezing-point methods are compared, it is found that they are closely similar, except in the case of the tetraionic salt, when the result calculated from the freezing-point is much the higher. The deviation is in such a direction as could be explained by the theory of Harkins[5] that all salts of this and other higher types give a considerable percentage of intermediate ions in solutions of about 0.1 normal concentration. It is of course realized that the apparent ionizations obtained by the two methods are in no cases directly comparable, since we do not know the variation in the activity of the different components.



While many of the cobaltammines are unstable and hydrolyze rapidly at higher temperatures, at zero degrees no trouble was experienced, except with dinitrotetrammine cobalt chloride when it was exposed to a bright light, and with sodium cobaltic nitrite.

The writers wish to express their indebtedness to the National Academy of Sciences for a grant of two hundred dollars from the Wolcott Gibbs Fund, which has been used to purchase the apparatus for this work. The complete paper will be published in the *Journal of the American Chemical Society.*

---

[1] Werner and Herty, *Zs. physik. Chem.*, **38**, 331 (1901).

[2] Petersen, *Zs. physik. Chem.*, **22**, 410 (1897); **39**, 249 (1902).

[3] Harkins, *J. Amer. Chem. Soc.*, **33**, 1807–73 (1911).

NATIONAL RESEARCH COUNCIL

REPORT OF THE FIRST MEETING OF THE COUNCIL

The first meeting of the Research Council was held in New York City on September 20, 1916.

Dr. M. I. Pupin, as Temporary Chairman, called the meeting to order at 3.10 p.m., and directed a roll-call of the members of the Council. There were present the following members: Messrs. Carty, Dunn, Goss, Hale, Herschel, Holmes, Keen, Manning, Marvin, Millikan, Noyes, Pickering, Pupin, Rand, Skinner, Squier, Stratton, Swasey and Vaughan.

The Temporary Chairman then called for nominations for Permanent Chairman. Dr. George E. Hale was nominated and unanimously elected. Dr. Hale then took the Chair and presided for the remainder of the meeting. Dr. Charles D. Walcott was elected First Vice-Chairman, and Mr. Gano Dunn, Second Vice-Chairman.

Dr. Hale, as Chairman of the Organizing Committee of the Council, announced an agreement between the National Academy of Sciences and the Engineering Foundation by which the Engineering Foundation has placed its funds at the disposal of the Council for a period of one year and has given the services of its Secretary, Dr. Cary T. Hutchinson, to the National Research Council, to serve as its Secretary. Dr. Hale announced that in accordance with this agreement the National Academy of Sciences has appointed Dr. Hutchinson Secretary of the National Research Council. Dr. Hutchinson was present and acted as Secretary to the meeting.

The Chairman then gave an extended account of the work done by the Organizing Committee during the summer months, dwelling particularly upon his trip to England and France, and upon the results that have been reached there by similar organizations of the men of science.

At the conclusion of the Chairman's remarks the organization of the Council was discussed; and the motion was made that an Executive Committee to consist of a chairman and nine members be appointed by the Chair, with the Chairman of the Council and President of the Academy as additional members ex officiis, this Committee to have full authority in the interim between meetings of the Council to carry out the purposes described in the Preliminary Report of the Organizing Committee (published in the August number of these PROCEEDINGS), it being understood that, in carrying out the general plan of work of the Council as there outlined, the Executive Committee shall not be limited to a narrow interpretation of the objects, but shall have full power to undertake similar or related work, even though not specifically included in that report. This motion was carried unanimously.

The Chairman then requested the members of the Council to express their views on the proposed work of the Council, and in particular to make suggestions covering definite lines of work that might fall within the sphere of activity of the Council.

Dr. Keen reported that upon notification of his selection to the Council he had made a careful investigation of the attitude of the colleges and medical schools in the vicinity of Philadelphia and that all approved the objects of the Research Council and would give their best support to it; also that the American Philosophical Society would co-operate with the Research Council in every particular. Professor Pickering dwelt on the opportunity for co-operation in astronomical work. Dr. Pupin spoke at length on the great value of co-operation in industrial research, as evidenced by the work of the Research Laboratory of the General Electric Company; he emphasized the difficulties in securing men adequately trained. Professor Conklin cited the results accomplished by co-operative effort in zoological work at the Woods Hole Biological Station, which he stated has passed resolutions endorsing the work of the Research Council and promising co-operation, as follows:

> WHEREAS, the National Academy of Sciences at the request of President Wilson has taken the initiative in bringing into co-operation existing governmental, educational, industrial and other research organizations and has brought about the establishment of a National Research Council which shall be representative of these various organizations, and whose object shall be the promotion of scientific research with especial reference to national welfare; and
>
> WHEREAS, the Marine Biological Laboratory is a striking illustration of the value of co-operation in the promotion of scientific research; and
>
> WHEREAS, we believe that human progress depends upon increasing knowledge of nature and that national welfare can most effectively be advanced by the co-operation of scientific investigators;
>
> *Now, Therefore, be it resolved,* that the Trustees of the Marine Biological Laboratory approve of the establishment of a National Research Council and agree to co-operate in all practicable ways in its work of co-ordinating and promoting scientific research.

Dr. Noyes called attention to the need of urging universities and colleges to interest more men in research work and to train them for it more effectively. Dr. Carty dwelt on the essential identity between pure and applied research, pointing out that they do not differ in kind but merely in the objects to be accomplished; that industrial research has as its objective commercial or industrial development, and that scientific research has no such immediate purpose. Colonel Squier spoke at some length on his experience in England during the past four or five months in communication with the War Department; he dwelt on the vital importance of pure scientific research and predicted that the United States will probably find it desirable to appoint as a cabinet officer a Secretary of Sciences in the not distant future. Dr. Stratton emphasized the growing need of more and better training of men for research work. Dr. Goss deprecated the distinction between pure and applied science, claiming that such distinction did not in reality exist and that all was science. Dr. Vaughan believed that much good could be done by the Council in stimulating the Congress of the United States to make greater appropriations to help pure science, and instanced the mean support given by the Government in many cases. Mr. Manning dwelt briefly upon the work of the Bureau of Mines and outlined the assistance given to the Bureau of Mines by the great chemical and smelting companies; he suggested similar assistance in the development of pure scientific research.

Dr. Millikan then presented a preliminary report for the Committee on the Newlands' Bill, which provides for the appropriation by the Government of $15,000 annually to each of the states, to be applied to research in engineering and applied science. After discussion the report was referred back to the Committee for further consideration with instructions that it be then referred to the Executive Committee of the Research Council.

A recess was then taken to enable the members of the Council to attend a dinner given in their honor at the University Club by Dr. George E. Hale.

The Council was called to order at 9.30 p.m. by the Chairman. The same members were present as at the afternoon meeting. Mr. Melville Stone, manager of the Associated Press, was also present as a guest of the Council.

At the invitation of the Chairman, the members discussed special features of the work that the Council might undertake.

Dr. Keen emphasized the necessity for making available the advances in surgery during the war. Dr. Vaughan emphasized the importance of this. Dr. Pupin spoke generally on the value of the proposed work of the Council. Dr. Holmes spoke of the possibilities of development in anthropological work and promised to present his views in writing later. Prof. Marvin believed it to be desirable to conduct a campaign of education to bring to congressmen a realization of the importance of scientific research.

Dr. Noyes made a general report on the work done by the Nitrate Supply Committee, of which he is the Chairman. He thought also that much good could be accomplished by encouraging the publication of a journal of popular science which should emphasize the relations of science, and especially of scientific research, to the public welfare, and that this might be done in co-operation with the American Association for the Advancement of Science.

Mr. Rand dwelt upon the essential need of co-operation with the great industrial research organizations, instanced the assistance that the Research Laboratories of the U. S. Steel Corporation had rendered to the Institute of

Mining Engineers, and expressed the belief that the co-operation of the U. S. Steel Corporation with the Research Council could readily be secured. Mr. Skinner outlined the work of the Westinghouse Research Laboratory and suggested specifically investigations on fuel for internal engines, on the prevention of corrosion, and on vocational diseases. Mr. Herschel pledged the support of the American Society of Civil Engineers, Dr. Goss dwelt on the necessity of co-operation in institutions of the Middle West, and suggested a committee on relations of the universities to the National Research Council.

Dr. Vaughan called attention to the need of proper means of publicity, and Mr. Melville Stone, the guest of the Council, made a forceful address on the relations of the Associated Press to men of science and pledged the firm support of his organization to the objects of the Research Council.

Mr. Dunn outlined in considerable detail the relations of the Engineering Foundation to the National Research Council and praised the generosity of Mr. Swasey in giving his liberal support to the Foundation. Mr. Swasey in reply minimized his own share in the development of the Foundation. He suggested that a committee on optical and commercial glass would find a large field for useful work.

The Chairman announced the appointment of the following six members of the Executive Committee: Messrs. J. J. Carty, E. G. Conklin, Gano Dunn, A. A. Noyes, M. I. Pupin and V. C. Vaughan. He stated that the other four members would be appointed after further consideration.

It was voted that the Council should meet at the call of the Executive Committee.

A vote of thanks to the Chairman for his invaluable services was unanimously adopted.

After further general discussion the meeting adjourned at 11.30 p.m.

CARY T. HUTCHINSON, *Secretary.*

REPORTS OF MEETINGS OF THE EXECUTIVE COMMITTEE

The first meeting of the Executive Committee was held in New York on September 21, 1916. There were present Messrs. J. J. Carty, E. G. Conklin, Gano Dunn, G. E. Hale, A. A. Noyes, Michael Pupin and V. C. Vaughan, and C. T. Hutchinson the Secretary of the Council.

Dr. J. J. Carty was elected Chairman; and Dr. C. T. Hutchinson, Secretary of the Committee.

It was voted that the terms of the present members of the Research Council and of the Executive Committee be deemed to expire on January 1, 1918.

The following resolution was adopted, expressing the general policy to be followed by the Council in the promotion of research.

Resolved, that the efforts of the Research Council shall be uniformly directed to the encouragement of individual initiative in research work, and that co-operation and organization, as understood by the Research Council, shall not be deemed to involve restrictions or limitations of any kind to be placed upon research workers.

The following resolution was adopted, inviting the American Association for the Advancement of Science to co-operate with the Research Council.

Resolved, that the American Association for the Advancement of Science be informed that the National Research Council has been organized by the National Academy of Sciences at the request of the President of the United States for the purpose of bringing into co-operation existing governmental, educational, industrial, and other research organizations, with the object of encouraging the investigation of natural phenomena, the increased use of scientific research in the development of American industries, the employment of scientific methods in strengthening the national defense, and such other applications of science as will promote the national security and welfare; and that the Association, which has itself established the Committee of One Hundred on Research, be invited to co-operate with the Research Council in the promotion of research, and that to this end it be asked to appoint a committee of three to meet with a similar committee of the Research Council to consider how such co-operation can be made most effective.

As members of this Committee on behalf of the Research Council, Dr. Welch, president of the Academy, and Messrs. Conklin and Noyes were appointed.

The following committees were also appointed:

A committee to arrange for an office for the Research Council in the Engineering Societies Building, consisting of Messrs. Pupin and Dunn.

A Committee on Rules and Procedure, consisting of Messrs. J. J. Carty (Chairman), E. G. Conklin and A. A. Noyes.

A Committee on Publication, of which Mr. Gano Dunn was made Chairman and given power to name his associates.

A Committee on Research in Educational Institutions, consisting of G. E. Hale, Chairman, J. S. Ames, R. H. Chittenden, J. M. Coulter, G. N. Lewis, G. H. Parker, Harold Pender, C. R. Van Hise and F. J. E. Woodbridge; this committee to consider general plans for the promotion of research in educational institutions, and to have power to arrange for local committees in each institution.

A Committee on the Promotion of Industrial Research, Dr. J. J. Carty, Chairman, with functions in its field somewhat similar to those of the Committee on Research in Educational Institutions.

A Committee on a National Census of Research, Dr. Stratton, Chairman, to prepare a national census of equipment for research, of the men engaged in it, and of the lines of investigation pursued in co-operating Government bureaus, educational institutions, research foundations, and industrial research laboratories.

The second meeting of the Executive Committee was held in New York on September 29, 1916. Messrs. J. J. Carty, E. G. Conklin, Gano Dunn, G. E. Hale, A. A. Noyes, Michael Pupin, V. C. Vaughan, W. H. Welch and the Secretary were present.

Mr. Dunn reported that the United Engineering Society had granted for a period of one year from October 1, 1916, free of assessment, rooms number 901 and 902, for the use of the Engineering Foundation, to serve as New York office for the National Research Council, these rooms being those about to be vacated by the Naval Consulting Board.

After considerable discussion of the financial affairs of the Council it was voted that a Committee on Finance, to consist of three members, should be appointed by the Chairman to consider the question of obtaining additional funds for the Research Council and the method of handling such funds as it has or as it may acquire.

It was voted to recommend to the President of the Academy that Dr. Marston T. Bogert, of Columbia University, Dr. Russell H. Chittenden, of Yale University, and Dr. Raymond Pearl, of the Maine Agricultural Experiment Station, be invited to become members of the Council.

It was voted that joint committees on research in various branches of science be formed in cooperation with the corresponding national scientific societies.

Dr. Vaughan was also asked to report on the need of research in the preparation of therapeutic agents.

Dr. S. W. Stratton of the Bureau of Standards, and Dr. Raymond Pearl of the Maine Agricultural Station were appointed members of the Executive Committee by the Chairman of the Research Council.

CARY T. HUTCHINSON, *Secretary.*

ORGANIZATION OF THE RESEARCH COUNCIL

The National Research Council was formally organized at a meeting held in New York City on September 20, 1916. It was established by the National Academy of Sciences at the request of the President of the United States. The members of this Council have been appointed by the President of the Academy, after consultation with the presidents of leading national scientific societies. The representatives of the Government were appointed by the President of the United States. The Council is to be gradually enlarged by the addition of new members who are to serve as chairmen of important committees or who are otherwise to engage in some special capacity in the work of the Council.

The organization of the Council is at present as follows:

OFFICERS AND EXECUTIVE COMMITTEE

Chairman, GEORGE E. HALE.

Vice-Chairmen, CHARLES D. WALCOTT and GANO DUNN.

Secretary, CARY T. HUTCHINSON.

Executive Committee, JOHN J. CARTY (Chairman), WILLIAM H. WELCH (ex officio), GEORGE E. HALE (ex officio), EDWIN G. CONKLIN, GANO DUNN, ARTHUR A. NOYES, RAYMOND PEARL, MICHAEL I. PUPIN, S. W. STRATTON, V. C. VAUGHAN (others to be appointed).

MEMBERS

Dr. L. H. Baekeland, Yonkers, N. Y.
Dr. Marston T. Bogert, Professor of Organic Chemistry, Columbia University.
Dr. John A. Brashear, Allegheny, Pa.
Dr. John J. Carty, Chief Engineer, American Telephone and Telegraph Co.
Dr. Russell H. Chittenden, Director, Sheffield Scientific School, Yale University.
Dr. Edwin G. Conklin, Professor of Zoology, Princeton University.
Dr. John M. Coulter, Professor of Botany, University of Chicago.
Major-General William Crozier, Chief of Ordnance, U. S. A.
Mr. Gano Dunn, President, The J. G. White Engineering Corporation.
Dr. Simon Flexner, Director, Rockefeller Medical Institute.
Brigadier-General William Crawford Gorgas, Surgeon-General, U. S. A.
Dr. W. F. M. Goss, Dean of Engineering, University of Illinois.
Dr. George E. Hale, Director, Mt. Wilson Solar Observatory.
Mr. Clemens Herschel, President, American Society of Civil Engineers.
Dr. William H. Holmes, Curator, United States National Museum.
Dr. W. W. Keen, President, American Philosophical Society.
Mr. Van H. Manning, Director, Bureau of Mines.
Prof. Charles F. Marvin, Chief, United States Weather Bureau
Prof. A. A. Michelson, Director, Ryerson Physical Laboratory, University of Chicago.
Dr. Robert A. Millikan, Professor of Physics, University of Chicago.
Dr. Arthur A. Noyes, Director, Research Laboratory of Physical Chemistry, Massachusetts Institute of Technology.
Dr. Raymond Pearl, Director, Maine Agricultural Experiment Station.
Prof. E. C. Pickering, Director, Harvard College Observatory.
Dr. Michael I. Pupin, Professor of Electro-Mechanics, Columbia University.
Mr. Charles F. Rand, President, United Engineering Society.
Dr. Theodore W. Richards, Director, Walcott Gibbs Memorial Laboratory, Harvard University.
Mr. C. E. Skinner, Director, Research Laboratory, Westinghouse Electric and Manufacturing Co.
Lieutenant-Colonel George O. Squier, Chief of Aviation, U. S. A.
Dr. S. W. Stratton, Director, Bureau of Standards.
Mr. Ambrose Swasey, Cleveland, Ohio.
Chief Constructor David W. Taylor, U. S. Navy.
Dr. Elihu Thomson, Swampscott, Mass.
Dr. C. R. Van Hise, President, American Association for the Advancement of Science.
Dr. Victor Clarence Vaughan, Director, Medical Research Laboratory.
Dr. Charles D. Walcott, Secretary, Smithsonian Institution.
Dr. William H. Welch, President, National Academy of Sciences.
Dr. W. R. Whitney, Director, Research Laboratory, General Electric Co.

CONTENTS

Page

ASTRONOMY.—PRELIMINARY RESULTS ON THE COLOR OF NEBULAE . . . . *By Frederick H. Seares* 553

CHEMISTRY.—THE ACTION OF ALKALI IN THE PRODUCTION OF LIPOLYTICALLY ACTIVE PROTEIN . . . . *By K. George Falk* 557

BOTANY.—THE EXCRETION OF ACIDS BY ROOTS . . . . *By A. R. Haas* 561

ASTRONOMY.—SPECTROGRAPHIC OBSERVATIONS OF RELATIVE MOTIONS IN THE PLANETARY NEBULAE . . . . *By W. W. Campbell and J. H. Moore* 566

BOTANY.—NEW DETERMINATIONS OF PERMEABILITY . . . . . *By S. C. Brooks* 569

MATHEMATICS.—POINT SETS AND CREMONA GROUPS. PART III. *By Arthur A. Coble* 575

PHYSICS.—THE INTERFERENCES OF SPECTRA BOTH REVERSED AND INVERTED . . . *By Carl Barus* 576

GENETICS.—SEX INTERGRADES IN A SPECIES OF CRUSTACEA . *By Arthur M. Banta* 578

MATHEMATICS.—SOME PROBLEMS OF DIOPHANTINE APPROXIMATION: A REMARKABLE TRIGONOMETRICAL SERIES . . . . . *By G. H. Hardy and J. E. Littlewood* 583

CHEMISTRY.—STERIC HINDRANCE AND THE EXISTENCE OF ODD MOLECULES (FREE RADICALS) . . . . *By Gilbert N. Lewis* 586

MATHEMATICS.—NEWTON'S METHOD IN GENERAL ANALYSIS . *By Albert A. Bennett* 592

CHEMISTRY.—THE COBALTAMMINES . . . . *By William D. Harkins, R. E. Hall and W. A. Roberts* 598

NATIONAL RESEARCH COUNCIL.—REPORT OF THE FIRST MEETING 602

——. REPORTS OF MEETINGS OF THE EXECUTIVE COMMITTEE 605

——. ORGANIZATION OF THE RESEARCH COUNCIL 607

VOLUME 2 NOVEMBER, 1916 NUMBER 11

PROCEEDINGS

OF THE

NATIONAL ACADEMY OF SCIENCES

OF THE

UNITED STATES OF AMERICA

EDITORIAL BOARD

ARTHUR A. NOYES, *Chairman*
EDWIN B. WILSON, *Managing Editor*
ARTHUR L. DAY, *Home Secretary*
GEORGE E. HALE, *Foreign Secretary*

J. J. ABEL
W. B. CANNON
J. McK. CATTELL
J. M. CLARKE
E. G. CONKLIN
J. M. COULTER
C. B. DAVENPORT
SIMON FLEXNER
E. B. FROST
R. G. HARRISON
W. H. HOLMES
J. P. IDDINGS
R. A. MILLIKAN
E. H. MOORE
ALEXANDER SMITH

Publication Office: Williams & Wilkins Company, Baltimore
Editorial Office: Massachusetts Institute of Technology, Cambridge
Home Office of the Academy: Washington, D. C.

Entered as second-class matter at the Post Office at Baltimore, Md., under the Act of August 24, 1912

INFORMATION TO CONTRIBUTORS

The Proceedings is the official organ of the Academy for the publication of brief accounts of important current researches of members of the Academy and of other American investigators, and for reports on the meetings and other activities of the Academy. Publication in the Proceedings will supplement, not replace, that in journals devoted to the special branches of science. The Proceedings will aim especially to secure prompt publication of original announcements of discoveries and wide circulation of the results of American research among investigators in other countries and in all branches of science.

Articles should be brief, not to exceed 2500 words or 6 printed pages. Technical details of the work and long tables of data should be reserved for publication in special journals. But authors should be precise in making clear the new results and should give some record of the methods and data upon which they are based. The viewpoint should be comprehensive in giving the relation of the paper to previous publications of the author or of others and in exhibiting where practicable, the significance of the work for other branches of science.

Manuscripts should be prepared with a current number of the Proceedings as a model in matters of form, and should be typewritten in duplicate with double spacing, the author retaining one copy. Illustrations should be confined to text-figures of simple character, though more elaborate illustrations may be allowed in special instances to authors willing to pay for their preparation and insertion. Particular attention should be given to arranging tabular matter in a simple and concise manner.

References to persons should include initials; citations of literature, numbered consecutively, should be placed at the end of the article; and short footnotes should be avoided. Indicate 'subject' for 'Contents.'

Papers by members of the Academy may be sent to Edwin Bidwell Wilson, Managing Editor, Mass. Institute of Technology, Cambridge, Mass. Papers by non-members should be submitted through some member.

Proof will be sent to authors only on their request, in order to avoid delay in publication. Authors are therefore requested to make final revisions on the typewritten manuscripts. The editors cannot undertake to do more than correct obvious minor errors.

Reprints should be ordered at the time of submission of manuscript. They will be furnished to authors at cost, approximately as follows:

Reprints of - -	2 pp.	4 pp.	6 pp.	8 pp.	Covers extra
Charge for first 100 copies	$1.10	$1.45	$2.50	$2.50	$2.50
Charge for additional 100s	.35	.60	1.10	1.10	1.00

INFORMATION TO SUBSCRIBERS

Subscriptions at the rate of $5.00 per annum should be made payable to the National Academy of Sciences, and sent to Williams & Wilkins Company, Baltimore, or Arthur L. Day, Home Secretary, National Academy of Sciences, Smithsonian Institution, Washington, D. C. Single numbers, $0.50.

Copyright, 1916, by the National Academy of Sciences

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES
Volume 2 NOVEMBER 15, 1916 Number 11

PATH DIFFERENCES WITHIN WHICH SPECTRUM INTERFERENCES ARE OBSERVABLE

By Carl Barus
DEPARTMENT OF PHYSICS, BROWN UNIVERSITY

Received by the Academy, September 9, 1916

1. *Reflecting systems.*—Certain earlier results (Carnegie Publications, No. 249, 1916, §56) made it seem plausible that the path differences within which interferences are obtained (i.e., the *apparent* lengths of uniform wave trains) increase as the dispersion, to which the incident collimated white light (L figure 1) is subjected, is made continually greater. With this quest in view the aim is to produce the interferences by one and the same method, but with a successive variation of the dispersion of the spectra. The method (figure 1) was first selected for this purpose (M, N, P', mirrors; T, telescope; s,s', screens), inasmuch as the use of prisms or gratings of different dispersive power at P meets the requirements, while spectra of the first and second order are equally available.

In work of this kind the spectra must be bright; otherwise the fine lines will escape detection. Deficient values will thus be obtained if the spectra are too dark. Moreover the results can not furnish data of precision, since the exact instant at which fringes, continually decreasing in size, have actually vanished, can not be fixed; and it is the fine fringes which furnish a considerable amount of the displacement. The differences, however, are so large, that orders of values are apparent, more than sufficient to substantiate the argument.

It is possible that the method (figure 1) gives the half ranges only, since the efficient pencils of light, C C', can not cross each other when M is displaced. The methods applied will nevertheless be trustworthy, since they are identical, the same telescope and other appurtenances being used throughout. Later, the grating method is to be suitably modified for corroborative experiments.

The first series of measurements was obtained with a 60° prism at P, the dispersive power $d\theta/d\lambda$ being computed (approximately) from Cauchy's equation, so that in wave length λ

$$d\delta/d\lambda = 4B \sin \tfrac{1}{2} \varphi/\lambda^3 \cos \tfrac{1}{2} (\varphi + \delta),$$

φ being the prism angle (60°) and δ the angle of minimum deviation. The dispersion constant B was estimated to be 4.6×10^{-11}.

In the remaining series with a grating at P, $d\theta/d\lambda = 1/D \cos \theta$, the usual expression, θ being the angle of diffraction and D the grating space. The dispersive power thus increases from about 800 to 17,000, over twenty times. Throughout this whole enormous range good fringes were obtained.

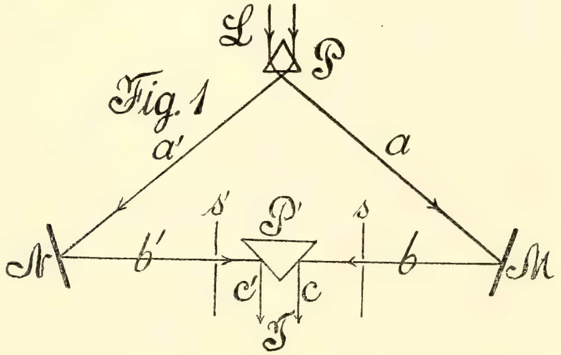


Fig. 1

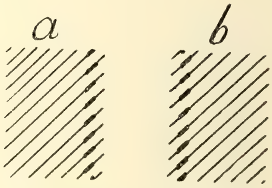


Fig. 2

The values, e, show the normal displacements of the opaque mirror M, during the presence of fringes, and of the opaque mirror N, as specified. Of these, e_M is systematically larger than e_N for reasons due to residual curvature in the mirrors and surfaces, whereby fringes on the left (N) vanish sooner than those on the right (M). The datum, y, is the displacement of the right angled reflecting prism P', parallel to the component rays bb'. This value is necessarily smaller than e, as will be shown elsewhere. All measurements were frequently repeated and the means finally taken for comparison with $d\theta/d\lambda$.

In the experiments with a ruled grating at P and a concave grating reflecting at P', the phenomenon of figure 2 was observed. A wide field of faint fringes was visible, enormously accentuated and clear in the narrow strip of the linear phenomenon. As the micrometer mirror at M moves forward, these faint fringes shift bodily across the stationary bright linear strip, beginning therefore with the pattern a

and ending with b. The faint fringes follow the rules of displacement interferometry.

TABLE 1

RANGE OF DISPLACEMENT, e, y, VARYING WITH DISPERSION

METHOD	$e_M \times 10^3$	$e_N \times 10^3$	$y \times 10^3$	θ	$d\theta/d\lambda$
	cm.	cm.	cm.		
60° prism at P	28	24	21	49°45′	760
Grating ($D = 352 \times 10^{-6}$) at P	161	136	108	9°39′	2,880
Same. 2d order	250	230	155	19°34′	6,030
Film grating ($D = 167 \times 10^{-6}$) at P	302	236	190	20°40′	6,400
Same. 2d order	470	420	440	44°56′	16,900

In addition to the data of the table, a large number of miscellaneous tests were made with the reflecting prism in different positions. Unless brought too far to the rear, when the beams are lost at the edge and e too small, the results for fine and coarser fringes were of the same order.

The data for e are not sufficiently regular in the dispersive powers above 1000 for graphic treatment (it is probable that at 16,900 the sliding along the prism surface is interfered with) but the data for the path difference, $2y$, are available. All the data, in consideration of their limitations, bear out the inference that the range of displacement within which fringes are seen, increases in marked degree with the dispersion, the average initial ratio $2y/(d\theta/d\lambda)$ is about 60×10^{-6} cm.

A very surprising result in these experiments is the efficiency of the film grating in series IV and V, not only in the first but in the second order of spectra.

After these experiments an attempt was made to obtain similar results with the more comprehensive method of two gratings G and G' (transmitting G at P and reflecting G' at P') with an appropriate change of the angle δ. But here the choice of gratings with satisfactory constants was limited and with high double dispersion the fields were apt to be too dark. Good results were obtained with the 60° prism and concave grating and with the ruled grating together with the latter. The data again showed marked increase of displacement with the dispersion $d\theta/d\lambda$.

2. *Diffraction at M, N, replacing reflection.*—The present method of observing interferences in the zeroth, first, second, third and even fourth order, successively, without essential change of the parts of the apparatus, is noteworthy. In figure 3, the incident light L from the collimator is separated into two component beams a and a' by the 60° prism, P. This is essential here, as an abundance of light is needed

(sunlight should be focused by a large lens of long focus—5 feet—on the slit). The rays a, a', are then either reflected or diffracted in any order, by the identical plane reflecting gratings G, G', into the collinear rays b, b'. These are reflected by the silvered right angled prism P' and observed in a telescope at T. G and G' and also P' should be on micrometers, so that the corresponding displacements, e, e', normal to G and G' and y in the direction $b\,b'$, may be registered.

The adjustments, if symmetry were demanded, would be cumbersome; for in addition to precise modification of the position and orientation of the prisms P, P', the grating requires fine adjustment and a means of securing parallelism of the rulings. But an approximate adjustment does very well and no pains were taken to secure symmetry. The spectra were brilliant in the low order work; but even in the fourth order the light was adequate. One may note that the gratings enhance the dispersion of the prism P, which is relatively small. Table 2 is an example of results.

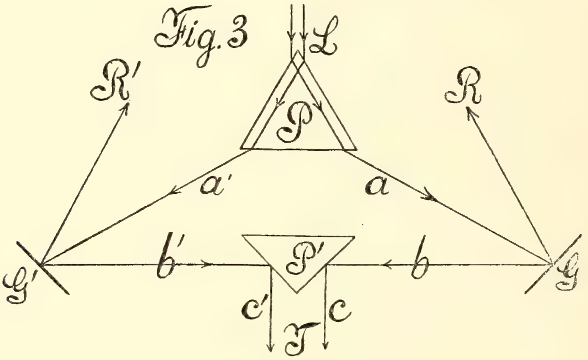


Fig. 3

TABLE 2

RANGES OF DISPLACEMENT, e, VARYING WITH DISPERSION. PAIRED GRATINGS (SPACE $D = 200 \times 10^{-6}$ cm) AND 60° PRISM. $\theta = 46°$, $\delta = 44°$. PATH DIFFERENCE $x = 2e \cos \delta/2$

ORDER	OBSERVED		
	$e \times 10^3$	$x \times 10^3$	$d\theta/d\lambda$
	cm.	*cm.*	
0	38	70	760
1	190	350	3,490
2	420	780	6,440
3[1]	520	962	9,930
4[1]	580	1,070	14,800

[1] Fringes too faint.

The fringes in the zeroth order were good and strong not inferior to any of the others, but unfortunately too shortlived. In the fourth order the fringes are weak (although the enormous sodium doublets stand out clearly), doubtless from excess of extraneous light. Here also it is difficult to prevent the beam from vanishing at the edge of the prism P'. Hence the anomalously small displacement, a discrepancy already quite manifest in the third order.

The present experiments furnish a striking example of the uniform breadth of the strip of spectrum carrying the fringes, quite apart from the dispersion of the spectra. In the prism spectrum, where the sodium doublets are indicated by a hair line just visible, to the fourth order spectra, where they stand apart like ropes, the linear interference phenomenon has the same width.

The computation of the dispersive power in these cases is peculiar. It will be seen from figure 3, that the angle ($\delta = 44°$) between the incident ray a and the diffracted ray b is constant and is $\delta = \theta + i$ in the first and second and $\delta = \theta - i$ in the third and fourth order. Hence in succession (i changing sign after the 2d order)

$$\sin(\delta - i) - \sin i = \lambda/D,$$
$$\sin(\delta - i) - \sin i = 2\lambda/D, \text{ etc.,}$$

from which equations the angle i may be computed. I did this with sufficient accuracy graphically.

Since $d\theta = di$, apart from sign, it follows that the dispersing power is

$$-d\theta/d\lambda = n/D\,(\cos i + \cos(\delta - i))$$

where n is the order of the spectrum. With the values of i given, the data for $d\theta/d\lambda$ were finally computed. The dispersive power of the prism was computed as above and is to be added to all the succeeding dispersive powers. The path difference, $x = 2e \cos \delta/2$ here corresponds to $2y$ above. Through the second order the rate $x/(d\theta/d\lambda)$ is about 120×10^{-6}. This value is larger than the former owing to incidental conditions. Similarly the proportionate effect of dispersion breaks down in the third and fourth orders, as already stated. The spectra themselves were still adequately bright, but the fringes were faint for some reason and I failed to make them stronger.

NON-REVERSED SPECTRA OF RESTRICTED COINCIDENCE

By Carl Barus

DEPARTMENT OF PHYSICS, BROWN UNIVERSITY

Received by the Academy, September 15, 1916

In figure 1 the white ray L from the collimator is diffracted by the grating G and the two spectra a and a' thereafter reflected by the parallel opaque mirrors M and N, to be again diffracted by the grating G'. The rays are observed by a telescope at T. If the gratings, G, G' have the same constant, it is obvious that the field of the telescope will show a sharp white image of the slit, for each mirror. If M N G G' are adjusted for symmetry by aid of the adjustment screws on each and the rulings are parallel, the two white slit images will coincide horizontally and vertically. If now a direct vision spectroscopic prism, or a direct vision prism grating G'' is placed in front of the telescope, the superposed white slit images will be drawn out into overlapping non-reversed spectra, which will usually show a broad strip of interference fringes.[1]

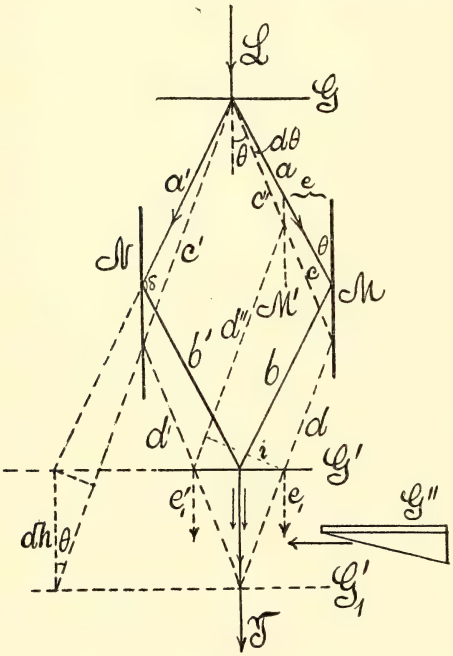


FIG. 1.

When first found, the fringes were very fine parallel lines, filling an irregular strip in the orange-yellow region, and it was already obvious that an enormous play of the micrometer screw at M would be permissible.

A number of film gratings were tested and the best samples selected ($D = 175 \times 10^{-6}$), although the dispersion was still too large and the D lines not clear. To secure more light, a beam of sunlight 15 cm. in diameter was condensed to a focus on the slit by a large lens of about 2 meters in focal distance. The illumination was now adequate and the fringes were found at once. These fringes, in view of more accurate adjustment, were also larger than before.

Figure 1 shows, if ab, $a'b'$ and cd, $c'd'$ are pairs of corresponding rays of the same order but different wave lengths, λ and λ', respectively, that for the given position of G and G', only the rays a a' issue coincidently at T. The rays cd, $c'd'$ issue at e_1, e'_1 and though brought to the identical focus by the telescope, the distance e_1 e'_1 may be too large to admit of appreciable interference. Hence the colored strip within which interferences occur will comprise those wave lengths which lie very near λ, whereas the colors lying near λ', etc., will be free from interference.

If the mirror M is displaced parallel to itself to M' by the micrometer screw, the rays c'' d'', and c' d' will now coincide at e'_1, whereas the rays from a b and $a'b'$ will no longer issue coincidently and may not interfere. Thus the interferences are transferred as a group from rays lying near λ to rays lying near λ'. It is obvious therefore that with the displacement of M, the strip carrying interferences will shift through the spectrum and that a relatively enormous play of the micrometer slide at M will be available without the loss of interferences. In fact a displacement, e, of over 3 cm. of M normal to itself, produced no appreciable change in the size or form of fringes, but they passed from the green region into the red. The fringes as seen with a fine slit were straight parallel strong lines. They did not thin out to hair lines at their ends, nor show curvature, as one would be inclined to anticipate. On the contrary, they terminated rather abruptly at the edges of a strip occupying about one-fourth of the visible length of the spectrum.

It follows, from figure 1, that the displacement of M does not change the lengths of rays; for they are enclosed between parallel planes, as it were. Since the double angle of reflection is here $\delta = 180° - 2\theta$, where θ is the angle of diffraction of G and G', the displacements of M over a normal distance e will shorten the path at M in accordance with the equation

$$n\,\lambda = 2e \cos \delta/2 = 2e \sin \theta \tag{1}$$

where n is the number of fringes passing at wave length λ.

This equation[2] is not obvious, as for constant λ, the distance between G and G' measured along a given ray, for any position of M or N, is also constant. The equation may be corroborated by drawing the diffracted wave front i for instance, which cuts off a length $2e \sin \theta$ from d''.

Since $\sin \theta = \lambda/D$ if D is the grating space, the last equation becomes

$$n = 2e/D$$

or per fringe

$$\delta e = D/2,$$

a remarkable result, showing that the displacement of the mirror M per fringe is independent of wave length and equal to half the grating space. An interferometer independent of λ and available throughout relatively enormous ranges of displacement is thus at hand. It appears that it is also independent of the angle of incidence at G.

In case of the given grating and sodium light $\theta = 19° 37'$. Hence if δe is the displacement per fringe[2]

$$e = \lambda/2 \sin \theta = 10^{-6} \times 88 \text{ cm.}$$

Actuating the micrometer at M directly by hand the following rough data were successively obtained from displacements corresponding to 10 fringes:

$$10^6 \times \delta e = 65 \quad 95 \quad 90 \quad 80 \quad 60 \text{ cm.}$$

Without special precaution the fine fringes can not be counted closer than this, so that the data are corroborative.

The fore and aft motion of G' produces no separate shift of fringes while the fringe bearing strip is displaced as a whole in mean wave length. Figure 1 shows at once that if G' were displaced to G'_1, the λ rays b b' would lose their coincidence in T, while that property would now be possessed by the λ' rays d d'. But the same path difference is added to both d and d'. The ratio of corresponding displacements at M and G' is $\tan \theta : 1$. Equation (1) is of interest in interferometry, in view of the very long ranges of displacements available. For such purposes gratings of lower dispersion (preferably ruled gratings or else prisms) may be used to obtain greater luminous intensity in the spectrum. Path difference may also be introduced by compensators. If a thin sheet of mica is moved in either b or b' there is a lively skirmish of fringes, but they do not change size appreciably. A plate 2.8 millimeters thick with strong fringes horizontal in the yellow, if placed in the b' pencil produces hair lines inclined toward the left in the red; if placed in the b pencil, hair lines inclined to the left in the green; etc. Plates were tested up to 2 cm. To fully exhibit their effect it is necessary to produce the elliptic fringes, presently to be referred to. The shift from red to green, if produced without compensators by the displacement of M, shows scarcely any variation of fringes, either as to size or inclination.

To change the size of fringes it is easiest to rotate the grating G' (relatively to G) on a horizontal axis normal to itself. They then both rotate and grow coarser, usually attaining the maximum of size when the fringes are vertical. Fringes quite large and black may be obtained in this way, which are naturally much more sensitive. Size may also be changed by compensators and this method is usually more available.

In addition to the above experiments, work was done at some length with homogeneous light, with gratings of different constants, etc., which cannot be detailed here. The most interesting result was obtained with a *wide* slit and white light. It was shown that the fringes are ultimately nearly confocal ellipses of enormous eccentricity and with the major axis vertical. To produce and center them, the refraction (dispersion) of plates is advantageous, if not necessary.

It is now of interest to turn to the displacement of G', normal to itself, and to consider the resolving power of the system. For the latter bears a close analogy to the experiments made in a preceding paper. (Carnegie Publication, No. 249, § 37 et. seq., 1916) on the behavior of crossed rays. If G' is displaced to G'_1, over a distance $e' = dh$ (see figure 1 where h is the distance apart of G and G'), the rays λ' meeting in T will now be in the same condition as were originally the rays λ. In other words, e'_1 and e'_1 have become coincident at G'. If we assume that rotationally the same type of fringe results in these cases, and if $\lambda' - \lambda = d\lambda$, $\theta - \theta' = d\theta$, (for the passage of bb' into dd' is in the direction from red to violet)

$$d\,\theta = dh \sin\theta \cos\theta/h, \qquad \text{nearly.}$$

Since $\lambda = D \sin\theta$ and $d\,\lambda = -D\cos\theta\, d\,\theta$, this may be changed to

$$d\,\lambda/\lambda = dh\,(1 - \lambda^2/D^2)/h$$

when D is the grating constant.

The present method, apart from any practical outcome, is worth pursuing because of the data it will furnish of the width of the strip of spectrum carrying interference fringes, under any given conditions. For here the spectra are not reversed or inverted and the latitude of interference or diffraction throughout λ is much broader than in case of reversed spectra. But for this purpose films will not suffice and rigid refracting systems must be devised, and the grating constants must be quite identical.

---

[1] This article is a note from a Report to the Carnegie Institution of Washington.

[2] The equation is also true for oblique incidence. But for this and the use of homogeneous light with a wide slit, the availability of gratings of different constants, etc., the report mentioned may be consulted.

THE EQUILIBRIUM BETWEEN ACIDS AND BASES IN SEA WATER

By Lawrence J. Henderson and Edwin J. Cohn

WOLCOTT GIBBS MEMORIAL LABORATORY, HARVARD UNIVERSITY

Received by the Academy, October 4, 1916

The alkaline reaction of sea water and the chemical equilibria that define it concern not only the oceanographer but also the geologist and the biologist. For information upon this subject must lead to further knowledge regarding geographical variations in the composition of the sea, the chemical reactions which have taken place during the existence of the ocean, the present movements of carbonic acid upon the earth, and the environmental relations of sea water to marine flora and fauna.

In the present investigation it has been sought, first, to determine the influence of carbon dioxide tension and of salinity upon the hydrogen ion concentration of sea water and, secondly, to construct a solution of exactly known composition, which shall reproduce the behavior of sea water when subjected to changes in carbonic acid concentration. This task has been accomplished with such accuracy as existing methods of investigation afford. The results define the acid-base relation between marine organisms and their environment, the fluctuations in this relationship, and the means by which such fluctuations are controlled. They also yield evidence concerning the geological history of the ocean. When combined with the observations of Palitzsch, they lead to a general description of the geographical variation in the chemical equilibria of sea water and to information concerning the gradients along which carbon dioxide is at present moving in the sea and in the air.

1. *Hydrogen Ion Concentration and Carbon Dioxide Tension of Sea Water.*—A large number of estimations of the hydrogen ion concentration of sea water, when saturated with carbon dioxide at tensions between 0 and 760 mm., have been made. The hydrogen ion concentrations were determined colorimetrically, following exactly the procedure of Palitzsch,[1] and at first with his kind assistance,[2] so that the results are strictly comparable with his. The standard solutions were in all cases analyzed with the hydrogen electrode. Carbon dioxide tensions, after equilibrium had been established, were determined either by chemical analysis of the gas phase or with the manometer. The following table corresponds to the curve which best fits the data of these experiments.

Salinity per cent	*Temperature degrees*	*Hydrogen ion concentration*	*CO_2 tension mm.*
31.92	20 ± 1	135×10^{-7}	760
31.92	20 ± 1	121×10^{-7}	700
31.92	20 ± 1	112×10^{-7}	600
31.92	20 ± 1	93×10^{-7}	500
31.92	20 ± 1	76×10^{-7}	400
31.92	20 ± 1	55×10^{-7}	300
31.92	20 ± 1	36.2×10^{-7}	200
31.92	20 ± 1	17.0×10^{-7}	100
31.92	20 ± 1	12.1×10^{-7}	75
31.92	20 ± 1	8.7×10^{-7}	50
32.45	20 ± 1	5.24×10^{-7}	25
32.45	20 ± 1	2.19×10^{-7}	10
32.45	20 ± 1	1.74×10^{-7}	7.5
32.45	20 ± 1	1.20×10^{-7}	5.0
32.45	20 ± 1	0.66×10^{-7}	2.5
32.45	20 ± 1	0.27×10^{-7}	1.0
32.45	20 ± 1	0.195×10^{-7}	0.75
32.45	20 ± 1	0.121×10^{-7}	0.5
32.45	20 ± 1	0.078×10^{-7}	0.25
32.45	20 ± 1	0.057×10^{-7}	0.1

2. *Effect of Salinity upon the Hydrogen Ion Concentration of Sea Waters at the same Carbon Dioxide Tension.*—Such variations of salinity as occur in the sea, although they do not ordinarily sensibly affect the proportions of the several components, must obviously exert a certain influence upon the hydrogen ion concentration. The following data indicate approximately the magnitude of this effect.

Number	*Depth*	*Salinity*	*CO_2 tension mm.*	*Hydrogen ion concentration*
10313	20 meters	30.73	0.3	0.093×10^{-7}
10301		31.58	0.3	0.087×10^{-7}
10305		32.05	0.3	0.085×10^{-7}
10305		32.34	0.3	0.085×10^{-7}
1000	20 ft.	32.45	0.3	0.087×10^{-7}
10299	210 meters	33.62	0.3	0.081×10^{-7}
10304		34.16	0.3	0.083×10^{-7}
10288	225 meters	34.60	0.3	0.079×10^{-7}

3. *Effect of Temperature upon the Hydrogen Ion Concentration of Sea Water.*—A more important effect upon hydrogen ion concentration is exerted by the temperature. This depends, first, upon the change in the absorption coefficient of the gas with change of temperature and, secondly, upon a simultaneous change in the ionization constants of the several substances involved in the equilibrium. Certain observations upon the apparent hydrogen ion concentrations of sea water at different temperatures, to which no value as quantitative estimations can at

present be attached, illustrate the general character of this effect, which is dependent mainly upon the first factor mentioned above.

Temperature degrees	*Salinity*	*Hydrogen ion concentration*
0	32.45	0.141×10^{-7}
10	32.45	0.107×10^{-7}
20	32.45	0.085×10^{-7}
30	32.45	0.060×10^{-7}
40	32.45	0.0446×10^{-7}

These measurements indicate that under ordinary circumstances, when unaffected by the products of metabolism or the constituents of river water, the reaction of the sea water is fully determined by (1) the tension of carbonic acid, (2) the concentration of water, or salinity, and (3) the temperature. This relationship suggests a method of determining the carbon dioxide tension of sea water, which will be published later.

4. *Hydrogen Ion Concentration of Natural and Artificial Sea Water.*—So far as the acid-base equilibrium is concerned, sea water may be exactly imitated by a system consisting only of water and sodium chloride together with carbonic acid, and boric or silicic acid, partly combined with alkali. Thus, if a solution be prepared containing

NaCl	35.0 g. per liter
Na_2CO_3	0.1035 g. per liter
H_3BO_3	0.0620 g. per liter
$Na_2B_4O_7$	0.0253 g. per liter

and set up in series with sea water of salinity 32.45, the two systems, after saturation with carbon dioxide at *any* tension between 0 and 760 mm., will possess the same hydrogen ion concentration, as estimated by indicators. This result, however, in that it is comparative, eliminates all estimations of the acidity of the solution. And accordingly the most trustworthy result of the present investigation is proof of the equivalence, in respect to hydrogen ion concentration, of sea water with this artificial system containing boric acid in a concentration 0.0015 molal, carbonic acid in an amount determined by the tension of its gas, and a single base in a concentration 0.0022 molal, distributed between boric and carbonic acids.

It is now evident that the ocean, which, because of the presence of free carbonic acid, was originally acid, and which has been becoming more alkaline from the accumulation of basic material, is at present in an epoch where the growing alkalinity is checked by the *buffer* action of acids of approximately the strength of boric acid. These acids are probably chiefly silicic acid and boric acid, though the conversion of bicarbonates into carbonates is also of importance.

These buffers regulate the reaction of sea water at the present time in a manner quite similar to the way in which bicarbonates and phosphates regulate the reaction of blood. The principal effect of the products of metabolism upon sea water is defined by the data concerning variation in the carbon dioxide tension.

5. *Geographical Variation in Carbon Dioxide Tension.*—Taking account of the facts as stated above, certain conclusions regarding geographical variations in the composition of sea water may be deduced from Palitzsch's data.

First, the uniform increase in acidity of sea water with increase of depth, proves that the concentration of free carbonic acid everywhere varies in like manner. This may be illustrated by one example.

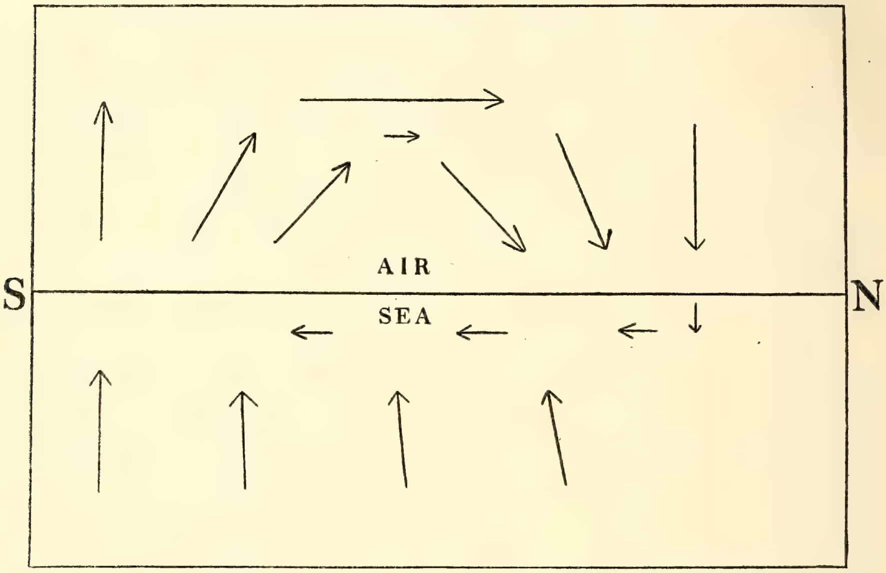
Station 81.

Depth meters	*Temperature in situ*	*Salinity %*	*Hydrogen ion concentration at room temperature*	*Relative concentration of free carbonic acid*
0	17.94	35.43	0.060×10^{-7}	1.0
50	13.56	35.52	0.066×10^{-7}	1.3
100	12.56	35.63	0.074×10^{-7}	1.8
400	11.01	35.57	0.091×10^{-7}	2.5
800	11.25	36.02	0.098×10^{-7}	2.8
1000	10.84	36.02	0.098×10^{-7}	2.8
1200	9.89	36.00	0.104×10^{-7}	3.1
1500	6.97	35.50	0.112×10^{-7}	3.4
2000	4.20	35.10	0.112×10^{-7}	3.4

Disregarding a conceivable effect of pressure, it seems to follow from these facts that, upon the whole, (i.e., in most places and at most seasons) carbonic acid is escaping from the sea into the air.

Secondly, the well marked variation in hydrogen ion concentration with latitude and therefore according to the temperature, in the case of surface waters, indicates that the concentration of free carbonic acid at the surface steadily increases in the direction of the pole. This, no doubt, depends upon the variation of the absorption coefficient of the gas with change in temperature. And, taking account of Krogh's[3] observations upon the high tensions of carbon dioxide, and the uniformly greater tension of atmospheric than of surface carbon dioxide in high latitudes, it appears to be certain that in cold marine regions carbonic acid is passing from air to sea. It does not appear likely that this absorption of carbon dioxide by the cold oceans can balance its escape from the warm oceans, first, because Palitzsch found, even in cold regions, an increase of hydrogen ion concentration with the depth, and, secondly, because in the Mediterranean, where no deep arctic current flows, carbonic acid is escaping from the sea.

The special features of this movement of carbon dioxide can only be discovered through very extensive investigations, but its general character seems to be indicated by the following diagram.



MOVEMENTS OF CARBON DIOXIDE IN A VERTICAL NORTH-SOUTH PLANE, IN THE NORTHERN HEMISPHERE

These results afford one more instance of similarity between regulatory processes in the organism and in the environment.

We are greatly indebted to Dr. Palitzsch for furthering this investigation by his invaluable advice.

---

[1] Sven Palitzsch, Measurements of the hydrogen ion concentration in sea water, *Report on the Danish Oceanographical Expeditions 1908–10 to the Mediterranean and Adjacent Seas*, vol. 1, p. 237.

[2] During the early part of this investigation Dr. Palitzsch, as *Scandinavian Scholar*, was at work in the Gibbs Laboratory.

[3] August Krogh, The abnormal CO_2 percentage in the air in Greenland and the general relations between atmospheric and oceanic carbonic acid, *Meddelelser om. Grönland*, No. 25–26, p. 434, 1904.

AN APPARENT CORRESPONDENCE BETWEEN THE CHEMISTRY OF IGNEOUS MAGMAS AND OF ORGANIC METABOLISM

By Henry S. Washington

GEOPHYSICAL LABORATORY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, October 5, 1916

In a recent paper[1] it was shown that potassium and magnesium on the one hand, and sodium and iron on the other, tend to vary together in igneous magmas and the minerals formed from them. That is, igneous rocks and minerals which are high in potassium contain, (of the two), much magnesium and but little iron; while if the rock or mineral is dominantly sodic iron will be high and magnesium low, if these are present. This relation seems to be independent of the silicity (amount of silica) and but slightly modified by the amount of calcium.

The object of the present note is to call attention to what appears to be a congruous relation of these pairs of elements in the organic world —briefly stated, it would appear that, of the four, iron and sodium are necessary for animal metabolism, (magnesium and potassium very much less so); while magnesium and potassium are essential to vegetable metabolism, the other pair being of minor importance.

This correspondence is indicated by the following considerations, all of which are well-known to biologists, so that it is only in their collocation and the pointing out of an apparent congruity in the chemistry of the mineral with the animal and vegetable kindgoms that there is any novelty.

It is well-known that the blood pigments of the higher animals, hemoglobin and its congeners and derivatives, are protein bodies which contain iron as an essential constituent, and it has been suggested[2] that this element acts as the oxygen carrier in the vital processes in which the blood plays the all-important part. In the blood plasma sodium is an important constituent, its amount being several times that of the potassium, though the latter element is, apparently, more important in the cells. The amount of magnesium in the plasma is also very small.

As stated by one authority[3] it would appear that "potassium compounds are replaced in the [animal] organism by sodium chloride," and that as a consequence "common salt is, not only a condiment, but a necessary addition to the food." The universal and necessary use of sodium chloride in food by man and many of the higher animals need only be mentioned.

In plants, on the other hand, the active metabolic agent which enables the organism to assimilate carbon from the CO_2 of the atmosphere is chlorophyll, and it is now well-known through recent researches[4] that this is a protein compound which contains magnesium as an essential constituent. In this connection it is of especial interest to note that chlorophyll is very closely related chemically to hemoglobin, the formulae and apparently the constitution of the two being almost identical;[5] except that the animal metabolic agent is an iron compound and the vegetable agent is one of magnesium, and that the former contains slightly more oxygen than the latter. In this similarity we are reminded of the association and mutual isomorphous replaceability of iron and magnesium among minerals.

In plants again, in contradistinction to animals, of the two alkali metals, "potassium, unlike sodium, is essential to plant life."[6] Water culture experiments have abundantly proved that, of the cations potassium, magnesium, and calcium are necessary to plant life, though a small amount of iron is requisite to the formation of chlorophyll, in which connection the close similarity between this substance and hemoglobin becomes of special interest. Sodium is not necessary to most plant existence.

Although of subsidiary importance, the relative toxicity of the several cations to animal and vegetable life is of interest, as bearing in the same way as the facts set forth above. It must, of course, be understood that toxicity is largely a matter of concentration and that we have to deal, here very briefly and inadequately, with the *relative* toxicities of approximately similar concentrations of the several elements. It is also to be understood that we shall consider only the four cations (Fe, Na, Mg, K), which here concern us, and disregard others, as Ca, as well as the anions or acid radicals.

To the animal organism the magnesium salts appear to be mildly toxic, or at least are very readily eliminated when ingested into the system, which is only true of the iron salts when taken in much greater amounts. That potassium is largely replaced in the animal organism by sodium has already been noted, and experiments by Loeb[7] and others, which can be but barely alluded to here, show that to many animal organisms potassium is distinctly toxic, its ill effects being neutralized by immersion in solutions of sodium salts.

To the higher plants, at least, the iron salts, especially those of ferrous[8] iron, are distinctly toxic when present in the soil in much more than the small amounts needed for its (possibly catalytic) action in the formation of chlorophyll. Likewise sodium salts are, especially relatively to those of potsssium, decidedly toxic toward most vegetable life, though some plants, even of the highest genera, have become halophytic and have adapted themselves to soils and waters containing relatively large concentrations of sodium salts. Even in these, however, it would appear that potassium is the one of the alkali elements necessary to the plant's existence.

Briefly put, therefore, these relations of activity and toxicity of the two pairs of salts are antagonistic in the two kingdoms; that which is active in metabolism and chiefly necessary for life in the animal being relatively toxic in the vegetable, and vice versa.

Finally it may be pointed out that, according to Reichert,[9] hemoglobin is entirely absent from the lower orders of animals, such as the protozoa, porifera and coelenterata, is rare in the echinodermata, and that chlorophyll is present in some of these, as the protozoa. Thus it might almost be suggested that the nearer the animal organism approaches the vegetable the less the influence of iron and the greater that of magnesium in assimilative metabolism.

The facts thus very briefly and imperfectly set forth seem consistently and collectively to lead to the conclusion that, in general, of the four elements discussed, iron and sodium are, relatively to the other two, jointly necessary to animal metabolism and existence, while, per contra, magnesium and potassium replace them as essentials for plant life. Here we have, then, though opposed in the two types of organic metabolism, the same congruity between the two pairs of elements that has been shown to exist in minerals and igneous magmas. Whether this apparent state of affairs is merely coincidental and due to such external factors as conditions of environment, or whether it is based on some fundamental chemical characters or relations of the elements involved, can not be adequately discussed here.

Thus it might be argued in favor of the first supposition that the presence of abundant sodium in the blood plasma is due to the fact that this may represent the sea water in which the present day animal's ancestors once lived; that the presence of iron in the blood pigments may be ascribed to the existence of this element in two valencies, ferric and ferrous iron, and their ready transformation from one state of oxidation to the other;[10] that the presence of potassium rather than sodium in plants may reasonably be connected with the greater resistance to decomposition and greater insolubility of the potassium than the sodium minerals, and the consequent greater tendency of the former to accumulate in soils; and, lastly, that the fact that chlorophyll is a magnesium compound may be correlated with the ability of this element to act as it does in Grignard's reaction, as has been suggested by Willstätter.[11] The relatively greater toxicity of potassium over sodium, as in Loeb's experiments with Fundulus, may be ascribed to selective absorption or membrane permeability. The connection between these pairs of elements in magmas and minerals may be ascribed to differences in solubility.

Each one of these or other such explanations, taken individually and separately, may be rational and valid; but taken collectively, especially in view of the congruity of the four elements in the mineral and organic kingdoms, they make it appear possible and indeed reasonable to ascribe the correspondence to some fundamental chemical relation between the elements involved, of which such proximate causal nexuses as have been suggested are the consequences. No hypothesis can be offered as to what may be the nature of this relation; only the mere possibility of its existence, based on the correspondences set forth in this and the preceding paper, can be suggested.

[1] H. S. Washington, these PROCEEDINGS, **1**, 574 (1915).

[2] Cf. Reichert and Brown, *Carnegie Inst. Publ.*, No. 116, 24 (1909).

[3] O. Hamarsten (Transl. by Mandel), *Textbook of Physiological Chemistry*, p. 564 (1900).

[4] Cf. S. B. Schryver, *Science Progress*, **3**, 440 (1909).

[5] Reichert and Brown, op. cit., 1; Schryver, op. cit., 444.

[6] E. Strasburger et al. (transl. by Porter), *Text-book of Botany*, 173 (1898).

[7] Cf. J. Loeb these PROCEEDINGS, **1**, 473 (1915); **2**, 511 (1916).

[8] It is suggested that the lesser toxicity of the ferric salts may be ascribed to their ready hydrolysis and the consequent formation of very difficultly soluble basic compounds.

[9] Op. cit., 2.

[10] Copper, likewise with two valencies, exists in the blood of certain lower animals, as mollusca, limulus, helix and sepia.

[11] The dependence of plant life on magnesium and of animal life on iron, and the difference in their functions, is expressed by Willstätter as follows: "There are fundamentally two kinds of life, which develop alongside of each other; the anabolic life with magnesium and the katabolic life (of animals) with iron, that is a reducing and an oxidizing life." *See Liebig's Ann. Chem.*, **250**, 65 (1906).

THE OAKS OF AMERICA

By William Trelease

DEPARTMENT OF BOTANY, UNIVERSITY OF ILLINOIS

Received by the Academy, October 13, 1916

For a number of years I have been engaged in a study of the oaks of tropical America. These have not been treated comparatively for a generation, with the result that the extensive collections made within that time have gone into herbaria largely unnamed or wrongly named.

My feeling has been that the only way to unravel the difficulties was to begin with the study of types of the earlier species, passing to those of later date and deferring examination of unauthentic collections until the end. Though it was not possible to carry this plan out in all detail, it has been my privilege to see most of the types of tropical American species, and to photograph in natural size representative parts of them whenever found, so that whatever errors may have slipped in, they scarcely include a misapprehension as to what is meant by the earlier used names except for one or two of Née's species of which no identifiable material is known to remain.

It was not until this study of the forms that occur to the south of us had been essentially finished, that it seemed best to include in my treatment those of the United States. These have been so long and so repeatedly studied and for the most part figured that little would seem to have been left undone with them. Yet within the year Professor Sargent has pointed out a serious misapprehension as to the proper Latin names of the rock chestnut oak and the cow oak, and has made it very questionable whether what we know as red oak in the northern States is what Linnaeus called *Quercus rubra*. As I finish my manuscript, in which for completeness the northern oaks are included summarily, I have a feeling that more uncertainty still attends several of these polymorphic species than perhaps any one which occurs in the tropics; and unfortunately this uncertainty for the most part cannot be removed by reference to types, which do not exist for the most puzzling of these northern species.

A careful analysis of the characters presented by wood, inflorescence and flowers leads me to believe that the Fagaceae are far from being the primitive plants that they are commonly taken for, and I am disposed to conclude that their affinities are with such seemingly advanced and certainly specialized but still really simple orders as the Ranunculales and Rosales, from the type of which they have receded.

On this continent, the oaks (excluding *Pasania* as a distinctly separable genus) seem to fall into three subgenera or main groups instead of two as usually understood, Leucobalanus, the white oaks, typical of *Quercus;* Erythrobalanus, the red or black oaks; and Protobalanus, a more ancient type as I conceive it, comprising the protean intermediate assemblage clustering about *Q. chrysolepis*.

Summarized, my study of American materials contained in the principal herbaria of the world leads to the recognition of 354 species, of which 158, or very nearly one-half, are described as new in the manuscript which I am now prepared to submit to the Academy for publication, in which 183, or slightly over one-half of the whole, are figured for the first time. As in our immediate flora, white and red oaks occur in approximately equal numbers for the American flora as a whole: 170 species of the former, and 179 of the latter, only 4 species of Protobalanus being known.

In variety as well as in actual number of species, the countries to the south of us are much richer in oaks than the Atlantic United States—a result to be expected from the more rugged configuration and greater meteorologic differences in those countries. The principal facts of the distribution of American oaks by countries are indicated in the following table. A very few species occur in more than one country, and therefore are counted for both.

COUNTRY	LEUCOBALANUS	PROTOBALANUS	ERYTHROBALANUS	TOTAL
United States....................	43	2	26	71
Mexico........................	121	2	125	248
Central America................	20	0	35	55
South America..................	0	0	4	4
Antilles.......................	1	0	0	1
Pacific Islands.................	0	1	0	1

A glance at this table shows that, rich as the United States are in oaks, they are nearly equaled by the small Central American countries and far surpassed by Mexico. In the West Indies only a single species of white oak, doubtfully distinct from the live oak of our Gulf States, occurs, and this unquestionably derived from our mainland. In South America there are only four closely related species, of the red oak group, and these are clearly allied with some of the Costa Rican species.

The genus *Quercus* is conceded to have existed in Cretaceous time though many Cretaceous and Tertiary fossils formerly referred to this genus are now placed in *Dryophyllum*, which is taken for the ancestral stock of the Fagaceae rather than of *Quercus* alone. Paleobotanists now admit 150 species of American fossil oaks pretty evenly distributed through the Cretaceous and the Eocene and Miocene divisions of the Tertiary. Few Pliocene fossils are known for North America; but in South America where a few others have been made known for the Argentine, 4 species have been described from Pliocene deposits of equatorial Brazil. It is noteworthy that the few existing South American oaks are confined to the Andes of Colombia.

So far as I know, none of the earlier species survived from one to another of the periods of geologic time except for *Q. furcinervis americana* (perhaps two distinct species) in Eocene and Miocene deposits, and that what has been taken for the existing *Q. chrysolepis*—the type of the subgenus Protobalanus—occurred in the Miocene; but this identification might well be questioned by a critic. A proper understanding of the affinities of existing groups of species undoubtedly calls for a just appreciation of their connection with these ancestral forms. This I do not profess to have formed. I find the occurrence of a few aberrants—among them the South American species—in the subgenus Erythrobalanus puzzling, but can see no satisfactory evidence that red- or black-oaks are recognizable in any of the older fossils; and the group is certainly exclusively American today. Notwithstanding this, a resemblance is observable between the white oaks of Europe and those of western rather than eastern America, that proves puzzling.

On the whole, I am unable to trace any existing groups to those of Tertiary time. Pleistocene species, of which 18 are recognized for the United States, as is to be expected are scarcely different from those of today, though they are sometimes given distinctive names. In the early Pliocene should be sought definitely recognizable ancestral forms of these and their living descendants.

Considering the multitudinous—and in their extremes very diverse—types of such an existing assemblage as that of the Rocky Mountains, in which Engelmann and other excellent botanists have been unable to see more than a single polymorphic species, and the comparable range—to which the keen von Ettingshausen refers the manifold European oak types of today—in the fossil *Q. Palaeo-Ilex,* I am unable to see that all of the existing American oaks may not have descended from a single synthetic type of this kind, such as the Miocene species that has been held to be identical with the existing intermediate oak *Q. chrysolepis.* Out of this type, rightly or wrongly, I have built the present divergent branches of white and red oaks as I understand their affinities, believing that the European and American white oaks have no direct connection and that on each continent the manifold and often parallel forms of today have been independently derived from distinct late Tertiary types not closely related to one another in descent.

A SET OF INDEPENDENT POSTULATES FOR CYCLIC ORDER

By Edward V. Huntington

DEPARTMENT OF MATHEMATICS, HARVARD UNIVERSITY

Received by the Academy, October 16, 1916

There are four types of order which are important in geometry and other branches of mathematics: (1a) linear order with a definite sense along the line (theory of *serial order*); (1b) linear order without distinction of sense (theory of *betweenness*); (2a) circular order with a definite sense around the circle (theory of *cyclic order*); and (2b) circular order without distinction of sense (theory of *separation of pairs of points*).

The present note is concerned with type (2a): cyclic order.

Let us consider a *class* K of elements A, B, C, . . . , and a *triadic relation* $R(ABC)$. The class K may be said to be *cyclically ordered* by the relation R if the following postulates are satisfied:

I. *If A, B, C, are distinct, then ABC implies BCA.*

II. *If A, B, C are distinct, then at least one of the orders ABC, BCA, CAB, CBA, ACB, BAC is true.*

III. *If A, B, C are distinct, then ABC and ACB cannot both be true.*

IV. *If ABC is true, then A, B, and C are distinct.*

V. *If A, B, X, Y are distinct, and XAB and AYB, then XAY.*

From these postulates it follows that any three distinct elements A, B, C, in the order ABC, divide the class into three sections, such that (1) the three sections, together with the dividing elements, exhaust the class; (2) no element belongs to more than one section; (3) if X, Y, Z are elements taken one from each section, so that AXB and BYC and CZA, then XYZ. The details of the proof will be given in a later paper.

It will also be shown that the postulates are independent of each other.

From I, II, and V we can prove, as a theorem,

VI. *If A, B, X, Y are distinct, and AXB and AYB, then either AXY or YXB.*

But if we should replace postulate I by the following postulate:

I′. *If A, B, C are distinct, then ABC implies CBA,*

then VI would become an independent postulate, and the set of six postulates, I′, II, III, IV, V, VI, would then define not cyclic order, but betweenness, and would be, in fact, identical with one of the sets of independent postulates for betweenness obtained in a forthcoming paper by E. V. Huntington and J. R. Kline. The transition from the theory of cyclic order to the theory of betweenness may thus be made by merely interchanging two letters in the first postulate; postulates II–VI are true in both theories.

A NEW METHOD OF STUDYING IDEATIONAL AND ALLIED FORMS OF BEHAVIOR IN MAN AND OTHER ANIMALS[1]

By Robert M. Yerkes

PSYCHOLOGICAL LABORATORY, HARVARD UNIVERSITY

Received by the Academy, October 20, 1916

Despite widespread interest in the evolution of reasoning, the comparative study of ideational behavior has been neglected. Only a few methods of research have been devised, and these have seen scant service.

Thorndike[2] is responsible for the problem or puzzle-box method (used by him in the study of cats, dogs, and monkeys); Hamilton,[3] for the method of quadruple choices (by which he has studied cats, dogs, horses, monkeys, rats, gophers, and men); Hunter,[4] for the method of delayed reaction (applied by him to rats, raccoons, dogs, and children).

I have perfected and applied a new method—that of multiple choices—for the detection of reactive tendencies and the study of their rôle in the attempted solution of certain types of problem. The method involves the presentation to the subject of a problem or series of problems whose rapid and complete solution depends upon ideational processes.

The apparatus consists of twelve, or, in some forms, nine identical reaction-mechanisms, of which any number may be used for a given experimental observation. In the type of apparatus originally used for human subjects, these mechanisms are simple keys; in that which has been used for lower animals, they are boxes arranged side by side, each with an entrance door at one end and an exit door at the other, which may be raised or lowered at need by the experimenter through the use of a system of weighted cords. Under the exit door of each box is a receptacle in which some form of reward for correct reaction may be concealed until the door of the appropriate box is raised by the experimenter.

It is the task of the subject to select from any group of these boxes whose entrance doors are raised that one in which the reward (food, for example) is to be presented. The experimenter in advance defines the correct box for any group of boxes which may be used as that which bears a certain definite spatial or numerical relation to the other members of its group. Definitions which have actually been employed (problems presented) are the following: (1) the first box at the left end of the group (as faced by the subject); (2) the second box from the right end of the group; (3) alternately, the box at the left end and the box at the right end of the group; (4) the middle box of the group.

The boxes are presented in varying groups in accordance with a prearranged plan. The subject is punished by confinement in the box selected every time it makes an incorrect choice and is then allowed to choose again, and so on until it finally selects that box which is by definition the correct one. It is then rewarded with food and permitted to pass through the box and return to the starting point, where it awaits opportunity to respond to a new group.

The experimenter keeps a precise record of the subject's choices and of various important aspects of behavior. These data include the nature of the choices from trial to trial, series to series, day to day; the appearance and fate of specific reactive tendencies or methods of attempting to solve a problem; and the final outcome, in success or failure, of prolonged effort.

The essential statistical features of the results obtained with certain types of subject may be summarized briefly thus:

(1) Crows quickly solve problem 1 (first mechanism at one end of the group), with 50 to 100 trials.

Problem 2 (second from the end) they fail to solve in 500 trials. No consistent improvement appears, although there are four conspicuous reactive tendencies: (a) to go to the end box because of previous training in problem 1; (b) to go to the first box at the left and then to the one next in order, which in the particular experiment happened to be the correct one; (this is the most nearly adequate tendency exhibited by the crow); (c) to reënter whichever box happened to be chosen first and to choose next the second box from the left (correct); (d) to enter a box at or near the right end of a group, and on emerging, to turn to the right and enter the box directly in front, and so on until the correct box is located.

In the method of multiple choices, the crow gives no convincing evidence of ideational behavior. Its general intelligence is clearly indicated by alertness, keenness of perception, emotional responsiveness, and rapid adjustment to various essential features of the experimental situation. It appears to be temperamentally ill-suited to the kind of task presented by this method of studying reactive tendencies.

(2) White rats solve problem 1 in from 170 to 350 trials on the basis

of certain acquired motor tendencies, one of which may be described thus. The rat follows the wall of the reaction compartment to the entrance to the box at the right end of the series of boxes. It then turns sharply to the left and passes along close to the boxes until the first open door is reached. This it enters. Kinaesthetic, tactual, and visual data constitute the basis for the motor habits by which rats solve this simple relational problem.

A single individual exhibited reactive tendencies less obviously describable in motor terms, but it is by no means certain that this indicates ideational ability sufficient for the solution of relational problems.

Problem 2 was not solved by rats within 800 trials, and there is no indication in the data that solution is possible to them.

Whereas the influence of training in problem 1 disappeared quickly when problem 2 was presented to the crow, it persisted in the case of the rat for about 100 trials. There are numerous other evidences, in the experimental data, of the higher intelligence of the crow. The rat is distinctly less versatile and markedly less responsive to slight changes in environment.

(3) Pigs solve problem 1 with 50 trials or less; problem 2 with 390 to 600 trials; problem 3 with 420 to 470 trials. Problem 4 is not solved in 800 trials.

These successes as well as the varied reactive tendencies manifested place the pig much higher in the scale of adaptive capacity than the rat or crow.

The data obtained with these three types of subject proves the method of multiple choices to be a feasible means of eliciting reactive tendencies which are characteristic of various points in ontogeny and phylogeny.

Results which have been obtained with monkeys, apes, and men will be presented in separate communications.

[1] Yerkes, Robert M., The study of human behavior, *Science*, **39**, 625–633 (1914).

Coburn, Charles A. and Yerkes, Robert M., A study of the behavior of the crow, *Corvus Americanus* Aud., by the multiple-choice method, *J. Animal Behavior*, **5**, 75–114 (1915).

Yerkes, Robert M. and Coburn, Charles A., A study of the behavior of the pig, *Sus scrofa*, by the multiple-choice method. *J. Animal Behavior*, **5**, 185–225 (1915).

Burtt, Harold E., A study of the behavior of the white rat by the multiple-choice method, *J. Animal Behavior*, **6**, 222–246 (1916).

Yerkes, Robert M., The mental life of monkeys and apes: a study of ideational behavior, *Behavior Monographs*, **3**, Serial number 12 (1916).

[2] Thorndike, E. L., *Animal Intelligence*, New York, 1911.

[3] Hamilton, G. V., A study of trial and error reactions in mammals, *J. Animal Behavior*, **1**, 33 (1911).

[4] Hunter, W. S., The delayed reaction in animals and children, *Behavior Monographs*, **2**, serial number 6 (1913).

ELECTRICAL CONDUCTION IN DILUTE AMALGAMS

By Gilbert N. Lewis and Thomas B. Hine

DEPARTMENT OF CHEMISTRY, UNIVERSITY OF CALIFORNIA

Received by the Academy, October 17, 1916

The modern theory of electrical conduction in metals, according to which the metal is dissociated to give a positive ion of low mobility, which is characteristic of the metal, and a negative carrier of high mobility, which is common to all metals and presumably to be identified with the electron, has given rise to certain misgivings, despite the complete analogy between this theory and the accepted theory of electrolytic dissociation. It is true that in the case of solid metals the crystalline forces, which lead to the formation of the various components of the metal into symmetrical space-lattices, produce conditions which have no counterpart in liquid electrolytes, but in the case of a liquid metal like mercury we must assume that it differs from any electrolyte —we are tempted to say from any other electrolyte—only in as far as one of its ions has properties which differ very greatly in degree, but probably not in kind, from the properties of other ions. If therefore we should ultimately find that the Arrhenius theory of ionization is not applicable to a liquid metal we should be inclined to believe that it is not completely valid in the case of electrolytes.

This theory of electrolytic dissociation, which was originally proposed to explain the properties of aqueous salt solutions, has since been successfully applied without essential modification to many types of non-aqueous solution. Moreover the extremely important work of Kraus,[1] which unfortunately has not yet been published in full, demonstrates the applicability of the same theory to solutions of metals in non-metallic solvents. Thus, when metallic sodium dissolves in liquid ammonia, he shows conclusively that it dissociates into sodium ions and into electrons which, like other ions, are to a considerable extent combined with the molecules of the solvent.

Since therefore the theory of ionization has been successfully applied to electrolytes and to solutions of metals in electrolytes, it seems desirable to attempt a further extension of these ideas by several methods to solutions of metals in metallic solvents, in particular to solutions of the alkali metals in liquid mercury, and thus eventually to pure mercury itself. It has been shown by Lewis, Adams, and Lanman[2] that the transference of matter with the electric current, which had previously been regarded as a distinguishing characteristic of electrolytic conductors, can be detected and measured in sodium and potassium amalgams. It is to be hoped that further knowledge of the transference numbers so obtained, together with accurate information concerning the electrical conductivity of amalgams, may furnish important evidence as to the nature of electrical conduction in dilute amalgams and in pure mercury.

Important investigations of the conductivity of amalgams and liquid alloys have been conducted by Bornemann and his collaborators,[3] but they have confined their studies to high concentrations. If from transference number and conductivity we propose to obtain information concerning the nature of the carriers in the amalgams and in pure mercury, analogous to that which has been derived from a similar study of aqueous solutions, we may expect the most significant results from an investigation of dilute amalgams, ranging from zero to one atom per cent. We shall describe in this paper the results of an investigation of the conductivity of dilute amalgams of sodium, potassium, and lithium.

Since the conductivity of these amalgams differs but little from that of pure mercury, the difference itself can be determined with accuracy only by methods of the highest precision. By means of a simple potentiometer method it was found possible to compare the conductivity of an amalgam with that of pure mercury with an accuracy of two or three parts in one hundred thousand. (Full details of the potentiometer method, together with the method of preparing, transferring and analyzing the amalgams, are contained in a paper which will shortly be offered for publication in the *Journal of the American Chemical Society.*)

The experimental results are given in the three following tables, in each of which the first column gives the atom per cent of alkali metal, and the second the specific resistance divided by that of pure mercury at the same temperature. The measurements were all made at about 20°C.

TABLE I
Lithium Amalgam

ATOM PER CENT	RESISTANCE REFERRED TO MERCURY
0.03269	0.999468
0.09988	0.998280
0.3080	0.994006
0.4844	0.990105
0.7103	0.985603
0.8221	0.983097

TABLE II
Sodium Amalgam

ATOM PER CENT	RESISTANCE REFERRED TO MERCURY
0.09325	1.00071
0.2905	1.00204
0.3851	1.00274
0.8646	1.00535
1.197	1.00657
1.670	1.00800
2.113	1.00867
2.409	1.00893
2.644	1.00824
3.661	1.00657
4.916	1.00621

TABLE III
Potassium Amalgam

ATOM PER CENT	RESISTANCE REFERRED TO MERCURY
0.05831	1.00220
0.1791	1.00574
0.4068	1.01767
0.5590	1.02623
0.6502	1.03032
0.8777	1.04147
1.186	1.05341

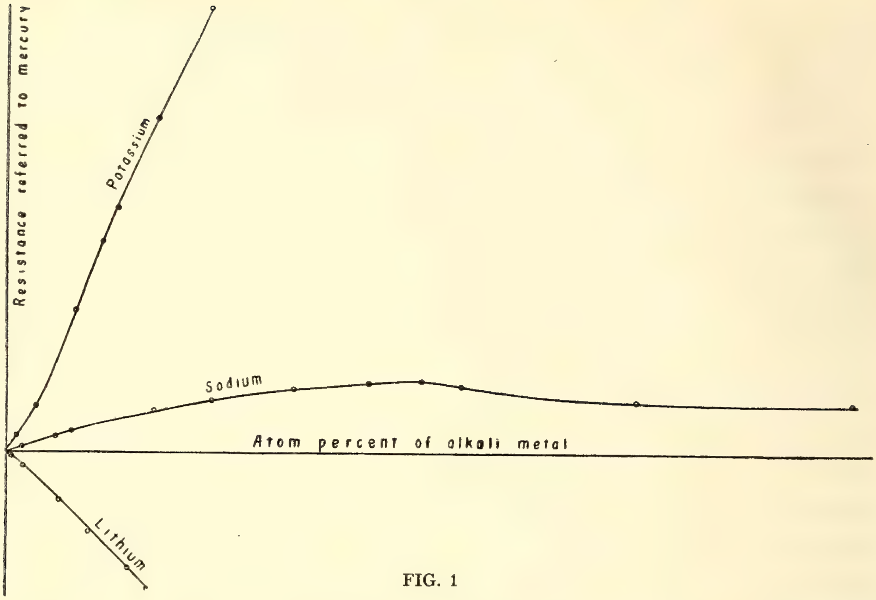


FIG. 1

The extraordinary difference between the three types of amalgam is shown in figure 1, from which it will be seen that a small addition of potassium increases the resistance of mercury, an equivalent amount of sodium produces a less increase in the resistance, while the addition of lithium diminishes the resistance.

It might have been expected, at first sight at least, that in accordance with the theory of electrolytic dissociation the strongly electropositive alkali metals would, owing to a greater dissociation, increase the conductivity of mercury. But when we consider the well substantiated fact that all these metals are, when dissolved in mercury, combined with the solvent to form hydrargyrates of greater or less complexity, the results cannot be considered to disprove this theory. Nevertheless it is important to observe that in interpreting the conductivity and transference number of dilute amalgams, the generalizations which can be drawn and the analogies which suggest themselves are of a very different character from those which have been found useful in the interpretation of the phenomena of aqueous solutions.

Thus in the paper of Lewis, Adams, and Lanman it has been shown that, contrary to expectation, the strongly electropositive metals sodium and potassium, when dissolved in mercury, wander in the direction not of the positive but of the negative current. They show further that this phenomenon is closely correlated with the increase in resistance produced in mercury by the solution of these metals; for if the increased resistance be ascribed to a greater resistance to the motion of the electron in those regions of the mercury which contain particles of the dissolved metal, these particles or regions, according to the law of action and reaction, will be impelled in the direction in which the electrons are moving. This explanation is borne out by the fact that potassium, which produces a greater increase in the resistance of mercury than an equivalent amount of sodium, is also transferred to a greater extent in the direction of the negative current.

We may also suggest here another relationship of considerable interest. Attention has been called by Kraus[4] to the very large influence upon the conductivity of liquid mercury of a change of volume through pressure. Now it has been shown by Maey[5] that the addition of the alkali metals produces a very considerable change in the volume of mercury. This change in volume follows the same order as the electrical resistance of the amalgams given in the preceding tables. It is interesting therefore to see how the resistance of mercury would change upon the addition of the three alkali metals, if this addition were made, not at constant pressure, but with such change of pressure as would keep constant the average atomic volume, that is, the volume occupied by 1 gram of solvent and solute together.

If R is the resistance of pure mercury, V and V' the atomic volumes respectively of pure mercury and of amalgams, P the pressure in kilograms per square centimeter, and N the atomic fraction of the dissolved metal, then, according to the measurements of Bridgman,[6] $d\ln R/dP = -3.34 \times 10^{-5}$, while from the measurements of Richards, of Buchanan, and of Bridgman,[7] $d\ln V/dP = -3.8 \times 10^{-6}$. Hence $d\ln R/d\ln V = 8.79$. In Table IV the second column gives the value of $d\ln V'/dN$ calculated from the work of Maey, and the third column the values of $d\ln R/dN$, which are obtained from the second column through multiplication by 8.79. These figures therefore show the change in the resistance of mercury which would be produced by the three metals, assuming that the effect of the dissolved metal is due solely to the accompanying volume change. The fourth column gives the values of $d\ln R/dN$ obtained directly from our measurements, by finding the slope of the resistance curve at $N = 0$. These figures will vary somewhat according to the relative weight given to the measurements at the lowest concentration. The fifth column gives the values of $(d\ln R/dN)_{V'}$ namely, the fractional change in the resistance per gram-atom of dissolved metal when the average atomic volume is kept constant. These figures are obtained by subtracting the second column from the third.

TABLE IV

	$\frac{d\ ln\ V'}{dN}$	$\frac{d\ ln\ R}{dN}$ (calc.)	$\frac{d\ ln\ R}{dN}$ (obs.)	$\left(\frac{d\ ln\ R}{dN}\right)_{V'}$
Lithium	−0.534	−4.70	−0.82	+3.88
Sodium	−0.014	−0.12	+0.92	+1.04
Potassium	+0.643	+5.66	+2.73	−2.93

It is evident that when added at constant atomic volume lithium would increase and potassium decrease the resistance of mercury, the very reverse of that which happens at constant pressure. The parallelism between the figures of the second and the third columns is striking; we need not, however, consider the change in volume and the change in resistance to be directly related as cause to effect, but rather we shall regard them as concomitant effects of some condition in the immediate neighborhood of the dissolved particles, a condition which probably is closely connected with what is known as the solvation of the dissolved metal. The amount of this solvation or combination with the solvent undoubtedly increases in the same order as the electrical resistance, being least in the case of lithium and greatest in the case of potassium. Thus, to cite one of the most direct pieces of evidence, it is shown by Lewis and Keyes[8] that the heats of solution in mercury of lithium, sodium, and potassium are respectively 19,600, 19,800 and 26,000 calories.

In drawing attention to the difference between the phenomenon of conductivity in dilute amalgams and in dilute electrolytic solutions, it is only just to remark that, since we are considering solutions in a solvent which is itself a good conductor, the case is not entirely analogous to that of salts dissolved in a poor conductor like water, but rather to a solution of a salt in another molten salt, a system which has not as yet been intensively studied.

[1] Kraus, *J. Amer. Chem Soc.*, **29**, 1557 (1907); **30**, 653 (1908); **30**, 1197 (1908); **30**, 1323 (1908); **36**, 864 (1914).

[2] Lewis, Adams, and Lanman, *J. Amer. Chem. Soc.*, **37**, 2656 (1915).

[3] Bornemann, *Metallurgie*, **7**, 730 (1910); **9**, 473 (1912).

[4] Kraus, *Physic. Rev.*, Ser. 2, **4**, 159 (1914).

[5] Maey, *Zs. physik. Chem.*, **29**, 119 (1899).

[6] Bridgman, *Proc. Amer. Acad. Arts Sci.*, **44**, 221 (1909).

[7] Landolt, Börnstein, and Roth, *Tabellen.*

[8] Lewis and Keyes, *J. Amer. Chem. Soc.*, **35**, 340 (1913).

IDEATIONAL BEHAVIOR OF MONKEYS AND APES

By Robert M. Yerkes

PSYCHOLOGICAL LABORATORY, HARVARD UNIVERSITY

Received by the Academy, October 20, 1916

The reactive tendencies of two monkeys and a young orang utan have been studied by means of the method of multiple choices described in a previous communication to the Academy[1] and by certain supplementary methods.

Four multiple choice problems were presented: (1) the problem of choosing from among any group of mechanisms the one at the left; (2) the second from the right end; (3) alternately, the first at the left end and the first at the right end; (4) the middle mechanism.

Each of the three primates in question solved problem 1. One monkey (P. rhesus) required 70 trials; the other (P. irus), 132 trials. The ape succeeded only after 290 trials. The behavior of these three animals was most interesting and illuminating. P. irus was erratic, easily fatigued or discouraged, and apparently of low grade intelligence. P. rhesus, on the contrary, was alert, businesslike, intent on his task, and direct in his attack on experimental devices. The orang utan was childlike in his desire for assistance, as also in his resentment of annoyances or disappointments. He was given to settling down to a simple routine.

The accompanying figure 1 presents the curves of error for these animals as constructed from the data in connection with problem 1.

P. rhesus (Sobke) quickly and regularly eliminated mistakes and completely solved the problem. His disposition as well as his achievement, is pictured by the curve of errors.

P. irus (Skirrl) exhibited marked irregularities of performance, and the curve indicates his variable attention and effort as well as his slow progress toward success.

The orang utan (Julius) reacted uniquely, as his curve suggests. At the very outset, he developed a definite habit of response which, as it happened, was inadequate for the solution of the problem but yielded constantly 60% of correct first choices. The habit or reactive tendency was that of choosing each time the box nearest to the starting point.[2] Julius continued to use this method without variation for eight successive days. Then a break occurred, but after a few days he settled back into the old rut. At the end of 230 trials, it was decided to try to destroy the ape's unprofitable habit. This attempt was made by using

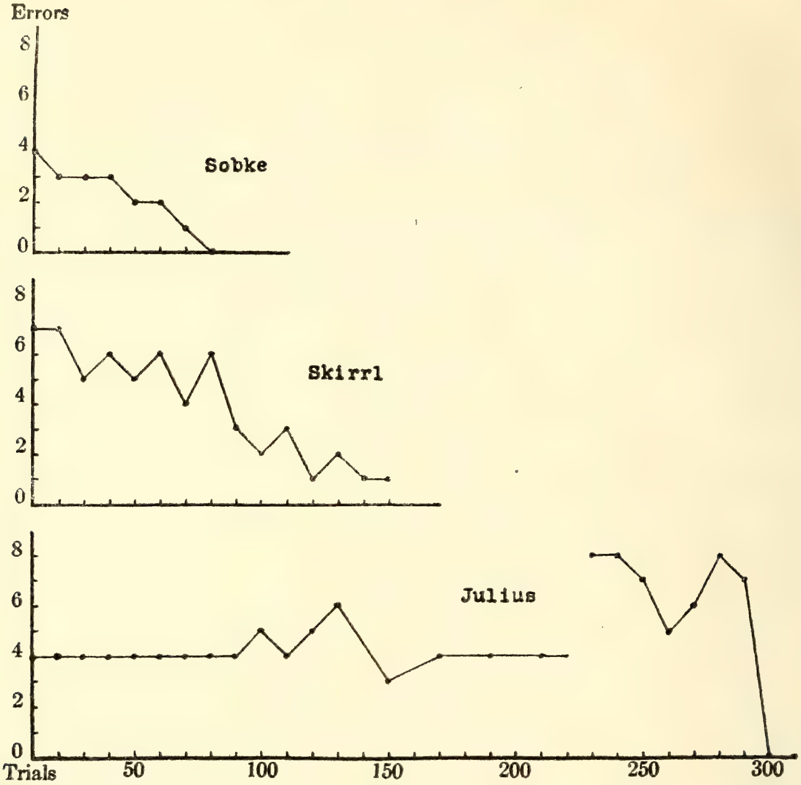


FIG. 1

as correct boxes only those to the left of the middle box of the series. The nearest box, in such case, was never the correct box. Consequently, this modification of method greatly increased, as the curve of errors shows, the number of mistakes.

For a few days after this change was made, no improvement in reaction appeared. On May 10, in a series of 10 trials, 7 were incorrect, but the following day and thereafter only correct choices appeared. Thus, suddenly and without warning, the ape solved his relational problem.

Is this the result of ideation? If not, what happened between the poor performance on May 10 and the perfect series on May 11? Because of varied results obtained in other experiments with this ape, I suspect that ideational processes developed.

The two monkeys succeeded in solving problem 2; the P. irus after 1070 trials, and the P. rhesus after 400 trials. The orang utan failed utterly, although he was given 1380 trials. Ultimately, he ceased to try to select the right box and followed the path of least resistance.

In addition to the method of multiple choices, the chiefly significant results of which cannot well be summarized, several supplementary methods of studying the adaptive behavior of monkeys and apes were employed. Chief among these are (1) a box stacking test; (2) a box and pole experiment; (3) a form of draw-in experiment.

The ape, although failing to stack boxes spontaneously in order to obtain a banana which was suspended from the roof of his cage, did so readily and skilfully when shown how to do it by the experimenter. His imitative activity was convincingly purposive. Previous to the opportunity to imitate the experimenter, he exhibited various methods of trying to get the banana. His attention was surprisingly constant, and his activity, although varied, was for the most part definitely directed toward the food. In the controlling influence of the prospective reward and in the precision of execution of his various acts, Julius differed markedly from the monkeys.

Neither monkey made systematic and sustained attempts to obtain the banana by the use of boxes. Neither imitated the experimenter and neither attended to the prospective reward more than a few seconds at a time. These statements indicate a vast gulf, psychologically, between monkey and ape.

In the box and pole experiment, the banana was so placed in the middle of a long box that it could be obtained only by the use of a pole. The ape quickly, of his own initiative and with few useless motions, succeeded in obtaining the food. The monkeys never succeeded in obtaining it by any method and failed to use the pole at all as a tool.

Similarly, in an experiment which gave the animals opportunity to obtain food by drawing it into the cage with a stick, the ape quickly and repeatedly adapted means to ends by using the stick, whereas the monkeys never once attempted to use it.

The specimen of P. irus (Skirrl) had a penchant for the manipulation of objects as tools. It is therefore surprising that he failed in the above experiment. When given a board, hammer and nails, he drove the nails into the board skilfully and persistently only to draw them out again and repeat the performance, for the activity was its own reward. In all probability, this use of hammer and nails was not imitative, since no other monkey or ape under observation showed any inclination to use them as did Skirrl. Quite as assiduously and with evident satisfaction, he used lock and key, and saw, or any other object which happened to fall into his hands. In the use of a saw, he persistently refused or failed to imitate the experimenter, but finally hit upon a use for the instrument which clearly gave him great satisfaction. By holding it, teeth uppermost, with his feet and rubbing a nail or spike over the teeth rapidly, he succeeded in producing a noise which apparently delighted him. Skirrl, although pronounced feeble-minded on the basis of various studies of problem-solving ability and reactive tentencies, proved himself to be a mechanical genius.

The general conclusions which may be deduced from this limited experimental study of two monkeys and an orang utan are that the ape exhibits various forms of ideational behavior, whereas the reactive tendencies of monkeys are inferior in type and involve less adequate adaptation to factors remote in space or time. I suspect, from data now available, that both monkeys and apes experience ideas, and I believe that it will shortly be possible to offer convincing evidence of the functioning of representative processes in their behavior.

The original account of the results which have been summarized in this communication presents also a plan for a research station to be devoted to the study of the primates. It is pointed out that without scientific conscience we have permitted race after race of primitive man to disappear, unstudied by psychologist, sociologist, or anthropologist, or at best inadequately studied. The pertinent question of the comparative psychologist is "Are we also to permit the gorilla, chimpanzee, orang utan, and gibbon to disappear before we make them yield their incalculably valuable contribution to human enlightenment, welfare, and the general progress of science?"

---

[1] A new method of studying ideational and allied forms of behavior in man and other animals, these PROCEEDINGS, 2, 631, (1916).

[2] In the apparatus used for these observations, the boxes were placed in a straight line instead of on the arc of a circle. Consequently, the distance from the starting point increased from the center of the series toward the ends.

THE OSMOTIC PRESSURE AND LOWERING OF THE FREEZING-POINT OF MIXTURES OF SALTS WITH ONE ANOTHER AND WITH NON-ELECTROLYTES IN AQUEOUS SOLUTIONS

By William D. Harkins, R. E. Hall, and W. A. Roberts

KENT CHEMICAL LABORATORY, UNIVERSITY OF CHICAGO

Received by the Academy, August 26, 1916

In the work of this laboratory upon the abnormalities in behavior of salts in aqueous solutions with respect to their ionization, it seemed desirable to determine the effect of mixing salts with common ions. Since in this laboratory we have a potentiometer system which gives temperature readings to 0.00005°, the work was carried out by the freezing-point method.

Freezing-point determinations at various concentrations were made upon solutions of mannite, erythrite, potassium iodate, sodium iodate, magnesium sulphate, potassium sulphate, barium chloride, and cobalt chloride, and upon mixtures of potassium chloride and potassium nitrate, potassium chloride and mannite, potassium and sodium sulphates, and of potassium and sodium iodates. The method used was similar to that of Adams, and the same apparatus was used. Most of the analyses for the determination of the concentration of the solutions, were made by the use of a Haber-Rayleigh-Zeiss water-interferometer.

The results are expressed most clearly in the form of curves, which are presented in figures 1–3. These figures show the values of the freezing-point lowerings divided by the equivalent concentrations ($\Delta T/N$) on the Y-axis, and the logarithms of the concentrations in milli-equivalents per liter on the X-axis. Figure 1 gives the set of curves for potassium nitrate, potassium chloride, and their equimolal mixture. The curve for the mixture will be seen to lie about half-way between those which represent the salts separately. Figure 2 gives the results for sodium iodate, a uni-univalent salt, and for magnesium sulphate, a bi-bivalent salt. Potassium iodate, and an equimolal mixture of it with sodium iodate were also studied, but the two curves are not given as they are almost coincident with that for sodium iodate. Figure 3 gives those for potassium sulphate, barium chloride, and cobaltous

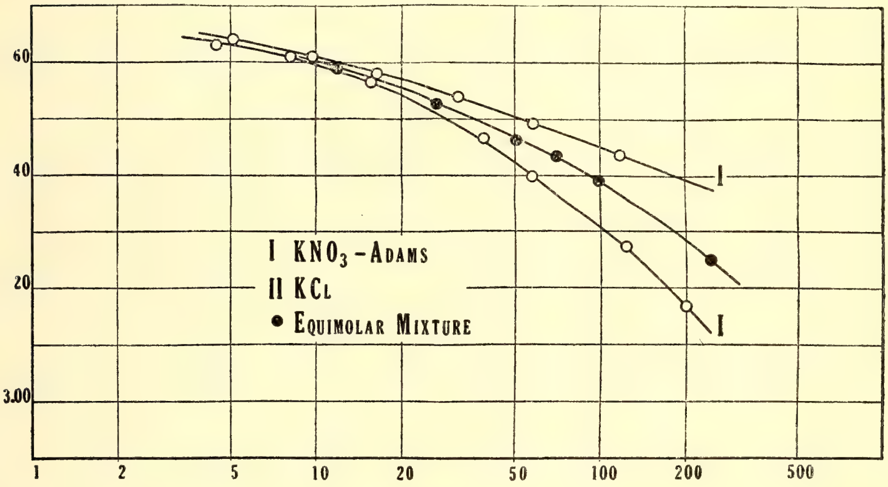


FIG. 1

chloride, three uni-bivalent salts. Lanthanum nitrate, a tri-univalent salt, was also investigated.

The general result obtained with the mixtures investigated up to the present time is that the lowering of the freezing-point of the mixture is very nearly that which would be calculated on the basis that each salt produces a lowering of the freezing-point proportional to its own concentration and to the mol-number (van't Hoff coefficient i) which it has

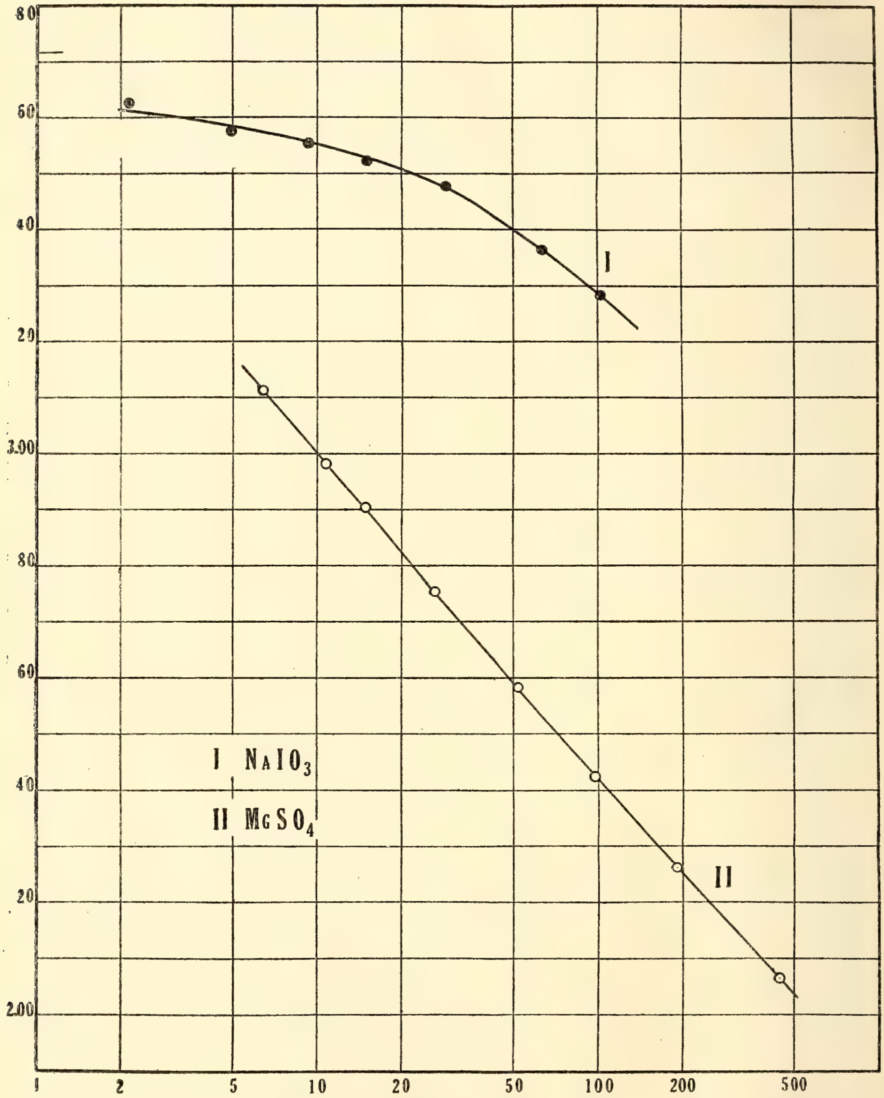


FIG. 2

when present alone in a solution of the same total salt concentration. In the 1 : 1 mixtures the freezing-point curve for the mixture lies midway between the curves for the two salts taken separately. However, it would not be surprising if later results on other salts should show somewhat different relations; for the salts thus far used have been chosen because on the whole they act normally in other respects.

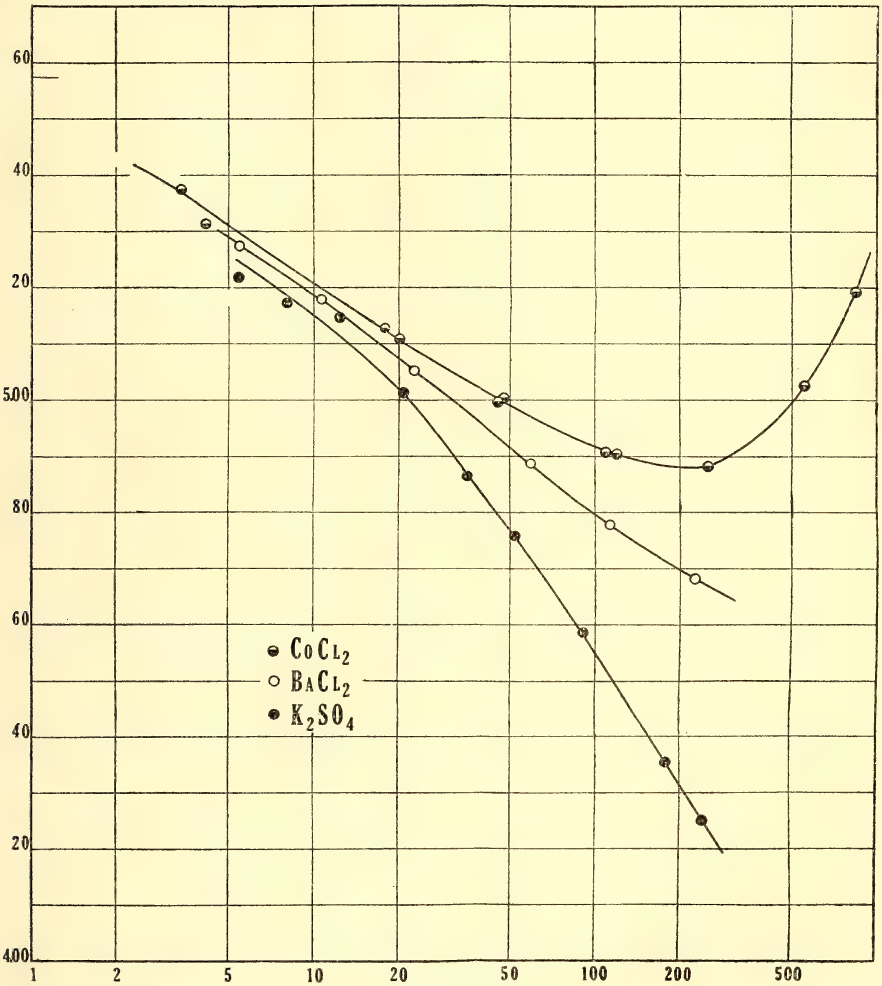


FIG. 3

The results obtained on a mixture of 1 mol of mannite and 2 mols of potassium chloride are given in the following table, which shows how closely the effect is additive. The 'deviation' given in the last column shows how much the sum of the freezing-point-lowerings caused by the separate components exceeds that observed for the mixture.

TABLE 1.

LOWERING OF THE FREEZING-POINT IN A MIXTURE OF MANNITE AND POTASSIUM CHLORIDE

CONC. MANNITE	CONC. KCl	MIXTURE	FREEZING-POINT LOWERING		DEVIATION
			Mannite	KCl alone	
0.00493	0.00987	0.04479	0.00927	0.03557	0.00005
0.01071	0.02153	0.09676	0.02012	0.07670	0.00006
0.02183	0.04367	0.19400	0.04061	0.15343	0.00004
0.04067	0.08134	0.35700	0.0757	0.2813	0.0000

So far as the writers have been able to find, this is the first work on mixtures of salts which has been carried out with an accurate temperature measuring system. One set of measurements on mixtures of a salt with a non-electrolyte has been carried out by Osaka [*Zs. physik. Chem.*, **41**, 560 (1902)], but his results show a greater deviation than that illustrated by the preceding table.

We wish to express our indebtedness to the National Academy of Sciences for a grant from the Wolcott Gibbs Fund, which was used for the purchase of the temperature measuring system, to W. P. White for the design of a special potentiometer, and to L. H. Adams and John Johnston of the Geophysical Laboratory of the Carnegie Institution for the loan of the freezing-point apparatus.

CERTAIN GENERAL PROPERTIES OF FUNCTIONS

By Henry Blumberg

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF NEBRASKA

Received by the Academy, October 14, 1916

Let $g(x)$ be a real, one-valued, bounded, continuous or discontinuous function defined in the interval (a, b). The functional values of $g(x)$ *in the subinterval* (α, β) of (a, b) have a least upper-bound, a greatest lower-bound and a saltus, which we denote respectively by

$$u(g, \alpha\beta),\ l(g, \alpha\beta) \text{ and } s(g, \alpha\beta) = u(g, \alpha\beta) - l(g, \alpha\beta).$$

The upper-bound, the lower-bound[1] and the saltus of $g(x)$ *at the fixed point* x of (a, b) are defined respectively as the (greatest) lower-bound of $u(g, \alpha\beta)$, the (least) upper-bound of $l(g, \alpha\beta)$ and the (greatest) lower-bound of $s(g, \alpha\beta)$ for all possible subintervals (α, β) of (a, b) that contain x as interior point.[2] With the given function $g(x)$, there are thus associated three new functions of x, *the upper-bound function*, *the lower-bound function* and *the saltus function*, which we denote by

$$u(g, x),\ l(g, x) \text{ and } s(g, x)$$

respectively.[3]

It is seen that

$$s(g, x) \equiv u(g, x) - l(g, x).$$

Let us, for the sake of brevity, write

$$s(g, x) \equiv s'(x), s(s', x) \equiv s''(x), s(s'', x) \equiv s'''(x), \dots .$$

Sierpiński[4] has proved the

Theorem. $s''(x) \equiv s'''(x) \equiv s^{\mathrm{IV}}(x) \dots .$

The chief aim of the present paper is to communicate a number of companion propositions to Sierpiński's theorem. The new results are based on the definition of other types of saltus, which immediately suggest themselves and arise from the one described above, when certain specified subsets of the range of the independent variable may be neglected.

The first new type arises when *finite* subsets may be neglected. For every subinterval (α, β) of (a, b), there evidently exists a number, which we denote by $u_f(g, \alpha\beta)$, uniquely characterized by the following double property:[5] For every $\epsilon > 0$, on the one hand, the set of points of the interval (α, β) for which $g(x) > u_f(g, \alpha\beta) + \epsilon$ is finite, while on the other hand, there exists an infinite set of points of (α, β) at which $g(x) > u_f(g, \alpha\beta) - \epsilon$. This number $u_f(g, \alpha\beta)$ is the lower-bound of all possible upper-bounds that $g(x)$ may have in the interval (α, β), in case a finite set of points may be neglected. Likewise, there is a number, which we denote by $l_f(g, \alpha\beta)$ characterized by the property that for every $\epsilon > 0$, the set of points of (α, β) where $g(x) < l_f(g, \alpha\beta) - \epsilon$ is finite, whereas, the set of points of (α, β) where $g(x) < l_f(g, \alpha\beta) + \epsilon$ is infinite. $l_f(g, \alpha\beta)$ is the upper-bound of all possible lower-bounds of $g(x)$ in (α, β), when a finite number of points may be neglected. Finally, we denote by $s_f(g, \alpha\beta)$ the lower-bound of the saltus of $g(x)$ in (α, β), in case a finite number of points may be neglected. Evidently

$$s_f(g, \alpha\beta) = u_f(g, \alpha\beta) - l_f(g, \alpha\beta).$$

We shall designate the numbers

$$u_f(g, \alpha\beta), l_f(g, \alpha\beta) \text{ and } s_f(g, \alpha\beta)$$

as '*the f-upper-bound,*' '*the f-lower-bound*' and '*the f-saltus*' of $g(x)$ *in the interval* (α, β). As in the case where no point may be neglected, we now define '*the f-upper-bound,*' '*the f-lower-bound,*' and '*the f-saltus*' of $g(x)$ *at the fixed point* x of (a, b), as the lower-bound of $u_f(g, \alpha\beta)$, the upper-bound of $l_f(g, \alpha\beta)$ and the lower-bound of $s_f(g, \alpha\beta)$ for all possible subintervals (α, β) of (a, b) that contain x as interior point. With the given function $g(x)$, we have thus associated '*the f-upper-bound function,*' '*the f-lower-bound function*' and '*the f-saltus function,*' which we denote by

$$u_f(g, x),\ l_f(g, x) \text{ and } s_f(g, x)$$

respectively.

In the second place, the subsets that may be neglected are *denumerable.* As before, there exists a number, which we denote by $u_d(g, \alpha\beta)$, uniquely characterized by the property that, for every $\epsilon > 0$, the set of points of (α, β) where $g(x) > u_d(g, \alpha\beta) + \epsilon$ is denumerable, whereas the set of points where $g(x) > u_d(g, \alpha\beta) - \epsilon$ is non-denumerable; this number $u_d(g, \alpha\beta)$ we call '*the d-upper-bound*' of $g(x)$ in (α, β). Precisely as before we define the related numbers

$$l_d(g, \alpha\beta),\ s_d(g, \alpha\beta),\ u_d(g, x),\ l_d(g, x)$$

and '*the d-saltus function*' $s_d(g, x)$.

In the third place, '*exhaustible*' sets (i.e., sets of first category[6]) may be neglected. We then obtain '*the e-saltus function*' $s_e(g, x)$, together with the related numbers.

Finally, sets of (Lebesgue) *zero measure* may be neglected. We then obtain '*the z-saltus function*' $s_z(g, x)$, together with the related functions.

As in the case where no point may be neglected, we write

$$s_f(g, x) \equiv s_f'(x),\ s_f(s_f', x) \equiv s_f''(x),\ s_f(s_f'', x) \equiv s_f'''(x),\ .\ .\ .\ ;$$

and similarly for the d-saltus, the e-saltus and the z-saltus functions.

Having defined the new types of saltus we had in view, we may now state the corresponding analogues of Sierpinski's theorem. The most interesting and least obvious results are the following two theorems.

Theorem. $\quad s_d'''(x) \equiv s_d^{\text{IV}}(x) \equiv s_d^{\text{V}}(x) \equiv .\ .\ .\ .$

Theorem. $\quad s_z'''(x) \equiv s_z^{\text{IV}}(x) \equiv s_d^{\text{V}}(x) \equiv .\ .\ .\ .$

Moreover, as examples show, $s_d''(x)$ and $s_z''(x)$ may be different from $s_d'''(x)$ and $s_z'''(x)$ respectively.

In the case of the f-saltus, the functions $s_f^{(n)}(x)$ $(n = 1, 2, .\ .\ .\ .\ )$ *may all be different.*

In the case of the e-saltus, while $s_e''(x)$ may be $\pm\, s_e'''(x)$, we have

Theorem. $\quad s_e'''(x) \equiv 0.$

A generalization of our result for the d-saltus is as follows:

Theorem. If $s_{\aleph}'(x)$, $s_{\aleph}''(x)$, $s_{\aleph}'''(x)$ . . . *represent the successive saltus functions that arise when*, instead of neglecting denumerable sets (i.e., sets of cardinal number $\aleph_0$), *we neglect sets of cardinal number* $\aleph$, *where* $\aleph$ *is any cardinal number* $< c$, the cardinal number of the continuum, *then*

$$s_{\aleph}'''(x) \equiv s_{\aleph}^{\text{IV}}(x) \equiv s_{\aleph}^{\text{V}}(x) \equiv .\ .\ .\ .$$

Because of our negative result in the case of the f-saltus, we are naturally led to define $s_f^{(\beta)}(x)$ for transfinite β's. Our result will show that it is sufficient to confine ourselves to transfinite numbers belonging to Cantor's second class. If β is not a limiting number, $\beta - 1$ exists, and we define $s_f^{(\beta)}(x)$ as equal to $s_f(s_f^{(\beta-1)}, x)$. For our purpose, therefore, all we have to do now is to define $s_f^{(\beta)}(x)$, in case β is a limiting number, in terms of the functions $s_f^{(\nu)}(x)$, where $\nu < \beta$. This we do simply by means of the equation

$$s_f^{(\beta)}(x) = \lim_{\nu_n \to \beta} s_f^{(\nu_n)}(x),$$

where $\{\nu_n\}$ is a sequence of numbers less than β. It is seen that this limit always exists and is independent of the particularly chosen sequence $\{\nu_n\}$.

Our positive result for the case of the f-saltus may now be stated as follows:

Theorem. There exists a number β of the first or the second class, such that

$$s_f^{(\beta)}(x) \equiv s_f^{(\beta+1)}(x).$$

Furthermore, it is shown by means of an example, that *if β is a given number of the first or the second class, then the functions $s_f^{(\nu)}(x)$ $(1 \leqq \nu \leqq \beta)$ may all be different, whereas $s_f^{(\beta)}(x) \equiv s_f^{(\beta+1)}(x)$.*

The following interesting connection exists between the d-saltus and the f-saltus.

Theorem. For every function $g(x)$ for which $s_f^{(\beta)}(x) \equiv s_f^{(\beta+1)}(x)$ —— and according to the preceding theorem there always exists such a β of the first or the second class ——, *we have*

$$s_d(s_f'', x) \equiv s_f^{(\beta)}(x).$$

The results may be easily extended to the case of many-valued, bounded, or unbounded functions of several variables, or of infinitely many variables; and by means of simple postulate systems, to more abstract situations.

The above is essentially the introduction of a paper, which is to be offered to the *Annals of Mathematics.*

[1] Throughout the paper, we use the expressions, 'the upper-bound' and 'the lower-bound,' in the sense of 'the least upper-bound' and 'the greatest lower-bound,' respectively.

[2] Of course, it will be understood, that in case $x = a, b$—and only then—we permit α, β to coincide with x. This remark applies also to similar situations below.

[3] The functions $u(g, x)$ and $l(g, x)$ are often, though not quite unobjectionably, called the 'maximum' and the 'minimum' functions belonging to $g(x)$. Cf., for example, Hobson, *The Theory of Functions of a Real Variable* (1907), art. 180.

[4] *Bull. Acad. Sci., Cracovie* (1910), 633–634.

[5] Cf. Baire, *Acta Mathematica,* **30,** 21 and 22 (1906).

[6] Cf. Denjoy, *J. math. Paris,* Ser. 7, **1,** 122–125 (1915).

SPHENACODON MARSH, A PERMOCARBONIFEROUS THEROMORPH REPTILE FROM NEW MEXICO

By Samuel W. Williston

WALKER MUSEUM, UNIVERSITY OF CHICAGO

Received by the Academy, October 23, 1916

Thirty-eight years ago, O. C. Marsh gave the name *Sphenacodon ferox* to some jaws and vertebrae collected by David A. Baldwin near Arroya de Agua in Rio Arriba County, New Mexico. From his brief description it was suspected that the form was nearly allied to the remarkable reptiles known as *Dimetrodon* Cope, from the Permian of Texas, and a study of the type collections in the Yale Museum enabled me a few years ago to confirm the suspicion. The locality whence the type specimens came had almost been forgotten till five years ago, when Mr. Paul Miller, Prof. E. C. Case and I rediscovered the original bone-bed of Baldwin about a mile and a half above the confluence of the Poleo creek with the Puerco, and near the Mexican village of Arroya de Agua. Numerous specimens were obtained from this deposit, fixing the identity of the forms with those of other remains obtained from a bone-bed in the same horizon and a half mile nearer the mouth of the Poleo. From this bone-bed or 'bone-quarry,' discovered by Mr. Miller, numerous isolated bones of *Sphenacodon* and *Ophiacodon* Marsh, together with a very perfect skeleton of the latter, were obtained, but lack of time prevented a thorough exploitation of the deposit.

The past season my son, Samuel H. Williston, and I spent a few weeks in the same region, securing some very interesting and important specimens, a detailed description of which will be published later, including the material upon which the accompanying figure is based.

The 'Miller bone-bed' is in a hard, dark brown clay, which is worked with some difficulty because of the heavy embankment over it. It extends for about 100 feet along the foot of the high hills bordering the Poleo, and bids fair to yield not a few more good skeletons. About 20 feet east of the spot that yielded the remarkable skeleton of *Ophiacodon* that has been described, a femur of *Ophiacodon* was observed protruding from the excavation made by us in 1911. Upon following it up, two skeletons were discovered, one of *Ophiacodon*, lying immediately upon another of *Sphenacodon*, with their ends reversed. The extreme tip of the skull of *Sphenacodon* had been removed and lost in our previous excavations. It is altogether possible that I am the one who is responsible for the mutilation, but I am quite willing to share the responsibility with Mr. Miller and Prof. Case!

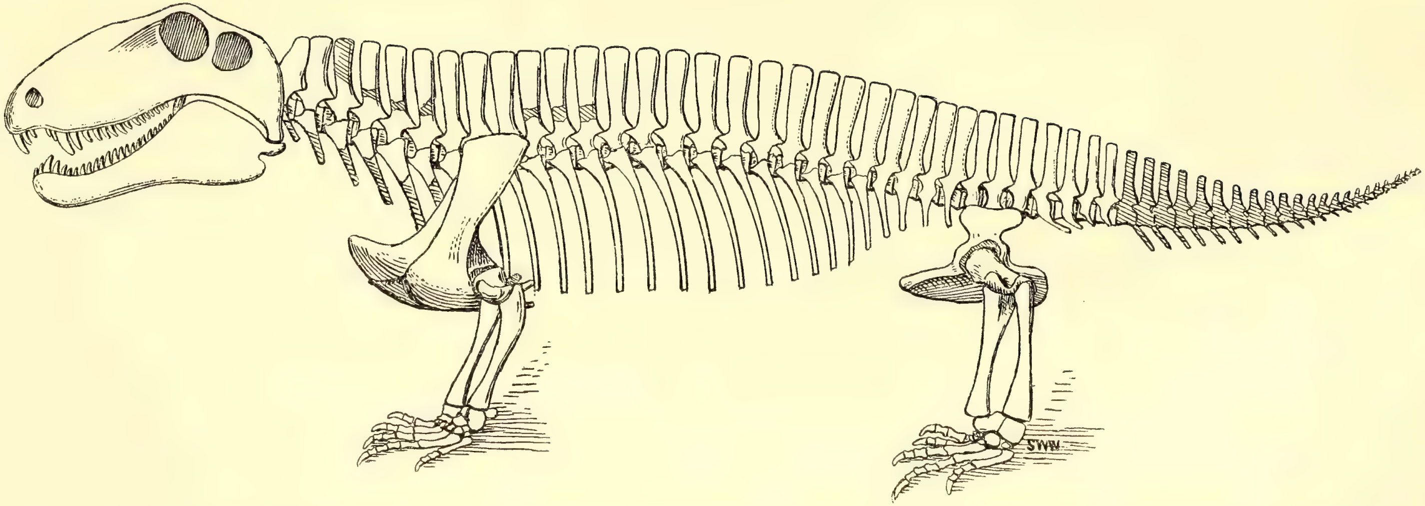


FIG. 1.—SPHENACODON MARSH, RESTORATION OF SKELETON, ONE-EIGHTH NATURAL SIZE. THE SKELETON IS REPRESENTED IN A WALKING POSTURE.

Otherwise the skull is very complete and but little distorted. Connected with the skull was a series of twenty-five presacral, three sacral and four caudal vertebrae, all closely united save for a break back of the eleventh vertebra, where there was an interval of a few inches, in which was found a single centrum and two connected spines, fixing the series as not less than twenty-seven presacral vertebrae. An additional spine, or twenty-eight in all was collected with the specimen, but inasmuch as isolated vertebrae are not rare in the deposit it is possible that the twenty-eighth spine does not belong in this series, but its close agreement with the adjoining spines makes it probable that it does, making twenty-eight presacral in all. In the accompanying restoration, however, only twenty-seven have been given, the number so commonly found in allied genera.

In collecting the vertebrae lying in the loose shales near the surface of the embankment, some fragments were lost connecting the spines, which, however, were attached to each other by the thin hard matrix with which all the bones were covered. These missing parts, which will doubtless be recovered in the excavated material next season, have been represented by oblique lines.

With this series of vertebrae, extending to the fourth caudal, were found parts of the girdles and limb bones which had been more or less lost by exposure, or the previous excavations. The preserved parts, however, are sufficient to determine their close agreement in size and characters with more perfect specimens found isolated near by. Of the feet nothing has been determined with certainty in this quarry, but the general agreement of the other parts of the skeleton with corresponding parts of *Dimetrodon*—except the vertebrae—make it practically certain that these parts are not appreciably different. As in *Dimetrodon*, the tail is, for the most part, unknown, and, as also in *Dimetrodon* the parts actually preserved in position leave little or no doubt of its relative shortness.

The spines of *Sphenacodon* are of nearly uniform length to the base of the tail. Those of the neck increase a little, both in thickness and length to the beginning of the dorsal region above the scapula, where they are the longest and stoutest in the whole series. They decrease very slightly in length posteriorly and grow thinner and less stout. With the first presacral they become markedly thinner and narrower and with the caudals grow rapidly more slender and shorter, leaving little doubt of the weakness and shortness of this part of the body. The vertebrae here also are shorter and smaller than those in front of them.

The ends of the cervical ribs have been lost, but with those exceptions they are preserved throughout the column, and not a few of them have been mended more or less completely. They are relatively slender for the size of the animal, and but little curved. Their unusual length was unexpected. The rib of the fifteenth vertebrae, for instance, measures more than ten inches. Inasmuch as the depth of the body is fixed by the girdles these ribs show decisively a broad body. The width back of the fore-legs could not have been less than twenty inches, or nearly a half of the length of the body to the sacrum, and may have been more. The length of the twenty-second, or fifth presacral, rib is seven inches. In the cervical and dorsal region they articulate by two well separated heads, the tubercle with the diapophysis, which is prominent, the head with the intercentral space. Near the middle of the back the broad head is less markedly dichocephalous, and their movability was less. With the seventh or eighth presacral they begin to show sutural union with the diapophysis and a facet on the front end of the centrum. At the fifth presacral the sutural union is obliterated and the ribs were fixed and immovable. As in *Dimetrodon*, the ribs extend back quite to the sacrum, the first presacral pair having an expanse of not less than six inches. Perhaps this unusually broad body is another reason for the short tail.

In a previous paper it was observed that the articular surface at the proximal end of the humerus in *Ophiacodon* has a spiral form, as is usual in the pelycosaurs, and that, in life the humerus could not have been depressed much below the horizontal without dislocation from the socket. Mr. Watson has called attention to this peculiarity of the American theromorphs and cotylosaurs, and has suggested that it explains the unusual expansions of the ends of this bone in these animals. It has been assumed that the remarkably broad and stout humeri of the cotylosaurs and pelycosaurs meant burrowing habits, but I have shown the error of this assumption in the fact, that, for the most part, the front legs are short, the fingers scarcely reaching as far forward as the end of the beak. Certainly an animal that could not reach in front of its nose would have difficulty in excavating a hole for the body to enter, unless it temporarily removed its head! The legs were carried, for the most part, with the elbow bent at a right angle, and the movement of the foot backward and forward in this position would account for the spiral motion in the glenoid socket. Furthermore, such a position of the limbs would require, for the support of the body, strong and broad muscular attachments on the humeri, a sufficient explanation of their shape.

Nearly every detail of the skull has been worked out and will be described and illustrated later. Suffice it to say here that the presence of quadratojugals, dermosupraoccipitals and tabulars has been definitely determined, and the supratemporals almost as definitely, thus giving to the genus nearly every skull element found in the most primitive cotylosaurs. The mandibles may be a little less slender and there may be fewer teeth but one can say with assurance that were an isolated skull of *Sphenacodon* found in Texas it would be referred unhesitatingly to *Dimetrodon*. And this similarity extends to other parts of the skeleton, with the exception of the spines of the vertebrae.

Many and various have been the conjectures offered to explain the extraordinary spines in *Dimetrodon*. In my opinion nothing better has been suggested than that of Professor Case, namely, that they represent an ornamental or senile character of the race, of no profound importance in the life economy of the animals. Certainly, as I have previously observed, if the spines of *Dimetrodon* had been of important use to their possessors they must have produced correlated differences in other parts of the skeleton. The differences in the spines have no more than a generic value.

There are doubtless several species of *Sphenacodon* in the New Mexican beds, but the known remains from the Baldwin and Miller bonebeds seem all to belong to one species. From different horizons on the Puerco, however, large specimens are known, now preserved in the Yale Museum. The present specimen, which doubtless belongs to *S. ferox* Marsh, was nearly five feet long as figured. The Yale specimens indicate a form one-half larger, or about seven and a half feet in length, or of about the size of most specimens referred to *Dimetrodon incisivus*. *Sphenacodon* is definitely known only from the valley of the Puerco and its tributaries in New Mexico; not a fragment of it has been found in the El Cobre deposits scarcely a score of miles away. However, there are specimens in the Chicago collection from Texas that suggest very strongly its occurrence there, but definite proof is lacking.

ON VOLUME IN BIOLOGY

By Lawrence J. Henderson

CHEMICAL LABORATORY OF HARVARD COLLEGE

Received by the Academy, October 28, 1916

In one respect this subject is time-honored. Herbert Spencer and many others since his day have recognized the biological importance of the principle of similitude. Bulk increases as the cube of length; surface as the square. Hence the limitation of the size of cells, the minute canalization of the body, and the prodigious jumps of the flea, as well as the variation in metabolism with the size of the organism. Yet even this subject is by no means exhausted. For example (a fact which appears to be of some importance in describing the internal regulation of temperature) the difference in temperature between center and surface of a sphere which is producing heat uniformly throughout its whole mass, when equilibrium has been established with a surrounding liquid medium of constant temperature, is proportional to the square of the radius.[1]

But apart from this great principle and certain superficial discussions of the nature of oedema and similar phenomena, the regulation of volume has remained without any physico-chemical analysis. Yet, from the standpoint of physical science, this is perhaps the most universal and fundamental of all organic regulations. I believe that this strange neglect may be traced to three facts. In the first place, the chemist is accustomed to vary the volume of his systems to suit his convenience. This is a justifiable practice, because, if the phases are so large that capillary phenomena may be disregarded, and so small that gravity need not be taken into account, the division of a phase into two parts does not change its energy. Thus, volume hardly enters into our calculations except as an indirect expression for that which is regarded as the true variable, viz., concentration. This, however, is to disregard the real question as it presents itself in biology. Secondly, *when equilibrium has been established* in a heterogeneous system, as Willard Gibbs rigorously proved, the volume of the phases—capillary and gravitational phenomena being absent—is not relevant to the state of the system. But it may be at once observed, first, that until equilibrium has been attained the volume is of great moment, and, secondly, that equilibrium is never attained within the organism. Finally, the ordinary conception of the process of diffusion is based upon a mathematical discussion, which, though leading to a consistent description of the phenomena, is nevertheless a false representation of the actual occurrences. And nearly all physiological changes of volume depend upon diffusion.

Contrary to a general though vague belief, the regulation of volume is theoretically independent of osmotic pressure regulation. For example, if a kidney produces a liter of urine of the same freezing point as blood, it must have diminished the volume of the body and left the osmotic pressure sensibly unchanged.

For the purpose of discussion, the activity of the kidney may be reduced to the following fiction: First there must be an excretion of all the dissolved constituents of blood plasma, including water itself, in such amounts as to bring the composition of the blood in all respects to a hypothetical normal composition; secondly, a certain portion of this normal plasma, minus the colloids, must be removed in order to regulate the volume.

Statistically the volume of the urine must vary with the magnitude of the volume regulation, although, in particular cases, it need bear no relation to this quantity. Now sodium chloride is the principal constituent of blood plasma. Hence, statistically the ratio Δ: NaCl (i.e. the ratio of freezing point depression to sodium chloride concentration in urine) will be small when the volume is large, and large when the volume is small. Thus we arrive at a theoretical deduction *a priori* of Korányi's coefficient.[2] This coefficient tells us, therefore, nothing about the mode of action of the kidney mechanism. It has no bearing on the question of the functions of glomeruli and tubules. For the ratio is seen to be, as existing evidence proves, necessarily liable to indefinite fluctuations in particular cases. There is involved merely a statistical truth, expressing the conditions under which any kidney must operate in case sodium chloride is the chief constituent of the blood.

The practical importance of this theoretical discrimination of volume regulation from the excretion of the several urinary constituents appears to be established by the fact that in pathological conditions the daily volume of urine may be constant during variations in amounts of water or salt ingested, even when such experiments lead to fluctuations in the physico-chemical properties of the urine.[3] For this shows that the regulation of volume may be deranged more or less independently of the proper excretory functions.

If the final stage of volume regulation has been neglected, the intermediate stages have been generally misconceived. Neither the swelling of colloids nor the pressure which results from osmosis can furnish the basis for an analysis of such phenomena. In their stead we must turn to the kinetic theory and to Willard Gibbs' thermodynamics. But it should be first remarked that osmotic 'pressure' and colloidal swelling 'pressure' hardly act as important sources of mechanical tension, in the living organism. Even oedema involves very small magnitudes of such pressures. Secondly, changes in volume merely consist in the passage of material from one phase to another (except in so far as physical and chemical changes within a phase may produce very slight fluctuations in volume). Finally, apart from the operation of special secretory mechanisms which we do not understand, such processes consist in the diffusion through phases and across interfaces from points of higher to points of lower potential. In the internal phenomena no less than in the exchanges with the environment, and in the internal phenomena of a single phase no less than in the heterogeneous process, water is quite as much in question as the dissolved substances.

This fact has been generally overlooked, by the physicists even more than by the physiologists. We may consider a simple diffusion experiment in which a layer of water is placed above a layer of sugar solution. It is true that the sugar must diffuse up into the water, but it is also true that the water must diffuse down into the solution. Yet the phenomena of diffusion have always been discussed with the help of the mathematical theory of heat conduction as exactly defined by Fourier. Thus Fick's theory of diffusion overlooks the rôle of the solvent. This has been possible because the process takes place *as if* the solvent were inert and the dissolved substance possessed a higher diffusibility. But the diffusion of the solvent is probably greater than that of the solute, in that water is, with a few exceptions, the most diffusible of substances. In their neglect of the relativity of motion, contemporary accounts of diffusion involve an old fallacy that occurs even in the astronomy of Ptolemy.

It may perhaps be objected that throughout the organism water exists at a uniform potential. This, however, is both untrue and beside the point. For, if two phases are free to exchange material, an exchange involving all their components will take place unless, at the outset, all their components possess the same potential in both phases. Thus, as Höber[4] has shown, isotonic solutions of sodium chloride and magnesium sulphate change in volume when brought into contact. For the chloride diffuses faster than the sulphate. Thus the sulphate solution becomes more concentrated and, as a result, water diffuses into it from the chloride solution. In like manner differences of pressure and of temperature influence the potential of water and of all dissolved substances. Finally, the processes of metabolism are continually altering the concentrations of solutions, and therefore the potential of water.

When these facts are taken into account, it becomes clear that the chief physical factor in the internal regulation of volume is water, through its distribution between the infinite assemblage of phases which make up the organism. The general concepts of the phase rule reveal the several elements of this process—except those which depend upon so-called selective activities—but the physical theory of diffusion, and therefore the whole kinetic description, has been developed from a false representation of the process, which neglects the movements of water itself. Only when this is immediately evident, as in osmotic phenomena, is it at all taken into account. But even here the current explanations are often incomplete. And the general theory of the diffusion of water is almost useless for the purposes of physiology. Yet there seems to be no reason to doubt that, in the organism, this is the most important process of diffusion.

---

[1] I am indebted to Prof. W. E. Byerly for kindly calculating the value of this difference which is $p/k. r^2/6$, where p is rate of heat production, k conductivity, and r the radius.

[2] A. v. Korányi, *Zs. Klin. Medizin*, **33**, 1892, and Korányi und Richter, *Physikalische Chemie und Medizin*, Leipzig, 1908, **2**, pp. 133–224.

[3] *Ibid.*, pp. 165, 166.

[4] *Pflüger's Archiv*, **74**, 225 (1899).

CONTENTS

Page

PHYSICS.—PATH DIFFERENCES WITHIN WHICH SPECTRUM INTERFERENCES ARE OBSERVABLE . . . . *By Carl Barus* 609

PHYSICS.—NON-REVERSED SPECTRA OF RESTRICTED COINCIDENCE . *By Carl Barus* 614

CHEMISTRY.—THE EQUILIBRIUM BETWEEN ACIDS AND BASES IN SEA WATER . . . *By Lawrence J. Henderson and Edwin J. Cohn* 618

BIOLOGY.—AN APPARENT CORRESPONDENCE BETWEEN THE CHEMISTRY OF IGNEOUS MAGMAS AND OF ORGANIC METABOLISM . . . . . . *By Henry S. Washington* 623

BOTANY.—THE OAKS OF AMERICA . . . . *By William Trelease* 626

MATHEMATICS.—A SET OF INDEPENDENT POSTULATES FOR CYCLIC ORDER . . . . *By Edward V. Huntington* 630

PSYCHOLOGY.—A NEW METHOD OF STUDYING IDEATIONAL AND ALLIED FORMS OF BEHAVIOR IN MAN AND OTHER ANIMALS . . . . *By Robert M. Yerkes* 631

CHEMISTRY.—ELECTRICAL CONDUCTION IN DILUTE AMALGAMS . . . . *By Gilbert N. Lewis and Thomas B. Hine* 634

PSYCHOLOGY.—IDEATIONAL BEHAVIOR OF MONKEYS AND APES. *By Robert M. Yerkes* 639

CHEMISTRY.—THE OSMOTIC PRESSURE AND LOWERING OF THE FREEZING-POINT OF MIXTURES OF SALTS WITH ONE ANOTHER AND WITH NON-ELECTROLYTES IN AQUEOUS SOLUTIONS . *By William D. Harkins, R. E. Hall, and W. A. Roberts* 642

MATHEMATICS.—CERTAIN GENERAL PROPERTIES OF FUNCTIONS . *By Henry Blumberg* 646

PALEONTOLOGY.—SPHENACODON MARSH, A PERMOCARBONIFEROUS THEROMORPH REPTILE FROM NEW MEXICO . . . . *By Samuel W. Williston* 650

BIOLOGY.—ON VOLUME IN BIOLOGY . . . . *By Lawrence J. Henderson* 654

VOLUME 2 DECEMBER, 1916 NUMBER 12

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES
OF THE
UNITED STATES OF AMERICA

FOUNDED 1863

EDITORIAL BOARD

Arthur A. Noyes, *Chairman* Edwin B. Wilson, *Managing Editor*
Arthur L. Day, *Home Secretary* George E. Hale, *Foreign Secretary*

J. J. Abel
W. B. Cannon
J. McK. Cattell
J. M. Clarke
E. G. Conklin
J. M. Coulter
C. B. Davenport
Simon Flexner
E. B. Frost
R. G. Harrison
W. H. Holmes
J. P. Iddings
R. A. Millikan
E. H. Moore
Alexander Smith

Publication Office: Williams & Wilkins Company, Baltimore
Editorial Office: Massachusetts Institute of Technology, Cambridge
Home Office of the Academy: Washington, D. C.

Entered as second-class matter at the Post Office at Baltimore, Md., under the Act of August 24, 1912

INFORMATION TO CONTRIBUTORS

THE PROCEEDINGS is the official organ of the Academy for the publication of brief accounts of important current researches of members of the Academy and of other American investigators, and for reports on the meetings and other activities of the Academy. Publication in the Proceedings will supplement, not replace, that in journals devoted to the special branches of science. The Proceedings will aim especially to secure prompt publication of original announcements of discoveries and wide circulation of the results of American research among investigators in other countries and in all branches of science.

ARTICLES should be brief, not to exceed 2500 words or 6 printed pages. Technical details of the work and long tables of data should be reserved for publication in special journals. But authors should be precise in making clear the new results and should give some record of the methods and data upon which they are based. The viewpoint should be comprehensive in giving the relation of the paper to previous publications of the author or of others and in exhibiting where practicable, the significance of the work for other branches of science.

MANUSCRIPTS should be prepared with a current number of the Proceedings as a model in matters of form, and should be typewritten in duplicate with double spacing, the author retaining one copy. Illustrations should be confined to text-figures of simple character, though more elaborate illustrations may be allowed in special instances to authors willing to pay for their preparation and insertion. Particular attention should be given to arranging tabular matter in a simple and concise manner.

REFERENCES to persons should include initials; citations of literature, numbered consecutively, should be placed at the end of the article; and short footnotes should be avoided. Indicate 'subject' for 'Contents.'

PAPERS by members of the Academy may be sent to Edwin Bidwell Wilson, Managing Editor, Mass. Institute of Technology, Cambridge, Mass. Papers by non-members should be submitted through some member.

PROOF will be sent to authors only on their request, in order to avoid delay in publication. Authors are therefore requested to make final revisions on the typewritten manuscripts. The editors cannot undertake to do more than correct obvious minor errors.

REPRINTS should be ordered at the time of submission of manuscript. They will be furnished to authors at cost, approximately as follows:

Reprints of - -	2 pp.	4 pp.	6 pp.	8 pp.	Covers extra
Charge for first 100 copies	$1.10	$1.45	$2.50	$2.50	$2.50
Charge for additional 100s	.35	.60	1.10	1.10	1.00

INFORMATION TO SUBSCRIBERS

SUBSCRIPTIONS at the rate of $5.00 per annum should be made payable to the National Academy of Sciences, and sent to Williams & Wilkins Company, Baltimore, or Arthur L. Day, Home Secretary, National Academy of Sciences, Smithsonian Institution, Washington, D. C. Single numbers, $0.50.

Copyright, 1916, by the National Academy of Sciences

PROCEEDINGS

OF THE

NATIONAL ACADEMY OF SCIENCES

Volume 2 DECEMBER 15, 1916 Number 12

THE ORIGIN OF VEINS OF THE ASBESTIFORM MINERALS

By Stephen Taber

DEPARTMENT OF GEOLOGY, UNIVERSITY OF SOUTH CAROLINA

Received by the Academy, November 4, 1916

In its strict application the name asbestos is limited to the fibrous varieties of the monoclinic amphiboles. The term is commonly applied, however, to those minerals having a highly developed fibrous (asbestiform) structure. When such minerals occur in veins the fibers are parallel, and run transverse to the strike of the veins. Hence they have been called cross-fiber veins.

Microscopic examination shows that bundles consisting of thousands of fibers of chrysotile or amphibole asbestos behave as crystal units, an entire bundle exhibiting the same optical properties as any one of the component fibers. All asbestiform minerals have prismatic cleavage, and this suggests that the fibrous structure may be due to an abnormal development of the cleavage, or at least that the separation of the fibers takes place largely along cleavage planes. The highly fibrous structure of the asbestiform minerals is not, however, a crystallization phenomenon in the sense that it is due solely to the inherent physical properties of the crystal molecule, for all minerals having asbestiform varieties occur in non-fibrous as well as fibrous forms.

A comparative study of the occurrence of all commonly fibrous minerals indicates that the peculiar structure of asbestiform minerals is due to the accentuation of a normal prismatic habit and cleavage by physical conditions which have limited crystal growth to a direction parallel to the principal axis. *Many minerals possessing perfect prismatic cleavage do not have fibrous varieties*, but they are always minerals that crystallize from fusion or solution under conditions permitting of growth in more than a single direction. All asbestiform minerals are second-

and for the most part they are limited to metamorphic rocks. The primary minerals of igneous rocks are never fibrous. *Many minerals without prismatic cleavage have fibrous varieties.* They differ from asbestos in having fibers that are usually coarser, more brittle and not so easily separable. With fibers of equal size the flexibility and tensile strength are probably determined chiefly by chemical composition. When minerals do not have a columnar or prismatic habit, the fibrous structure is obviously due to mechanical conditions which have prevented growth except in one direction.

The shape of a growing crystal is controlled by one or more of three independent factors, namely: (1) the tendency to assume a regular polyhedral form because of the forces of surface tension and molecular orientation; (2) the relative and absolute magnitude of the external forces resisting growth in different directions; and (3) the accessibility of the material from which the crystal is built. As a result of the first factor some substances normally crystallize in slender acicular or hair-like forms, but an asbestiform structure is never produced without the assistance of one of the other two factors. Crystal growth under unequal pressure undoubtedly explains the development of fibrous structure in some instances. The solubility of most substances increases with the pressure and therefore crystal growth may be limited to the direction of least pressure. This hypothesis is not applicable, however, to fibers that develop normal to the walls of a vein. Moreover the veins frequently intersect and run in all directions through a given rock mass. Such occurrences of abnormal fibrous structure must therefore be attributed largely to the circumstance that the material for growth was accessible only in one direction.

The efficacy of this method of producing fibrous structure in crystals has been proved experimentally.[1] Cups of porous porcelain were partly immersed in concentrated solutions of copper sulphate, alum, and other salts, thus permitting the solutions to be drawn by capillary attraction into the upper and exposed walls of the cups. When the solutions occupying the small pores were supersaturated by evaporation or cooling so that crystallization began, fibrous crystals grew slowly outward from the surfaces of the cups.

Crystals also developed at favorable places within the walls of the cups, which were ruptured by the pressure as growth continued. The fractures thus formed were gradually extended and widened by the growth of fibrous veins, closely resembling in structure the cross-fiber veins found in rocks. The crystals have a fibrous structure because additions of new material were made only at their base. Using alum solutions it was found possible to change the color of the material by adding chrome alum, and so prove experimentally that the fibers grew only at their base.

The development of the fibrous structure is probably aided by small adjustments or slips along the surfaces of the fiber prisms and cleavages when the latter are present.

Fibers of copper sulphate and alum are brittle and therefore difficultly separable, but some were obtained that had a length of several millimeters and a thickness of less than 0.001 mm. Such fibers are slightly flexible and somewhat elastic. Slender acicular forms are never found in crystals of copper sulphate or alum that have grown normally in free contact with solutions, because this form is not the most stable. The total surface energy of one of the acicular crystals is great as compared with the mass, and therefore they are dissolved by solutions that are supersaturated with respect to more stable forms.

Many theories[2] have been advanced in explanation of the origin of cross-fiber veins, particularly those of chrysotile. Most of these theories presuppose the existence of open fissures in which the vein minerals were deposited. It is conceivable that some cross-fiber veins may have been formed in pre-existing fissures, but in most cases this is mechanically impossible.

Dresser[3] has pointed out the absurdity of this theory as applied to the chrysotile deposits of southern Quebec, Canada, where the veins run in all directions from vertical to horizontal, occasionally reaching a length of 100 feet, and in places occupy over 10% of the entire rock. According to Pratt[4] and Diller[5] the chrysotile veins in the Grand Canyon, Arizona, over 4000 feet below the rim, extend horizontally parallel to the bedding of the enclosing rocks for distances of 150 feet or more.

In many instances there is evidence that the formation of chrysotile veins and the alteration of the enclosing rock to form massive serpentine were contemporaneous processes; but the alteration of a rock to serpentine is usually accompanied by an increase in volume sufficient to close all appreciable openings. The lenticular shape of many veins is an argument against the theory of deposition in open fissures of mechanical origin.

It has been suggested that chrysotile veins may have been formed by some process of replacement, but no one has explained why serpentine should replace serpentine of the same chemical composition. More over, chrysotile veins never contain pseudomorphs or show any trace of an inherited structure. Replacement veins are characterized by great irregularity in width and lack of definite boundaries, while chrysotile veins are sometimes remarkably uniform in width, and always have well defined walls that are easily separable from the veins.

The last objection applies likewise to the hypothesis that chrysotile veins represent portions of the serpentine that have crystallized *in situ*. Recrystallization is a common phenomenon in rocks, but it is never confined to a narrow vein-like zone with sharply defined walls. If chrysotile can be formed by such a process, why is the ratio of chrysotile to massive serpentine limited? Why is the entire mass of serpentine never recrystallized to form chrysotile?

Not one of the theories previously cited satisfactorily explains the presence of one or several partings in some veins and their absence in others. They do not explain the angular inclusions of massive serpentine that frequently mark the partings in chrysotile veins and are also found irregularly distributed through them. They do not explain why the fibers are normal to the vein walls in some instances and oblique in others. They do not explain the presence of sharp bends in the fibers, nor veins of cross fiber that grade into slip fiber. In view of all these facts it must be concluded that previous theories are inadequate to explain the origin of fibrous veins.

As noted above, cross-fiber veins with structural features similar to those found in rocks have been produced in the laboratory where their formation and growth could be observed. The evidence thus obtained supplements that furnished by veins of fibrous minerals, and makes inevitable the conclusion that cross-fiber veins are formed through a process of lateral secretion the growing veins making room for themselves by pushing apart the enclosing walls.

The force that enables the growing veins to push apart their walls is not due to the tendency of a crystalline substance to assume a regular polyhedral form, for the columnar or fibrous structure of most minerals occurring in cross-fiber veins is not a crystallization property, but is caused by the conditions of growth.[6] Under similar conditions the fibrous structure will develop in substances that crystallize in any of the systems of crystallization. In most cases it is not the normal habit, and therefore is unstable. The writer believes that the force is due chiefly to the expansion in volume which accompanies the separation of most solids from solution, for, as yet, he has obtained no pressure effects during the crystallization of substances that separate from solution with decrease in volume. When a substance separates from solution with increase in volume, the pressure developed depends on the magnitude of the forces resisting expansion, and may be much greater than the force required to crush the substance. It is improbable that pressure alone can expel solutions occupying subcapillary pores in rocks; and in serpentine and other rocks found enclosing cross-fiber veins the openings are almost entirely subcapillary in size. In such cases the transfer of material to the growing vein is probably due to diffusion rather than to circulation.

A central parting is formed whenever fibers start to grow from both walls of a fracture, and apparently this always happens when veins develop along pre-existing fractures unless growth is limited to one side only. In laboratory experiments, growth is frequently limited to one side by the development of the fracture in such a direction as to cut the other side off from solutions furnishing material for growth. When the formation of the fracture and the beginning of vein growth are contemporaneous, the parting is absent because of simultaneous growth at both ends of the fibers—a fact proved in the laboratory by changing the color of the solutions so as to produce symmetrically banded veins. Partings may also result from a pause in the process of growth or from a slight displacement of the walls. The irregularity of some partings is due to the more rapid growth of groups of fibers most favorably situated to receive additions of new material.

The inclusions of wall rock found along central partings represent fragments formed when rupture occurred, and their position is due to the growth of the vein on both sides as new material was added through the walls. Occasionally vein matter begins to crystallize along an incipient fracture or line of weakness close to the vein, and in this way a fragment is gradually separated from the wall and included in the growing vein.

The fibers are always parallel to one another and extend in the direction in which the walls moved as they were pushed apart by vein growth. In most veins the fibers are normal to the walls because the latter have been forced directly apart, but when the walls have had also a lateral displacement because of the simultaneous growth of adjacent non-parallel veins or other causes, the fibers grow in the direction of the resultant motion. If the course of the vein is not straight, the fibers may be normal to the walls at one place and oblique at another. As long as the relative motion of the walls of a growing vein is in a straight line, the fibers are straight; any change in the direction of motion is immediately recorded by the slowly lengthening fibers. If the change in the direction of relative motion is gradual and continuous the fibers are curved; if abrupt, it results in the development of sharp bends. Sometimes the fibers of a chrysotile vein record several changes in the relative movement of the walls, and this gives a banded appearance due to the unequal reflection of light where the fibers run in different directions.

The structural phenomena discussed in the preceding paragraphs all furnish evidence tending to confirm the writer's theory of the origin of cross-fiber veins, and they are difficult or impossible of explanation under any other hypothesis of vein formation hitherto advanced.

Conclusions. Cross-fiber veins are formed through a process of lateral secretion, the growing veins making room for themselves by pushing apart the enclosing walls. The fibrous structure is to be attributed largely to the mechanical limitation of crystal growth through the addition of new material in only one direction. In the case of the asbestiform minerals the fibrous structure is accentuated by a normal prismatic habit and cleavage.

---

[1] The experiments herein referred to are described in my more extended paper on "The Genesis of Asbestos and Asbestiform Minerals" which will appear in an early number of *Bull. Amer. Inst. Min. Eng.*

[2] For a review of these theories, see Fritz Cirkel, Chrysotile asbestos, its occurrence, exploitation, milling and uses, Ed. 2, *Canada Dept. of Mines, Mines Branch, Report No. 69* (1910); O. B. Hopkins, A report on the asbestos, talc and soapstone deposits of Georgia, *Geol. Survey Ga., Bull.* No. 29 (1914); and J. A. Dresser, Preliminary report on the serpentine and associated rocks of Southern Quebec, *Canada Dept. Mines Geol. Survey Mem.* No. 22 (1913),

[3] Dresser, J. A., *Ibid.*, p. 65.

[4] Pratt, J. H., Asbestos, *U. S. Geol. Survey Mineral Resources*, 1904, pp. 1137–1140 (1905).

[5] Diller, J. S., Asbestos, *U. S. Geol. Survey Mineral Resources*, 1907, pt. II, pp. 720–721 (1908).

[6] Taber, Stephen, The growth of crystals under external pressure, *Amer. J. Sci.*, ser. 4, **41**, 532–556 (1916).

A NEW TEST OF THE SUBSIDENCE THEORY OF CORAL REEFS

By Reginald A. Daly

DEPARTMENT OF GEOLOGY AND GEOGRAPHY, HARVARD UNIVERSITY

Received by the Academy, October 27, 1916

According to Charles Darwin's much-discussed theory, a barrier reef represents the upgrowth of a coral reef which originally fringed a sinking (generally volcanic) island; and an atoll reef represents a further upgrowth, completed in typical form after the central island has sunk below sea-level. Between the upgrowing reef and the subsiding, central island is a concavity or 'moat.' Through the accumulation of detritus and shells and skeletons of organisms inside the reef, the moat is slowly filled. Above the detrital covering of the moat surface is the water of the 'lagoon.' The subsidence is supposed to have progressed until the deepest part of each moat has sunk scores, hundreds, or possibly thousands, of meters (see fig. 1).

The observed general flatness of the lagoon floors of atolls shows that there the hypothetical moats must have been completely filled and obliterated as distinct submarine forms. The depression occupied by the lagoon water inside each barrier reef has the position of the hypothetical moat, but in every case the lagoon floor is about as flat as an atoll-lagoon floor. For each kind of lagoon, therefore, the filling of the moat must be assumed to be far advanced. The inferred maximum depths of calcareous detritus are to be estimated in scores, hundreds, or possibly thousands, of meters. The mechanism of this filling has never been adequately discussed; yet its analysis cannot fail to indicate a valuable test of the subsidence theory of reefs. To the writer's knowledge, no earlier statement of the test has been published.

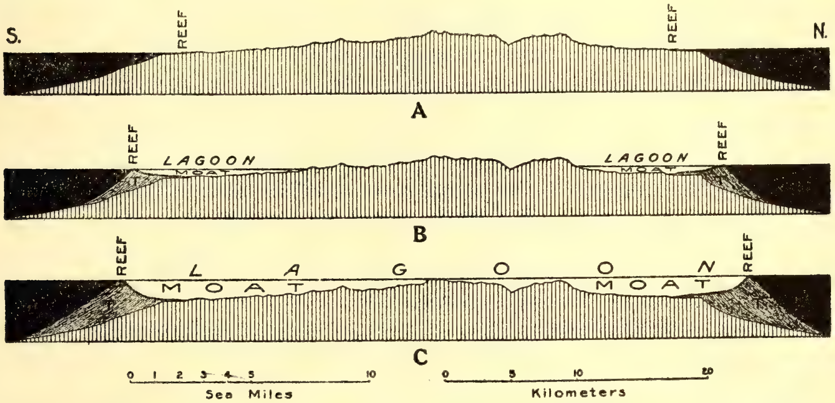


FIG. 1

Sections illustrating the subsidence theory as applied to the wider barrier and atoll lagoons; also illustrating the relations of moat, moat filling, lagoon, and central island. There is no exaggeration of the vertical element, and the depth of water in the deepest existing lagoons is no greater than that shown, to scale, by the width of the black line drawn below the word 'Lagoon.' Water indicated by solid black.

A. North-south section of the island of Kauai, chosen as a large, deeply eroded, volcanic island, which may be fairly presumed to typify the ancestor of one of the larger atolls, according to the subsidence theory. Fringing reefs indicated.

B. Section of the same island after subsidence, with the growth of each fringing reef (now become a barrier) upward and outward on its own talus (*T*). Between the reef and central island is the moat, the upper part of which is occupied by the water of the lagoon. The detrital filling of the moat is not shown.

C. Section of the same island after further drowning, with widening of the moat and ultimate development of an atoll.

The filling mechanism involves two factors: sediment, and active transportation of that sediment. Neither has been thoroughly discussed by Darwin or by any of his supporters; yet serious attention to both factors is the manifest, special duty of all those who advocate the subsidence theory.

Darwin stated that the filling of the concavity, here called a moat, is to be referred to "the growth of the delicately branched corals within the reef, and to the accumulation of sediment" (*The Structure and Distribution of Coral Reefs*, London, 2d ed., 1874, p. 132). He did not precisely indicate the nature or origin of the sediment. In part it is derived from the erosion of the central island, though at a rate rapidly diminishing as the island sinks; in part from the reefs, inner and outer; and in part from the remains of non-coral organisms, both free-swimming and bottom species. The rain of planktonic and nektonic shells and skeletons to the bottom will, of itself, be nearly uniform in the lagoon and not directly competent to obliterate the moat, until the lagoon is filled almost to sea-level. Modern field studies, like those at Funafuti, have shown that bottom lagoon organisms, including corals, are very largely restricted to depths less than 30 meters; that is, to shallows immediately adjoining the main reef, the coral knolls of the lagoon, and the shores of the central island, if present. In general the lagoon floor is 'sandy' and barren of growing coral. Dana held that the open-ocean waves throw into the lagoon, over the main reef, 'a large part' of the débris eroded from the growing, outer face of that reef. However, the detritus thrown over the main reef by waves of the open ocean is practically nil in all those cases where the reefs form dry land; and there are evidences, not here to be detailed, that the relative amount of lagoon sediment so added to the deposits in the average lagoon is actually small. The sediment brought into the lagoon through the occasional ship channels may be more important, but most of it is deposited, in delta form, just inside the main reef.

Thus, whatever be the various origins of the lagoon sediments, the detritus necessary to fill the hypothetical moat must be found principally in the shallower places within the lagoon. If the detritus were as fine as mud, its transfer to the deeper part of the moat might be comparatively rapid. Observation shows, on the other hand, that lagoon floors are characteristically more sandy than muddy, even in depths of 50 meters or more. Marine sand is transported on the bottom. It follows, then, that, if the floor of the hypothetical moat were carried down by subsidence to depths greater than those where waves and currents can stir bottom sands, the moat could be obliterated only after the detrital shelves, advancing from the shallows, have become confluent. Obliteration of the moat by this process is extremely slow for narrow lagoons, still slower for lagoons 20 to 50 kilometers wide.

Incidentally a question, not directly the subject of the present communication, arises. Is it possible to credit this explanation of all the hundreds of atoll lagoons, every one of which exhibits no trace of the hypothetical moat? The entire absence of moat concavities in the world's atolls and the correlated flatness of barrier lagoons can only mean prolonged crustal stability in the reef-charged areas.

The agents most responsible for the transportation of sediment into the depths of the hypothetical moat must be the waves and currents set going in the lagoon itself. The normal atoll or barrier reef, which reaches nearly or quite to sea-level, almost perfectly protects the lagoon from the powerful waves and currents of the open ocean. Some water is thrown over into the lagoon when great rollers break on low parts of the reef, with the effect of forming currents in the lagoon; but such currents cannot be other than weak. Tidal currents may be strong in narrow ship channels; inside the reef they are weak. The advance of detrital shelves, so as to fill the hypothetical moat, must, then, be very largely attributed to the activity of waves and currents originating in the lagoon. Except possibly during the rare hurricanes, these agents are incompetent to brush sand over the general bottom at the depths commonly ruling in atoll and barrier lagoons. But, assuming (in accordance with the hypothesis) that the transporting agents have been competent to fill the moats, an important effect on the form of the lagoon floor should be expected; the moat might in time be nearly filled, but *the lagoon floor should not be level.*

That feature ought to be found especially in lagoons situated in the trade-wind belts and not affected by monsoons or other strong winds from westerly quadrants. In these cases the lagoon waves and currents must set in a westerly direction, and chiefly in that direction sediment should be transported. Hence the hypothetical moat, like its existing lagoon remnant, should be filled at different rates on the windward and leeward sides respectively. According to the degree of enclosure of the lagoon by continuous reefs, the leeward part of its floor should now be higher or lower than the windward part. To use a technical expression, the lagoon area should be differentially aggraded. The same tendency ought to be seen in lagoons subject to variable winds, among which is a dominant system of winds blowing from one quarter of the compass.

To this reasoning it might be objected that back-running currents would reverse the direction of transportation for bottom débris. 'Banking' of water against the leeward section of the main reef would naturally cause return currents, if the reef were there continuous. Even in that case the currents must be comparatively feeble and unable to stir sand at depths of 40 or more meters. However, the leeward part of each atoll and barrier reef is usually discontinuous, the gaps in the reef wall being deep enough and wide enough to permit the ready escape of water blown thither by the wind. Hence, return or compensatory currents are not likely to be efficient in undoing the work of the direct, wind-driven waves and currents.

Consulting the charts and meteorological records, one finds scores of typical atolls located in belts where the trade winds cause the dominant waves and currents. The floor of each of these lagoons is strikingly level and not higher, more aggraded, either to leeward or to windward of the lagoon center. An example is the best known of all atolls, Funafuti, in the Ellice group of the Pacific (fig. 2). Funafuti has occasional strong winds from westerly quadrants, but records show the dominance of the trade winds. Since the main reef at Funafuti is continuous for long stretches on the leeward side, its lagoon should, according to the subsidence theory, be distinctly shallower there than on the windward side. The charted soundings, actually much more numerous than those entered in figure 2, demonstrate that the floor of the lagoon is nearly level and that the variations in depth have no systematic relation to the trend of the trade wind.

The lagoon floors of many other Pacific atolls, including those of the Marshall and Paumotu groups, show similar levelness. Most barrier lagoons of the Fiji and other archipelagoes in the trade-wind belts are also characterized by practically the same depths to windward and leeward of their respective central islands. Exceptional lagoons in the southwest Pacific have been shallowed or deepened, uniformly or differentially, by recent, local warpings of the earth's crust; in these instances the test cannot be applied with conclusiveness. Nor is it easily applied to the Maldives and neighboring atolls of the Indian ocean, where the monsoons change direction, through an angle of 180°, every few months.

Nevertheless, in spite of the quite special weather conditions required by the test, very many atoll and barrier lagoons are known to be favorably situated, and yet none shows systematic lack of symmetry in the submarine profiles crossing its lagoon from windward to leeward. So also are the cross-profiles of the windward and leeward reefs themselves respectively of the same quality. The windward reefs tend to be more developed, but their inner slopes are of nearly the same steepness as the inner slopes of the corresponding leeward reefs.

In conclusion, observed facts do not seem to agree with a legitimate deduction from the subsidence theory of coral reefs. On the other

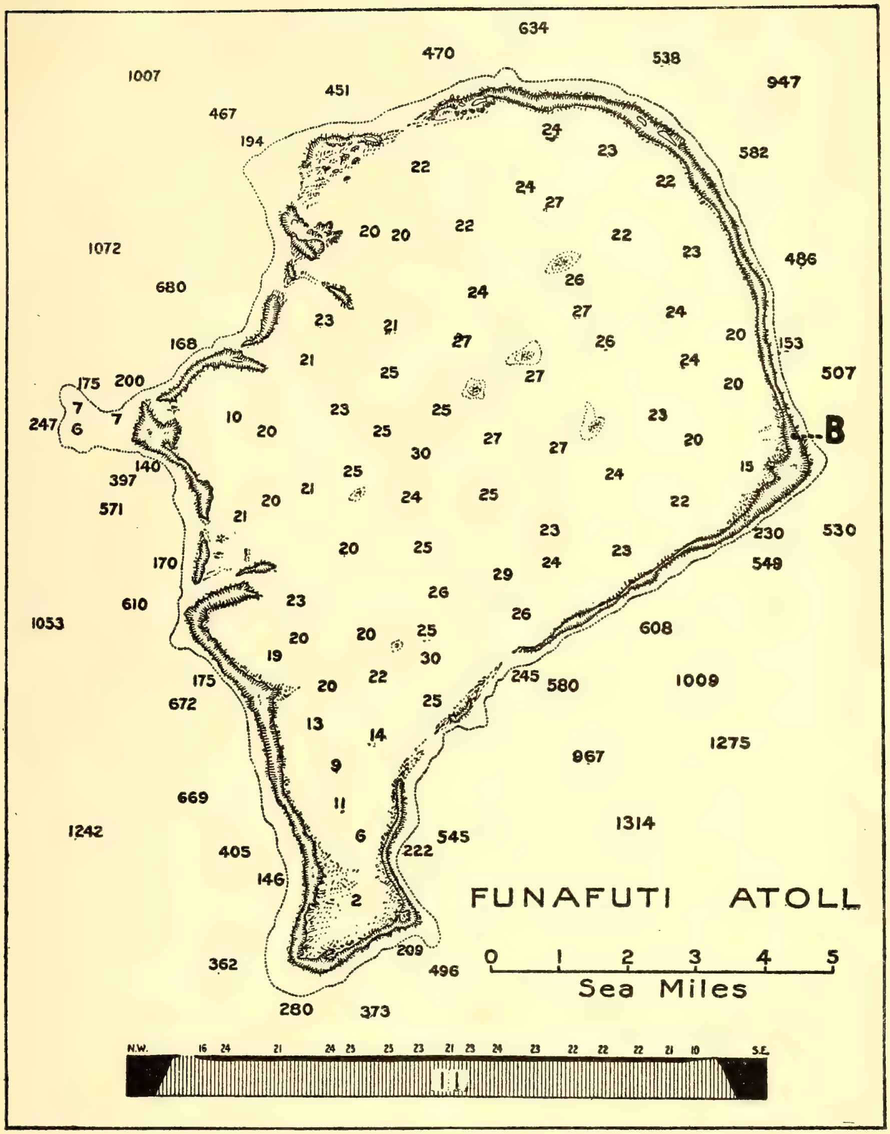

634
470
538
1007
451
947
467
194
24
23
582
22
24
27
22
20
20
22
1072
22
23
486
680
26
24
23
21
27
24
27
26
20
153
168
21
25
24
27
507
175
200
20
7
B
247
6
7
10
20
23
25
25
23
27
30
27
20
140
397
25
15
21
24
25
24
20
571
21
22
23
530
230
170
20
25
23
24
549
29
26
610
23
1053
26
608
20
20
25
19
30
175
245
580
672
20
22
1009
25
13
14
1275
967
9
11
669
1314
6
545
1242
405
222
146
FUNAFUTI ATOLL
2
209
0 1 2 3 4 5
362
496
Sea Miles
280
373
N.W.
16 24 21 24 25 25 23 21 23 24 23 22 22 22 21 10
S.E.

FIG. 2

Chart and section of the Funafuti atoll. Depths in fathoms (1 fm. is 1.83 m.). In the section water is shown in solid black; vertical scale three times the horizontal. The section is run near the middle of the atoll and in the general direction of the trade wind.

hand, a study of the symmetrical profiles of the reef-lagoon structures here considered has furnished one more direct indication that the existing coral reefs are new upgrowths on platforms, which have been formed before, and independently of, the reefs. The volumes of the reefs suggest that these began to grow not many thousands of years ago. The platforms, on which the reef crowns rest, seem to have been finally smoothed, and prepared as foundations, during the Glacial period. In any case, the submarine topography of each reef-platform structure *as a whole* and the elementary principles of oceanography together declare against the assumption that the forms and spatial relations of atoll and barrier reefs are due to the sinking of the earth's crust.

A NEW THERMOMETER SCALE

By Alexander McAdie

BLUE HILL OBSERVATORY, HARVARD UNIVERSITY

Received by the Academy, October 23, 1916

In the *Physical Review* (Vol. VI, No. 6, December, 1915), I urged the importance of adopting without further delay, in aerophysics, units that have a scientific basis. At Blue Hill Observatory for the past two years we have published summaries of air pressure also vapor pressure in *kilobars* or force units, temperature in degrees absolute centigrade, and wind velocities in meters per second. Wind direction is given in degrees, starting from the West; rainfall in millimeters and evaporation in force units.

The British Meteorological Office, under the progressive leadership of Sir Napier Shaw has published since 1915 results in similar units. Unfortunately the European definition of the megadyne atmosphere, the *bar* so-called, is misleading, a more scientific definition being *megabar*. The use of the smaller unit does away with the inconsistency of defining a millibar as 1000 bars, a self-evident contradiction, and allows us to define the bar, the basic unit as the force which would impart to a gram an acceleration of 1 cm. per second per second. A *kilobar* or 1000 bars becomes then the natural and consistent unit for all measurements of pressure in force units.

With regard to temperature, a concept all important in atmospherics, it is regrettable that we have no available method of expressing gain or loss of heat in terms of molecular motion. The ordinary mercurial thermometer is certainly a crude instrument and so far as thermometer scales go, there has been no real improvement since Linnaeus reversed the order of the Celsius marking. It will be generally conceded that the Fahrenheit scale has now outlived its usefulness. Some meteorologists however still oppose the use of the centigrade scale and also the absolute, on the ground that the scale division even when read to tenths is too large for meteorological purposes.

To meet this objection and for other reasons, I suggest a new scale to be known as the New Absolute, or briefly, New. The zero of the new scale will be the same as that of the absolute, approximately 491° below freezing on the Fahrenheit and 273° below on the centigrade. The other fiducial point is the temperature of melting ice at a pressure of 1000 kilobars and is marked 1000. (The degree sign is omitted as it has been decided to reserve this symbol for angular measure.) The scale divisions are thus 0.366 of the centigrade, even smaller than the Fahrenheit and permit of any desired refinement of reading. Some other advantages of the new scale are:

1. The abolition of all minus signs. In upper air work temperatures are far below freezing. At a height of 10 kilometers readings may be lower than those recorded by Scott in the Antarctic. Given such a reading as −66.0°F., or −54.0°C or 219.0°A.; on the new scale this is read 800. Or again, some of the surface winter temperatures are confusing unless the minus sign is emphasized. The new scale has an advantage here as when we write 900 in place of −17.5°F. or −27.0°C or even 245.7°A.

2. The grand division of *warm* and *cold* as experienced by the general public in the every-day affairs of life is characteristically marked. Warm is any reading above 1100; cold is any reading below 1000.

3. In published tabular work there is a saving in typographical composition. There is also a saving of time in computation and increased accuracy.

4. But of even greater importance is the fact that the new scale makes for clearer conceptions of the nature and magnitude of temperature changes. It is astonishing how indefinite and vague are the ideas of most students regarding heat and the significance of a given temperature. And furthermore it is a difficult matter with the present scales to present clearly to the student a picture of the process of energy transfer available as heat.

5. The new scale starting as it does from the temperature of no molecular motion and laying stress upon the temperature of change of form of the most familiar substance, water to ice, has it would seem to me, a certain educational value, which the Fahrenheit and centigrade certainly do not have and which the absolute scale owing to the awkwardness of the fraction 1/273 loses.

6. Finally in problems of thermodynamics, there would seem to be no valid objection to its use. In the characteristic equation for a pure gas, the product of the new reading and the gas constant give as before a measure of the kinetic energy of translation of the molecules. Similarly in Avogardo's law, we have an even more definite conception of the temperature function; and also in the Stefan-Boltzmann law of black body radiation.

ON THE IMMUNITY COLORATION OF SOME NUDIBRANCHS

By W. J. Crozier

BERMUDA BIOLOGICAL STATION FOR RESEARCH, AGAR'S ISLAND, BERMUDA*

Read before the Academy, November 14, 1916. Received, November 8, 1916

Certain large nudibranchs, notably species of *Chromodoris*, exhibit a combination of brilliancy of pigment pattern and boldness of habits sharply distinguishing them from related nudibranchs which appear to be efficiently protected by their concealing coloration and behavior. It has long been suspected that the strikingly colored chromodorids achieve an immunity from predatory enemies by virtue of some distasteful quality, according to the classical conception of 'warning coloration.' The experimental evidence for this belief, as presented by Herdman and by Crossland, has remained, however, very fragmentary and is indeed far from carrying conviction.

For several years I have been interested in the pigmentation of the large Bermudian *Chromodoris zebra* Heilprin, mainly because its blue integumentary pigment is available as an intracellular indicator of acidity. On the basis of field observations, continuous throughout the year, I was led to undertake a series of experiments intended to throw light upon the possible biological meaning of this animal's coloration. The outcome seems reasonably conclusive, and it leads to the rejection of the idea that a warning significance attaches to the pigmentation of *Chromodoris*. By inference, one is inclined to suspect that more complete study will reveal analogous conditions with reference to other brightly colored nudibranchs.

The pigment pattern of *C. zebra* is, in brief, an irregular streaking of yellow or orange upon a field of blue. The foot, the ventral surface of the mantle, and various less conspicuous parts are of a pure blue. The animals occur at various stations, ranging in depth from about 10 fathoms to the surface of the water. They are most abundant, though, in quite shallow water, except during a portion of the summer and immediately after severe storms at other seasons. Their coloration is always conspicuous, not only because of its brilliancy but also by reason of the freedom with which they expose themselves in bright sunlight upon bare open bottoms or (a favorite situation) on eel grass in tidal 'creeks.' The animals are positively phototropic. Their conspicuousness is entirely independent of their being viewed form above the water surface.

---

*Contributions from the Bermuda Biological Station for Research, No. 51.

Less than two per cent of the specimens collected are found to show evidence of mutilation. As these nudibranchs regenerate very slowly (Child, and my own observations), this evidence is significant.

Experiments in feeding entire, undamaged specimens of this nudibranch to other animals, both in natural surroundings and under favorable conditions in aquaria, have shown that to all the animals usually associated with *Chromodoris* its flesh is repugnant. This is true of fishes as well as invertebrates, including anemones, crustacea, various worms, and starfish. Some fishes which feed by night (squirrel fishes), and others which hunt their prey by sight (snappers), will make several attempts to bite a *Chromodoris* when it is first dropped into their aquarium tank, but thereafter will not approach it; after the nudibranch has sunk to the bottom and begun its normal creeping, it is never seriously molested.

The unpleasant quality is associated with the skin, since the internal organs are greedily devoured. The blue pigment is not the unpleasant substance, for the intensely blue rhinophores are eaten readily, while the blood, which I have elsewhere shown to contain the blue skin pigment (presumably as a respiratory chromogen of some sort), evokes positive food-taking responses from various fishes, crabs, and anemones. The yellow pigment is not the responsible agent, as it is absent from the edge of the mantle, a region which comparative tests indicate to be the most 'unpleasant' part of the animal's body.

Small pieces of the skin of *Chromodoris* will be snapped at several times by a fish (over twenty species have been tested), while an intact nudibranch will usually be merely approached, 'nosed,' and perhaps bitten once, if the fish swims up suddenly, before it is left alone. The explanation of this behavior is found in the fact that when the intact nudibranch is locally disturbed by being handled severely, bitten, cut, stung by nematocysts, or stimulated with induction shocks, it excretes from the irritated area, into the sea water, a bluish-white material which causes immediate negative reactions in fishes and in all classes of marine invertebrates which I have tested. Isolated portions of the *Chromodoris* skin, however, give the reaction with difficulty, if at all, and then only to a slight extent.

It is significant that the secretion comes mostly from the ventral surface of the mantle. When this nudibranch is disturbed, its first reaction is a general contraction, in which the mantle edge is extended and made prominent by internal fluid pressure; the edge of the mantle is invariably the region which is bitten at by fishes, and it is only the part which is damaged in the small number of mutilated specimens found in the field. There exists here, I believe, an interesting case of adaptive correlation.

As excreted, the repelling material contains globules of the blue skin pigment, which seems to be accidentally included in the discharged substance; but it is mainly a coagulated white substance holding oily globules. It is this substance which is the essential repellant.

There is still another phase of the matter, which is the most significant of all. A striking characteristic of these nudibranchs is the curiously penetrating, disagreeable odor which attends them even when they are undisturbed. To this is to be assigned responsibility, I believe, for the facts, (1) that cloth bags containing *Chromodoris* are avoided by fishes; (2) that blinded fishes avoid them; and (3) that animals never normally in contact with *Chromodoris*, such as the dogfish, remora, and the hawksbill turtle, will approach this nudibranch closely, then quickly retreat without touching it. The olfactory locus of these reactions seems probable, but it does not necessarily involve the idea of 'warning odor.'

In the present instance we have, in my opinion, positive proof for the validity of the theory of immunity coloration developed by Reighard. According to this view, startling colors are in many (if not in most) cases "conceived to have arisen through internal forces under immunity of the organism from the action of selection on its characters." The immunity of *Chromodoris zebra* is determined by the repugnatorial character of skin secretions under the control of its nervous system, and is further made effective by a repelling odor, which we have reason to believe stimulates the olfactory organs of fishes at least. Its success in maintaining itself as a plentiful inhabitant of the shoal waters of a 'coral' reef region is conditioned by this immunity and by the unpalatable nature of the jelly surrounding its egg strings; the slow rate of development of its eggs is counterbalanced by the fact that the animals reproduce throughout the year.

The coloration of *C. zebra* is a metabolic accident, at least in relation to its protection, for a single experience with a normally colored specimen is sufficient to cause snappers, turbots, and groupers to have nothing to do with subsequent individuals offered to them, even though these individuals are stained red or blue.

Conceivably, its color may at times serve to warn predatory foes, but the evidence herein adduced makes it clear that the pigmentation of *Chromodoris* plays no necessary rôle of this kind. We are consequently at liberty to infer that the conspicuous coloration did not develop, as the result of selection, according to the scheme proposed by the theory of warning coloration.

An illustrated account of this work will be submitted for publication later.

SOME EFFECTS OF THE CONTINUED ADMINISTRATION OF ALCOHOL TO THE DOMESTIC FOWL, WITH SPECIAL REFERENCE TO THE PROGENY

By Raymond Pearl

BIOLOGICAL LABORATORY, MAINE AGRICULTURAL EXPERIMENT STATION[1]

Read before the Academy, November 13, 1916. Received, November 13, 1916

1. In a recent number of these PROCEEDINGS there was presented[2] a summary statement of the results which had been obtained up to February 1, 1916, in a series of experiments designed to test the effect upon the progeny of the continued daily administration, by the inhalation method, of either ethyl alcohol, methyl alcohol, or ether to the parents, the domestic fowl being the form used in the work. The chief result was that the offspring of parents which had inhaled the vapor of one or another of the substances mentioned, for one hour each day over an average period of 210 days each, were measurably *superior* to the offspring of untreated parents in respect of the following characters or attributes: prenatal mortality, postnatal mortality at all ages, weight at hatching, adult weight, rate of growth, and percentage of abnormalities or defects of development. The offspring of untreated parents were superior to those of treated parents only in respect of their number; the untreated parents produced a larger total number.

It is the purpose of the present paper to report in brief and condensed, but numerical, form the results which have been obtained since the data at which the former report ended. The data included are those obtained in the breeding and rearing season of 1916. The length of the period over which the parents had been given daily treatments was of course longer by one year than in the 1915 breeding season. Thus the total germ dosage index[3] for the 1916 matings ranged from 458 to 1138, with a mean value of 562.9 days, for the parental generation, and from 0 to 266, with a mean of 40.6, for the grandparental generation.

Owing to various reasons, which cannot be gone into here, ether has been dropped from the list of experimental agents, ethyl and methyl alcohols alone being used now.

A detailed account of the experimental methods used and the precautions taken to ensure accuracy and trustworthiness of results is given in the complete reports now in press.

2. The general results of the 1916 matings, so far as concern the chief characters which may be taken as indices of vigor and vitality, are set forth in Table 1.

TABLE 1

SHOWING THE MORTALITY (PRENATAL AND POSTNATAL) OF CHICKS OF ALCOHOLIC ANCESTRY, AS COMPARED WITH THOSE OF NON-ALCOHOLIC ANCESTRY

Matings in 1916

NATURE OF ANCESTRY	NUMBER OF MATINGS	TOTAL EGGS SET	TOTAL ZYGOTES FORMED	ZYGOTES FORMED	PRENATAL MORTALITY	POST-NATAL (180 DAY) MORTALITY	ZYGOTES ALIVE AT 180 DAYS OF AGE
				per cent	*per cent*		*per cent*
Both parents alcoholic	5	82	18	21.95	11.11	25.00	66.67
Dam only alcoholic	3	63	5	7.94	80.00	0	20.00
Sire only alcoholic	13	512	274	53.52	47.08	13.79	45.62
Sire and one or more grandparents alcoholic	3	233	158	67.81	46.84	28.38	39.87
One or more grandparents alcoholic	3	187	113	60.43	46.02	6.56	50.44
All of alcoholic ancestry	27	1077	568	52.74	45.95	16.50	45.42
All of non-alcoholic ancestry	28	1333	1060	79.52	55.94	21.20	34.72
Differences	*1*	*256*	*492*	*26.78*	*9.99*	*4.70*	*10.70*
Alcoholic better = + Alcoholic poorer = −		−	−	−	+	+	+

From this table we note the following points:

a. Considering the totals, and comparing chicks with any alcoholic ancestry, as a group, with chicks having no alcoholic ancestry, as a group, it is clear that there are fewer of the former than of the latter from a given number of eggs. In every 100 eggs from birds in the alcoholic series approximately 53 zygotes (embryos) were formed as against 80 in every 100 eggs in the non-alcoholic series.

b. The total number of offspring zygotes dealt with in the experiments (1628) is respectable, and such as to lead to reasonable confidence in the results, especially in view of the fact that these results are in every essential particular in full accord as to their sense with those obtained in the 1915 experiments, and even more pronounced in degree.

c. The prenatal mortality is approximately 10%, and the postnatal mortality (to 180 days of age) is approximately 5% *lower* in the chicks with alcoholic ancestry than in those of non-alcoholic ancestry.

d. While from a given number of eggs 26.78% *fewer* zygotes are formed in the alcoholic series than in the non-alcoholic, 10.7% *more* of those zygotes are alive when adult age (180 days) is reached if their ancestry was alcoholic than if it was non-alcoholic.

e. The great reduction in zygote formation (partial sterility) observed in the alcohol treated series is preponderantly due to the effect of the alcohol upon the germ cells of the *female*. While there is some diminution in the effective fecundity of the male as a result of the treatment, it is much smaller than in the female. In the first two lines of the table, where the dam is alcoholic, we have 22 and 8% of the eggs forming embryos, as against values of from 54 to 68% where the sire is alcoholic and the dam not. The lower value for the percentage of zygote formation in the case of matings where the dam only is alcoholic as compared with those in which both parents are alcoholic, almost certainly does not represent a significant difference, but is an accidental result of the small number of matings in the class. The significant thing is the fact that the percentage of zygote formation steadily rises as we compare the following groups: (1) dam alone or dam and sire alcoholic, (2) sire alone or sire and some other ancestry not including the dam alcoholic, (3) no ancestry alcoholic.

3. In the season of 1916 every fertile egg in which the embryo died on or after the tenth day of incubation was opened, and an examination was made of the embryo in each case, in order to determine whether any structural abnormality or malformation was present. It was not found practicable to deal with embryos under ten days of age. A detailed record was kept of all such congenital abnormalities, in both the dead embryos and hatched chicks. Every discernible departure from perfect structural normality was taken account of, the range including at one extreme such slight and unimportant things as the congenital absence of one toenail in an otherwise perfectly normal chick, and at the other extreme acraniate and eyeless monsters. A later complete report will give the detailed data for the different classes of abnormalities. Here I wish simply to report the general results. These are set forth in Table 2.

TABLE 2

SHOWING THE NUMBER AND PROPORTION OF CONGENITAL ABNORMALITIES AND MALFORMATIONS FOUND AMONG THE CHICKS OF ALCOHOLIC AND NON-ALCOHOLIC ANCESTRY IN THE SEASON OF 1916

OFFSPRING OF	NO. OF MATINGS	MEAN GERMINAL DOSAGE INDEX		DEAD IN SHELL AFTER THE TENTH DAY OF INCUBATION		ABNORMAL KILLED AT HATCH	HATCHED CHICKS		TOTAL ZYGOTES SURVIVING TO THE TENTH DAY OF INCUBATION AND BEYOND	TOTAL ABNORMAL	PER CENT ABNORMAL
		P.	G. P.	Normal	Abnormal		Normal	Abnormal			
Treated parents or grandparents or both	27	562.9	40.6	78	19	4	267	20	397	43	*10.8*
Untreated control parents	28	0	0	190	23	14	372	42	641	65	*10.2*

From this table the following points should be noted:

a. The apparent discrepancy between Tables 2 and 1 in the figures for total zygotes arises from the fact that in Table 2 account is taken only of zygotes which survived for 10 or more days after fertilization. Out of a total of 568 zygotes formed in the alcoholic ancestry series 171 died before the tenth day of incubation.

b. The proportionate number of abnormalities of development is substantially identical in both series, amounting to about 10% of all zygotes formed. If this seems a high proportion let it be remembered that the category of abnormality is here very broadly defined and includes a number of developmental deviations which in no way reduce the survival value of the individual possessing them.

c. It is entirely certain that in these experiments there was ***no significantly higher proportion of abnormalities of development among the zygotes of alcoholic ancestry than among those of non-alcoholic ancestry.*** This result is in striking contrast to those which have been reported by Stockard,[4] and would appear to indicate a rather fundamental difference between guinea-pig and fowl germ plasms in respect of their susceptibility to alteration by alcohol.

d. While lack of space prevents the presentation of the detailed evidence here, it may be stated that an analysis of the data shows that the average relative gravity or intensity of the 10.8% of developmental abnormalities observed among the zygotes of alcoholic ancestry is probably not significantly different from that of the 10.2% in the non-alcoholic series.

4. It has been suggested in objection to all experiments where alcohol is administered by the inhalation method used by Stockard, MacDowell, the present writer, and others, that the animal gets no appreciable amount of alcohol into the system. In view of all the facts this position is an extraordinary naive one and could only be held by one who had had no experience with the administration of alcohol by the inhalation method. It is true that it is practically impossible to induce by the inhalation method in animals habituated to alcohol that state of muscular incoördination which is usually, but by no means always, the most striking objective symptom of the condition of being drunk. On the other hand, it is extremely easy to kill an animal with ethyl or methyl alcohol administered by the inhalation method, quite regardless of whether it is or is not habituated to the substance. If the animal 'gets no appreciable amount of alcohol in its system' in the course of one hour's sojourn in a tank containing alcohol vapor it is extraordinarily difficult to understand how it manages to accumulate a fatally toxic dose of alcohol by staying in the same tank under the same conditions for from 20 minutes to half an hour longer. This is not only true for my fowls, for which the above time relations hold, but also, with other time relations, for mammals, as lately shown by Tyson and Schoenberg[5] in the case of methyl alcohol. Thirty-eight years ago Poincaré[6] demonstrated extensive and grave lesions in animals subjected to the continuous inhalation of methyl alcohol vapor.

It appears to be impossible, in the present development of the technique, to state the exact dosage per kilogram of body weight of alcohol administered by the inhalation method. I am now working on an indirect method of measuring the dosage, logically based upon the same reasoning as is used in the so-called 'physiological assay' of drugs. As a first contribution in this direction I wish to present some data regarding the immediate physiological effect of the administration of ethyl and methyl alcohol to fowls thoroughly habituated to these substances. These observations have to do with respiration rate (respirations per minute) and rectal temperature (in degrees Fahrenheit). They are set forth in Table 3. Owing to lack of space this table gives mean values only, observations will be reported in a complete paper later. The means undoubtedly understate the true differences between the different groups because they include all the observations, some of which were made on cold days and before the apparatus for maintaining a constant temperature for the evaporation of the alcohol had been perfected. On such days the dosages, and in consequence the physiological differences, were reduced. By 'tanked controls' in the second line is meant birds which were put for one hour in a tank precisely like the alcohol tanks, but in which there was no alcohol or alcohol vapor.

TABLE 3

SHOWING THE EFFECT OF THE INHALATION OF ETHYL AND METHYL ALCOHOL VAPOR ON THE TEMPERATURE AND RESPIRATION OF FOWLS HABITUATED TO THESE SUBSTANCES

GROUP AND TREATMENT	RECTAL TEMPERATURE BEFORE TREATMENT F°	RECTAL TEMPERATURE AFTER TREATMENT F°	DIFFERENCE	RESPIRATION PER MINUTE BEFORE TREATMENT	RESPIRATION PER MINUTE AFTER TREATMENT	DIFFERENCE	NUMBER OF BIRDS IN GROUP	NUMBER OF OBSERVATIONS
Mean of normal controls.......	107.14			23.77			12	48
Mean of tanked controls........	107.07	107.13	*+0.05*	22.89	23.67	*+0.66*	4	16
Mean of all ethyls.............	107.07	106.43	*−0.64*	18.24	27.86	*+9.62*	7	29
Mean of ethyl ♀ ♀............	107.00	106.36	*−0.64*	18.24	28.76	*+10.52*	6	25
Mean of all methyls............	107.14	105.91	*−1.23*	18.56	22.50	*+4.31*	4	16
Mean of methyl ♀ ♀..........	107.53	106.45	*−1.08*	19.00	23.75	*+4.75*	3	12

This table shows that:

a. Those birds which, by long continued daily treatment, extending over periods varying from six months to two years, have become thoroughly habituated to alcohol have a *slower respiration rate* than normal untreated birds of the same breeds, the difference amounting on the average to 4 or 5 respirations per minute. There appears to be no steady constant difference between chronic alcoholists and normals in body temperature.

b. The tanked control birds show no significant change in temperature or respiration as a result of an hour's sojourn in an empty tank.

c. On the contrary the birds which stay one hour daily in a tank containing alcohol vapor exhibit marked and definite changes in their physiological condition at the end of the hour. These changes consist of a *drop in temperature*, amounting on the average to nearly 1¼°F. in the case of methyl treatment, and about half as much in the case of ethyl, and a coincident *increase in respiration rate*, amounting on the average to about ten respirations per minute, or a 50% increase in the ethyl, and to about four and one-half respirations, or a 25% increase in the methyl.

d. These immediate physiological changes are precisely the same in kind and comparable in amount to those which have been shown by Völtz and Baudrexel[7] to follow the taking of moderate to large doses of ethyl alcohol by drinking, in the case of dogs and men.

5. I have interpreted the results on parental alcoholism which have been obtained by myself and other workers as due to a selective action of the agent upon the germ cells, in the sense of killing, or at any rate

inactivating, the weak germ cells and permitting only the strong, vigorous, and resistant germ cells to form zygotes. This results, in forms like the fowl having germ cells rather highly resistant to alcohol, in a small but superior progeny from alcoholists. On the other hand, in forms like the guinea-pig, whose germ cells are known to have a relatively low resistance to alcohol[8] it results in the formation not only of a relatively small progeny, but also a weak and abnormal one, since the germ cells which are not entirely inactivated by the agent are still not sufficiently resistant to be uninjured by it.

A rather striking confirmation of this interpretation was obtained in an experiment carried out this past summer. I reasoned that if the above interpretation is correct, alcohol ought to act as a selective agent in the same sense on the very young zygotes, killing the weak and permitting the survival only of the strong provided the appropriate dosage could be found. To test this idea the following experiment was performed. Two incubators were used, each containing at the outstart 390 eggs. These eggs were selected with the utmost care to ensure likeness of age, of strain, and of other characteristics in the two lots. In one incubator 40 cc. of 95% ethyl alcohol were evaporated beneath the eggs daily. In the other incubator no alcohol was used. At the end of 7 days 130 eggs, designated as Lot 1, were removed from the alcohol incubator and allowed to complete their development in a normal non-alcoholic incubator. At the end of fourteen days the remaining eggs (after testing out infertiles) of Lot 2, which originally contained 130 eggs, were removed from the alcohol incubator and finished their development without further dosage of alcohol in a normal incubator. Finally Lot 3, originally containing 130 eggs, was subjected throughout the 21 days of incubation to the daily dosage of alcohol fumes. At hatching all the normal chicks from both incubators were put together in the same house and brooder and given throughout life the same treatment as to feed, etc. Careful account was kept of mortality, each chick, being marked so that it could be told from which lot it came. The conditions of brooding were purposely made bad so to obtain a maximum severity of post-embryonic environmental conditions with a consequent high absolute mortality rate. The results are set forth in Table 4.

TABLE 4

DATA ON HATCHING AND MORTALITY OF CHICKS FROM ALCOHOL TREATED EGGS

Eggs set May 15. Hatched June 5

CHARACTER	LOT 1	LOT 2	LOT 3	CONTROLS
	Alcohol 1 week	Alcohol 2 weeks	Alcohol 3 weeks	No alcohol
Eggs set	130	130	130*	390
Eggs infertile	34	35	41	111
% infertile	*26.2*	*26.9*	*32.3*	*28.5*
All prenatal deaths	41	41	52	108
% of fertile eggs	*42.7*	*43.2*	*60.5*	*38.7*
Number good chicks in brooder	53	52	32	138
Dead in first 30 days	15	11	12	47
% chicks in brooder	*28.3*	*21.2*	*37.5*	*34.1*

* Three eggs were broken during incubation from this lot.

From this table we note the following points:

a. The original sorting of the eggs in the different lots was very fair and equal as is indicated by the nearly equal percentages of infertility.

b. The prenatal mortality was *higher* in all treated lots than in the controls, the differences ranging from 4% to 21.8%.

c. The postnatal mortality was *lower* in Lots 1 and 2, where the eggs had been alcohol treated for one and two weeks of incubation respectively. The decrease in postnatal mortality in Lot 2 amounted to 12.9%.

d. In Lot 3 the dosage was evidently too severe and the chicks which hatched had been injured by the treatment during incubation, with the result that they showed a somewhat higher postnatal mortality than the controls.

e. Synthesizing the results we may say that the more alcohol the embryos received during incubation the higher was the prenatal mortality, but until the dosage became so prolonged as to injure *all* the zygotes, as in Lot 3, the prenatal mortality was *selective*, since the higher the prenatal death rate the lower was the postnatal mortality among the hatched chicks.

f. The general result set forth in the preceding paragraph is contrary to the usual experience of poultrymen with ordinary untreated eggs, which is that if a lot of eggs hatch badly, many dying during incubation, a relatively high postnatal mortality among the chicks is also to be expected. It is believed that the present results may have an important practical application in the poultry industry.

In summary it may be said that the results during the 1916 breeding season confirm and extend the earlier results regarding the effects of parental alcoholism on the progeny in poultry. The additional data make still more certain the conclusions, first, that the progeny of alcoholized parentage (in poultry) while fewer in numbers is made up of individuals superior in physiological vigor, and, second, that this result is due to a selective action of the alcohol upon the germ cells.

---

[1] Papers from the Biological Laboratory of the Maine Agricultural Experiment Station, No. 105.

[2] Pearl, R., *Proc. Nat. Acad. Sci.*, **2**, 380–384 (1916). *Cf.* also for a fuller statement of the results *Proc. Amer. Phil. Soc.*, **55**, 243–258 (1916). The complete report on the earlier stages of the work is in press in *J. Exper. Zool.*

[3] This term is defined in my former papers as "the total number of days during which the two gametes making the offspring zygote have been exposed to alcoholic influence while sojourning in the body of the treated individuals."

[4] For summary and bibliography of earlier papers see Stockard, C. R., and Papanicolaou, G., A further analysis of the heredity transmission of degeneracy and deformities by the descendants of alcoholized mammals, *Amer. Nat.*, **50**, 65–88, 144–177 (1916).

[5] Tyson, H. H., and Schoenberg, M. J., Experimental researches in methyl alcohol inhalation, *J. Amer. Med. Assoc.*, **63**, 915–922 (1914).

[6] Poincaré, L., Sur les dangers de l'emploi de l'alcool methylique dans l'industrie, *Paris, C. R. Acad. Sci.*, **87**, 682–683 (1878).

[7] Cf. for example Völtz, W., and Baudrexel, A., Ueber die vom tierischen Organismus unter verschiedenen Bedingungen ausgeschiedenen Alkoholmengen. II. Mitth., *Pflügers Arch.*, **142**, 47–88 (1911); and other papers in the same series. Also note similar data in Völtz, W., Förster, R., and Baudrexel, A., Ueber die Verwertung des Bierextraktes und des Bieres im menschlichen und tierischen Organismus. *Ibid.*, **134**, 133–258 (1910).

[8] Cf. Ivanov, J., Action de l'alcool sur les spermatozoïdes des mammifères (Première communication), *C. R. Soc. Biol., Paris*, **74**, 480–482 (1913).

AN IONIZATION MANOMETER

By O. E. Buckley

RESEARCH LABORATORY, AMERICAN TELEPHONE AND TELEGRAPH COMPANY AND WESTERN ELECTRIC COMPANY

Received by the Academy, November 13, 1916

Heretofore the only manometers available for measuring extreme vacua have been the Knudsen manometer and the Langmuir molecular gauge. Both of these have serious disadvantages due to their delicate construction and slowness of action. A new manometer free from these objections and with a greater range of presssure than either has been developed. This manometer makes use of the ionization of gas by an electron discharge.

The manometer consists of three electrodes sealed in a glass bulb which serve as cathode, anode, and collector of positive ions. The cathode may be any source of pure electron discharge such as a Wehnelt cathode or a heated tungsten or other metallic filament. The exact forms of the electrodes are not of great importance. The collector is preferably situated between the other two electrodes and of such form as not to entirely block the electron current to the anode. A milliammeter is used to measure the current to the anode and a sensitive galvanometer to measure the current from the collector which is maintained negative with respect to the cathode so as to pick up only the positive ions.

If there were no gas at all in the space between the electrodes a pure electron current would flow from cathode to anode and no current would flow to the collector. However, if gas is present positive ions are formed by collision in amount proportional to the electron current and the number of gas molecules in the space. Since the collector is negative with respect to the cathode a certain proportion of the positive ions, depending on the form, dimensions, and potentials of the electrodes, will flow to the collector. Hence the ratio of the collector current to the anode current is proportional to the pressure and may be used to measure the pressure when the constant of proportionality has been determined.

This relation has been tested experimentally with air over a pressure range from 10^{-3} to 4×10^{-6} mm. of mercury by comparison with McLeod and Knudsen manometers. The actual apparatus used consisted of a glass bulb 6 cm. in diameter enclosing three parallel, V shaped filaments of thin platinum strip, each about 3.5 cm. long, placed 5 mm. apart, the collector being between the other two. Leads from both ends of each filament were brought through the glass. This arrangement permits glowing the electrodes to free them from occluded gases. An oxide coated filament was used for the cathode. The bulb was sealed to a large glass reservoir which was connected to a high vacuum pump and either the Knudsen or McLeod manometers. When the latter was used a liquid-air trap served to keep the mercury vapor of the McLeod manometer out of the ionization manometer.

Currents from 0.2 to 2.0 milliamperes were used with from 100 to 250 volts between cathode and anode. The collector was held at 10 volts negative with respect to the cathode. The resulting current to the collector at a pressure of 10^{-3} mm. was about one-thousandth the current to the anode and at lower pressures was proportionately less. Hence at a pressure of 10^{-6} mm. with a current of 2.0 milliamperes to the anode a collector current of 2×10^{-9} amperes could be obtained. With a sensitive galvanometer much lower pressures could easily be measured.

Experiments with hydrogen and with mercury vapor in place of air gave constants of proportionality nearly the same as with air.

The advantages of this type of manometer are readily apparent. Its range compared to that of other high vacuum gauges is very large, extending from more than 10^{-3} mm. to as low pressures as can be obtained, without any change of apparatus. On account of its simplicity of construction it is inexpensive and exactly reproducible. Since there are no moving parts there are no difficulties due to vibration. The pressures of vapors which would not be registered on the McLeod gauge are measured by the ionization manometer. One of the greatest advantages is the rapidity and ease with which measurements of a varying pressure may be made since only the reading of a galvanometer need be followed.

Many applications for which other manometers cannot readily be used at once suggest themselves, such as the measurement of vapor pressures of metals, etc. Since the device may be made with extremely small volume the pressure of very small quantities of gas may be measured. It would also be useful to measure pressure changes over a long period of time for which more expensive manometers could not well be employed.

A number of interesting physical measurements other than the measurement of pressure can be made with devices operating on the principle of this manometer, among which is that of the removal of occluded gases by electron bombardment. It is also hoped that experiments with various gases will give some information as to the relative cross sectional areas which different kinds of molecules present to the electron discharge, for although the constant of the manometer was found approximately the same for hydrogen, air, and mercury vapor, more exact measurements might show differences due to different molecular diameters.

PHYSIOLOGICAL STUDIES ON RHIZOPHORA

By Howard H. M. Bowman

DEPARTMENT OF BOTANY, UNIVERSITY OF PENNSYLVANIA, AND TORTUGAS LABORATORY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, November 10, 1916

Research on the physiology and ecology of the red mangrove, *Rhizophora mangle*, which has engaged my attention for the past few years, was continued at the Tortugas Laboratory of the Carnegie Institution during the summer of 1916. The phase of physiologic investigation most emphasized was that relating to the transpiration rate of *Rhizophora* seedlings grown in solutions of different concentrations of salt water and in various soils. The mangrove, growing as it does in such peculiar conditions, in salt and fresh water alike, offers a rich field for studies on absorption and transpiration rates, the effects of chemical substances on these phenomena, and the physical relations of density of media. A brief account is here given of some of the more important observations made on the effects of these changes in media on the transpiration.

The material used was *Rhizophora* seedlings. These were selected as the most easily handled and the most readily procurable. Seedlings one to two years old, growing on shores of keys from 3 to 75 miles from the Tortugas, were secured and planted, some in Tortugas sand (a coarse sand composed of broken shell, corals, and other calcareous débris) and others in a ferruginous sandstone soil brought down from New Jersey. Another series also was planted in fine calcareous mud taken from the moat at Fort Jefferson. These cultures compose the soil series of experiments.

Another group of plants was planted in the Tortugas sand and kept in water of the following concentrations: 100, 75, 50, 20, 10, and 5% fresh, and 100% salt water. In a previous season records were taken on plants grown in hyperconcentrated sea-water, 140% salt. The record for these plants gave a very slow rate of growth, and after lingering for a few weeks, the plants yellowed, dropped the leaves, and died. These experiments were not repeated.

In all these cultures the plants were grown in large 10-inch beakers and the water was siphoned off every day and a fresh solution was put on. This was found to be necessary to keep algæ and mosquito larvæ out of the cultures and also to simulate the tidal action in daily bringing a fresh supply of water to the plants, as in their natural habitat.

Still another group of cultures was made of plants kept in various light intensities as well as moisture conditions. One class was kept in New Jersey soil in partial shade with 100% salt water; another in partial shade, New Jersey soil, 50% salt water; a third class, kept in partial shade in shell sand, merely moistened with 100% salt water; a fourth class was kept in partial shade in New Jersey soil, merely moistened with 100% salt; and a fifth class in full sun all day long, planted in shell sand and kept moist with 100% salt water.

This last class was in a condition which most nearly approximates the natural environment of *Rhizophora* seedlings cast up by the waves on a coral strand in the Gulf region. The tides bury the hypocotyl or it bores a resting-place by the radicular end twirling in the water-current and often becomes entirely covered with sand. Leaves are put out as long as the source of moisture in the sand is constant and there is enough reserve food in the hypocotyl, but the tiny leaves are soon burned up by the fierce sunlight and the drying winds. This putting out of leaves happens repeatedly until the little seedling is exhausted and it succumbs to the hard conditions on the beach. The same thing occurred in the full-sun cultures, so that no leaves could be secured on which to take transpiration records.

The method of securing the records was that of Stahl—that is, a colorimetric method. A Ganong leaf-clasp was employed and the rate of transpiration was measured in minutes and seconds until the cobalt chloride paper in the clasp was changed to a uniform pink, due to the water given off through the epidermis of the leaf, chiefly that of the lower surface, as there are no stomata on the upper surface. Potometer records were also taken, but on account of the limited amount of cultures this method was not feasible. The plants in the soil and water concentration cultures could have repeated tests taken on them by the Stahl method, whereas for a potometer record a plant would have had to be sacrificed for every reading. The readings were taken mostly during the middle portion of the day, to secure as uniform conditions as possible. Some potometer records were made to check up results and to get some quantitative idea of the amounts of water really transpired.

In tabulating the results of this work it is found that by plotting these transpiration readings (given below in part) with the time intervals as ordinates and the concentration percentages as abscissæ, a parabolic curve is described. The following is a statement of the solution concentrations and transpiration intervals:

Series A, 100 per cent fresh, 1.6 minutes
Series B, 75 per cent fresh, 1.7 minutes
Series C, 50 per cent fresh, 2.4 minutes
Series D, 20 per cent fresh, 2.8 minutes
Series E, 10 per cent fresh, 3.2 minutes
Series F, 5 per cent fresh, 3.9 minutes
Series G, 100 per cent salt, 4.1 minutes

By applying the formula $Y = CX^n$, the constant being the initial quantity of water needed to affect the transpirometer, it is also deduced that the additional time required for the transpiration of equal quantities of water is due to the retarding effect of progressively higher salt concentration of the medium—that is, the time factor is a function of the square of the concentration divided by a constant. In other words, the rate of transpiration varies directly with the concentration of the medium in which the *Rhizophora* plants grow.

For the soils series of experiments the data show that the most striking phenomenon is the accelerating effect on the transpiration due to the soil from Maplewood, New Jersey. It is assumed that this accelerating effect is due to chemical action, the New Jersey soil having a larger number of elements in it, and others in greater amount (viz., iron, aluminium, and silica) than in the shell sand of the Tortugas, in which calcium carbonate preponderates. Though, unfortunately, the range of data is not wide enough on the New Jersey soil to produce a full curve, there is enough to show that the segment paralleled the parabola of the shell sand curve, which curve is expressed with the concentration of the water being constant for both soils.

Another interesting point, illustrated in this latter plotting, particularly of the data secured on the cultures which were kept merely moist with the salt water, was the demonstration of the physiological law that the smaller the quantity of available moisture the slower the transpiration. Of course, this was only to be expected, but the clarity of the evidence presented in this case was a pleasant surprise to the investigator, inasmuch as he was only indirectly concerned with the water available for absorption.

Casual mention may here be made also of a series of biochemical tests conducted on *Rhizophora* hypocotyls this summer, to determine the relations of the amounts of tannic acid and dextrose in these organs; and also to demonstrate, if possible, the presence of the enzyme tannase —broadly, to learn something of the rôle of tannin, which is so abundant in *Rhizophora* in nearly all its tissues. This work, while merely started, presents a very interesting field of investigation, since it is supposed that some tannins of the plastic group probably contribute to the nutrition of the plants containing them. It is hoped to continue work on this problem, since the knowledge of the function of tannins in the plant economy is rather obscure, and generally they are supposed to be merely excretory products. On account of the paucity of data in this research the writer does not feel justified in making a definite statement regarding the relation of the amounts of tannic acid and dextrose in the *Rhizophora* hypocotyl; at present, however, it may be safely said, on the strength of over 50 tests for tannase, conducted according to the usual methods of enzyme tests, that this enzyme could not be detected in the hypocotyl of *Rhizophora*.

ON THE HYDROGEN ION CONCENTRATION OF SEA WATER, AND THE PHYSIOLOGICAL EFFECTS OF THE IONS OF SEA WATER

By J. F. McClendon

DEPARTMENT OF PHYSIOLOGY, UNIVERSITY OF MINNESOTA, AND TORTUGAS LABORATORY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, November 10, 1916

Both natural sea water at Tortugas, Florida, of salinity of 35–36 per kilo and artificial sea water of the following composition were studied:

		Normal solutions	or	Isotonic solutions.
$CaCl_2$	0.5 m.	22.07 cc.	0.38 m.	29.0 cc.
$MgCl_2$	0.5 m.	50.21 cc.	0.37 m.	67.9 cc.
$MgSO_4$	0.5 m.	57.09 cc.	0.975 m.	29.5 cc.
KCl	1.0 m.	10.23 cc.	0.577 m.	17.7 cc.
NaCl	1.0 m.	483.65 cc.	0.568 m.	852.0 cc.
NaBr	1.0 m.	0.80 cc.	0.565 m.	1.4 cc.
$NaHCO_3$	1.0 m.	2.40 cc.	0.93 m.	2.58 cc.
H_2O		373.63 cc.		0.00 cc.
		1000.00 cc.		1000.00 cc

The mixture was aerated many hours or until it reached the proper alkalinity. It was tested by placing the most delicate marine organisms in it, which were found to live and behave normally.

The excess of non-volatile base over mineral acid in Tortugas sea water was found by Dole to be 0.002265–0.00253 N. per kilogram, and my own observations fall within these limits. That of various artificial sea waters studied was 0.00225–0.00245 Normal per kilogram. This variation in natural sea water is independent of the salinity and is apparently associated with the variation in the lime content.

The hydrogen ion concentration will be indicated by the minus logarithm or PH, the PH of pure water, or neutral solution, at 23° being 7, that of *n.* HCl ca. 0, and that of *n.* KOH ca. 14. The PH was determined by the hydrogen electrode and standardized instruments, the special apparatus being described by McClendon and Magoon[1] All measurements were made at 30°. The PH of Tortugas sea water was found to vary from 8.1 to 8.22 and this variation was found to be partly due to the variation in the excess base but largely due to variations in the CO_2 tension.

The PH of sea water at known CO_2 tensions was studied and it was found that, as the CO_2 tension decreased, the ratio of change in PH with change in CO_2 tension increased so that the PH could be much more accurately measured than the CO_2 tension. The CO_2 tension of the sea varied from about 0.03–0.05% of an atmosphere, but these measurements will be made more accurately next summer. One estimation of the CO_2 of the air was 0.033%. It seems probable that CO_2 is passing from the sea to the air at Tortugas, but this question also will be studied more accurately next summer. From Fox's data and the above measurements, the following estimations at 30° may be made:

CO_2 content in cc. per litre of sea water	44.5	46.25	47.5
CO_2 tension in percent of an atmosphere	0.03	0.04	0.05
PH	8.22	8.15	8.1

From this table the CO_2 content may be estimated from the PH and hence in respiration may be made by estimating the PH of water containing animals, at certain periods. For this reason rapid methods of estimating the PH were calibrated by comparing vials of Sörensen's borate mixtures colored with thymolsulphonephthalein[2] with a similar vial of sea water to which is added the same quantity of thymolsulphonephthalein, and the PH read off colorimetrically. It was found that when the sea water was tested with the same hydrogen electrode used in standardizing the borate mixtures its PH was found to be 0.3 lower than that estimated colorimetrically, owing to the salt action on the indicator, consequently 0.3 must be subtracted from the result obtained colorimetrically. The salt error could not be determined more accurately with these tubes since they read down only to 0.1 on the PH scale but I understand that Lubs and Clark are studying this question in detail and may have more exact data.

Some general experiments on the physiological significance of the ions of sea water were made, but since they were intended to be general, no extremely exact quantitative data were recorded. Hence these experiments cannot be compared with those of A. G. Mayer on the same subject in which he has made some exact determinations of the effect of H and OH ions at various concentrations. Since I have shown that ions may change the permeability of irritable cells, and excitation seems evidently to be associated with increase in permeability to ions and I have also shown that it is accompanied by increase in electrical conducitivity; the question arises whether all ions may be divided into two classes: those which increase permeability and those which inhibit this change. Experiments on the pulsation of the jelly fish, *Cassiopea*, and the heart of the Conch, *Strombus*, indicate that OH', $Na^{\cdot}$, and $K^{\cdot}$ increase permeability, and that $H^{\cdot}$, $Mg^{\cdot\cdot}$, and $Ca^{\cdot\cdot}$ inhibit increase in permeability. Since a certain increase in permeability puts the cell in the stimulated condition which is the same as the refractory period, $K^{\cdot}$ may inhibit stimulation by giving the cell a single stimulus which continues and is manifested by the continuance of the refractory period as long as the potassium ion is present. $Ca^{\cdot\cdot}$ on the other hand is very toxic and when too liberally applied causes death accompanied by increase in permeability and death rigor, which may simulate the stimulated state.

OH', $Na^{\cdot}$, and $K^{\cdot}$ lower the threshold for excitation of the jellyfish and conch heart, and $H^{\cdot}$, $Mg^{\cdot\cdot}$ and $C^{\cdot\cdot}$ raise it. When cautiously applied, OH', $Na^{\cdot}$, and $K^{\cdot}$ stop the conch heart in systole, and $H^{\cdot}$, $Mg^{\cdot\cdot}$, and $Ca^{\cdot\cdot}$ stop it in diastole, but the action of $Ca^{\cdot\cdot}$ may seem the reverse when not carefully studied. This seemingly uncertain condition is probably due to the organism being composed of several kinds of irritable elements, each requiring a balance of ions of the two classes. A balance for all these elements cannot be maintained with one pair of antagonistic ions alone.

Since the plasma membrane or cell surface is very probably composed of emulsoids, or hydrophile colloids,[3] the change in permeability must be due to a colloid change. Although suspensoids are easily coagulated or precipitated, emulsoids have too great an affinity for water to be easily separated from it except in a most gradual manner unless they are changed by certain reagents or temperatures so as to approach the character of suspensoids. We should therefore not look toward the processes of aggregation and dispersion as seen in suspensoids for an explanation of change in permeability. The changes in an emulsoid membrane are to be regarded as swelling and shrinkage as seen in gelatine or collodion plates. Water soluble substances diffuse easily through collodion gels until the shrinkage reaches a certain point, beyond which the permeability rapidly decreases with shrinkage and the collodion finally becomes impermeable to electrolytes. This is probably due to the approximation of the collodion molecules until the pores are too small to admit of free diffusion of hydrated ions through them. It is to be distinguished from mere approximation of colloid particles, since we may have diffusion through a continuous sheet that has swelled in water, and the diffusion probably takes place in the bound or hydrate water itself. Bartell and Bigelow have shown that porcelain becomes rapidly semipermeable when the pores decrease below 0.1 micron.

It was shown by H. N. Morse that KCl and NaCl increase the permeability of colloidal copper ferrocyanide membranes, but it is a generally known fact that Ca and Mg salts do not increase the permeability, at least not sufficiently for their own diffusion or of that of sugar. Na and K salts (especially chlorides) increase the swelling and permeability of gelatine plates and Mg and Ca salts, at least under certain conditions, retard the swelling and may even cause shrinkage. OH' increases swelling and $H^{\cdot}$ decreases swelling if the reaction of the medium is on the alkaline side of the isoelectric point for the colloid, which seems to be the case in regard to blood and animal proteins within the body or in sea water.

We may therefore conclude that OH', $Na^{\cdot}$, and $K^{\cdot}$ increase the permeability of the plasma membrane by causing it to swell and that $Ca^{\cdot\cdot}$, $Mg^{\cdot\cdot}$, and $H^{\cdot}$ (at least on the alkaline side of the isoelectric point) inhibit increase in permeability by inhibiting swelling.

---

[1] *J. Biol. Chem.*, **25**, 669 (1916).

[2] These vials were obtained from Hynson, Westcott and Company, Baltimore, Maryland.

[3] There is reason to support the view that the cell surface is composed of emulsoids rather than suspensoids.

SOME INTERRELATIONS BETWEEN DIET, GROWTH, AND THE CHEMICAL COMPOSITION OF THE BODY

By Lafayette B. Mendel and Sarah E. Judson

SHEFFIELD LABORATORY OF PHYSIOLOGICAL CHEMISTRY, YALE UNIVERSITY

Read before the Academy, November 13, 1916. Received, November 14, 1916

Changes which normally occur in the water, ether extract, and ash content of the body during its most active growth have been determined for the white mouse. Based on eighty-eight analyses of the entire body at different stages of growth, the following results were obtained: (*a*) increase in solids from 16% at birth to a maximum of 35% at fifty days with a subsequent decrease to 33%; (*b*) decrease in the proportion of water in the fat-free substance from 85.5% at birth to 73% in the adult mouse; (*c*) rapid increase in fat during the first twelve days from 1.85% to about 10%, followed by a slow increase to about 12%; (*d*) absolute and relative increase in ash content from 25 mg., 1.86%, in a mouse weighing one and a half grams at birth, to 950 mg., 3%, of ash in the adult mouse.

The ash content of mice growing normally on an artificial food mixture of milk powder, casein, starch, salts, and butter fat, (protein 30%, fat 32%, and ash 5.5%), is uniformly higher at corresponding stages of growth than the ash content of mice fed on a ration of mixed grain, dog bread, and small amounts of milk. It is not known to what this difference is due, but it is not related to differences in the amount of protein or salts in the food. Since the ash content of normal animals may be thus increased by differences in diet alone, it would seem that conclusions as to the development of the skeleton during arrested growth which are based on an increased ash content of the body, should depend upon comparison not with the 'normal animal,' but with the normal animal on the same diet.

When abundance of fat is furnished in the diet, but not enough protein to maintain normal growth, the fat content of the animal is greater than when the food contains an adequate amount of protein with the same proportion of fat. There seems to be a tendency to protect the limited amount of protein by increasing the available supply of fat in the body. This does not occur when growth is arrested by lack of lysine, as in the use of gliadin as the only protein in the diet, since in this case the limiting factor lies not in the amount but in the nature of the protein.

By underfeeding, mice have been completely arrested in growth, as far as growth is expressed in gain in weight, and have been maintained at a constant body weight of 12 grams for thirty and sixty days. At the end of the thirty-day period, control mice of the same initial weight as the experimental animals, and on the same diet, weigh 22 grams. Comparison of the composition of the stunted mice with that of mice growing normally shows that the proportion of fat in the stunted animal is about the same as in the normal mouse of the same weight (but younger), while the percentage of water in the fat-free substance corresponds to the water content of a normal mouse of the same age (but heavier). That there is no evidence of a general replacement of fat by water, such as is often reported in underfed animals, may be due to the large proportion of fat used in this diet. The ash content, both absolute and relative, of the stunted mouse is greater than that of the normal mouse of the same weight, confirming in this the results reported by Aron for the rat, and indicating continued growth of the skeleton under conditions which prevent the animal from gaining in weight. The tendency of the skeleton to develop under such adverse conditions does not appear to be as strong in the mouse, however, as in the rat, if the few data reported for the rat are representative.

Retardation of growth by other means—reduction of protein or of salts in the food, or substitution of gliadin for other proteins—affects the ash content of the animal in the same way as simple underfeeding.

If young mice which have been maintained for a time at constant weight are given sufficient food again, they grow at a greatly accelerated rate which enables them to overtake control mice which have grown uninterruptedly. The ash content, however, does not increase at the same rate as the body weight; and the development of the skeleton, as represented by changes in the total ash of the body, which proceeded for a time at the expense of other tissues while the animal was held at constant weight, does not now, in the active growth which accompanies refeeding, keep pace with this rapid gain in body weight, and consequently in a few days the normal relation is almost reestablished.

FURTHER STUDY OF THE ATOMIC WEIGHT OF LEAD OF RADIOACTIVE ORIGIN

By Theodore W. Richards and Charles Wadsworth, 3d
WOLCOTT GIBBS MEMORIAL LABORATORY, HARVARD UNIVERSITY

Read before the Academy, November 14, 1916. Received, November 27, 1916

The recent independent and almost simultaneous investigations upon the atomic weight of lead from radioactive minerals have proved with very little room for doubt that the substance derived from this source has a much lower atomic weight than ordinary lead.[1] This conclusion is so important in its theoretical relations that its every aspect should be carefully investigated. Accordingly, the present paper represents further research in this direction, embodying determinations of the atomic weight of new samples of varied origin. The outcome entirely supports the earlier conclusion.

Four samples from widely separated sources were studied in the present research, namely, lead from Australian carnotite, from American carnotite, from Norwegian cleveite, and Norwegian bröggerite.

The first of these samples was obtained in large quantity through the kindness of Mr. S. Radcliff and Mr. E. R. Bubb, of New South Wales.

The preliminary purification of the sample was carried out in Australia.[2] Our subsequent purification was briefly described in a recent paper,[3] but some additions to the account are needed. The metallic lead was dissolved in nitric acid, leaving practically no residue. A portion of the nitrate thus obtained was precipitated with 20% hydrochloric acid from dilute solution. Lead sulphide was precipitated from the warm acidified solution of this chloride by pure hydrogen sulphide, and after separation and washing was dissolved in pure nitric acid. The small portion oxidized to sulphate during this process was boiled with sodium carbonate and the lead carbonate, washed free from sodium, was dissolved in nitric acid and united to the main portion of the nitrate, which was recrystallized four times from pure water and precipitated as chloride from a warm solution in a quartz dish with hydrogen chloride. The precipitate was centrifuged, dissolved in water and recrystallized four times, each time having added a few drops of hydrochloric acid to prevent the formation of basic salt. The second mother liquid gave no test for nitrate. This chloride formed Sample A, and after two additional recrystallizations a similar specimen constituted Sample B.

The second portion of the original nitrate made from the Australian sample was purified in a much simpler fashion. Avoiding the troublesome precipitation with hydrogen sulphide, the nitrate was recrystallized five times successively by adding concentrated nitric acid to its aqueous solution; then the lead was converted into chloride, and by precipitation with excess of hydrochloric acid this salt also was recrystallized five times. This was Sample C of the chloride, a portion of which before the last crystallization had served to prepare Sample D of the metallic isotopic lead used for determining the density.[4]

The next sample, designated F, was prepared from American carnotite and came to us in a considerably purified state through the kindness of Dr. C. H. Viol of Pittsburgh, of this company, and of Prof. W. D. Harkins. The method of further purification was essentially like that just described.[5]

Two other samples of especial value and significance were obtained through the kindness of Dr. Ellen Gleditsch, of Kristiania. Both came from primary rocks—Norwegian pegmatite dykes. The purification of one of these, from cleveite, has already been described;[6] the source of this material, which occurred in cubic crystals and was carefully selected, was near Langesund, Norway. It was recrystallized first as nitrate and then as chloride three times, each in the usual manner, and the pure substance was designated as Sample G.

Yet another sample, designated H, was prepared from lead sulphide, also kindly sent by Dr. Gleditsch, obtained from selected crystals of Norwegian octahedral bröggerite from Roade, near Moss, Norway. This was purified in precisely the same way as lead from cleveite.

In addition to these four samples containing isotopic lead enough ordinary lead was carefully purified to serve as the basis of control analyses. The purest "test lead" of commerce, free copper, was dissolved in nitric acid and recrystallized four times as nitrate and four times as chloride.

Throughout this work the usual care taken in atomic weight investigations was not forgotten. The nitric acid, hydrochloric acid, water and silver were all purified in methods already often described, and throughout the work on the nitrate and chloride of lead, except in the preparation of Sample A, the material was treated exclusively in vessels of platinum or quartz. All the weighings were reduced to the vacuum standard and all other precautions usual in this sort of work were carefully maintained.

The analysis was essentially similar in every way to the method described so often in Harvard contributions. The lead chloride was fused in a platinum boat in pure hydrochloric acid; this gas was displaced with nitrogen while the substance was cooling; and finally the pure dry salt in its boat was pushed into the weighing bottle, stoppered in pure dry air with the help of the familiar Harvard "bottling apparatus" and weighed at leisure. The weighed salt was placed in a large Erlenmeyer flask with glass stopper very carefully ground. Enough water was then added to form a fiftieth normal solution of the salt and the flask and contents, with the addition of a drop of pure nitric acid to prevent the formation of basic salt, were gently warmed on an electric stove, at about 50°C., until complete solution was obtained. The boat was then removed, and the residue filtered off, both boat and residue being carefully washed and the filtrate being collected directly in the precipitating flask.

The chlorine contained in this solution was then precipitated in the usual fashion by an amount of silver calculated from preliminary trials to correspond with it as nearly as possible. Any slight deficiency or excess was corrected by adding silver nitrate or chloride, and testing in the nephelometer; and the finally corrected weight is given in the table. The precipitation was carried on in a dark room, under red light, and all the usual precautions were taken.

The first two determinations of ordinary lead were only preliminary, in order to gain practice with the method, and are not included in the table below. They yielded values for the atomic weight respectively 207.15 and 207.16. All the other analyses which were brought forward to conclusion are recorded in the tables, which are self-explanatory.

The atomic weight of ordinary lead

SAMPLE	CORRECTED WEIGHT $PbCl_2$	CORRECTED WEIGHT Ag ADDED	RATIO $PbCl_2$: Ag.	ATOMIC WEIGHT Pb
3	3.72918	2.89325	1.28892	207.179
4	5.35111	4.15151	1.28896	207.188
			Average	207.183

The atomic weight of "isotopic" lead

SAMPLE	CORRECTED WEIGHT $PbCl_2$	CORRECTED WEIGHT Ag ADDED	RATIO $PbCl_2$: Ag	ATOMIC WEIGHT Pb
1 A Carnotite, Australia.......	4.64010	3.61118	1.28493	206.318
2 B Carnotite, Australia.......	5.35517	4.16711	1.28512	206.359
3 B Carnotite, Australia.......	6.15608	4.79072	1.28500	206.334
4 C Carnotite, Australia.......	4.14770	3.22748	1.28512	206.359
			Average	206.342
5 F Carnotite, U. S. A.........	5.31585	4.12670	1.28816	207.015
6 F Carnotite, U. S. A.........	4.65899	3.61707	1.28806	206.994
			Average	207.004
7 H Bröggerite, Norway........	4.29104	3.34187	1.28402	206.122
8 G Cleveite, Norway..........	3.92736	3.05913	1.28382	206.079
9 G Cleveite, Norway..........	4.45270	3.46818	1.28387	206.090
			Average	206.084

The results of these analyses show that the different samples containing isotopic lead all give lower values for the atomic weights than ordinary lead, but that the material from each source gives a different value, precisely as had been previously found in the earlier investigations in this and other laboratories. Ordinary lead gave the maximum value (essentially equal to that found by Baxter[7]) and isotopic lead from Norwegian cleveite gave the minimum value (essentially equal to that found by Hönigschmid in bröggerite[8] 206.06).

It seems reasonable to suppose that the other samples were composed of mixtures of these two kinds of lead. The value 206.34 would be given approximately by a mixture of three parts of isotopic lead like that obtained from Norwegian cleveite with one part of ordinary lead—a reasonable supposition, since the Australian carnotite was known to have contained galena. The American carnotite, Sample F, had an atomic weight which would be given by a mixture of only one part of the pure isotope with 5 or 6 of ordinary lead,—a condition which seems to indicate the admixture of very large amounts of galena with the sample in question.

Two physical properties of the several preparations under consideration have especial interest, namely, the magnitude of the radioactivity and the nature of the spectrum. Both were studied in the present research. We confirmed the outcome of earlier work[1] that the radioactivity is not proportional to the decrease in atomic weight in samples containing isotope coming from different sources. If the radioactivity were dependent upon the presence of the isotope radium

G (the supposed-end product of the decomposition, the present form of isotopic lead), the rate of fall in the gold leaf electroscope for the lead from cleveite would be the greatest, and that from the American carnotite the least; but our results were precisely the other way about. There can be little question, then, that radioactivity is not due to the isotope which gives the low atomic weight. Probably it is due to radium E, since in all of our samples the half-value of the radioactivity was obtained in about five days, the half life of radium E. The maximum value, approached asymtotically, is nearly reached in a month.

The spectrum of isotopic lead as thus far studied by Hönigschmid, by Merton and by Harkins, as well as by Baxter (who kindly photographed the ultraviolet spectrum of one of the samples of lead prepared by one of us with the help of Dr. Lembert) has always been found to be essentially like that of ordinary lead. In the present research we thought it worth while once more to test this question and especially to extend the inquiry to the visible portion of the spectrum—most of the tests in the past having been made in the ultraviolet region. We found difficulty in eliminating traces of copper, silver and calcium too small to be detected by ordinary analysis. Although these were far too diminutive to affect the atomic weight, we continued the purification, and after eight recrystallizations as nitrate and two as chloride, the product gave in every respect precisely the same spectrum as the purest ordinary lead prepared by Baxter and Grover, except for a vanishingly small trace of two of the most prominent silver lines.

The photographs of the ultraviolet region in the Féry spectrometer were very kindly made by Professor Baxter. Photographs of the visible portion of the spectrum were made by us in the Gibbs Laboratory on specially prepared plates sensitive as far as wavelength 7800. All the work of loading and developing had to be done in complete darkness. To make assurance doubly sure, a further study of the visible spectrum was made, in collaboration with Mr. Norris F. Hall, with the help of the Hilger wave-length spectrometer, comparing visually in the same field of view the spectra of pure ordinary lead and the best purified specimen from Australian carnotite. Every line was scrutinized between the range 4000 and 7600, especial pains being taken in the red and yellow portions, the least satisfactory from a photographic point of view. No discernible differences between the two spectra were observed.

Because no lines were detected between wave-lengths 7800 and 2200 in any of the samples which were not due either to ordinary lead or to unimportant traces of well-known impurities, one of the alternative conclusions previously reached by Richards and Lembert is supported, namely, that this isotopic lead possesses the same spectrum as ordinary lead. The present outcome is especially interesting because the isotopic lead obtained from the preparations of Dr. Gleditsch was probably almost pure—certainly much purer than that examined in 1913.

Since the atomic weight is variable, but the spectrum and atomic volume[9] of these samples all the same, one can hardly avoid concluding that a part of the atom exercising an important effect upon the atomic weight is without influence upon the spectrum or volume. The dual nature thus postulated is, of course, in accord with the interesting hypothetical assumptions which have been advanced by various authors concerning the possible makeup of the atom; but our present research can go not further than support the idea of duality without defining exactly of what the two parts may consist.

We are glad to acknowledge our indebtedness to the Carnegie Institution of Washington for generous support in this investigation.

Summary.—In this paper the atomic weight of four different samples of isotopic lead not hitherto tested, as well as one sample of ordinary lead (used to control the others), was determined. The results were as follows:

Ordinary lead	207.18
Isotopic lead (Carnotite, Colorado)	207.00
Isotopic lead (Carnotite, Australia)	206.34
Isotopic lead (Bröggerite, Norway)	206.12
Isotopic lead (Cleveite, Norway)	206.08

That the most carefully selected sample should give the lowest result is strong (although not absolutely conclusive) evidence that the higher results obtained from other samples were due merely to the accidental admixture of ordinary lead. No new lines were found either in the ultraviolet or visible spectrum of any of these samples. Each, except the ordinary lead, possesses radioactivity, but the magnitude of this radioactivity seemed to bear no relation to the lowering of the atomic weight.

---

[1] Richards and Lembert, *J. Amer. Chem. Soc.*, **36**, 1329 (1914); Hönigschmid and St. Horovitz, *Paris, C. R. Acad. Sci.*, **158**, 1798 (1914); M. Curie, *Ibid.*, **158**, 1676 (1914); Soddy and Hyman, *London, J. Chem. Soc.*, **105**, 1402 (1914); also especially, Hönigschmid, *Sitz. k. Akad. Wiss.*, *Wien*, IIa (Dec., 1914).

[2] A description of the details is to be found in the paper by S. Radcliff, *J. Proc. R. Soc.*, New South Wales, **47,** 145 (1913).

[3] *J. Amer. Chem. Soc.*, 38, 223; these PROCEEDINGS, 2, 505 (1916).

[4] Loc. cit., these PROCEEDINGS, **2,** 505 (1916).

[5] A fuller description is given in our paper published in the December, 1916, number of *J. Amer. Chem. Soc.*

[6] Richards and Wadsworth, *J. Amer. Chem. Soc.*, **38**, 1659 (1916).

[7] Baxter and Grover, *J. Amer. Chem. Soc.*, **37**, 1027 (1915).

[8] Hönigschmid and St. Horovitz, *Sitz. k. Akad. Wiss., Wien*, **123**, IIa, 1 (2407) (Dec., 1914). He gives the name 'Uranblei' to this form of lead. The name is appropriate if, as seems probable, the substance may ultimately be traced back genetically to uranium, according to Boltwood's brilliant and well-supported hypothesis. Because the relation of this form of lead to radium is somewhat less remote than that to uranium, we used the term 'radiolead' for it in a previous paper; but this term has also been applied to Radium D and is, therefore, not distinctive. The time for a final nomenclature of these substances has probably not yet come, but the expression "isotopic lead," based upon Soddy's word 'isotope,' is certainly safe as applied to every substance fitting into this place in the Periodic System. We venture to suggest that the present permanent isotope of lead be called 'isolead,' because it is now by far the best known of these isotopes. If any other permanent leads are verified, they might be called 'meta' and 'para.' For the highly transitory isotopes these names would be inappropriate, since they do not resemble lead in one of its most characteristic properties, namely, permanence. These might be called 'pseudo-leads,' giving them Greek ordinal prefixes to distinguish between them. But we offer these suggestions without any desire to be insistent, and have not even adopted this nomenclature in the present paper. It may well be best to retain the present nomenclature, especially as regards the transitory isotopes.

[9] See Richards and Wadsworth, these PROCEEDINGS, **2**, 505 (1916).

ON SOME ANOMALIES IN GEOGRAPHIC DISTRIBUTION OF PACIFIC COAST MOLLUSCA

By William Healey Dall

SMITHSONIAN INSTITUTION, WASHINGTON, D. C.

Received by the Academy, November 9, 1916

The islands, usually called the Santa Barbara Islands, lie off the coast of southern California from 15 to 50 miles roughly parallel to the shore of the mainland, from which they are separated by a depth of about 400 fathoms. They range in latitude from 33° to 34°N. That they were formerly connected with the mainland seems probable from general considerations, as well as the fact that the fossil tooth of an elephant is reported to have been found on Santa Catalina.

As we go south we find other islands or banks farther off shore and separated from the mainland by an increasing depth averaging more than 1500 fathoms. The Cortez Bank lies about 120 geographical miles south of Santa Cruz Island of the Santa Barbara group, in latitude 32° 20′N. It is 80 miles west of the mainland and separated by a depth of 1090 fathoms.

Next comes Guadelupe Island in latitude 29° N., 200 miles south of the Santa Barbara islands and 150 westward from the nearest Lower California mainland, from which it is separated by a depth of 1500 fathoms.

Six hundred miles south of Guadelupe is Socorro Island and its neighbors, forming the group of the Revillagigedo Islands. It is 330 miles west of the Mexican mainland in latitude 18° 50′ N., separated by a depth of about 1500 fathoms.

In a general way the sea near the coast of the mainland, west of the edge of the continental shelf, is considerably deeper than it is a relatively short distance seaward; that is to say a sort of trough in the sea-bottom lies near and roughly parallel with the mainshore, for a distance of nearly 1000 miles. The soundings are not numerous enough to warrant a more definite statement, but are sufficient to indicate the above generalization.

Investigations into the distribution of the Molluscan faunas of the Pacific coast of the Americas have been carried on for many years by the writer and are gradually reaching a point where positive conclusions can be drawn. We find, for instance, the Panamic or tropical fauna extending from Point Aguja on the Peruvian coast to Cape St. Lucas and the Gulf of California, and on the western shore of the peninsula of Lower California to Cerros Island, and thence (gradually diluted by the Californian fauna) as far as Point Conception, which however is reached by extremely few of the really tropical species.

There is a certain number of littoral mollusks which live only in shallow water near the tidal boundaries and hence are more subject to the influences of climate than those species which inhabit deeper water. Their limits of distribution are strongly marked.

Now it is a fact that many of these littoral species belonging to the Northern or Oregonian fauna actually reach the island of Guadelupe though stopping far short of it on the coast of the mainland.

More surprising still, we find a number of these northern species living on the island of Socorro, where the normal fauna should be wholly tropical: while on Clarion Island in nearly the same latitude and 220 miles west of Socorro there is Indo-Pacific tropical fauna with conspicuous species like *Voluta deshayesii.*

Socorro has, as far as we know, a strictly American tropical fauna with little Indo-Pacific adulteration, though it is 110 miles nearer to Clarion Island than to the American mainland. The other islands of the Revillagigedo group have not been visited by a collector.

Work has not gone far enough to enable a complete list of the northern species now living on Socorro Island, Latitude 18° 20′ N., to be made, but the following species are included in it.

Mytilus californianus Conrad. Southernmost reported locality on mainland in latitude 24° 40′ N., Magdalena Bay, Lower California.

Pachydesma stultorum Mawe. Southernmost mainland locality latitude 21° 36′ N., San Blas, Mexico.

Protothaca staminea Conrad. Southernmost mainland locality latitude 24° 40′ N., Magdalena Bay, Lower California.

Pholadidea sagitta Stearns. Southernmost mainland locality latitude 32° 24′ N., San Diego Bay, California.

There are also several gastropods, but on none of the islands cited has an exhaustive collection been made, so that a complete list is still a desideratum.

Mr. W. H. Ochsner, naturalist of the Galapagos Expedition of the California Academy of Sciences was the first to discover and collect from fossiliferous sedimentary strata on those islands.

In the case of the recent fauna of Socorro Island, we have at least four characteristically Californian species surviving on the island from 200 to 844 geographical miles south of their southernmost known extension on the mainland.

As the distances are calculated only from the difference in latitude and in geographical miles, while the coast line trends in a southeasterly direction, the actual distances are very considerably larger.

What explanation can be offered of these remarkable anomalies? The facts here presented, with others not yet fully worked out, have led to the framing of the following hypothesis.

Suppose the islands and banks here briefly referred to are the remnants of a peninsula or chain of islands once existing, parallel to the present peninsula of Lower California and connected to the mainland of southern California. During the existence of this peninsula, the cold current down the coast which now gives rise to fogs as far south as Cerros Island, in Pleistocene time, especially during the Glacial Epoch, might have carried its normal fauna far to the southward; this peninsula at the same time protecting from the cold current the waters, normally warm, of the sea between it and Lower California as well as of the Gulf of California, so that the tropical fauna might creep up to the extreme northern limit of these enclosed waters.

With the subsidence of the water barrier most of the northern fauna would have perished, but on the undrowned peaks a few of the more adaptable of the northern forms might have been able to persist or have left their traces in the subsequently slightly elevated Pleistocene sediments.

Our present knowledge of the seabottom west of the inshore deep trough previously referred to, is insufficient to add support to this hypothesis, but the stupendous changes of elevation on the California coast in very late geological time (see *Proceedings U. S. National Museum,* 1, 3, 1878) add a certain plausibility to it, taken in connection with the facts above recorded of the geographical distribution of the shore mollusca.

It should be noted that the Pleistocene fossils collected on the Galapagos Islands by Mr. Ochsner contain no traces of the northern fauna, though plainly of American derivation and with hardly any intrusion of Indo-Pacific forms. Also that the northern species mentioned appear in the Pleistocene and even older beds of the California coast from San Pedro to San Diego.

A somewhat analogous overlapping of faunas occurs on the southeastern coast of Alaska where a considerable number of boreal mollusks, as well as one of the Arctic hair seals, extend southward in the cold waters of the inland passages of the Alexander archipelago, following the low temperature of the water induced by the drainage from glaciers and the shadows of the highlands bordering these narrow passages which, with the prevalent fogs, cut off most of the sunlight. On the outer fringe of the archipelago where the more normal sea temperatures occur, numerous representatives of the Californian fauna find their way northward at least as far north as the latitude of Sitka, and very likely to Cross Sound.

SOME PSYCHO-PHYSIOLOGICAL PROCESSES AS AFFECTED BY ALCOHOL

By W. R. Miles

NUTRITION LABORATORY, CARNEGIE INSTITUTION OF WASHINGTON

Read before the Academy, November 14, 1916. Received, November 16, 1916

The difficulty of adequately measuring the effects of small amounts of ethyl alcohol upon the human organism is indicated by the number of investigators who have worked in this field, by the variety of methods that have been employed, and more especially by the contradictions in the results. It is impossible to harmonize the published data because of the many and unknown variables. If two investigators would independently employ the same apparatus, methods, doses, subjects, and general conditions, the two sets of results would be comparable and seemingly of unique importance. It is the object of this paper to report a comparison of measurements made under such conditions.

Subsequent to the experimentation which is the basis of the extended report by Dodge and Benedict[1] arrangements were made with one of their normal group (Number VI) for a second series of measurements.

This subject had served during parts of twenty days, in all approximately seventy hours well distributed over a period of six months. The fact that he had showed the smallest positive effect of the alcohol was an especial reason for selecting him for the later tests.

During this second series of six consecutive days (June 29 to July 4, 1914, inclusive) the subject, who was a medical student,[2] was free from the usual duties of the school year. Although serving part of the time as an intern in a nearby hospital, he regarded the period as a vacation and could thus carefully regulate his daily habits. The last food before each experimental session was a hearty meal taken seven hours previously from which coffee and tobacco were excluded. No alcohol was taken during the week other than that given in the laboratory. The experiments began at 8 a.m. and continued until 1 p.m., thus providing a total session of five hours as contrasted with the three hour periods employed by Dodge and Benedict. In the former series the subject was working intensely and an effort was made to guard against any weekly rhythm that might exist on this account by having all the experiments come on the same day of the week. In our experiments there seemed no such logical reason for extending the sessons over a period of weeks; consequently the measurements were made on consecutive days, the alcohol being given on the second, fourth and sixth days. The giving of alcohol doses in this frequency may present the possibility of an additive effect. However, such an effect would apply even more to the normal days and thus theoretically a smaller resulting difference would be shown.

The alcohol for each day was given in a single portion. The dose used was that designated by Dodge and Benedict as 'Dose A,' which contained 30 cc. of absolute alcohol, 7 cc. of orange infusion,[3] 1 cc. of a strong infusion of quassia, a slight amount of saccharine for sweetening, and water to bring the volume to 150 cc. A mixture of the same volume and composition, but without the alcohol, served as the control dose for each of the three normal days. In giving the two mixtures, every effort was made to prevent the subject from distinguishing between the alcohol and control days and he had no way of knowing before the session what the dose for that particular day was to be. As the alcohol frequently produced sensations of warmth in the stomach and flatulency, the subject never failed to identify the mixture before the experiment was over, although he received no confirmatory information regarding his impression. Unfortunately only the 30 cc. dose could be used in the second series. This was in accordance with the expressed desire of the subject who insisted that the 45 cc. dose, which he had taken three times in the previous series, made him feel uncomfortable for the rest of the day.

For the detailed technique of the various measurements used reference must be made to the careful description by Dodge and Benedict. There were but few minor changes in procedure since it was the intention to repeat the measurements of the first series with the same apparatus and as nearly the same conditions as practicable. In outline the experimental cycle was as follows: (1) Preliminary adjustments; placing of the electrodes for electrocardiograms from body leads; taking of data concerning the subject's general condition. (2) Patellar reflexes stimulated by two pendulum hammers of known weight; separation interval 0.5 seconds, latency and amplitude recorded from the thickening of quadriceps muscle. (3) Sensory threshold for Faradic stimulation measured in β-units, method of Martin. (4) Reaction time in reading isolated 4-letter words these being the same set of 24 words that was previously employed. (5) Finger movements, a modified form of the tapping test recorded photographically. (6) Voluntary tetanus designed to produce rapid fluctuations in the pulse rate. (7) Memory test for series of twelve 4-letter words. The same lists of words were used on each day and in the same order. (8) Eye reaction to peripherally appearing stimuli, photographically recorded. (9) Eye movement speed and accuracy in looking successively at marks separated by 40 degrees. (10) Protective lid reflexes stimulated by two sharp noises, separation interval 0.5 seconds, the photographic record being the shadow of the eyelash movement. (11) Pulse records (electrocardiograms from body leads) each approximately 15 seconds in length. Eleven such records were taken during each experimental cycle.

At the beginning of the second experimental period the subject drank the dose for the day, i.e., either the alcohol mixture or the control mixture. Then followed the patellar reflexes and other measurements in rotation as outlined. These were repeated in the same order in each experimental period. Theoretically, for purposes of comparison, the periods on the experimental days should be equal in number and length, but this ideal of regularity is hard to attain. During the six days, however, there was no accidental interruption which caused the omission of any measurements or enforced a delay of more than ten minutes for the adjustment or repair of apparatus. To complete a period required about one hour. This was approximately twice as long as the periods employed by Dodge and Benedict as in their series all of the measurements were not made on the same day.

In addition to providing for an equal number of normal and alcohol days, the first period of each day was also normal since the dose was not given until the beginning of the second period. Thus the first period values for each particular day serve to indicate the neural condition of the different processes measured. In the results presented emphasis is placed on the differences between the values obtained for the first period of the day and those obtained for succeeding periods of the same day as indicating the effect produced by the alcohol. To obtain the average difference for any particular day and any particular measurement, therefore, the value for the first period is deducted from the values for the succeeding periods, and the algebraic sum of the individual differences is divided by the number of periods succeeding the taking of alcohol. Such a method, while seemingly indirect, presents the effects of alcohol in a less contaminated form than does the simple comparison of the average values for the different days, as the experimental days are subject to changes in the level of nervous excitability. It cannot, however, be assumed that changes will not occur in the 5-hour period, although they are likely to be smaller than between different days.

With this subject the effects of alcohol do not always retain one direction throughout the experimental day, for a depression during the the first two periods after taking alcohol may change to a facilitation during the last hour or more of the session. To average either the measurement values or the differences between the first and succeeding periods under these conditions would therefore mask important tendencies. The results must be presented in a more analytic form.

A summary of the findings for the different experimental periods following alcohol together with the comparable averages found by Dodge and Benedict both for their normal group and for Subject VI individually are shown in table 1. The values given are the percentile effects of alcohol on the various processes in question, a plus showing increase and a minus decrease, which signs must be interpreted according to the nature of the measurement. Similar processes have been grouped in the table by sections. The first value in Period 2, Section 1, + 3.4 shows that the patellar reflext latency (L′, first reflex) was lengthened 3.4% in the first measurements made after taking alcohol as compared with the condition that existed in the period immediately after the taking of the control mixture. This lengthening of the reflex time was also present in the third and fourth experimental periods (+0.9 and +1.5) but not in so great a degree. In the fifth and sixth periods the sign changes, representing a shortened reflex time.

Since in the later series slightly more than 4 hours experimentation followed the drinking of the dose it is obvious that an average for Periods

2 and 3 is most directly comparable to the former results. A column of such averages is included in the table to aid comparison. This average for the patellar reflex, L′, is +2.1 as compared with +11.1, the average for the Dodge and Benedict normal group. The earlier result for Subject VI alone is +0.3, which although not large shows him to be in line with the normal group and also with his later percentages for periods 2, 3 and 4.

TABLE 1

THE PERCENTILE EFFECTS OF THE INGESTION OF 30 CC. OF ABSOLUTE ALCOHOL ON A RELATED GROUP OF PROCESSES

PROCESSES MEASURED	EXPERIMENTAL PERIODS FOLLOWING THE TAKING OF ALCOHOL					AVERAGE OF PERIODS 2 AND 3	RESULTS FOUND BY DODGE AND BENEDICT WITH EQUIVALENT DOSE	
	Period 2	Period 3	Period 4	Period 5	Period 6		Average for normal group	Average for Subject VI
Section I								
Patellar reflex								
Latency, L′	+3.4	+0.9	+1.5	−7.4	−3.4	+2.1	+11.1	+0.3
Latency, L″	+0.8	+4.0	+5.8	−15.1	−10.6	+2.4	+4.0	
Lid-reflex								
Latency, L′	+13.6	+5.8	−2.9	+6.4	+4.3	+9.7	+5.9	−4.5
Latency, L″	+15.1	+1.8	−7.5	−3.8	+3.8	+8.4	+6.8	−10.7
Eye reaction time	+15.5	+16.4	+14.7	+11.8	−1.9	+15.9	−5.4	−9.3
Word reaction time	+3.2	−8.2	−7.4	−10.6	−20.8	−2.5	−1.0	−26.0
Faradic sensory threshold	+31.0	+20.4	+18.2	+24.9	−8.3	+25.7	+21.0	+48.0
Eye movement velocity. Rt. movements	+19.3	+6.4	+2.1	−1.4		+12.8	+3.4	+25.8
Section II								
Patellar reflex								
Amplitude, A′	−2.5	+3.3	−22.7	−42.0	+5.0	+0.4	−60.3	+13.0
Amplitude, A″	−24.8	−38.0	−24.0	+62.0	+81.0	−31.4	−38.0	+173.0
Lid-reflex								
Amplitude, A′	−38.0	−33.6	+13.3	−17.7	−16.8	−35.8	−10.7	−18.0
Amplitude, A″	−57.0	−42.8	+25.7	−68.5	+65.8	−49.9	−103.9	−27.0
Progress in memorizing	+10.9	+2.5	−5.9	+27.8	+57.0	+6.7	+2.5	+13.3
Number of finger movements	−3.9	−4.0	−3.1	−3.5	+7.1	−3.9	−8.9	−15.2
Section III								
Pulse Cycle Length								
Immediately before exercise	−11.6	−3.9	−14.5	−5.8		−7.7	−3.1	−4.0
During severe exercise	+1.3	−4.0	−4.0	−2.7		−1.3	−4.1	+6.0
Immediately after exercise	−2.5	−8.7	+2.5	−1.2		−5.6		
Average of other pulse data	−12.4	−12.2	−14.8	−12.9	−19.4	−12.3	−3.0	−3.5

Referring to one of the measurements placed in Section II of the table,—for example, the number of finger movements performed in 8 seconds,—we find that the percentages given under Periods 2 and 3 are −3.9, and −4.0 (average −3.9), showing a decrease in the number of finger oscillations on alcohol days as compared with the normal days. This decrease in approximately the same degree is found in Periods 4 and 5, while in 6 there is a change in sign, thus representing an increase in the number of finger oscillations. In comparison Dodge and Benedict find −8.9% and −15.2% the averages for the normal group and for Subject VI respectively. These values indicate that Subject VI in both series of experiments conforms to the group.

The percentile changes in the length of the pulse cycle are nearly all minus, indicating a decrease in the length of the individual pulse cycle which of course is equivalent to an increase in pulse rate per minute. The effect of Alcohol in producing a faster pulse is more pronounced with Subject VI than for the average of the normal subjects.

In general the various periods may be compared on the basis of the preponderance of plus or minus signs. In Section I, Period 2, all of the values are plus and range from 0.8 to 31.0, with a central tendency above 12%. In Section II the signs for Period 2 are with one exception minus and show a central tendency of about 19%. It should be borne in mind that plus in Section I and minus in Section II have the common meaning of inferior performance or depression. Period 3 is clearly more like 2 than it is like 4, while 2 and 3 averaged may be compared logically with the percentages for the normal group. The signs agree here almost exactly, measurement for measurement, although the percentages are naturally different. In the one clear case of disagreement, that is, eye reaction time, −5.4% for the normal group as against +15.9% for Subject VI in the second series of experiments, and −9.3% in the first series, it should be stated that Dodge and Benedict for this measurement found a decided practice effect for their subjects which masks the influence of the 30 cc. dose of alcohol. In the first series Subject VI is in harmony with the group. In the second series no practice effect is shown and the reaction time is lengthened. The plus and minus signs in the averages for Subject VI otherwise agree for the most part with his averages for Periods 2 and 3. Detailed discussion of the individual measurements cannot be given here but will appear in a fuller report soon to be published by the Carnegie Institution of Washington.

The two series of measurements for Subject VI taken together unmistakably indicate as a result of a dose of 30 cc. of absolute alcohol a lengthened reflex latency with a decrease in the amplitude of movement, slower reactions, slower coördinated movements, less sensitiveness to stimulation and an increase in pulse rate. The memory and word reactions, as in the earlier results, were improved after the alcohol.

Some attention should be given to the fact that in Period 6, and to a less extent in Period 5, in Sections I and II of the table, the signs are in the majority of cases the opposite of those in Periods 2 and 3 for similar measurements. This means a superior performance and is in contrast to the earlier condition of general depression. There are some indications in the data previously published that this facilitation following the alcohol depression may not be a peculiarity of Subject VI, but a characteristic phenomenon of the alcohol effect.

---

[1] Dodge and Benedict, Psychological effects of alcohol, *Carnegie Inst. Washington, Pub.* 232, 1915; these PROCEEDINGS, **1**, 605 (1915).

[2] For family and personal history see Dodge and Benedict, op. cit., p. 277.

[3] Rivers, *The influence of alcohol and other drugs on fatigue*, London, 1908.

THE INFLUENCE OF THE MARGINAL SENSE ORGANS ON METABOLIC ACTIVITY IN CASSIOPEA XAMACHANA BIGELOW

By L. R. Cary

DEPARTMENT OF BIOLOGY, PRINCETON UNIVERSITY, AND DEPARTMENT OF MARINE BIOLOGY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, November 20, 1916

The results of my earlier studies[1] have shown a marked influence of the marginal sense-organs on the rate of regeneration in *Cassiopea,* when halves of the same specimen are used for comparison. Further experiments on regeneration have confirmed these results, and have shown that when two halves of any medusa disk are subjected to the same operation the amount of regeneration from the two halves is identical in extent within the limits of error of measurement employed

Since the influence of the sense-organs on the rate of regenerations is most marked in the earlier stages of any experiment, several series of disks were (1) separated into halves, and (2) the sense-organs were then removed at different intervals of time after the first operation as shown in the following table:

TABLE 1

Series No.	*First operation Disk cut into 2 half disks*	*Second operation S. O. removed from one-half disk*	*Result*
1	9.45 a.m. July 22	7.00 a.m. July 23	Half with S. O. fastest
2	8.00 a.m. July 24	7.00 p.m. July 24	Half with S. O. fastest
3	7.15 a.m. July 25	8.00 a.m. July 26	Reg. equal
4	8.00 a.m. July 25	4.00 p.m. July 25	Half with S. O. fastest
5	7.00 a.m. July 26	8.00 a.m. July 27	Reg. equal

When the sense-organs were removed from one half disk in less than 24 hours regeneration occurs just as if they had been removed at the time of the first operation. After about 26 hours, however, the removal of the sense-organs from one of the half disks had no apparent influence on the rate of regeneration, which was the same from both halves of any disk.

Influence of marginal sense-organs on loss of weight in starving.—Mayer[2] found that when Cassiopea was starved in sea water from which all food organisms had been removed by careful filtration, the loss of weight could be expressed mathematically by the formula $y = W\ (1-a)^x$, in which W = the original weight, x the number of days of starvation and a a constant, the 'coefficient of negative metabolism.' The value of a in the equation above differs in experiments involving varying conditions as regards light and darkness, presence or absence of regeneration, etc., but in all cases the formula gives a very close approximation to the observed loss of weight.

In my experiments the two halves of a series of disks were compared after they had been subjected to operations which made possible the comparison of the halves of the disks upon one of which the sense-organs remained, while these were removed from the other half disk (active and inactive series); second, the comparison of half disks on one of which the sense-organs remained, while the other, from which all sense-organs were removed, had its muscles activated by a circuit wave of contraction maintained in an endless labyrinth of subumbrella tissue (active and activated series); third, half disks from which all sense-organs were removed, while one of them was activated by a circuit wave of contraction (activated and inactive series).

TABLE 2

	Loss of weight in active and inactive half disks		*Loss of weight in active and activated half disks*		*Loss of weight of activated and inactive half disks*	
Days after operation	*Weight of half with sense-organs*	*Weight of half without S. O.*	*Weight of half with sense-organs*	*Weight of activated half*	*Weight of activated half*	*Weight of half without S. O.*
0	100.00	100.00	100.00	100.00	100.00	100.00
1	75.34	81.81	76.29	79.41	79.47	81.97
2	66.72	71.27	67.18	70.58	70.63	71.84
3	58.54	64.09	59.76	61.99	62.03	63.91
4	55.27	55.41	55.88	57.18	57.21	57.23

In every instance the results follow closely those obtained when the regeneration was used as the standard of comparison. The results of the entire series of experiments are shown in the following tables, in which the original weight of each series of disks is taken as 100 so that the results read as percentages of the original weight.

In the activated and inactive series as well as in the series composed of activated and inactive half disks the visible activity, i.e., muscular contraction was much greater in those members of each series in the tissues of which the circuit wave of contraction was maintained. When compared with half disks under the control of the sense-organs, the rate of pulsation of the activated disks was from 3.5 times as great at the beginning of any experiment to 10 times as great at the end of the first day. During this interval the rate of the half disks with the sense-organs fell to scarcely more than half the original rate while, on the contrary, the rate of the activated half disk always increased.

Simultaneous kineograph records of the pulsations of the two halves of the same disk, one with its sense organs and the other activated by a circuit wave of contraction, were made to measure the amount of muscular work done by each half under the given experimental conditions. In all these determinations it was found that the amplitude of the contraction as recorded on the drum depended upon the character of the operation that had been performed upon the half disk. When the active half remained with its subumbrella muscles undisturbed, the resulting contraction was more extensive than that of the activated mate. When, however, the same operation had been made upon each of the two halves of a disk, from one of which the sense organs were removed, the amplitudes were equal for each half, so that the rate of pulsation is apparently a true measure of the work done.

The comparison of the loss of weight shown by activated and active, as well as by the activated and inactive pairs of half-disks, shows very clearly, just as when regeneration is used as the basis for comparison, that muscular activity is a relatively unimportant factor in determining the metabolic activity of *Cassiopea.*

Influence of marginal sense-organs on total metabolism.—To measure the total metabolism of half disks of *Cassiopea* under the several operative conditions involved in the regeneration and starvation experiments, specimens prepared in the manner previously described for these experiments were placed in closed jars containing known volumes of fresh sea water, and after different intervals of time the amount of CO_2 given off was determined for each specimen. In making these determinations, the records were kept in terms of increased hydrogen ion concentration, the values of which were later determined by adding known volumes of CO_2 to a volume of fresh sea water equal in amount to that contained by the jars used in the experiments with the medusa disks. In many of the experiments the disks were kept in the closed jars until one member of a pair had ceased to pulsate, because of the narcotizing effects of the CO_2, so that it was possible to obtain a measure of the CO_2 concentration necessary to bring about narcosis. In nearly all instances the activated half disk was the first to succumb, and when it had stopped it did not again pulsate until once more stimulated by an induction shock. The active half disks showed more resistance to CO_2, and when removed from the jars in which they had ceased to pulsate and put into fresh sea water would start pulsating again within one or two minutes, even when they had been inactive for several hours.

When the closed jars, in which the medusae were kept for these experiments, were allowed to remain in the light, the disks would continue to pulsate for several days as the CO_2 would be in part used up by the symbiotic algae which are very abundant in the tissues of *Cassiopea.*

The results of a typical experiment are shown in table 5, in which the half disk with sense-organs is designated "*a*" and the activated half disk "*b*."

TABLE 3

No. of Specimens		*Weight in grams*	*Pulsation Rate*			
			1.45 p.m.	*4.40 p.m.*	*7.30 p.m.*	*H· Concentration*
1	a	22.8	44	62	18	7.80
	b	23.0	128	136	120	7.90
2	a	30.0	36	34	22	8.00
	b	30.5	88	86	98	8.00
3	a	39.0	44	36	8	7.90
	b	41.0	130	158	126	7.90
4	a	28.00	56	22	25	7.90
	b	29.00	96	106	Out	8.00
5	a	29.25	84	32	32	7.90
	b	28.75	116	120	132	7.90

The hydrogen ion concentration of the sea water determined at the beginning of the experiment was PH. 8.10 (8×10^{-9}) so that the change brought about in its reaction on account of the activity of the several half disks was from 0.1 to 0.3 of the PH. unit. Using the same volume (1200 cc.) of fresh sea water, it was found that the addition of the 5 cc. of CO_2 would usually bring about a change of 0.2 in the PH. reading, so that each half disk had apparently given off approximately that volume of CO_2, in the 8 hours during which they were in the jars. When the specimens were left for a longer time in the jars the amount of CO_2 given off became proportionately less as time went on as the disks became more thoroughly narcotized, so that when the hydrogen ion concentration of the water had become 7.8 the disks had ceased pulsating. If left for some hours in this water, the nervous system became incapable of transmitting impulses and finally the sense-organs were rendered inactive.

[1] Cary, L. R., these PROCEEDINGS, **1**, No. 12; *J. Exper. Zoöl.*, **21**, No. 1.

[2] Mayer, A. G., *Publ. Carnegie Inst. Washington*, No. 183.

NEW EVIDENCE IN REGARD TO THE INSTABILITY OF HUMAN TYPES

By Franz Boas

DEPARTMENT OF ANTHROPOLOGY, COLUMBIA UNIVERSITY

Read before the Academy, November 14, 1916. Received, November 24, 1916

A number of years ago I carried on, under the auspices of the United States Immigration Commission, an investigation on the physical types of immigrants and of their descendants. One of the results of this inquiry was the establishment of the fact that there is a difference in appearance between the immigrants and their descendants. So far as the bulk of the body is concerned, this information was not new. Analogous phenomena had been observed in 1877 by H. P. Bowditch in Boston, and by Peckham in Milwaukee. It was new, however, that there is also a change in such features as the cephalic index and the width of the face. It was found that on the average the heads of descendants of immigrants of East European types are more elongated, and those of the descendants of South Europeans more rounded, than those of their parents. The data were obtained partly by a generalizing method, partly by a comparison between parents and children.

The results of this inquiry have been attacked by many writers, on the basis that they decline to believe that such changes can occur. I have not found any actual criticism of my method and of the results, except by Corrado Gini, who doubts the inferences drawn in regard to the populations of Italian cities which also show a modification of the cephalic index.

I think the hesitation of many authors to accept the results is due largely to a misinterpretation of their significance. I may be allowed to state concisely here what I think has been proved, and what inferences seem justifiable.

The investigation has a direct bearing upon the question of the classification of human local types, more particularly of European types. Many attempts have been made to give a satisfactory classification of the divergent types that occur in Europe. Pigmentation, stature, form of the head, and form of the face, show material differences in various parts of Europe, notwithstanding the fundamental sameness of the whole race. Authors like Deniker, and many others, have carried out on this basis an elaborate classification of European types in a number of 'races' and 'sub-races.'

In this classification the assumption is made that each race that we find at the present time in its particular environment is an hereditary type different from the others. In order to express this assumption, I should like to use the term that these races and sub-races represent, 'genetic' types—genetic in the sense that their characteristics are determined by heredity alone. The question, however, has not been answered, whether these types are really genetic types, or whether they are what I might call 'ecotypes,' in so far as their appearance is determined by environmental or ecological conditions. If we include in this term not only environmental conditions in a geographical and social sense, but also conditions that are determined by the organism itself, we might, perhaps, still better call them physiological types, in the same sense in which the biologist speaks of physiological races. My investigation then was directed to the question in how far a certain type of man may be considered a genetic type, in how far a physiological type. If there is any kind of environmental influence, it is obvious that we can never speak of a genetic type *per se*, but that every genetic type appears under certain environmental or physiological conditions, and that in this sense we are always dealing with the physiological form of a certain genetic type. The question, then, that demands an answer, is, in how far genetic types may be influenced by physiological changes.

I believe, that, on the basis of the material that I collected, we must maintain that the same genetic type may occur in various physiologically conditioned forms, and that so far as stature, head-form, and width of face are concerned, the differences between the physiological forms of the same genetic type are of the same order as the differences between the races and sub-races which have been distinguished in Europe. I must add, however, that these remarks do not refer to pigmentation, for, contrary to a widespread belief, we have no proof of environmental influences upon pigmentation. For this reason the classification of European races cannot be considered as proving genetic differentiation.

The whole investigation which I carried on, and certain comparable observations obtained from older literature, do not indicate in any way to what physiological conditions the observed changes may be due. The only physiological causes in regard to which evidence is available relate to the bulk of the body, and to a certain extent to the proportions of the limbs. The size of the body depends upon the conditions under which growth takes place. Growth depends upon nutrition, upon pathological conditions during childhood, and upon many other causes, all of which have an effect upon the bulk of the body of the adult. When these conditions are favorable, the physiological form of a certain genetic type will be large. If there is much retardation during early life, the physiological form of the same genetic type will be small. Retardation and acceleration of growth may also account for varying proportions of the limbs. On the other hand, we have no information whatever that would allow us to determine the cause of the physiological diminution in the size of the face that has been observed in America, nor for the change in the head-index that occurs among the descendants of immigrants.

Furthermore, there is nothing to indicate that these changes are in any sense genetic changes; that is to say, that they influence the hereditary constitution of the germ. It may very well be that the same people, if carried back to their old environment, would revert to their former physiological types.

In fact, it can be shown that certain features are strictly hereditary, and that, although the physiological form of a genetic type may vary, nevertheless the genetic type as such will exert its influence. Professor von Luschan has repeatedly called attention to this fact as revealed in the modern populations of Asia Minor, where, notwithstanding the mixture which has continued for at least four thousand years, the characteristic Armenian, Northwest European, and Mediterranean types survive in the mixed population. Similar examples may be observed in Italy. I have calculated the variability of the head-form that is found in different parts of Italy, based on the data collected by Ridolfo Livi. The head-form of the North Italians is excessively short. The head-form of the South Italians is decidedly elongated. In between we find intermediate forms. In the Apennines, we have, in addition to the mixture of these two Italian forms, a marked immigration from the Balkan Peninsula, which introduced another short-headed type. As a result of these long-continued mixtures, we observe low degrees of variability in northern and southern Italy, high degrees of variability in the central regions, particularly in the Abruzzi. These indicate permanence of the component types of the mixed population.

During the last few years some new data have been collected that confirm my previous observations. I have pointed out several times that changes of types have been observed in Europe wherever a careful comparison between city population and country population has been made. Generally the changes that occur there have been ascribed to selective influences; but the intensity of selection would have to be so great, that it does not seem plausible that they can be explained by this cause.

In conjunction with Miss Helene M. Boas, I have made a comparison between the head-forms of the city populations of Italy and of the rural population in the areas surrounding the cities, and compared these data with the information given in the Italian census in regard to the immigration into cities. I found throughout that the variability of head-form in each city is smaller than would be found in a population in which all the constituent genetic types were present without physiological modification. This result has been criticised by Corrado Gini, on the basis that in former times migration was less than what it is now. I grant this point; but nevertheless it is quite obvious, that, although no exact data are available, the mixture of population in a city like Rome or like Florence must be very great, since the political conditions for the conflux of Italians, and even of individuals from outside of Italy, have been favorable for a very long period. If this is true, we should expect a very high degree of variability in Rome, which, however, is not found.

Turning to new data, I wish to mention the observation made by Dr. Hrdlička, who, in a paper read before the Pan-American Scientific Congress, has stated that he found the width of face of Americans of the fourth generation—that is to say, of descendants of Europeans who had no foreign-born ancestor after the fourth generation back—was materially decreased as compared to the width of face found among European types. This conforms strictly with what I found among the descendants of immigrants of all nationalities.

A year ago I had the opportunity to make an anthropometric investigation of a considerable number of natives of Porto Rico. This work was carried on in connection with the Natural History Survey of Porto Rico organized by the New York Academy of Sciences. The population of Porto Rico is derived from three distinct sources—from people belonging to the Mediterranean type of Europe, from West Indian aborigines, and from Negroes. The Mediterranean ancestry of the Porto Ricans leads back to all parts of Spain; but among the more recent immigrants, Catalans, people from the Balear Islands and from the Canary Islands prevail. There are also a fair number of Corsicans. The Spanish immigration has been quite strong even up to the present time. Among the individuals whom I measured, 14% had Spanish-born fathers, some even Spanish-born mothers. From all we know about the history of the people of Porto Rico, we must consider them essentially as descendants of male immigrants who intermarried with native women. It is evident that in early times this must have led to the development of a Mestizo population, in which, however, the amount of Indian blood must have decreased very rapidly owing to the continued influx of Spanish blood, and the elimination from the reproductive series of the male Mestizo element. The Negro population is settled particularly on the outer coast of the island; while the amount of Negro blood in the interior is apparently not very great, except near the principal routes of travel.

According to European observations, the Spanish ancestors of this population, while living in Spain, are long-headed. The Negro element is of mixed provenience, from many different parts of Africa, but, on the whole, the Negro in Africa is also long-headed. The West Indian element, judging from the few prehistoric crania that have been recovered, represents a very short-headed type. The modern Porto Rican is short-headed to such a degree that even a heavy admixture of Indian blood could not account for the degree of short-headedness. If we apply the results of known instances of intermixture to our particular case, and assume stability of type, we find that, even if the population were one-half Indian and one-half Spanish and Negro, the head-index would be considerably lower than what we actually observe. There is therefore no source that would account for the present head-form as a genetic type; and we are compelled to assume that the form which we observe is due to a physiological modification that has occurred under the new environment. The head-form of those individuals whose fathers were born in Spain is noticeably more elongated than that of the individuals whose parents are both Porto Ricans. The head-index of the Mulatto population is intermediate between the index of the native Porto Ricans and that of those whose one parent is Spanish. The average index of the Porto Rican is 82.5. The average index of the Spaniard in Spain is less than 77. We find, therefore, an increase of five units here, which can in no way be accounted for by genetic considerations.

I may mention in this connection that the average stature of the Porto Ricans is apparently almost the same as that of the Sicilians in New York, and that throughout the period of growth the stature follows about the same curve as that represented by Sicilian children living in America. If anything, the stature is a little lower, and there is no indication of that acceleration of development which is so often claimed to be characteristic of a tropical environment. Undoubtedly poor nutrition, and probably also pathological causes, have a retarding influence here, which might easily be overcome by better hygienic conditions.

It is unfortunate that we have no accurate statistics of Porto Rican immigration and emigration, which would enable us to state with much greater definiteness what genetic type should be expected here. There is a popular belief in Porto Rico that in certain parts of the island, in the so-called 'Indiera,' Indian types have persisted to a greater extent than elsewhere. I have not been able to find any definite indication of a difference in type; but I have measured only a few individuals from these districts. The material that I have been able to study comes from all parts of the island, but principally from the western-central part. The phenomena here described occur with equal intensity in all parts of the island.

The question of the degree of instability of human types seems to my mind an exceedingly important one for a clear understanding of the problems of physical anthropology. It would be particularly desirable to study the problem among immigrants living in different rural communities of the United States, and it would be even more desirable to have information in regard to the types that develop among the East Europeans and South Europeans who return to Europe and settle in their old geographical environment.

A REVISION OF THE ATOMIC WEIGHT OF TIN

By Gregory Paul Baxter and Howard Warner Starkweather

COOLIDGE MEMORIAL LABORATORY, HARVARD UNIVERSITY

Read before the Academy, November 14, 1916. Received, November 23, 1916

A recent investigation upon the atomic weight of tin by Briscoe,[1] in which stannic chloride was compared with pure silver, yielded a very concordant series of results, with an average value 118.698 (Cl = 35.457). This value has been adopted by the International Committee on Atomic Weights in preference to that found by Bongartz and Classen,[2] 119.0, which has been in general use for some time. Since the electrodeposition of cadmium and zinc in a weighed mercury cathode has been found to be a process capable of great accuracy,[3] and since tin amalgam promised to be unusually well adapted for quantitative handling, this electrolytic method has been applied to the analysis of stannic chloride also.

In brief the method of operation was as follows: Pure tin was converted to tetrachloride by treatment with pure chlorine, and the tetrachloride was purified by fractional distillation. After weighed portions of the chloride had been dissolved in dilute hydrochloric acid, the metal was deposited electrolytically in a mercury cathode contained in a weighed glass cell similar to that previously described.[4]

Before converting the tin to tetrachloride it was freed as far as possible from acid-forming elements by twice transporting the metal electrolytically through an acid solution of stannous chloride, the anode being an ingot of tin. This served at the same time to eliminate a large part of the copper and lead which were the chief metallic impurities. The product was fused in hydrogen on an alundum boat.

In preparing the tetrachloride special pains was taken to avoid exposing the substance to moisture. The ingots of metal were placed in a glass tube connected at one end with an apparatus for producing dry chlorine and at the other with a glass bulb containing a small quantity of tin. After the tube and bulb had been exhausted dry, chlorine was admitted and passed over the metal until it had been almost completely converted to chloride. The product was allowed to stand in contact with tin for some time to remove excess of chlorine and reduce lead tetrachloride, and was then fractionally distilled in vacuum at room temperature by condensing the product in bulbs cooled either with alcohol and solid carbon dioxide or with liquid air. First the most volatile fraction was removed with liquid air and rejected. The remainder was four times distilled, with rejection of a residue of about 15 grams in each case. Then two more fractions were eliminated with liquid air. The remainder was finally collected for analysis in a series of small bulbs. In the course of the distillation each fraction was sealed off from the rest of the apparatus as soon as collected. Two series of preparations were made, the procedure differing only slightly in the two cases.

The whole apparatus was constructed of glass, with sealed joints, the only glass stopcock was located between the tube containing the tin and the chlorine generator, and was lubricated with syrupy phosphoric acid. In place of stopcocks special glass valves devised by Briscoe were employed, in which a fused seal was broken when it was desired to open the valve.

The experimental procedure of an analysis was as follows: The glass bulb containing the stannic chloride was weighed in air, and again under water to find its displacement. Next the bulb was broken under about 75 cc. of 0.03 normal hydrochloric acid. The solution was then filtered into the weighed cell containing about 200 grams of mercury, and the glass was thoroughly washed with hydrochloric acid of the above concentration. The filter with the glass was burned in a weighed platinum crucible. Blank tests showed no appreciable quantity of glass to be dissolved in the process. The more concentrated washings were added to the main solution in the cell, the more dilute were evaporated with nitric and sulphuric acids in quartz. After nearly all the tin had been electrolytically deposited from the main solution, the residual electrolyte was concentrated, together with the evaporated washings, and the whole further electrolyzed. At the completion of the electrolysis the electrolyte was displaced successively with water and pure alcohol, both of which had been freshly saturated with hydrogen. To dry the amalgam the cell was placed in a small desiccator and the pressure reduced as far as possible. The electrolyte was evaporated, at first in a quartz dish, later in a weighed platinum crucible, and the residue heated gently. The weight of tin, computed upon the assumption that the residue was stannic sulphate, was added to that found in the amalgam. Although complete precipitation of the tin was never secured, the residue usually did not exceed 2 mg. in weight, and was never as much as 4 mg. Repetition of the electrolysis, after dissolving the residue in dilute sulphuric acid and returning it to the cell, sometimes caused a slight decrease in the weight of tin recovered. In all cases constant weight within 0.1 mg. was finally secured.

As a check upon the accuracy of this method of determining tin, in two experiments a weighed ingot of pure tin was electrolytically transported from the anode to the mercury cathode of the weighed cell. In one case a gain of 0.01 mg., in the other a loss of 0.06 mg. was observed. Such small changes are less than the probable error of weighing the cells.

The experimental data and the results calculated from them (assuming the atomic weight of chlorine to be 35.457) are shown in the following table. The weights of the bulbs containing the stannic chloride, of the glass and of the tin dissolved in the mercury are all corrected to vacuum.

Series	*Fraction*	*Grams of* $SnCl_4$	*Grams of* Sn	*Ratio* $Sn:Cl_4$	*Atomic weight of* Sn
I	5	11.64269	5.30498	0.837050	118.717
I	7	13.5943	6.1935	0.83687	118.691
I	8	10.0897	4.5971	0.83696	118.705
I	9	11.4319	5.2080	0.83677	118.678
I	11	12.2869	5.5983	0.83699	118.709
I	12	12.20889	5.56286	0.837020	118.713
I	13	10.7469	4.8965	0.83695	118.703
I	14	11.54233	5.25846	0.836818	118.684
			Average........	0.836929	118.700
II	2	15.65437	7.13198	0.836852	118.689
II	3	16.23310	7.39664	0.837059	118.718
II	4	17.29151	7.87850	0.836980	118.707
II	7	15.04889	6.85695	0.837037	118.715
II	8	18.36074	8.36507	0.836869	118.692
II	9	21.58929	9.83676	0.836991	118.709
II	10	17.22210	7.84718	0.837040	118.716
II	11	15.70516	7.15589	0.837018	118.713
			Average........	0.836981	118.707
				0.836955	118.703

The results for the atomic weight of tin are arranged in the order in which the fractions were distilled, not in the order of analysis. Since only slight irregular variations are apparent, there can be little question that the different portions were identical in composition.

The final average, Sn = 118.703 (Cl = 35.457) is in very close agreement with the one obtained by Briscoe by comparison with silver, 118.698. The percent of tin in stannic chloride found by us is 45.562, while Briscoe found 54.439% of chlorine. The sum is 100.001%, a highly satisfactory and convincing outcome.

It is a great pleasure to express our gratitude to the Carnegie Institution of Washington and to the Elizabeth Thompson Science Fund for very generous assistance in providing indispensable apparatus.

---

[1] *Trans. Chem. Soc.*, **107**, 63 (1915).

[2] *Ber. D. Chem. Ges.*, **21**, 2900 (1888).

[3] Baxter and Hartmann, *J. Amer. Chem. Soc.*, **37**, 113 (1915). Baxter and Grose, *Ibid.*, **38**, 857, 868 (1916).

[4] Baxter and Hartmann, these PROCEEDINGS, **1**, 26 (1915).

FURTHER STUDIES OF NERVE CONDUCTION IN CASSIOPEA

By Alfred Goldsborough Mayer

DEPARTMENT OF MARINE BIOLOGY, CARNEGIE INSTITUTION OF WASHINGTON

Read before the Academy, November 13, 1916. Received, November 22, 1916

Studies of recent years have shown the importance of hydrogen ion concentration in determining the rate of nerve conduction in *Cassiopea.* Ordinary distilled water often remains acid even though air freed from CO_2 by means of soda-lime has been bubbled through it for 72 hours. Accordingly, Prof. George A. Hulett kindly arranged to have 144 litres of distilled water prepared in accordance with his well known method (Ueber die Reinigung des Wassers durch Distillation, *Zs. phys. Chem.*, **21**, 287, 1896) in his laboratory at Princeton University. This water was sealed in 144 pyrex glass flasks and thus transported to Tortugas. The hydrogen ion concentration of each flask was tested separately, the range being 0.8 to 1.0×10^{-6}, and the average being 0.9×10^{-6}, or 6.04 PH.

Fifty litres of this water were placed in a green glass carboy which had previously held Merck's distilled water; and air freed from CO_2 was bubbled through it at an active rate for 78 hours, after which the water in the carboy had a PH. of 8.0 which it maintained for eight days while 139 experiments were made with it. The alkalinity then declined to 7.5 PH. while 26 other experiments were made; the average for the series being 7.93 PH. or 1.17×10^{-8} hydrogen ion concentration. Its alkalinity was probably due to soda derived from the glass carboy, the balance being maintained by a tendency of the water itself to become acid through leakage of CO_2 from the air. Professor J. F. McClendon found the PH. of the Tortugas sea water to range from 8.1 to 8.22, and dilution with alkaline distilled water of 7.93 PH. maintained the normal hydrogen ion concentration of the sea water even when diluted with its own volume of such distilled water.

TABLE I

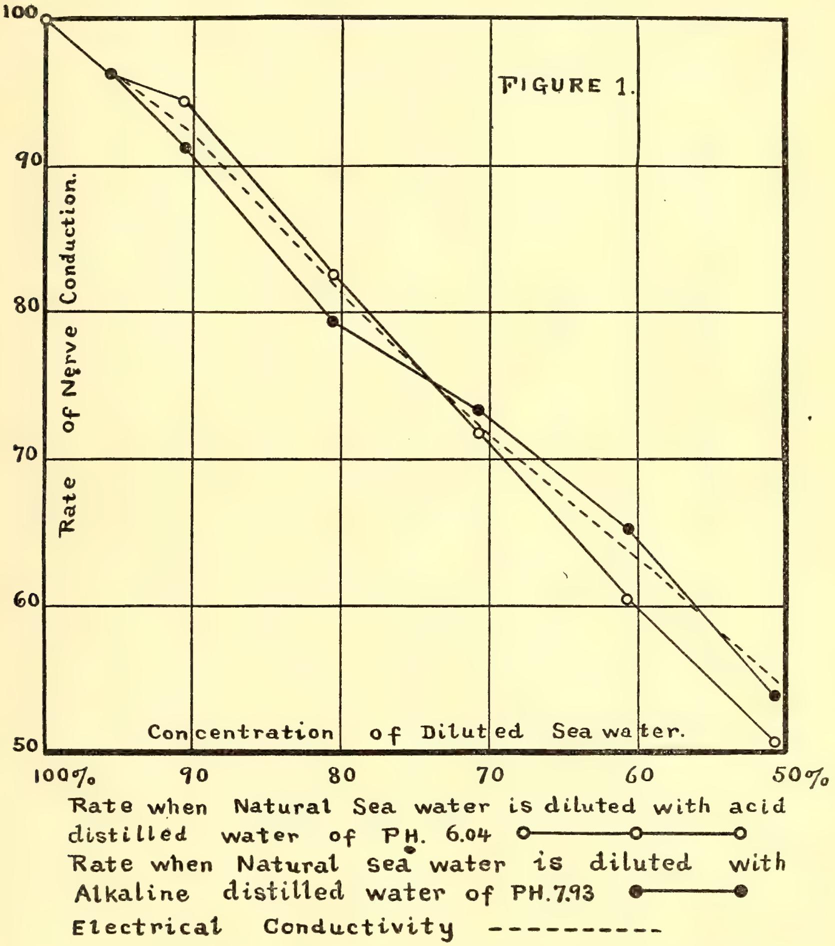
ILLUSTRATED BY FIGURE 1

COMPOSITION OF THE SOLUTION	I RATE OF NERVE CONDUCTION WHEN THE SEA WATER IS DILUTED WITH ACID DISTILLED WATER OF 6.04 PH. AT 30°C.	PROBABLE ERROR	II RATE OF NERVE CONDUCTION WHEN THE SEA WATER IS DILUTED WITH ALKALINE DISTILLED WATER OF 7.93 PH AT 30°C.	PROBABLE ERROR	III RELATIVE ELECTRICAL CONDUCTIVITY OF TORTUGAS SEA WATER DILUTED WITH DISTILLED WATER OF 7.8 PH. AT 30°C.
Natural sea water of 8.1 to 8.22 PH.	100.00		100.00		100.00
95 cc. sea water + 5 cc. distilled water	96.43	0.74	96.23	0.67	
90 cc. sea water + 10 cc. distilled water	94.38	0.88	91.44	1.00	92.16
80 cc. sea water + 20 cc. distilled water	82.68	1.16	79.51	1.18	81.38
70 cc. sea water + 30 cc. distilled water	71.95	1.13	73.91	1.16	71.53
60 cc. sea water + 40 cc. distilled water	60.41	0.59	65.72	1.01	64.26
50 cc. sea water + 50 cc. distilled water	50.83	0.90	54.16	0.79	54.08

It will be seen by comparing columns II and III of the above table that when alkaline distilled water of about 8 PH. is used to dilute the sea water the decline in rate of nerve conduction is practically the same as the decline in the electrical conductivity of the sea water when similarly diluted. We should remember, however, that the concentration of the sodium, calcium and potassium cations declines in practically the same ratio and thus the rate of nerve conduction may be proportional to the concentration of these cations rather than to the electrical conductivity of the diluted sea water as a whole.

However, Prof. Ralph S. Lillie is right in a recent paper (*Amer. J. Physiol.*, **41**, 123) wherein he states his belief that the rate of nerve conduction in *Cassiopea* in diluted sea water does not decline in accord with Freundlich's law of adsorption as I had erroneously supposed (these PROCEEDINGS, **1**, p. 270; **2**, p. 37).

The distilled water used in my previous experiments was slightly acid and thus stimulating in slight dilution and depressant in stronger dilutions (see Table I, column 1) thus giving the semblance of an adsorption curve. Nor did I realize the effects of slight changes in temperature in the various solutions, for the effect of 0.1°C. is readily appreciable.



Lillie advocates an extension of the theory of Faraday and de la Rive, that the transmission of the excitation state from the immediate site of activity to the adjoining resting area is dependent on an electrical local action of the same essential nature as that which is responsible for the etching or corrosion of non-homogeneous metallic surfaces, such as iron in contact with an electrolyte solution; and if this be true, the rate of nerve conduction should be a function of the electrical conductivity of the medium, and also of the surrounding fluid.

To further test this hypothesis, I determined the electrical conductivity of Tortugas sea water when heated or cooled, and compared it with the rate of nerve conduction at corresponding temperatures.

This shows that the rate of nerve conduction has a temperature coefficient about two and one-half times as high as that of the electrical conductivity of sea water; as will appear in Table II.

TABLE II

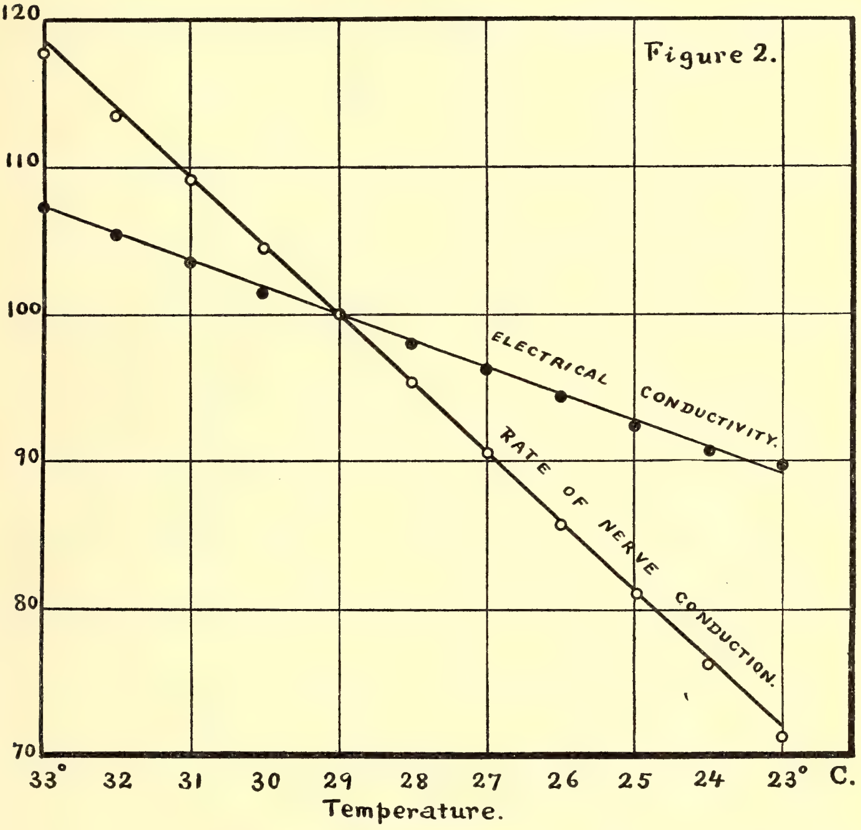
ILLUSTRATED BY FIGURE 2

TEMPERATURE C° OF THE SEA WATER	RATE OF NERVE CONDUCTION IN CASSIOPEA. MEAN OF HARVEY, MAYER, AND CARY'S OBSERVATIONS	RELATIVE ELECTRICAL CONDUCTIVITY OF TORTUGAS SEA WATER DETERMINED BY KOHLRAUSCH'S METHOD
23	71.3	88.9
24	76.3	90.7
25	81.17	92.6
26	85.8	94.4
27	90.74	96.2
28	95.47	98.0
29	100.00	100.0
30	104.47	101.6
31	109.2	103.5
32	113.4	105.3
33	117.8	107.1

It will be recalled that Harvey (1911, *Publ. Carnegie Inst. Washington*, No. 132) was the first to show that the rate of nerve conduction in *Cassiopea* augments in nearly a straight line as the sea water is heated until about 36° to 38°C. where it suddenly falls off, thus giving a curve resembling that of an enzyme reaction. This has been confirmed by the later work of Mayer, and Cary; and I find that when the rate has begun to fall off, the original rate is not recovered upon cooling. This may possibly mean that the hypothetical enzyme has been partially destroyed; for if this decline were due merely to asphyxiation the rate should recover to a greater degree when the medusa is replaced in sea water of normal temperature. Moreover, the high temperature coefficient of the rate of nerve conduction suggests that we may be dealing with a chemical reaction in which a compound is formed composed of sodium, calcium, and some proteid element; the degree of ionization of which is considerably affected by temperature in the manner suggested by Hardy, 1900; Quincke, 1902; and Bayliss, 1915, *Principles of General Physiology*, p. 77.

Possibly, also, the negative electrical potential associated with the wave of nerve conduction may increase the surface tension of the alkaline colloidal particles thus reducing their size, rendering them more soluble and thereby increasing the concentration of the reacting ions.

Contrary to Lillie's hypothesis, we have direct evidence that the rate of nerve conduction may be independent of the electrical conductivity of the solution surrounding the nerve, for I have found (these PROCEEDINGS, **1**, p. 270) that if sea water be diluted with 0.415 molecular $MgCl_2$ the rate of nerve conduction is only slightly more depressed than if the sea water is diluted with distilled water, or with dextrose; yet the $MgCl_2$ maintains a nearly normal electrical conductivity, while with distilled water or dextrose it declines in nearly the same ratio as the dilution. Nor do the experiments I have made with solutions containing some but not all the cations of sea water support Lillie's view. Thus if the rate of nerve conduction in 0.647 molecular NaCl is 55, it becomes 100 in 85.3 cc. of 0.6 molecular NaCl + 14.69 cc. of 0.39 molec-



ular $MgCl_2$. Here the electrical conductivity of the solution is somewhat reduced while the rate of nerve conduction is much augmented. This is, of course, a striking instance of Loeb's law of the antagonism between a univalent and a bivalent cation; even though the bivalent cation in this case is magnesium, well known to be a depressant especially for muscular activity in *Cassiopea.*

A full report will appear in papers from the Department of Marine Biology to be published by the Carnegie Institution of Washington.

THE EARLIEST FRESH-WATER ARTHROPODS

By Charles Schuchert

PEABODY MUSEUM, YALE UNIVERSITY

Received by the Academy, November 24, 1916

In the year 1900 Prof. T. C. Chamberlin published a prophetic paper under the title *On the habitat of the early vertebrates.*[1] In this paper he holds that the problem which he is considering admits of no other than hypothetical treatment, and proposes the hypothesis that "The chordate phylum is . . . . essentially from first to last a terrestrial race, whose main habitat was the land waters and the land itself, though still a race that sent its offshoots down to sea from time to time from the mid-Paleozoic onwards " (412). He arrives mainly at this conclusion on the basis that the fish form among animals could only have originated through mechanical genesis in swift streams and under a mode of life independent of the bottom. The most essential mechanical feature of rivers is their flow in a fixed direction, and to this insistent physical condition animal life had to adapt itself, for "otherwise the animal would be swept out to sea and its race be ended as a stream-dweller. It is different with ocean currents, for they return upon themselves and an animal may yield to them without losing its marine habitat" (406). This hypothesis is applied mainly to the origin of the fishes and less insistently to the eurypterids.

In regard to the origin of the eurypterids Chamberlin states: "From the occurrence of eurypterids first in marine beds apparently and later in fresh-water deposits it has been inferred that they were originally sea-dwellers and later became adapted to land waters, but the meagerness of their marine record on the one hand, and their abundance and fine preservation in the fresh-water deposits on the other, give point to the question whether their early marine record is anything more than the chance deposit of river forms borne out to sea" (403). But "it may be equally true that the fish and the eurypterids descended from the rivers to the sea in the mid-Paleozoic, though their remote ancestors may have ascended from it" (404). Finally, that the lands were clothed with vegetation even in Proterozoic time and that therefore there was an abundance of food in the rivers is attested in part by the great quantity of carbonaceous material in the strata of this time, plus the complete disintegration of the crystalline rocks as is shown by the fine aluminous muds, the clean sandstones, the great amounts of limestone, and the localized beds of iron-ore. Although arkoses are present (broken up and undecomposed granites), their mass is insignificant in comparison with the quantity of the other sedimentaries.

Early in 1916 Prof. Joseph Barrell also considered the probable first habitat of fishes, but from the standpoint of their occurrence as fossils in the rocks, in papers entitled *Dominantly fluviatile origin under seasonal rainfall of the Old Red Sandstone* and *Influence of Silurian-Devonian climates on the rise of air-breathing vertebrates.*[2] He agrees with Chamberlin and concludes that it is probable "that fishes arose in land waters" and "as such they constituted primarily a river fauna." The lung-fishes arose under semiarid climates and seasonal waters. "The exposure of the tidal zone alternately to water and to air had, then, nothing to do with the origin of lungs." "The evidence is regarded as strong that the air-bladder was originally developed as a supplemental breathing organ, although in modern fishes it has been mostly diverted to other uses. Among certain Devonian fishes, living under more and more strenuous climatic conditions of seasonal dryness, the use of the air-bladder for respiration became essential, and with the diminishing availability of the waters of certain regions the gills in those species which survived this crisis in evolution became correspondingly atrophied. The amphibians thus arose under the compulsion of seasonal dryness." Finally, "Climatic oscillation is a major ulterior factor in evolution" (388–391).

Later in 1916 appeared a suggestive memoir entitled *The habitat of the Eurypterida*, by Dr. Marjorie O'Connell,[3] which has stimulated the writing of these remarks. This work seeks to point out the actual habitat of eurypterids during the Paleozoic, as derived from a study of their entombment in all places Paleozoic, but chiefly in America and Europe. It is replete with paleontologic information, and all of the more important occurrences of these animals and their immediate associates are described in as much detail as the author was able to glean, in the main from the widely scattered literature, though she is also familiar with the actual field relations in the state of New York. There are in all the world about 130 described species in 14 genera, with the acme of development in the upper part of the Silurian where about 40 forms are known. There is very little evolution after Devonian time. On page 9 of Doctor O'Connell's memoir is given in italics the summation of her studies: *The eurypterids throughout their entire phylogenetic history lived in the rivers.* Lest the reader forget this thesis it is repeated on many subsequent pages. Though the book is argumentative and positive, and at times pleading in behalf of its theme, prying into and tearing apart all things eurypterid, the style remains essentially scientific.

It has been and still is widely held that eurypterids were marine animals previous to the Devonian, that toward the close of the Silurian they became euryhaline or able to live in both salt and brackish water, and that after Silurian time they probably became wholly restricted to the fresh water of the lands. Even though they were regarded as marine animals, living associated with trilobites and brachiopods, it has long been a difficult problem to explain why their remains are so very rare in the normal marine faunas of the Upper Cambrian, Ordovician, and Silurian, and even when they do occur are almost always in fragments. Although Doctor O'Connell points out this condition of occurrence, she does not make of it the strongest point in her argument, and yet it is the one fact that seemingly cannot be explained away. On the other hand, the entire specimen of *Strabops thacheri* at Yale has attached to the slab two specimens of the brachiopod *Obolus lamborni* and a poor head of a trilobite; the entombment is in a magnesian limestone, and because of the marine fauna of the same formation as listed by Beecher, one still retains the impression that this oldest and most generalized eurypterid may at least have been an inhabitant of the sea if not born of it. In the earliest deposits of the Silurian (Shawangunk) the eurypterids usually occur in fragments and in young specimens in thin and localized black shales, interbedded in a very thick and regularly bedded sandstone series, the material of a delta spreading into the epeiric sea. These beds are otherwise almost unfossiliferous, having associated with the eurypterids only Arthrophycus, a lobworm-like burrow that in the Medina formation is associated with marine fossils, showing that the Shawangunk is not a river flood-plain deposit but the sands of a delta under the influence of marine waters. On the other hand, in the latest Silurian formations of America, which are clearly not of normal marine waters, the eurypterids either have no marine associates or there are scattering specimens of cephalopods (Orthoceras, Trochoceras), bivalve crustaceans (Leperditia), and brachiopods (Lingula); in Bohemia, the middle Silurian, with a normal marine fauna of many species, has seven to eight forms of Pterygotus and one of Slimonia, but all of the specimens are in fragments. On Oesel, however, the three or four species of eurypterids are beautifully preserved in fine-grained gray dolomites or dolomitic limestones, with the original chitin of the animals still preserved; in fact, they have been etched out of the matrix and the tests mounted in Canada balsam. Here (at Attel) they are associated with an abundance of *Leperditia angelina*, *Orthoceras tenue*, and great stony heads, 2 feet across, of the hydroid Clathrodictyon (Twenhofel 1916). Surely these deposits are more of marine origin than of fresh waters.

All of this evidence, however, the present writer grants, does not necessarily prove that the eurypterids were born of the sea and continued to live in this environment until the close of the Silurian, for if this was their habitat, surely we should get good remains of them oftener than we do in the many normal marine formations of the Paleozoic, and especially in the Trenton and Niagaran formations. It would seem that, if the eurypterids were wholly marine animals in the Ordovician and especially in the Silurian, they should be as common and as well preserved as, for instance, the marine trilobites. This is not at all the situation. On the other hand, the evidence does not exclude the eurypterids from having also lived, at least at times, in brackish waters, the embayments of deltas, a conclusion that Doctor O'Connell will have none of. Nevertheless, we shall have to accept her main conclusion that the eurypterids were fresh-water animals, but with this modification, that they also appear to have lived at times in the brackish waters of more or less large bays and possibly in limited numbers even in the seas, just as some of the fresh-water fishes of the past have gone to sea. Whether they were born of the fresh waters or the ocean still remains a debatable question, though the evidence appears to favor strongly the former habitat.

It is a well-known fact that ceratiocarids and especially ancestral limulids and apodids are either very rare or occur locally in much restricted beds, and these, like the eurypterids, appear to have been entombed in the marine deposits near shore. The sporadic occurrences of the eurypterids and their great rarity as good specimens in normal marine deposits lead Doctor O'Connell to believe that these animals are of fresh-water habitats and that they were floated by the Paleozoic rivers into the open sea-ways just as we see the land animals and plants of today transported into the seas and oceans. On the other hand, her idea of delta deposits appears to be that they are either wholly of the land or of the sea, and her sketch of the late Silurian Bertie and Herkimer deltas (page 116) bears out this too rigid conclusion. Moreover, the illustrations of the Mississippi delta in our text-books are on so small a scale as to give an erroneous conclusion as to the amount of the river flood-plain area. A large-scale map shows that 50% or even more of this delta is under the waters of the Gulf of Mexico, and in areas like Lakes Ponchartrain and Borgne the largest of eurypterids may have dwelt. Hence it will always be very difficult to separate the deposits of such areas, with their mixed biotas, from those of the river flood-plains on the one side, and on the other from those of the more or less marine bays.

In this connection we should not neglect to state that nearly all the eurypterids occur in strata of times when the continents were largest and when the rivers were rejuvenated to stronger and longer flow into the interior seas. Also that Eurypterus lived from Ordovician into Permian time, and Pterygotus was distributed almost throughout the world during the Siluro-Devonian. However, at the times when the seas have their greatest spread we find almost no merostomes of any kind. These are significant facts bearing upon the probable habitats of the Paleozoic merostomes.

Since it appears that the eurypterids are probably in the main fresh-water animals, and as some of them take on the form of scorpions and may therefore be spoken of as river scorpions, this admission opens a most wonderful vista into the probable life of the land during early Paleozoic times, as far back at least as the Upper Cambrian. There are other merostomes of the order Limulava (two genera) in the Middle and Lower Cambrian, and these antennate forms are more directly related to the eurypterids. Of the Synxiphosura, forms ancestral to the Xiphosura, the Middle and Upper Cambrian have at least four genera. As all of these animals also appear to have lived in fresh water, it would seem that the rivers of Cambrian time were peopled by merostomes none of which exceeded 6 inches in length, but in late Silurian time they were not only far more varied in form, but Pterygotus at least, the most active and predaceous of them all, attained a length estimated at from 6 to 9 feet. Stylonurus, the great spider-like eurypterid of the Devonian rivers, may have attained a similar length. In regard to these and other eurypterids, Clarke and Ruedemann,[4] the authorities on these arthropods, think that most of our museum specimens are immature individuals, and often the casts of tests, conditions further favoring the theory that they are fresh-water animals whose thin, light tests were easily drifted by the rivers into the estuaries and the seas. The considerable variety and the great size attained by some forms means that there was an abundance of food, both animals and plants, in the rivers of Paleozoic time. Some of the animals were the smaller eurypterids, in the Ordovician and Silurian the armored fishes (the ostracoderms), in the Silurian the additional lung-fishes and diminutive primitive sharks (the acanthodians), and in the Devonian, large bivalve molluscs. However, there appear to have been still other arthropods present, such as the ceratiocarids, a stock of phyllocarids, the ancestral limulids (Synxiphosura), and probably also the branchiopod apodids. In the final analysis we must further admit that all were more or less directly dependent upon an abundance of plant food. and therefore that the early Paleozoic lands were clothed with vegetation, none of which is as yet certainly known to paleontologists. In this connection, however, we must also point out that today in no fresh waters are invertebrates of any kind known attaining to 9 feet long or even half that length, although it takes living Limulus eight years to attain to sexual maturity. This implies that at least some of the Silurian and Devonian eurypterids reached a great age, and, living in the rivers, and maybe at times in the brackish-water bays, were the monarchs of their environment. On the other hand, if the smaller merostomes had lived in the sea, as did the trilobites, they would also have been fed upon by the armored cephalopods, the nautilids, and we may have evidence of this preying in the common association of Orthoceras with the eurypterids. Having no enemies to feed upon them in the rivers, and none that were more agile, more powerful, or more cunning until Devonian time, the eurypterids of the Silurian and early Devonian continued to live on for a long time and so attained to a far greater size than any of the fresh-water invertebrates of today, which are dominated by the more active and intelligent river fishes. With the ascendancy of the fishes beginning in the Devonian, we see a diminution and lack of structural change in the eurypterids and the trilobites, and the vanishing of both stocks in the Permian.

The argument of faunal associates and their entombment, plus the nature of the deposits, used by Doctor O'Connell to prove that the eurypterids are fresh-water animals, will also apply to the ceratiocarids, the limulids, and the apodids. In the Middle Cambrian, Walcott has collected three genera of ancestral limulids and other merostomes (Limulava) in blue to black muds rich in kaolin, which were "probably laid down in a small bay or lagoon in close connection with the shallow Middle Cambrian sea." In these deposits trilobites of the genera Agnostus, Microdiscus, Neolenus, Ptychoparia, etc., are fairly common.

It may therefore be that the Limulava, the ancestral limulids, and the apodids were in this case also fresh-water animals drifted by the streams into the lagoon. If so, this is evidence of fresh-water life as far back as the Lower Cambrian. Further, in the uppermost Cambrian at one locality in Wisconsin the present writer has collected a few fragments and one entire specimen of the ancient limulid Aglaspis in association with an abundance of trilobites, fragmented and entire, and many well preserved brachiopods (*Westonia stoneana*). In all later Paleozoic deposits the limulids are almost always associated with eurypterids, and practically never in normal marine faunas. They are always very rare fossils until early Pennsylvanian time, when *Euproops danæ* is common in that most interesting mixed estuarine and terrestrial fauna preserved in the nodules of Mazon Creek, Illinois. The Triassic limulids also appear to have lived in fresh water, but since late Jurassic time Limulus has been in the sea, and is represented today by several living species in the Atlantic and Pacific oceans. Can, therefore, the annual return of these animals to the land to lay their eggs be a surviving instinct and an ontogenetic expression of their earlier, fresh-water habitat?

In this connection should be recorded a most interesting observation of Limulus made by Professor Barrell many years ago. He took home in the spring of the year a large female Limulus that he had picked up on the shore of New Haven Bay, and kept it in the grassed yard back of the house in which he lived. Under these conditions the animal was still able to crawl about feebly for two days and gave evidence of life for about a week, during all of which time it was out of the water. This clearly indicates that the gill-books are fairly easily adaptable to air-breathing, and the question may be asked whether in the Paleozoic during times of drought ancestral limulids of the fresh waters may not have lived over the dry season, as do the lung fishes of today, buried in the mud and breathing the air.

Attention should also be directed to the occurrences of the Apus-like branchiopods, the oldest of which is Protocaris of the Lower Cambrian, known in a single specimen associated with trilobites in a marine deposit laid down, however, near the shore. In the Middle Cambrian black shale lagoon deposits (Burgess), there are three genera, and then we have no record of them again until Apus is met with in the Triassic—henceforth a persistent synthetic genus, which in the living world is restricted to the more or less evanescent land waters of many continents.

If the eurypterids and limulids arose in the fresh waters, as appears probable, we can then the more readily explain why they and the terrestrial scorpions do not pass through a crustacean nauplius stage, for they had to adapt themselves to the lands or to waters limited in distribution and more or less evanescent in continuity, to the streams that flowed constantly in one direction and eventually into an unnatural habitat, the sea. They needed a more direct and quicker embryonic development, requiring, it would seem, the abandonment of the nauplius stage. It may therefore well be that the trilobites retaining the nauplius stage did not give rise to these stocks, as is sometimes assumed. We may have to look for this ancestral stock in one still more primitive, and the Protocaris-Apus line of branchiopods suggests itself, but whatever the stock, it would seem to have permanently invaded the rivers of the land either in Proterozoic time, or that postulated intermediate stage in the earth's history previous to the Cambrian but of which we have not a trace of direct evidence, the Lipalian time of Walcott.

[1] T. C. Chamberlin, *Jour. Geology*, 8, 400-412 (1900).

[2] J. Barrell, *Bull. Geol. Soc. America*, 27, 345-436 (1916).

[3] M. O'Connell, *Bull. Buffalo Soc. Nat. Hist.*, 11, No. 3, 1-277 (1916).

[4] J. M. Clarke and R. Ruedemann, The Eurypterida of New York, *Mem. New York State Mus.*, No. 14 (1912).

OBSERVATIONS UPON TROPICAL FISHES AND INFERENCES FROM THEIR ADAPTIVE COLORATION

By W. H. Longley

GOUCHER COLLEGE, BALTIMORE

Received by the Academy, November 24, 1916

The conception that species have been multiplied by divergent evolution of related strains is based upon a great body of verifiable observations. Sound judgment has not been exercised consistently, however, in the attempt to establish the fact that their development has been directed throughout by natural selection.

If the Darwinian hypothesis is true, the characters of organisms should be largely of an adaptive sort, but its adherents have failed, upon the whole, to distinguish between shadow and substance, and have been content to support their position by *imputing* utility to structures and habits, when nothing less than rigorous proof of the fact will suffice. It is not demonstrated, for example, that any class of markings serves for purposes of recognition, or for signalling between individuals of one species. Neither is it proved that some color combinations warn off possible enemies, nor, indeed, that any type of pigmentation is functionally conspicuous. The last assumption, nevertheless, underlies a series of suggestions whose apparent conformity with its terms is held to support the hypothesis of natural selection.

Under the auspices of the Tortugas Marine Laboratory of the Carnegie Institution of Washington, I have been enabled to study many of the bright-hued fishes of the West Indian region, and have attempted to place the facts regarding their coloration upon an objective basis.

Countershading appears almost universally upon these animals. That is to say, their pigments which are externally visible are definitely graded from darkest on the mid-dorsal, or upper, to lightest on the mid-ventral, or lower line. Exceptions to the rule occur only among species of unusual habit or peculiar form.

The systems of pigmentation indicated is no immediate effect of exposure to light, although it involves the production of dark shades in any region in direct proportion to the average intensity of the illumination of that part. This is shown clearly by certain cases of sexual dimorphism in the color of crabs. In the *Portunidae* the abdomen of the ovigerous female is exposed in dorsal view. In correlation with this family character the sexes differ in the coloration of that organ long before sexual maturity, when for the first time its position differs in the two. The abdomen of the male agrees in color with its sternum, but when that of the female is elevated it extends the pigment of the carapace posteriorly over the egg-mass and down into the shadow with delicate countershading. To explain this fact one may apparently appeal only to natural selection, or to sex-limited inheritance of the effect of exposure through many generations.

Other points of interest concern the striking color changes of thirteen species of fishes, which depend upon the color of the objects surrounding the animals. Further study will extend the list. These statements are based upon many records, of which a few were made under laboratory conditions. The greater number by far refer, however, to unconfined specimens studied from a boat, or from the bottom with diving equipment. The various phases of those which will gather about food provided for them may be induced at will by leading the creatures from place to place whose dominant colors differ. Changes of others obey the same laws and may be forecast with precision though they are not demonstrable with the same ease in uncontrolled individuals. Pictures taken with a submarine camera in ten feet of water record some of the observed changes, and show that their general effect is to reduce the conspicuousness of the animals that display them.

The distribution of colors among the various species examined has been investigated in a third phase of the research, and evident correlation of color with habit has been demonstrated. The results obtained may be expressed in brief as follows:

Those fishes in whose coloration red normally predominates are nocturnal, and, in proportion to their numbers, are rarely seen by day.

Gray appears with very different frequency among reef-ranging species and those whose diurnal activity is centered among the coral heads. The ratio of its occurrence in the two cases is roughly commensurate with that of its appearance in the environment of the contrasted groups. Among thirty-one reef-rangers, to state the facts in detail, fourteen show evident adaptive gray color phases or have permanent gray markings, while the same is true of only three of twenty-one species which remain near the coral heads by day. This is equivalent to saying that according to present information gray markings or color phases are about three times as common among reef-ranging fishes as they are among those which live close to the coral heads.

Brown appears so frequently in combination with gray in the patterns of fishes which adapt their coloration almost instantaneously to gray or brown bottoms, whenever the character of their surroundings changes, that in default of special evidence to that effect no functional conspicuousness may be imputed to either color alone, or in combination with the other. The same seems to be true of yellow.

Among the Tortugas fishes the lighter blues at least are correlated with the habit of swimming habitually well above the bottom in water of moderate depth. These tints are peculiarly inconspicuous in the eyes of an observer at a lower level, and photographs of fishes banded with other colors show that the effect of the blue is to blot out its possessor's contour under that condition, since at a distance of a few feet the blue-gray elements in patterns are indistinguishable from the color of the watery background.

Finally, two-thirds of the species seined upon the green grass-flats along shore, and with any show of reason considered typical members of the bionomic association inhabiting such places, are wholly or largely of a green color, or regularly show a green color phase amid green surroundings. If the forms that swim at a high level in open water be excluded, no other such aggregation of green fishes as may be secured on the grass flats may be named from the entire fish fauna of the region, though this includes more than two hundred species.

It seems significant that the suggestion from the observations so far recorded is uniform. The obliterative effect of countershading is demonstrated by Thayer's experiments. That adaptive color changes are very common and minister to the same end, although they occur among bright colored species, is a fair inference from my own experience. The colors of the fishes, also, are correlated with their habits in such a way that, upon the average, their conspicuousness would apparently be increased if their pigments were very different from those they display.

Additional research demonstrates that in so far as this class of animals is concerned there is no ground for the belief that bright color is correlated in any way with armament or distastefulness. If a list of species possessing organs capable of inflicting painful bodily injury be compiled, their colors and ability to change them are found to differ in no essential respect from those of any other group of the same size selected at random. That no unpalatability is correlated with gaudy coloration is proved by the fact that forms unsurpassed in brilliancy constitute an important part of the food, and may be recovered regularly from the stomachs of snappers, which are among the commonest of the predaceous fishes of the Tortugas. Hence, in view of all the evidence, it seems improbable, to say the least, that the bright colors of some fishes differ in function from the dull hues of their more modest congeners. But since the most highly colored of these creatures vie with birds and butterflies in vividness of coloration, it becomes necessary to revise all hypotheses which postulate conspicuousness.

It is interesting to find that mimicry among insects may be explained much more consistently than is otherwise possible upon the assumption that even the colors of the most gaudy tend to reduce their visibility. Every authenticated fact adduced by the supporters of the mimicry hypotheses may be accounted for, and most, if not all of the criticism levelled against them may be met, if the matter be set forth as follows:

Mimicry has arisen through bionomic pressure applied first by indiscriminate feeders, which have forced upon their accustomed prey color combinations which most effectually conceal it in its normal environment. In addition, by chance, in a few of many thousands of cases in which colors appropriate to the surroundings and habits of their possessors have been evolved, patterns have appeared, sufficiently like one another to deceive enemies that exercise discrimination in their choice of food. From this point onward the evolution of resemblance has proceeded according to accepted formulae, without conspicuousness being involved in the process.

We may assume that the *Pieridae* and *Heliconidae*, for example, are usually distinctly different in habit, and that the coloration of typical members of each family is a combination of hues well suited upon the average to render them inconspicuous in such places as they frequent. If this be so, the initial step toward mimicry might be any one of many variations in nutrition or reproduction which would lead representatives of the first family to live after the manner of the second. Reason has already been given for supposing that convergence in color would accompany convergence in habit.

Whatever may eventually prove to be the case with the problem of mimetic resemblance, the observations presented in this abstract embody a great mass of fact whose theoretical significance is obvious. It undermines many speculative explanations of animal coloration in terms of natural selection, but, being itself consistent with the Darwinian hypothesis, it replaces them by something which may not be lightly dismissed from consideration. It emphasizes the common occurrence among animals of attributes of apparent advantage to them, and forces the issue between natural selection and the inheritance of acquired characters as the immediate cause of adaptation.

NOTICES OF BIOGRAPHICAL MEMOIRS

The following biographical memoirs have been published by the Academy since the last notices of such memoirs appeared in the November, 1915, number of the Proceedings.

George William Hill (1838–1914). By Ernest W. Brown. *Biographical Memoirs of the National Academy*, **8**, pp. 275–309.

This Memoir discusses the life-work of George W. Hill along the following outline: Boyhood, First papers; Influence of Delaunay and Hansen, Comet of 1858, Elements of Venus, the Years 1872–1875; the Great Decade 1875–1885, Hill's Mental Development, Astronomy before Hill, the Two Great Memoirs of 1877, Influence of Euler, the Periodic Orbit, Stability, Infinite Determinants, Relation to J. C. Adams' Work, Theories of Jupiter and Saturn, Estimate by F. R. Moulton; Residence in Washington, Application of the Methods of Delaunay, Hansen, de Pontécoulant, and Gauss; Concluding years, Hill's Characteristics, Estimates by Poincaré, R. S. Woodward, A. S. Flint, H. B. Hedrick, and H. Jacoby; Hill's Scientific Honors; Bibliography.

Theodore Nicholas Gill (1837–1914.) By Wiliam Healey Dall. *Biographical Memoirs of the National Academy*, **8**, pp. 313–343.

This Memoir recounts the life-work of Theodore N. Gill: Boyhood, Report on the Fishes of New York, Trips to the Antilles and Newfoundland; Connections with the Smithsonian Institution, the Library of Congress, and the U. S. Fish Commission; Editor of the *Osprey* Work on Mollusca; Estimates by the Commissioner of Fisheries, by the Director of the National Museum; Associations with George Washington University; Personal Characteristics; Bibliography.

NATIONAL RESEARCH COUNCIL

REPORT OF THE SECOND MEETING OF THE COUNCIL

The second meeting of the Research Council was held on November 13 and 14, 1916, in Cambridge, in connection with the Autumn Meeting of the National Academy of Sciences.

No formal actions were taken.

Dr. W. H. Welch, President of the Academy, reviewed the organization and earlier work of the Council; and Dr. G. E. Hale, chairman of the Council, reported on the recent activities of its committees, and discussed broadly the general purposes of the movement and the plans for the immediate future. He described in some detail the studies which he has recently made in England and France of the organizations of scientific men in those countries for the purpose of assisting their governments.

Addresses were made by members of the Council as follows:

Dr. S. W. Stratton, Director of the National Bureau of Standards, Washington. Target practice in the Navy and some of the research problems involved; illustrated with moving pictures.

Lieut. Col. George O. Squier, Chief of Aviation, U. S. Army. Scientific research for national defense, as illustrated by the problems of aviation.

Dr. Arthur A. Noyes, Massachusetts Institute of Technology. The nitrogen problem in war and in agriculture.

Various phases of the work of the Council were then discussed by several of its members.

Cary T. Hutchinson, *Secretary.*

REPORT OF MEETINGS OF THE EXECUTIVE COMMITTEE

The third meeting of the Executive Committee of the Research Council was held in New York City on October 16, 1916. Messrs. Carty, Conklin, Dunn, Hale, Pearl, Pupin, Stratton, and the Secretary were present.

The following resolutions adopted by the American Philosophical Society on October 6, 1916, were presented to the Committee:

Whereas as effort is being made to bring into co-operation existing governmental, educational, industrial, and other research organizations with the object of encouraging the investigation of natural phenomena, the application of scientific principles in American industries, the employment of science in the national defense, and such other objects as will promote the national welfare; and

Whereas, these objects are among those for which the American Philosophical Society exists.

Now, therefore, be it resolved that the American Philosophical Society hereby registers its approval of the co-ordination and federation of the research agencies of the country and expresses its willingness to join with and assist the National Research Council, organized by the National Academy of Sciences to accomplish the above federation.

A letter was presented from the National Canners' Association asking the co-operation of the Research Council in the conduct of investigations on toxic elements in canned foods: Raymond Pearl, V. C. Vaughan, and C. L. Alsberg were appointed a committee to confer with representatives of the Association in regard to the matter.

The matter of the ice patrol of the North Atlantic was presented by Dr. S. W. Stratton, and it was voted that a committee be appointed by the Council to co-operate with the government Committee on Ocean Surveys and Ice Patrol.

The question of agricultural research in this country was presented by Dr. Raymond Pearl and discussed by the committee. It was voted that Dr. Pearl and Dr. Hale be authorized to confer with the Secretary of Agriculture to ascertain from him how best the Research Council can co-operate with his Department and who would best represent his Department in the work of the Research Council.

Regarding the procedure of the committees, it was voted that the committees of the Council appointed for carrying out the various branches of the work first decide upon a general plan of procedure and submit this plan to the Executive Committee for its approval. The committees shall then proceed to carry on in detail the general plan as approved by the Executive Committee; but they shall not take any action connected with legislation or with questions of public policy without first consulting the Executive Committee. They shall make reports of progress to the Council at its annual meeting and to the Executive Committee at such times as it may direct.

The fourth meeting of the Executive Committee was held on October 21, 1916, in New York City. Messrs. Carty, Dunn, Hale, Noyes, and the Secretary were present.

A committee on the Utilization of Industrial Research Facilities for the National Defense was formed, and J. J. Carty was appointed chairman.

The rest of the meeting was devoted to a consideration of a report drafted by Dr. G. E. Hale to be presented to the President of the Academy on the work of the Research Council and to be transmitted by him to the President of the United States.

The fifth meeting of the Executive Committee was held on November 4, 1916, in New York City. Messrs. Carty, Dunn, Hale, Noyes, and the Secretary were present. Dr. Hale reported that under the authorization previously given by the Executive Committee he had proceeded with the organization of the Military Section of the Research Council. As members of that Section there have been appointed by the President of the United States three members from the Army, namely, Brigadier-General William Crozier, Chief of Ordnance; Brigadier General W. C. Gorgas, Surgeon General; and Lieut. Col. G. O. Squier, Chief of Aviation, and three representatives of the Navy, namely, Rear-Admiral D. W. Taylor, Chief Constructor; Rear-Admiral R. S. Griffin, Engineer-in-Chief; and Dr. J. D. Gatewood, Medical Director.

It was voted to appoint a committee consisting of V. C. Vaughan (chairman), C. B. Davenport, H. H. Donaldson, W. H. Holmes, and Raymond Pearl to consider the organization of research in anthropological and sociological sciences, and to report to the Executive Committee in regard to this matter.

A report was presented from the Chemistry Committee in regard to appointment of sub-committees. It was voted that such committees be appointed by the Research Council on nomination of the Chemistry Committee.

The sixth meeting of the Executive Committee was held on November 15, 1916, in Boston. Messrs. Conklin, Hale, Noyes, Pearl, Stratton, Vaughan, and Welch were present.

A report was presented from the Committee on Chemistry (M. T. Bogert, chairman) recommending the organization of a number of sub-committees. It was voted to defer action on these committees until the relation of certain of them to other branches of science could be properly considered.

The Committee then took up the consideration of the formation of general committees representing various branches of science. It was voted to organize the following general science committees (in addition to that on Chemistry previously established): Mathematics. Astronomy. Physics. Geology and Paleontology. Geography. Botany. Zoology and Animal Morphology. Physiology. Medicine. Hygiene. Agriculture. Psychology. Anthropology.

Cary T. Hutchinson, *Secretary.*

NATIONAL RESEARCH COUNCIL

SCIENTIFIC RESEARCH FOR NATIONAL DEFENSE AS ILLUSTRATED BY THE PROBLEMS OF AERONAUTICS

Abstract from an Address of Lieut. Colonel George O. Squier, U. S. A.

The following are some present problems connected with the development of Military Aviation and Aerostation.

1. *Aerodynamics.*—(*a*) Continue the development of the mathematical theory to explain the aerodynamic phenomena recorded in the aerodynamical laboratories, and to forecast further results.

(*b*) Obtain solutions for the speed and direction of flow of air about geometric and aerotechnic forms and develop experimental means to visualize or map the speed and direction of flow.

(*c*) Map the currents of the upper atmosphere which may be of most use in aerial navigation, and evolve simple practical rules for the guidance of pilots.

(*d*) Give fuller explanation of the phenomena of soaring, i.e., airplaning indefinitely without motive power.

(*e*) Develop equations and laws of comparison by which the behavior of large aircraft may be more accurately foretold from tests of models. Apply further the principle of dynamical similarity.

(*f*) Investigate more direct and effective methods of securing a lift or thrust in the air from the consumption of fuel.

(*g*) Complete theory of the air-screw.

2. *Engine problems requiring research.*—(*a*) Fuel. Possibly the most far-reaching problem is fuel. A fuel that will carry more power into an engine per unit volume will be a direct gain.

Attempts have been made to combine alcohol, gasoline, acetylene, picric acid, ether and other hydrocarbons with the above object in view. Questionable results have followed. There has been an increase of power, but nothing so far commercially or practically useful.

This question must be studied with the greatest of care and from a truly research standpoint.

(*b*) Solid fuel. Solid fuels that can be converted into liquid in small quantities just prior to use, are desirable for military aviation. In case of accident from shot or shock, leakage of liquid fuel is a danger. Solid fuel could be carried in quantity with less danger.

(*c*) Engine cooling. The problem of radiation is important. If some substance could be found that would circulate through the cooling system, at higher temperatures than water, it is probable that greater engine efficiencies would result. Oils, salt waters, and other materials have been tried with indifferent success.

(*d*) Liquid fuel pipes. Tubing that will resist vibration (causing rupture) is desired. An oil and gasoline proof rubber tubing is reported as used in Europe. This development is highly important, not only for tubing, but for containers in which to carry liquid fuel. Some sort of fabric and rubber tank that would really resist the action of gasoline, would be of the highest benefit.

A difficulty lies in the fact that the tanks are large (say 20 to 100 gallon capacity). The structural problems would be serious. The tanks now used are large and of metal. Vibration causes much difficulty and leakage.

(*e*) Metal coating. The protecting of the metal parts of an airplane, especially the fittings and cables, is a serious problem. A material is desired that would really prevent dangerous corrosion. Nickel plating over copper is very good, but will not suffice. Rust strikes through very rapidly. Baked enamel is the best coating. It is impossible to apply in many cases.

(*f*) Sound. The question of eliminating the noises involved in the operation of aircraft is one of importance. The peculiar note of the propeller of a Zeppelin can be heard for several miles, and is usually the first warning of its approach at night.

3. *Miscellaneous.*—(*a*) Physiological. Study the physiological and psychological effects of low density air at high altitudes on the performance of pilots.

(*b*) Transparent wing covering for airplanes. A wing covering which would answer the following general requirements would be of great value to military aviation:

Weight not more than 5 ounces per square yard.

It should present reasonably great resistance to flame.

It should be reasonably proof against action of salt water, moist air, extreme dryness, and quick temperature changes.

It should not stretch in any direction. Its ability to retain original form as placed on the airplane is very important.

It should have tensile strength of at least 75 pounds per inch width in any direction.

Its tendency to tear and split because of tack holes through it, or because of bullet holes, should be as small as possible.

(*c*) Development of light alloys for airplane construction. Pure aluminium or aluminium alloys. It is believed that a great deal can be done in this direction. So far no alloy has been developed, except possibly in Germany, which can compare with average Alaskan spruce in its 'specific tenacity.'

(*d*) The structure of gusts. It is believed that this is of sufficient importance to aviation to warrant considerable expense in its study.

Painstaking investigation of the character of eddy formations caused when wind strikes trees, hollows, cliffs, etc., and the character of disturbances created by canyons, swamps, deserts, etc., would be of great value to aviators.

This can be done not only by smoke and toy balloon work in the vicinity of obstructions such as the above, but also by photographic work in wind channels.

A set of simple rules laying down just what the aviator may expect on one side or another of canyons, cities, trees, lakes and swamps would be very helpful in aviation.

(*e*) Radio-apparatus for aircraft. The subject of radio-intercommunication between aircraft in flight, and between aircraft and the earth requires for its solution the highest possible efficiency and reliability combined with minimum weight.

A present tendency is to separate entirely the power plant from the main engine of an aircraft. The generator body in this case has a stream-line figure and a separate small air-screw is provided. Among other methods the oscillion is being tried as the actual source of continuous electromagnetic waves.

(*f*) Bullet proof gasoline tanks. Development of a material with which to line or construct tanks to contain the gasoline in an airplane in which a bullet hole will quickly close, entirely or partly at least. This would enable many a flier to get back to his own lines after having been fired upon.

(*g*) Development of a fabric as good as, or better than, Irish linen, for the covering of airplanes. There has not been manufactured in this country a fabric suitable for use in covering airplanes.

The fabric should answer all requirements laid down under *transparent wing covering*, and be, in addition, such as to shrink the proper amount without harm when cellulose solution is applied.

It is possible that long fiber cotton might be developed that would answer the purpose.

We must become independent in all lines affecting our military aviation. Today we depend entirely upon Ireland and England for our linen, and the supply is becoming very low in this country.

(*h*) Aviator's clothing. Much is still to be done in devising non-inflammable and protective clothing for aviators. This question is intimately connected with personal armor and safety in case of fall.

(*i*) Ground-speed indicator. An instrument which would measure the actual speed of an aircraft over the ground would be useful in the operation of military machines.

4. *Physics of the Air.*—A number of physical properties of air, important in the problems of avaiation, were also discussed.

RESEARCH GRANTS FROM THE TRUST FUNDS OF THE ACADEMY

Since the last report, these PROCEEDINGS **2**, 307, the following grants for the promotion of research have been made from the Trust Funds of the Academy.

GRANTS FROM THE BACHE FUND

No. 196, H. N. NORRIS, $100. For assistance in making numerous preliminary sketches and drawings, involved in the plotting from serial sections, the immediate work to be carried on with Cryptobranchus material. An investigation of the cranial nerves of the Amphibia, involving first, an analysis of the nerve components of representative and typical Modela, on the plan followed in the writer's papers on the cranial nerves of Amphinnea and Siren; second, a review of the nerve component in the Anura. Finally a monograph on the cranial nerves of the Amphibia as a whole.

No. 197, G. P. BAXTER, Harvard University, $300. For the purchase of platinum and quartz apparatus, instruments of precision, special glass apparatus and glass blowing, and special chemicals for the determination of atomic weights; namely, of arsenic and iodine, by iodimetric comparison of arsenic trioxide with iodine and iodine pentoxide, and of various metals by electrolysis of weighed amounts of salts, and by coulometric comparison.

No. 198, L. T. MORE, University of Cincinnati, $500. For the purchase of a high voltage (200,000 volts), large current (10 kilowatt) generator of the transformer synchronous commutator type, to examine the region of the radiation spectrum between the hardest known X-rays and the X-rays of radium and the characteristic K radiation and its absorption by various metals.

No. 199, F. P. REAGAN, $100. For an intensive study of haematopolsis in teleost embryos involving continual observation.

No. 200, H. W. NORRIS, Grinnell College, $250. For an investigation of the cranial nerve components of the common dogfish, squales, acanthias; an analysis of the cranial ganglia and the peripheral distribution of the component fibers.

No. 201, A. F. SHULL, $400. To discover the causes of the changes in the life cycles of rotifers, aphids, and thrips, with special reference to the determination of sex; also related cyclical phenomena in the aphids, primarily the sporadic production of wings.

GRANT FROM THE J. LAWRENCE SMITH FUND

No. 7, George Perkins Merrill, U. S. National Museum, $500. To further aid his studies of rare meteorites.

GRANT FROM THE MARSH FUND

John M. Clark, State Hall, Albany, $400. For a study of mutualism, symbiosis, and dependent life among animals of geologic time.

REPORT OF THE AUTUMN MEETING

Prepared by the Home Secretary

The Autumn Meeting of the Academy was held in the buildings of the Massachusetts Institute of Technology at Cambridge, on November 13, 14, 15, 1916.

Seventy members were present as follows: Messrs. C. G. Abbot, H. L. Abbot, Barus, Baxter, Becker, Boas, Bogert, Boltwood, Bumstead, Cannon, Castle, Cattell, Chittenden, W. B. Clark, J. M. Clarke, Comstock, Conklin, Crafts, Crew, Cross, Davenport, Davis, Day, Farlow, Fewkes, Folin, Goodale, Hale, E. H. Hall, Harper, Harrison, Hayford, Holmes, Howard, Howell, Iddings, Jackson, Leuschner, Lewis, Lindgren, Lusk, Mall, Mark, Mayer, Meltzer, Mendel, Michelson, Morley, E. S. Morse, A. A. Noyes, H. F. Osborn, T. B. Osborne, Parker, Pearl, Pickering, Prudden, Reid, Richards, Rosa, Scott, Story, Thomson, Van Hise, Walcott, Webster, Welch, Wheeler, D. White, Wilson, R. W. Wood.

BUSINESS SESSIONS

The Home Secretary announced the following deaths since the last Annual Meeting of the Academy: Cleveland Abbe, elected 1879, died October 28, 1916; Josiah Royce, elected 1906, died September 14, 1916; also Sir William Ramsay, Foreign Associate, elected 1904, died July 24, 1916.

The Home Secretary announced that the following publications had been issued since the Annual Meeting: Proceedings, vol. 2, numbers 5–11; *Memoirs*, volume 14, first memoir, by George P. Merrill, consisting of a Report on Researches on the Chemical and Mineralogical Composition of Meteorites, with Especial Reference to Their Minor Constituents; Biographical Memoirs of George William Hill by Ernest W. Brown, and of Theodore N. Gill by William H. Dall.

Under the rules of the Academy the following members of the Editorial Board of the Proceedings retire on December 1, 1916: W. B. Cannon, J. McK. Cattell, J. M. Coulter, S. Flexner, R. G. Harrison. The Home Secretary announced that the following members have been appointed by the Council to serve in their places until December 1, 1919; H. H. Donaldson, R. A. Harper, Graham Lusk, A. G. Mayer, A. A. Noyes. Raymond Pearl was appointed to succeed A. A. Noyes as Chairman of the Editorial Board.

The Home Secretary also announced that an Auditing Committee has been appointed consisting of W. H. Dall, chairman, A. L. Day, and F. W. Clarke;

and that a Committee on the Organization of the Scientific Resources of the Country for National Service was appointed by the Council as follows: G. E. Hale, chairman, Simon Flexner, E. G. Conklin, R. A. Millikan, A. A. Noyes. This last Committee presented a detailed report recommending the organization of the National Research Council.

The Home Secretary also announced that a Committee on Nitrate Supply was appointed at the request of the Secretary of War to present to the War Department "a report containing such information as will aid the department, to the greatest degree which the Academy finds practicable, in reaching a conclusion as to the best method to be followed in the manufacture of nitric acid by a process not involving dependence upon a foreign source of supply, the report to take into consideration the legislation contained in Section 124 of the National Defense Act approved June 3, 1916." The following men were appointed members of this Committee by the President of the Academy: Arthur A. Noyes, chairman, Leo H. Baekeland,* Gano Dunn, Charles H. Herty,* Warren K. Lewis,* Michael I. Pupin, Theodore W. Richards, Elihu Thomson, Willis R. Whitney.* Those against whose name an asterisk is placed were named by the President of the American Chemical Society.

The President announced that he had forwarded a preliminary report on the National Research Council to the President of the United States.

The following new members were presented to the Academy: M. T. Bogert, L. O. Howard, A. G. Mayer, Raymond Pearl.

The Treasurer reported that notice had been received from the attorneys for Mr. W. W. Farnam, the executor of the estate of Othniel C. Marsh, that he had filed in the New Haven probate court a supplemental account as executor which showed a balance of interest due the Academy of $6,921,84.

The resolution in regard to a section of engineering which was proposed by the Council and adopted at the Business Session of the Academy on April 17, 1916, was presented, and it was voted that a copy of the resolution be sent to each member of the Academy. The resolution is as follows:

> Voted that a section of engineering be developed which shall include men who have made original contributions to the science or art of engineering; that to this end the present section of physics and engineering be designated the section of physics, and that the Council, under the authority granted by section 4, article 4, of the Constitution, nominate to the Academy, after inviting suggestions from the members of the Academy, two or three engineers each year until such time as it shall seem advisable to establish a separate section of engineering, any engineers elected as the result of such nominations being in the meantime assigned to that one of the existing sections to which their work is most closely related.

The following amendments to the Constitution reported from the Council and returned to the Academy from the Committee of the Whole with favorable recommendation were considered:

> That article 3, section I, line 3, of the Constitution be amended by substituting the word "fourth" for "third" in the phrase specifying that the Annual meeting shall begin "on the third Monday in April."
>
> That the last sentence of section 5, article 4, of the Constitution be stricken out.

The first of these amendments was rejected and the second one was adopted.

A new amendment to the Constitution was offered providing that article III, section I, in regard to the meetings of the Academy be amended so as to read as follows:

> Article III, section 1. The Academy shall hold one stated meeting in each year, called the annual meeting, in the city of Washington and another, called the autumn meeting, both to be held at such place and time as the Council shall determine, provided that the annual meeting shall be held in April.

This proposed amendment was referred to the Council.

Mr. C. R. Van Hise presented the report of the Committee on the Panama Canal Slides. It was voted that the printing of the final report be referred to the Council with power.

It was voted that the Home Secretary be directed to send copies of the minutes of each Business Session of the Academy to all members of the Academy for approval and correction.

It was voted that papers by non-members appearing in the PROCEEDINGS be announced as 'communicated by' some member.

It was voted on recommendation of the Committee on the J. Lawrence Smith Fund that a grant of $500 be made from that fund to Mr. George Perkins Merrill to further aid his studies of rare meteorites.

It was voted on recommendation of the Committee on the Marsh Fund that a grant of $400 be made from that fund to Mr. John M. Clark for a study of mutualism, symbiosis, and dependent life among animals of geologic time.

SCIENTIFIC SESSIONS

Two public lectures were given on November 13, by President W. H. Welch on The Formation of the National Research Council at the Request of the President of the United States; and by S. W. Stratton, Director of the National Bureau of Standards, Washington, on Target Practice in the Navy and Some of the Research Problems Involved, illustrated with moving pictures. There followed a public Scientific Exhibit (see list of exhibits, below).

At a joint session of the Academy and of the National Research Council on November 14 three public lectures were given by George E. Hale, Chairman of the National Research Council, on The Work of the National Research Council and on Recent Observations of Organized Science in England and France; by Lieut. Col. George O. Squier, Chief of Aviation, U. S. Army, on Scientific Research for National Defense, as illustrated by the Problems of Aviation; and by Arthur A. Noyes, Massachusetts Institute of Technology, on The Nitrogen Problem in War and in Agriculture.

Two public scientific sessions were held on November 13 and 14 at which the following papers were presented (an asterisk denotes presentation only by title):

Raymond Pearl, Maine Agricultural Experiment Station: Some effects of the continued administration of alcohol to the domestic fowl, with special reference to the progeny.

Alfred G. Mayer, Marine Laboratory, Carnegie Institution: Further studies of nerve conduction.

E. G. Conklin, Princeton University: The share of egg and sperm in heredity.

Lafayette B. Mendel and S. E. Judson, Yale University: Some inter-relations between diet, growth, and the chemical composition of the body.

Charles P. Olivier, University of Virginia (read by E. W. Morley): The meteor system of Pons-Winnecke's comet. 129 parabolic orbits of meteor streams.

Alessandro Fabbri (introduced by A. G. Mayer): Micro-cinematographs of marine and freshwater organisms.

A. G. Webster, Clark University:* Practical tests of a new phonotrope.

Edward S. Morse, Salem, Mass:* Protoconch of Solemya.

Jacques Loeb, Rockefeller Institute:* Diffusion and secretion.

Henry L. Abbot, Cambridge, Mass:* Hydrology of the Isthmus of Panama.

John M. Clark, State Museum, Albany:*The Strand and the Undertow.

W. Lindgren, Mass. Inst. of Tech:, *Notes on the deposition of quartz, chalcedony, and opal.

W. M. Davis, Harvard University:*Sublacustrine glacial erosion in Montana.

H. F. Osborn, American Museum of Natural History: *The tyrant (Tyrannosaurus) and the ostrich (Struthiomimus) dinosaur. The relation of Newton's and Darwin's laws to the fundamental biologic law.

Edwin H. Hall, Harvard University: Electric conduction in metals.

Edward B. Rosa, National Bureau of Standards: The silver voltameter as an international standard.

R. W. Wood, Johns Hopkins University: One-dimensional gases and the reflection of molecules. Series in resonance spectra.

Elihu Thomson, Swampscott, Mass.: Inferences concerning auroras.

A. A. Michelson, University of Chicago: Report of progress in experiments for measuring the rigidity of the earth. The laws of elastico-viscous flow.

C. G. Abbot, Smithsonian Institution: On the preservation of knowedge.

Franz Boas, Columbia University: Further evidence regarding the instability of human types.

Ross G. Harrison, Yale University: Transplantation of limbs.

Chas. B. Davenport, Station for Experimental Evolution, Carnegie Institution: Heredity of stature.

F. R. Moulton, Chicago University: *On analytic functions of infinitely many variables.

Henry S. White, Vassar College, F. N. Cole and Louise D. Cummings:* Enumeration of all triad systems on fifteen elements.

William E. Story, Clark University: *Some variable 3-term scales of relation.

Edw. L. Nichols, Cornell University: *New data on the phosphorescence of certain sulphides.

G. P. Baxter and H. W. Starkweather, Harvard University: *A revision of the atomic weight of tin.

T. W. Richards and H. S. Davis, Harvard University: *Improvements in calorimetric combustion.

T. W. Richards and C. Wadsworth, 3d, Harvard University: *Further study of the atomic weight of lead of radioactive origin.

Gilbert N. Lewis, University of California. *Chemical affinity.

Wm. Trelease, University of Illinois: *The American oaks.

H. S. Jennings, Johns Hopkins University: *The numerical results of diverse systems of breeding, with relation to two pairs of factors, linked or independent.

W. R. Miles, Carnegie Nutrition Laboratory, Boston (introduced by F. G. Benedict): *Some psycho-physiological processes as affected by alcohol.

Walter B. Cannon, Harvard University: *Oscillatory variations in the contraction of rhythmically stimulated muscles.

Wm. H. Dall, Smithsonian Institution: *On some anomalies in the distribution of Pacific coast mollusca.

G. H. Parker, Harvard University: *The responses of hydroids to gravity.

W. M. Wheeler, Harvard University: *The phylogenetic development of subapterous and apterous castes in the Formicidae.

W. J. Crozier, Resident Naturalist, Bermuda Biological Station (introduced by E. L. Mark): *On cell penetration by acids: the chloracetic acids. *On the immunity coloration of some nudibranches.

Hovey Jordan, Bermuda Biological Station (introduced by E. L. Mark): *The rheotropism of the marine fish known as "hamlet" or "grouper" (*Epenephalus striatus*).

A. C. Walton, Harvard University (introduced by E. L. Mark): *The occurrence of *Ascaris triquetra*, Schrank, in dogs.

The following is the list of scientific exhibits prepared for the meeting of the Academy.

H. S. White, Vassar College: Graphic representations of triad systems.

Miss A. J. Cannon, Harvard College Observatory: Stellar spectra.

Leon Campbell, Harvard College Observatory: Visual observation of variable stars.

Miss H. S. Leavitt, Harvard College Observatory: Photographic magnitudes.

Solon I. Bailey, Harvard College Observatory: Variable stars in clusters.

A. G. Webster, Clark University: Acoustical measuring apparatus: standard phone, phonometer and phonotrope. Application of a drop chronograph for use in ballistics.

Charles A. Kraus, Clark University: A new vacuum pump and a new thermostat.

H. P. Hollnagel, Mass. Inst. Technology: Methods of isolating the infra-red region of the spectrum.

Alexander McAdie, Blue Hill Observatory: Cloud studies, wind structure and snow flakes.

Ellsworth Huntington, Milton, Mass.: The relation between solar changes and barometric gradients. Optimum temperature for the human race.

Robert DeC. Ward, Harvard University: Weather types of the United States, illustrated by composite weather maps and instrumental records.

R. A. Daly and H. Clark, Harvard University: Design for a deep-sea thermograph.

Frank Hall, Mass. Inst. Technology: A thermophone arranged so that direct comparison may be made with a magnetic receiver.

A. H. Gill, Mass. Inst. Technology: Tests of lubricating mineral oils.

F. G. Keyes and J. B. Dickson, Mass. Inst. Technology: Continuous flow calorimeter for measuring heats of reaction in solution.

C. L. Burdick, Mass. Inst. Technology: Determination of crystal structure by X rays.

R. E. Wilson, Mass. Inst. Technology: Apparatus for maintaining pressures of one-tenth micron or less, and the investigation of the mechanism of chemical reactions.

Henry Fay, Mass. Inst. Technology: Erosion of large guns.

Albert Sauveur, Mass. Inst. Technology and Harvard University: 1. Photomicrographic apparatus (original). 2. Photomicrographs of metals and alloys; charts and diagrams; specimens.

H. O. Hofman, Mass. Inst. Technology: 1. Jenny flotation machine. 2. A laboratory revolving horizontal roasting furnace heated electrically and rotated in the same way.

A. E. Kennelly and Associates, Mass. Inst. Technology: Researches in electrical engineering.

Alexander Klemin, Mass. Inst. Technology: Aeroplane models used in wind tunnel.

W. Lindgren and W. L. Whitehead, Mass. Inst. Technology: Photomicrographs of silver ores from Chile and Tintic.

C. H. Warren, Mass. Inst. Technology: 1. A graduated sphere for crystallographic work. 2. Photographs of spherulites in polarized light.

Charles Palache, Harvard University: Models showing gnomonic crystal projection.

Wallace W. Atwood, Harvard University: The former glaciers of the San Juan mountains of Colorado. The physiographic stages in the evolution of the San Juan mountains of Colorado.

J. B. Woodworth, Harvard University: Glacial map of Cape Cod and adjacent islands. A glypolith from Nantucket.

Laurence La Forge, U. S. Geological Survey: Recent topographic and geologic maps of New England and other parts of the United States.

U. S. Coast and Geodetic Survey: Recent charts of the coasts of the United States.

John M. Clarke, State Museum, Albany, N. Y.: Portfolio of paleontological plates, in press. Plates of "Wild flowers of New York," in press. Geological map of Ogdensburg, N. Y., and vicinity, in press.

H. W. Shimer, Mass. Inst. Technology: Evolution of some brachiopods.

Richard M. Field, Harvard University: Ordovician rocks and faunas of central Pennsylvania.

W. B. Scott, Princeton University: Proofs of plates for forthcoming report on paleontology of Patagonia.

W. J. V. Osterhout, Harvard University: Pigments produced by the oxidation of a colorless plant chromogen.

Charles W. Johnson, Boston Society of Natural History: Distribution and variation of Helix hortensis.

Joseph A. Cushman, Boston Society of Natural History: Some fossil and recent foraminifera.

Alfred G. Mayer, Marine Laboratory, Carnegie Institution: Yacht and laboratory of the Carnegie Institution at Tortugas, Florida.

Hubert Lyman Clark, Museum of Comparative Zoölogy, Harvard University: Echinoderms from Torres straits, Australia, with colored drawings and lithographs.

G. H. Parker, Harvard University: The suction efficiency of a Californian sea-anemone.

W. T. Bovie, Harvard University: Visible effects of Schumann rays on protoplasm. Effects of radium rays on permeability of protoplasm.

C. T. Brues, Bussey Institution, Harvard University: Specimens and charts illustrating insects as carriers of infantile paralysis.

W. E. Castle, Bussey Institution, Harvard University: Examples of Mendelian inheritance, reversion and variety formation in rats and guinea-pigs.

Francis G. Benedict, Nutrition Laboratory, Carnegie Institution: Respiration apparatus for animals.

T. B. Osborne, Connecticut Agricultural Experiment Station, and L. B. Mendel, Sheffield Scientific School, Yale University: Photographs representing the growth of chickens fed with definite mixtures of food stuffs under laboratory conditions which have heretofore not led to success.

I. Chandler Walker, Medical Service, Peter Bent Brigham Hospital: Proteid sensitization in relation to bronchial asthma.

H. S. Wells, Medical Service, Peter Bent Brigham Hospital: Electrocardiography, or the application of the string galvanometer to the study of cardiac cases.

Albert A. Ghoreyeb, Cancer Commission, Harvard University: Metal casts of heart and kidney blood vessels.

S. B. Wolbach, Harvard Medical School: Studies in Rocky mountain spotted fever.

Harvey Cushing and W. M. Boothby, Peter Bent Brigham Hospital: Apparatus of routine methods for clinical metabolism determinations.

E. W. Goodpasture, Peter Bent Brigham Hospital: An anatomical study of senescence, with especial reference to tumors.

E. E. Tyzzer and C. C. Little, Harvard Medical School: The inheritance of susceptibility to transplanted tumor.

W. Duane, Harvard Medical School: The technique of the preparation of radium for therapeutic purposes.

G. C. Whipple, School for Health Officers, of Harvard University and Mass. Inst. Technology: Charts showing organization and membership of the School.

W. T. Sedgwick, Mass Inst. Technology: 1. Diagrams and tables illustrating the investigations of Professor Weston and Mr. Turner upon "The digestion of sewage effluents in an otherwise unpolluted stream." 2. An investigation of the behavior of certain species of bacteria in various materials between zero Centigrade and zero Fahrenheit. 3. A field investigation of the sanitary environment of a suburban population.

S. C. Prescott, Mass. Inst. Technology: Diseases of the banana in Central America and their control.

Alfred M. Tozzer, Peabody Museum, Harvard University: Race-mixture in Hawaii.

Charles Peabody, Peabody Museum, Harvard University: Prehistoric specimens from caves of France and Palestine.

E. A. Hooton, Peabody Museum, Harvard University: Casts and reconstruction of ancient man skull of apes.

S. J. Guernsey, Peabody Museum, Harvard University: Cave explorations in northeastern Arizona.

Oric Bates, Peabody Museum, Harvard University: Prehistoric Libyan remains.

INDEX

Abbot, C. G., and Aldrich, L. B. The Pyranometer: An Instrument for Measing Sky Radiation, 333.
Absorption of light in space (Shapley), 12.
Absorption of soft X-rays (Miller), 441.
Acids, excreted by roots (Haas), 561.
Acids and bases, equilibrium in sea water (Henderson and Cohn), 618.
Activity of ions of hydrochloric acid (Ellis), 83.
Adams, F. D., and Dick, W. J. The Extension of the Montana Phosphate Deposits Northward into Canada, 62.
Adams, W. S. Investigations in Stellar Spectroscopy. I. A Quantitative Method of Classifying Stellar Spectra, 143.
——. Investigations in Stellar Spectroscopy. II. A Spectroscopic Method of Determining Stellar Parallaxes, 147.
——. Investigations in Stellar Spectroscopy. II. Application of a Spectroscopic Method of Determining Stellar Distances to Stars of Measured Parallax, 152.
——. Investigations in Stellar Spectroscopy. IV. Spectroscopic Evidence for the Existence of Two Classes of M Type Stars, 157.
Adams, W. S., and Shapley, H. The Spectrum of δ Cephei, 136.
Adsorption, its part in nerve-conduction (Mayer), 37.
Aeronautics, problems in (Squier), 740.
Aeroplane in gusts (Wilson), 294.
Aeroplanes, dynamical stability (Hunsaker), 278.
Albedo of planets and satellites (Russell), 74.
Albinism, human (Jenks), 164.
Alcohol, effect on psycho-physiological processes (Miles), 703.
Alcoholic solutions, ionization in (Keyes and Winninghoff), 342.
Alcoholism, effect of parental on progeny (Pearl), 380, 675.
Aldrich, L. B., see Abbot, C. G., 333.
Algae, fossil, in petroleum shales (Davis), 114.
Algae, marine Pacific (Farlow), 424.
Alkali in production of protein (Falk), 557.
Amalgams, conductivity of (Lewis and Hine), 634.
Ameba feeding, surface-tension theory of (Mast and Root), 188.
American oaks (Trelease), 626.
Ammines, cobalt (Harkins, Hall and Roberts), 598.
Analysis situs (Moore), 270.
Animals and man, method of studying behavior (Yerkes), 631.
Annual Meeting, Report of, 300.
Anomalous dispersion (St. John), 458; (King), 461.
Anthropological Explorations (Hrdlička), 32.
Arthropods, earliest fresh-water (Schuchert) 726.
Art, American aboriginal (Wissler), 224.
Asbestiform Minerals (Taber), 659.
Asymptotic expressions in linear differential equations (Milne), 543.
Atmosphere of sun, minute structure of (Hale and Ellerman), 102.
Atolls, Murray-Agassiz theory (Mayer), 28.
Atolls, origin of Fiji (Davis), 471.
Atomic theory, steric hindrance and free radicals (Lewis), 586.
Atoms, hydrogen-helium structure (Harkins), 216.
Atwood, W. W., and Mather, K. F. Geographic History of the San Juan Mountains since the close of the Mesozoic era, 177.

Banta, A. M. Sex Intergrades in a Species of Crustacea, 578.
Barrell, J. Dominantly Fluviatile Origin under Seasonal Rainfall of the Old Red Sandstone, 496.
——. The influence of Silurian-Devonian Climates on the Rise of Air-Breathing Vertebrates, 499.
Barus, C. Interferometer Methods Based on the Cleavage of a Diffracted Ray, 275.
——. Channeled Grating Spectra. Obtained in Successive Diffractions, 378.
——. The Interferences of Spectra Both Reversed and Inverted, 576.
——. Path Differences within which Spectrum Interferences are Observable, 609
——. Non-Reversed Spectra of Restricted Coincidence, 614.
Bauer, L. A. Concomitant Changes in Terrestrial Magnetism and Solar Radiation, 24.
Baxter, G. P., and Starkweather, H. W. A Revision of the Atomic Weight of Tin, 718.
Becker, G. F. A Possible Origin for Some Spiral Nebulae, 1
Behavior in man and animals, new method of studying (Yerkes), 631.
Behavior of monkeys and apes (Yerkes), 639.
Behavior of sea-anemones (Parker), 450.

BENNETT, A. A. Newton's Method in General Analysis, 592.
BERRY, E. W. Upper Cretaceous Floras of the World, 185.
BERRY, E. W., see Clark, W. B., 181.
Biographical Memoirs, notices of, 737.
Black Hills (S. D.) Pre-Cambrian granite (Paige), 113.
BLACKWELDER, E. The Geologic Rôle of Phosphorus, 490.
BLUMBERG, H. Certain General Properties of Functions, 646.
BOAS, F. New Evidence in Regard to the Instability of Human Types, 713.
BOSS, B. Systematic Motion among Stars of the Helium Type, 214.
BOWMAN, H. H. M. Physiological Studies on Rhizophora, 685.
Breeding, numerical results of diverse systems of (Jennings), 45.
BRIGGS, L. J. A New Method of Measuring the Acceleration of Gravity at Sea, 399.
BROOKS, S. C. New Determinations of Permeability, 569.
BUCKLEY, O. E. An Ionization Manometer, 683.

Cactus deserts of South America (Rose), 73.
Cambrian trilobites (Walcott), 101.
CAMPBELL, D. H. The Archegonium and Sporophyte of *Treubia Insignis* Goebel, 30.
——. Some Problems of the Pacific Floras, 434.
CAMPBELL, W. W., and MOORE, J. H. On the Observed Rotations of a Planetary Nebula, 129.
——. Spectrographic Observations of Relative Motions in the Planetary Nebulae, 566.
Canada, extension of Montana phosphate deposits into (Adams and Dick), 62.
Cancer and crown gall (Smith), 444.
CANNON, W. B. Studies of Ductless Glands by the Electrical Method, 319.
Carbon dioxide, movements in air and ocean (Henderson and Cohn), 618.
CARY, L. R. The Influence of the Marginal Sense Organs on Metabolic Activity in Cassiopea Xamachana Bigelow, 709.
Cassiopea xamachana, nerve-conduction (Mayer), 37, 721.
Cassiopea xamachama, influence of sense organs on Metabolism (Cary), 709.
CASTLE, W. E. Size Inheritance in Guinea-Pig Crosses, 252.
Cattle, American Jersey, inbreeding of (Pearl and Patterson), 58.
Cepheid, short period, with variable spectrum (Shapley), 132.
Cepheid variables, δ Cepheî (Adams and Shapley), 136.
Cepheid variables, eight spectra (Shapley), 208.
Chicks, effect of feeding pituitary body and corpus luteum (Pearl), 50.
Child, development among primitive people (Hrdlička), 32.
Chondriosomes, distribution to spermatozoa (Wilson), 321.
Circle geometry, Feuerbach's theorem (Morley), 171.
CLARK, W. B., BERRY, E. W., and GARDNER, J. A. The Age of the Middle Atlantic Coast Upper Crustaceous Deposits, 181.
CLAUSEN, R. E., and GOODSPEED, T. H. Hereditary Reaction-System Relations —An Extension of Mendelian Concepts, 240.
Clay from near City of Mexico (Hilgard), 8
Cobaltammines (Harkins, Hall, and Roberts), 598.
COBLE, A. B. Point Sets and Allied Cremona Groups (Part II), 244.
——. A Proof of White's Porism, 530.
——. Point Sets and Cremona Groups (Part III), 575.
COHN, E. J., see Henderson, L. J., 618.
Color of nebulae (Seares), 553.
Coloration, immunity. (Crozier) 672; (Longley) 733.
Colors of stars, method for determining (Seares), 521.
Colors of stars in globular systems (Shapley), 525.
Committee on Panama Canal, preliminary report of, 193.
Conductivity of amalgams (Lewis and Hine), 634.
CONKLIN, E. G. Effects of Centrifugal Force on the Polarity of the Eggs of Crepidula, 87.
Coral atolls, Murray-Agassiz theory (Mayer), 28.
Coral reefs, extinguished and resurgent (Davis), 466.
Coral reefs, test of subsidence theory (Daly), 664.
Coral seas, clift islands (Davis), 284.
Corals, ecology of Floridian and Bahaman (Vaughan), 95.
Corpus luteum, fed to chicks (Pearl), 50.
Creedite (Larsen and Wells), 360.
Cremona groups (Coble), 244, 575.
Crepidula, polarity of egg of (Conklin), 87.
CROZIER, W. J. On the Immunity Coloration of Some Nudibranchs, 672.
Crown gall and cancer (Smith), 444.
Cubic curves, seven points on (White), 337; (Coble), 530.
Culture of American aboriginal (Wissler), 224.
Culture sequences, Zuñi (Kroeber), 42.

DALL, W. H. On the Distribution of Pacific Invertebrates, 424.
——. On Some Anomalies in Geographic Distribution of Pacific Coast Mollusca, 700.

DALY, R. A. A New Test of the Subsidence Theory of of Coral Reefs, 664.

DAVIS, C. A. On the Fossil Algae of the Petroleum-yielding Shales of the Green River Formation of Colorado and Utah, 114.

DAVIS, W. M. Clift Islands in the Coral Seas, 284.

——. The Exploration of the Pacific, 391.

——. Extinguished and Resurgent Coral Reefs, 466.

——. The Origin of Certain Fiji Atolls, 471.

DEMPSTER, A. J. The Light Excitation by Slow Positive and Neutral Particles, 374.

Deserts of South America (Rose), 73.

Devonian System (Barrell), 496.

DeVriesian mutation in garden beans (Harris), 317.

Dextrose, effect of morphin on elimination of (Kleiner and Meltzer), 369.

DICK, W. J., see Adams, F. D., 62.

Diet and Growth (Mendel and Judson), 692.

Differential equations, asymptotic expressions (Milne), 543.

Differential equations, infinitely many variables (Hart), 309.

Differential geometry (Wilson and Moore), 273.

Diffusion of electrolytes through membranes (Loeb), 511.

DONALDSON, H. H. The Relation of Myelin to the Loss of Water in the Mammalian Nervous System with Advancing Age, 350.

Ductless glands (Cannon), 319.

Dynamical stability of aeroplanes (Hunsaker), 278.

Dynamics of aeroplane in gusts (Wilson), 294.

Ecology of Floridian and Bahaman shoal-water corals (Vaughan), 95.

Effectors of sea-anemones (Parker), 385.

Egg of Crepidula, polarity of (Conklin), 87.

EISENHART, L. P. Deformations of Transformations of Ribaucour, 173.

Electric field, effect on spectra (Howell), 528.

Electromotive force produced by acceleration (Tolman and Stewart), 189.

Electron theory of metals (Tolman and Stewart), 189.

Electrons exciting light (Dempster), 374.

ELLERMAN, F., see Hale, G. E., 102.

ELLIS, J. H. The Chemical Activity of the Ions of Hydrochloric Acid Determined by Electromotive Force Measurements, 83.

Embryo, effect of removal of pronephros (Howland), 231.

Emission quanta of characteristic X-rays (Webster), 90.

Equations, Newton's method of solution (Fine), 546.

Erythrocytes, localization of (Lamson), 365.

Ethnology and archaeology in the Pacific (Fewkes), 427.

Evolution of vertebrates (Barrell), 499.

Excretion of acids by roots (Haas), 561.

Exploration, recent anthropological (Hrdlička), 32.

Exploration of Pacific (symposium, Davis *et al*), 391.

Explorations at Pecos, N. M. (Kidder), 119.

Explorations in cactus deserts (Rose), 73.

Eye, median, in Trilobites (Ruedemann), 234.

FALK, K. G. The Action of Alkali in the Production of Lipolytically Active Protein, 557.

FARLOW, W. G. The Marine Algae of the Pacific, 424.

FENN, W. O. Salt Antagonism in Gelatine, 534

——. Similarity in the Behavior of Protoplasm and Gelatin, 539.

Feuerbach's theorem extended (Morley), 171.

FEWKES, J. W. The Pacific as a Field for Ethnological and Archaeological Investigation, 427.

FINE, H. B. On Newton's Method of Approximation, 546.

Fire, uses by man (Hough), 123.

Floras, Pacific (Campbell), 434.

Floras, Upper Cretaceous (Berry), 186.

Fossil algae of petroleum shales (Davis), 114.

Fossil reptile (Williston), 650.

Fowl, effect of alcohol on progeny (Pearl), 380, 675.

Fraunhofer lines, repulsion of (St. John), 458; (King), 461.

Free radicals and steric hindrance (Lewis), 586.

Freezing-point of mixtures (Harkins, Hall, and Roberts), 642.

Functional calculus (Bennett), 592.

Functions, general properties of (Blumberg), 646.

Functions, implicit (Hart), 309.

Functions, zeros of integral (Porter), 247, 335.

Funds, research grants, 307, 743.

GARDNER, J. A., see Clark, W. B., 181.

Gearksutite (Larsen and Wells), 360.

Gelatine, salt antagonism in (Fenn), 534.

Gelatine, similarity to protoplasm (Fenn), 539.

Germ-cells, mitoses in (Hegner and Russell), 356.

Glacial slates showing seasonal variations (Sayles), 167.

Glands, ductless (Cannon), 319.

GOLDSCHMIDT, R. A Preliminary Report on Further Experiments in Inheritance and Determination of Sex, 53.
GOODSPEED, T. H., see Clausen, R. E., 240.
Granite, Pre-Cambrian, of Black Hills (Paige), 113.
Gravity observations at sea (Hayford), 394; (Briggs), 399.
GREEN, G. M. On the Linear Dependence of Functions of Several Variables, and Certain Completely Integrable Systems of Partial Differential Equations, 209.
Groups, Cremona (Coble), 244, 575.
Group, transitivity of substitution (Miller), 61.
Growth and Diet (Mendel and Judson), 692.

HAAS, A. R. The Excretion of Acids by Roots, 561.
HALE, G. E., and ELLERMAN, F. The Minute Structure of the Solar Atmosphere, 102.
HALL, R. E., see Harkins, W. D., 598.
——, see Harkins, W. D., 642.
HARDY, G. H., and LITTLEWOOD, J. E. Some Problems of Diophantine Approximation: A Remarkable Trigonometrical Series, 583.
HARKINS, W. D. The Abundance of the Elements in Relation to the Hydrogen-Helium Structure of the Atoms, 216.
HARKINS, W. D., HALL, R. E., and ROBERTS, W. A. The Cobaltammines, 598.
——. The Osmotic Pressure and Lowering of the Freezing-point of Mixtures of Salts with one Another and with Non-electrolytes in Aqueous Solutions, 642.
HARRIS, J. A. Personal Equation and Steadiness of Judgment in the Estimation of the Number of Objects in Moderately Large Samples, 65.
——. De Vriesian Mutation in the Garden Bean, Phaseolus Vulgaris, 317.
HART, W. L. Differential Equations and Implicit Functions in Infinitely Many Variables, 309.
HAYFORD, J. F. The Importance of Gravity Observations at Sea on the Pacific, 394.
HEGNER, R. W., and RUSSELL, C. P. Differential Mitoses in the Germ-Cell Cycle of Dineutes Nigrior, 356.
Helium stars, systematic motion among (Boss), 214.
HENDERSON, L. J. On Volume in Biology, 653.
HENDERSON, L. J., and COHN, E. J. The Equilibrium between Acids and Bases in Sea Water, 618.
Hereditary reaction-system relations (Clausen and Goodspeed), 240.
HILGARD, E. W. A Peculiar Clay from near the City of Mexico, 8.
HINE, T. B., see Lewis, G. N., 634.
HOUGH, W. Man and Metals, 123.
HOWELL, J. T. The Effect of an Electric Field on the Lines of Lithium and Calcium, 528.
HOWLAND, R. B. On the Effect of Removal of the Pronephros of the Amphibian Embryo, 231.
HRDLIČKA, A. Brief Notes on Recent Anthropological Explorations Under the Auspices of the Smithsonian Institution and the U. S. National Museum, 32.
HUBBLE, E. P. Changes in the Form of the Nebula N. G. C. 2261, 230.
HULL, A. W., and RICE, M. The High Frequency Spectrum of Tungsten, 265
Human types, instability of (Boas), 713.
HUNSAKER, J. C. Dynamical Stability of Aeroplanes, 278.
HUNTINGTON, E. V. A Set of Independent Postulates for Cyclic Order, 630.
Hydantoin compounds, synthesis of (Johnson), 69.
Hydrochloric acid, chemical activity of ions (Ellis), 83.
Hydrogen-helium structure of atoms (Harkins), 216.

IDDINGS, J. P. The Petrology of Some South Pacific Islands and its Significance, 413.
IDDINGS, J. P., and MORLEY, E. W. A Contribution to the Petrography of Japan, 452.
——. A Contribution to the Petrography of the Philippine Islands, 531.
Inbreeding in American Jerseys (Pearl and Patterson), 58.
Indian Tribes, terms of relationship and social organization of (Michelson), 297.
Inheritance and determination of sex (Goldschmidt), 53.
Inheritance in fowls as affected by alcohol (Pearl), 675.
Inheritance of glume characteristics (Surface), 478.
Inheritance of size (Castle), 252.
Interference, non-reversed spectra (Barus), 614.
Interference, path differences for observable (Barus), 609.
Interference, reversed and inverted spectra (Barus), 576.
Interferometer methods (Barus), 475.
Intersexes (Banta), 578.
Intersexes in gypsy-moth (Goldschmidt), 53.
Intoxication, effect of parental on progeny (Pearl), 380, 675.
Invariant, integral, of projective geometry (Wilczynski), 248.
Invertebrates, Pacific (Dall), 424.
Ionization manometer (Buckley), 683.
Ionization of salts in alcoholic solvents (Keyes and Winninghoff), 342.

Ions, mobilities in electric fields (Loeb), 345.
Ions of hydrochloric acid, activity of (Ellis), 83.
Islands in the coral seas (Davis), 284.

Japan, petrography of (Iddings and Morley), 450.
Jenks, A. E. The Failure and Revival of the Process of Pigmentation in the Human Skin, 164.
Jennings, H. S. The Numerical Results of Diverse Systems of Breeding, 45.
Jersey cattle, degree of inbreeding (Pearl and Patterson), 58.
Johnson, T. B. Polypeptide-Hydantoins, 69.
Judgment, steadiness of, and personal equation (Harris), 65.
Judson, S. E., see Mendel, L. B., 692.
Jupiter, monochromatic photography of (Wood), 109.

Keyes, F. G., and Winninghoff, W. J. Change of the Ionization of Salts in Alcoholic Solvents with the Concentration, 342.
Kidder, A. V. Archaeological Explorations at Pecos, New Mexico, 119.
King, A. S. An Attempt to Detect the Mutual Influence of Neighboring Lines in Electric Furnace Spectra Showing Anomalous Dispersion, 461.
Kleiner, I. S., and Meltzer, S. J. The Influence of Morphin Upon the Elimination of Intravenously Injected Dextrose in Dogs, 369.
Kroeber, A. L. Zuñi Culture Sequences, 42.

Lamson, P. D. The Processes Taking Place in the Body by Which the Number of Erythrocytes per Unit Volume of Blood is Increased in Acute Experimental Polycythaemia, 365.
Land slides of Panama Canal, 193.
Larsen, E. S., and Wells, R. C. Some Minerals from the Fluorite-Barite Vein near Wagon Wheel Gap, Colorado, 360.
Lead, density of radio- (Richards and Wadsworth), 505, 694.
Lewis, G. N. Steric Hindrance and the Existence of Odd Molecules (Free Radicals), 586.
Lewis, G. N., and Hine, T. B. Electrical Conduction in Dilute Amalgams, 634.
Life, temperature coefficient of (Loeb and Northrop), 456.
Light excitation by slow positive and negative particles (Dempster), 374.
Limestone, sub-marine solubility (Mayer), 28.
Linear dependence (Green), 209.
Linkage in inheritance (Surface), 478.
Lipolytically active protein (Falk), 557.
Littlehales, G. W. In Relation to the Extent of Knowledge Concerning the Oceanography of the Pacific, 419.
Littlewood, J. E., see Hardy, G. H., 583.
Liver, reservoir of erythrocytes (Lamson), 365.
Liverwort, Treubia insignis Goebel (Campbell), 30.
Locomotion of sea-anemones (Parker), 449.
Loeb, J. The Sex of Parthenogenetic Frogs, 313.
——. The Mechanism of Diffusion of Electrolytes through Animal Membranes, 511.
Loeb, J., and Northrop, J. H. Is There a Temperature Coefficient for the Duration of Life? 456.
Loeb, L. B. On the Mobilities of Gas Ions in High Electric Fields, 345.
Longley, W. H. Observation upon Tropical Fishes and Inferences from their Adaptive Coloration, 733.
Lucas' Theorem (Porter), 247, 335.

Magnetism, terrestrial (Bauer), 24.
Magnitudes in star clusters (Shapley), 12, 15, 525.
Man and animals, method of studying behavior (Yerkes), 631.
Man and metals (Hough), 123.
Manometer, ionization (Buckley), 683.
Marvin, C. F. Marine Meteorology and the General Circulation of the Atmosphere, 421.
Mast, S. O., and Root, F. M. Observations on Ameba Feeding on Infusoria, and their Bearing on the Surface-Tension Theory, 188.
Mather, K. F., see Atwood, W. W., 177.
Mating, results of diverse systems of (Jennings), 45.
Mayer, A. G. Sub-Marine Solution of Lime-Stone in Relation to the Murray-Agassiz Theory of Coral Atolls, 28.
——. A Theory of Nerve-Conduction, 37.
——. Further Studies of Nerve Conduction in Cassiopea, 721.
McAdie, A. A New Thermometer Scale, 670.
McClendon, J. F. On the Hydrogen Ion Concentration of Sea Water, and the Physiological Effects of the Ions of Sea Water, 689.
Medals, award of, 307.
Meltzer, S. J., see Kleiner, I. S., 369.
Membranes, diffusions of electrolytes through (Loeb), 511.
Mendel, L. B., and Judson, S. E. Some Interrelations between Diet, Growth, and the Chemical Composition of the Body, 692.
Mendelian breeding, numerical tables (Jennings), 45.
Mendelian concepts, extension of (Clausen and Goodspeed), 240.

Mesa Verde, physiographic study of (Atwood and Mather), 177.
Metals, production of electromotive force by acceleration of (Tolman and Stewart), 189.
Metals and man (Hough), 123.
Meteorology, marine (Marvin), 421.
Meteors (Olivier), 372.
Methylaminomethyl-3, 4-dihydroxyphenylcarbinol, synthesis of $C_3H_4ON_2$ derived from (Rouiller), 464.
Michelson, T. Terms of Relationship and Social Organization, 297.
Middle Atlantic Coast, Upper Cretaceous deposits (Clark, Berry and Gardner), 181.
Miles, W. R. Some Psycho-Physiological Processes as Affected by Alcohol, 703.
Miller, C. D. The Absorption Coefficients of Soft X Rays, 441.
Miller, G. A. Upper Limit of the Degree of Transitivity of a Substitution Group, 61.
Millikan, R. A. Quantum Relations in Photo-Electric Phenomena, 78.
Millikan, R. A., and Souder, W. H. Experimental Evidence for the Essential Identity of the Selective and Normal Photo-Electric Effects, 19.
Milne, W. E. On Certain Asymptotic Expressions in the Theory of Linear Differential Equations, 543.
Minerals from fluorite-barite vein (Larsen and Wells), 360.
Mitoses in germ cell of dineutes nigrior (Hegner and Russell), 356
Mobilities of gas ions (Loeb), 345.
Mollusca, Pacific Coast (Dall), 700.
Monkeys and apes, ideational behavior (Yerkes), 639.
Moore, C. L. E., see Wilson, E. B., 273.
Moore, J. H., see Campbell, W. W., 129.
——., see Campbell, W. W., 566.
Moore, R. L. On the Foundations of Plane Analysis Situs, 270.
Morley, E. W., see Iddings, J. P., 452.
——., see Iddings, J. P., 531.
Morley, F. An Extension of Feuerbach's Theorem, 171.
Morphin and elimination of dextrose (Kleiner and Meltzer), 369.
Mutation, de Vriesian (Harris), 317.
Myelin, and age (Donaldson), 350.

National Research Council, report of organizing committee, 507.
——., report of meetings, 602, 738.
——., report of meetings of executive committee, 605, 738.
——., organization (membership), 607.
——., abstract of address by Lieut.-Colonel G. O. Squier, 740.
Nebula, rotations of a planetary (Campbell and Moore), 129.
Nebula Messier 101, internal motion of (van Maanen), 386.
Nebula N. G. C. 2261, changes of form (Hubble), 230.
Nebula N. G. C. 4594, rotation and radial velocity (Pease), 517.
Nebulae, color of (Seares), 553.
Nebulae, origin of spiral (Becker), 1.
Nebulae, relative motions in planetary (Campbell and Moore), 566.
Nerve-conduction (Mayer), 37., 721.
Nervous system, myelin in (Donaldson), 350.
Nervous transmission in sea-anemones (Parker), 437.
Neuromuscular structure of sea-anemones (Parker and Titus), 339.
Newton's method in general analysis (Bennett), 592.
Newton's method of approximation (Fine), 546.
Nichols, E. L. Note on the Phosphorescence of Uranyl Salts, 328.
Northrop, J. H., see Loeb, J., 456.
Nudibranchs, immunity coloration (Crozier), 672.

Oaks of America (Trelease), 626.
Oceanica, continental fracturing and diastrophism in (Schuchert), 407.
Oceanography of the Pacific (Littlehales), 419.
Olivier, C. P. The Work of the American Meteor Society in 1914 and 1915, 372.
Order, cyclic, postulates for (Huntington), 630.
Organic metabolism, correspondence with chemistry of igneous magmas (Washington), 623.
Origin of Spiral Nebulae (Becker), 1.
Osmotic pressure in mixtures (Harkins, Hall, and Roberts), 642.
Osterhout, W. J. V. The Nature of Mechanical Stimulation, 237.

Pacific Exploration (symposium, Davis *et al*), 391.
Paige, S. The Mechanics of Intrusion of the Black Hills (S. D.) Precambrian Granite, 113.
Panama Canal, Report on land slides, 193.
Parallaxes, spectroscopic method of determining (Adams), 147, 152.
Parker, G. H. The Effectors of Sea-Anemones, 385.
——. Nervous Transmission in Sea-Anemones, 437.
——. The Responses of the Tentacles of Sea-Anemones, 438.
——. Locomotion of Sea-Anemones, 449.
——. The Behavior of Sea-Anemones, 450.
Parker, G. H., and Titus, E. G. The Neuromuscular Structure of Sea-Anemones, 339.
Parthenogenesis and sex (Loeb), 313.

Partial differential equations (Green), 209.
Patterson, S. W., see Pearl, R., 58.
Pearl, R. On the Effects of Feeding Pituitary Body (Anterior Lobe) Substance, and Corpus Luteum Substance to Growing Chicks, 50.
——. The Effect of Parental Alcoholism (and Certain Other Drug Intoxications) upon the Progeny in the Domestic Fowl, 380.
——. Some Effects of the Continued Administration of Alcohol to the Domestic Fowl, with Special Reference to the Progeny, 675.
Pearl, R., and Patterson, S. W. On the Degree of Inbreeding which Exists in American Jersey Cattle, 58.
Pease, F. G. The Rotation and Radial Velocity of the Spiral Nebula N. G. C. 4594, 517.
Pecos, N. M., archeological explorations (Kidder), 119.
Periodic system of elements (Harkins), 216.
Permeability, new determinations (Brooks), 569.
Perrine, C. D. On Some Relations Between the Proper Motions, Radial Velocities, and Magnitudes of Stars of Classes B and A, 289.
——. Asymmetry in the Proper Motions and Radial Velocities of Stars of Class B and Their Possible Relation to a Motion of Rotation, 292.
——. An Apparent Dependence of the Apex and Velocity of Solar Motion, as Determined from Radial Velocities, upon Proper Motion, 376.
Personal equation and steadiness of judgment (Harris), 65.
Petroleum shales, fossil algae (Davis), 114.
Philippine Islands, petrography of (Iddings and Morley), 531.
Phosphate deposits in Canada (Adams and Dick), 62.
Phosphorescence of uranyl salts (Nichols), 328.
Phosphorous, geologic rôle of (Blackwelder), 490.
Photo-electric effects (Millikan and Souder), 19.
Photo-electric phenomena, quantum relations (Millikan), 78.
Photography, monochromatic, of Jupiter and Saturn (Wood), 109.
Pilsbry, H. A. Mid-Pacific Land Snail Faunas, 429.
Pituitary body, fed to chicks (Pearl), 50.
Planetary nebula, rotations of a (Campbell and Moore), 129.
Planetary nebulae, relative motions (Campbell and Moore), 566.
Planets, albedo of (Russell), 74.
Point sets (Coble), 244, 530, 575.
Point sets on cubic curves (White), 339.
Polarity of egg of Crepidula (Conklin), 87.
Polypeptide-hydantoins (Johnson), 69.
Porter, M. B. On a Theorem of Lucas, 247.
——. Note on Lucas' Theorem, 335.
Postulates for analysis situs (Moore), 270.
Postulates for cyclic order (Huntington), 630.
Pottery, Pecos, N. M. (Kidder), 119.
Pottery, Venezuelan (Spinden), 325.
Pottery, Zuñi (Kroeber), 42.
Projective geometry, integral invariant (Wilczynski), 248.
Pronephros, removal in amphibian embryo (Howland), 231.
Proper motion, dependence of solar apex and motion on (Perrine), 376.
Proper motions, radial velocities and magnitudes of A and B stars (Perrine), 289, 292.
Protein, action of alkali in production of (Falk), 557.
Protoplasm, permeability to salts (Brooks), 569.
Protoplasm, similarity to gelatine (Fenn), 539.
Pyranometer (Abbot and Aldrich), 333.

Quanta of characteristic X-rays (Webster), 90.
Quantum relations in photo-electric phenomena (Millikan), 78.

Radial velocities, proper motions, and magnitudes of A and B stars (Perrine), 289, 292.
Radiation, instrument for measuring sky (Abbot and Aldrich), 333.
Radio-lead, density of (Richards and Wadsworth), 505, 694.
Reaction-system relations, hereditary (Clausen and Goodspeed), 240.
Regeneration rates, effect of successive removal on (Zeleny), 487.
Regeneration rates in new and old tissue (Zeleny), 484.
Relationship, terms of, and social organization (Michelson), 297.
Report of the Annual Meeting, 300.
Report of the Autumn Meeting, 744,
Report on land slides of Panama Canal, 193.
Research Council, see National.
Research grants, 307, 743.
Rice, M., see Hull, A. W., 265.
Rhizophora (Bowman), 685.
Richards, T. W., and Wadsworth, C., 3rd. Density of Radio-Lead from Pure Norwegian Clevite, 505.
——. Further Study of the Atomic Weight of Lead of Radioactive Origin, 694.
Roberts, W. A., see Harkins, W. D., 598.
——., see Harkins, W. D., 642.
Rocks, igneous of Japan (Iddings and Morley), 450.
Rocks, Pre-Cambrian, of Black Hills (Paige), 113.
Rocks, relation of chemistry to organic metabolism (Washington), 623.

Rocks, of Philippine Islands (Iddings and Morley), 531.
Root, F. M., see Mast, S. O., 188.
Roots, excretion of acids (Haas), 561.
Roots of integral functions (Porter), 247, 335.
Rose, J. N. Recent Explorations in the Cactus Deserts of South America, 73.
Rotation of a nebula (Hubble), 230.
Rotation of nebula N. G. C. 4594 (Pease), 517.
Rouiller, C. A. Synthesis of the Base $C_8H_4ON_2$ Derived from Methyl-Amino-methyl-3, 4-Dihydroxyphenylcarbinol, 464.
Rowland's solar spectrum wave-lengths (St. John), 226.
Ruedemann, R. On the Presence of a Median Eye in Trilobites, 234.
Russell, C. P., see Hegner, R. W., 356.
Russell, H. N. On the Albedo of the Planets and Their Satellites, 74.

Salt antagonism in gelatine (Fenn), 534.
Salt effect in diffusion through membrane (Loeb), 511.
San Juan Mountains, Col., geographic history (Atwood and Mather), 177.
Sandstone, fluviatile origin of Old Red (Barrell), 496.
Saturn, monochromatic photography of (Wood), 109.
Sayles, R. W. Banded Glacial Slates of Permo--Carboniferous Age, Showing Possible Seasonal Variations in Deposition, 167.
Schuchert, C. The Problem of Continental Fracturing and Diastrophism in Oceanica, 407.
——. The Earliest Fresh-water Anthropods, 726.
Sea Water, equilibrium between acids and bases (Henderson and Cohn), 618.
Sea Water, physiological effects (McClendon, 689.
Sea-anemones, behavior of (Parker), 450.
Sea-anemones, effectors of (Parker), 385.
Sea-anemones, locomotion of (Parker), 449
Sea-anemones, nervous transmission in (Parker), 437.
Sea-anemones, neuromuscular structure of (Parker and Titus), 339.
Sea-anemones, response of tentacles (Parker), 438.
Seares, F. H. A Simple Method for Determining the Colors of the Stars, 521.
——. Preliminary Results on the Color of Nebulae, 553.
Seasonal variations, Permo-Carboniferous age (Sayles), 167.
Series, trigonometric (Hardy and Littlewood), 583.
Sex, inheritance and determination (Goldschmidt), 53.
Sex intergrades (Banta), 578.
Sex of parthenogenetic frogs (Loeb), 313.
Shapley, H. Studies of Magnitudes in Star Clusters, I. On the Absorption of Light in Space, 12.
——. Studies of Magnitudes in Star Clusters. II. On the Sequence of Spectral Types in Stellar Evolution, 15.
——. A Short Period Cepheid with Variable Spectrum, 132.
——. Discovery of Eight Variable Stellar Spectra, 208.
——. Studies of Magnitudes in Star Clusters. III. The Colors of the Brighter Stars in Four Globular Systems, 525.
——., see Adams, W. S., 136.
Size, inheritance of (Castle), 252.
Skin, failure and revival of pigmentation (Jenks), 164.
Sky radiation, instrument for measuring (Abbot and Aldrich), 333.
Smith, E. F. Further Evidence as to the Relation between Crown Gall and Cancer, 444.
Smithsonian Institution, anthropological explorations (Hrdlička), 32.
Snail faunas, mid-Pacific (Pilsbry), 429.
Soil chemistry, analysis of peculiar clay (Hilgard), 8.
Solar apex and motion (Perrine), 376.
Solar atmosphere, minute structure of (Hale and Ellerman), 102.
Solar radiation and terrestrial magnetism (Bauer), 24.
Solar spectrum wave-lengths (St. John) 226.
Souder, W. H., see Millikan, R. A., 19.
South Pacific islands, petrology of (Iddings), 413.
Spectra, channeled grating (Barus), 378.
Spectra, interferences of reversed and inverted (Barus), 576.
Spectra, non-reversed of restricted coincidence (Barus), 614.
Spectra, quantitative method of classifying stellar (Adams), 143.
Spectra of lithium and calcium, Stark effect (Howell), 528.
Spectroscopy, investigations in stellar (Adams), 143, 147, 152, 157.
Spectrum, path differences for observable interferences (Barus), 609.
Spectrum, variable in short period Cepheid (Shapley), 132.
Spectrum of δ Cephei (Adams and Shapley), 136.
Spectrum of tungsten (Hull and Rice), 265.
Spermatozoa, distribution of chondriosomes to (Wilson), 321.
Sphenacodon Marsh (Williston), 650.
Spinden, H. J. New Data on the Archaeology of Venezuela, 325.
Spiral nebula, internal motion of (van Maanen), 286.

Squier, G. O., Scientific Research for National Defense as Illustrated by the Problems of Aeronautics (abstract), 740.
St. John, C. E. The Situation in Regard to Rowland's Preliminary Table of Solar Spectrum Wave-Lengths, 226.
——. On the Suggested Mutual Repulsion of Fraunhofer Lines, 458.
Star classes, two of M type (Adams), 157.
Star clusters, magnitudes in (Shapley), 12, 15, 525.
Star colors, method for determining (Seares), 521.
Star streaming, for helium type (Boss), 214.
Stark effect for lithium and calcium (Howell), 528.
Starkweather, H. W., see Baxter, G. P., 718.
Stars, Cepheid variables (Shapley), 132, 208; (Adams and Shapley), 136.
Stars of classes B and A (Perrine), 289, 292.
Stellar evolution, sequence of spectral types (Shapley), 15.
Stellar spectra, eight variable (Shapley), 208.
Steric hindrance (Lewis), 586.
Stewart, T. D., see Tolman, R. C., 189.
Stimulation, nature of mechanical (Osterhout), 237.
Substitution group, transitivity (Miller), 61.
Sugar content of blood, effect of morphin on (Kleiner and Meltzer), 369.
Sun, minute structure of atmosphere (Hale and Ellerman), 102.
Surface, F. M. On the Inheritance of Certain Glume Characters in the Cross *Avena Fatua* × *A. Sativa* var. Kherson, 478.
Surface-tension theory of feeding of Ameba (Mast and Root), 188.
Surface-theory (Wilson and Moore), 273.
Surface-theory, Ribaucour transformations (Eisenhart), 173.

Taber, S. The Origin of Veins Asbestiform Minerals, 659.
Temperature coefficient of life (Loeb and Northrop), 456.
Tentacles of sea-anemones (Parker), 438.
Terrestrial magnetism and solar radiation (Bauer), 24.
Thermometer scale, new (McAdie), 670.
Tin, atomic weight (Baxter and Starkweather), 718.
Tissues, rates of regeneration (Zeleny), 484, 487.
Titus, E. G., see Parker, G. H., 339.
Tolman, R. C., and Stewart, T. D. The Electromotive Force Produced by the Acceleration of Metals, 189.
Transformations of Ribaucour, deformed (Eisenhart), 173.
Transitivity of a substitution group (Miller), 61.
Trelease, W. The Oaks of America, 626.
Treubia insignis Goebel (Campbell), 30.
Trigonometric series (Hardy and Littlewood), 583.
Trilobites, Cambrian (Walcott), 101.
Trilobites, median eye in (Ruedemann), 234.

Unit-characters (Castle), 252.
Upper Cretaceous deposits of the Middle Atlantic Coast (Clark, Berry, and Gardner), 181.
Upper Cretaceous floras (Berry), 186.
Uranyl salts, phosphorescence of (Nichols), 328.
U. S. National Museum, anthropological explorations (Hrdlička), 32.

van Maanen, A. Preliminary Evidence of Internal Motion in the Spiral Nebula Messier 101, 386.
Vaughan, T. W. The Results of Investigations of the Ecology of the Floridian and Bahaman Shoal-Water Corals, 95.
Venezuela, archaeology of (Spinden), 325.
Vertebrates, rise of air-breathing (Barrell), 499.

Wadsworth, C., 3d., see Richards, T. W., 505.
——., see Richards, T. W., 694.
Walcott, C. D. Cambrian Trilobites, 101.
Washington, H. S. An Apparent Correspondence between the Chemistry of Igneous Magmas and of Organic Metabolism, 623.
Wave-length in solar spectrum (St. John), 226.
Webster, D. L. The Emission Quanta of Characteristic X Rays, 90.
Wells, R. C., see Larsen, E. S., 360.
White, H. S. A Variable System of Sevens on Two Twisted Cubic Curves, 337.
White's Porism (Coble), 530.
Wilczynski, E. J. Interpretation of the Simplest Integral Invariant of Projective Geometry, 248.
Williston, S. W. Sphenacadon Marsh, A Permocarboniferous Theromorph Reptile from New Mexico, 650.
Wilson, Edmund B. The Distribution of the Chondriosomes to the Spermatozoa in Scorpions, 321.
Wilson, Edwin B. Theory of an Aeroplane Encountering Gusts, 294.
Wilson, E. B., and Moore, C. L. E. A General Theory of Surfaces, 273.
Winninghoff, W. J., see Keyes, F. G., 342.
Wissler, C. The Genetic Relations of Certain Forms in American Aboriginal Art, 224.
Wood, R. W. Monochromatic Photography of Jupiter and Saturn, 109.

X-ray spectrum of tungsten (Hull and Rice), 265.
X-rays, absorption of soft (Miller), 441.
X-rays, emission quanta of characteristic (Webster), 90.

Yerkes, R. M. A New Method of Studying Ideational and Allied Forms of Behavior in Man and Other Animals, 631.
——. Ideational Behavior of Monkeys and Apes, 639.

Zeleny, C. A Comparison of the Rates of Regeneration from Old and from New Tissue, 484.
——. The Effect of Successive Removal Upon the Rate of Regeneration, 487.
Zeros of transcendental functions (Porter), 247, 335.
Zuñi culture sequences (Kroeber), 42.

JAN 2 - 1917

CONTENTS

Page

GEOLOGY.—THE ORIGIN OF VEINS OF THE ASBESTIFORM MINERALS . . . . . *By Stephen Taber* 659

GEOLOGY.—A NEW TEST OF THE SUBSIDENCE THEORY OF CORAL REEFS . . . . *By Reginald A. Daly* 664

PHYSICS.—A NEW THERMOMETER SCALE . . . . *By Alexander McAdie* 670

ZOOLOGY.—ON THE IMMUNITY COLORATION OF SOME NUDIBRANCHS *By W. J. Crozier* 672

GENETICS.—SOME EFFECTS OF THE CONTINUED ADMINISTRATION OF ALCOHOL TO THE DOMESTIC FOWL, WITH SPECIAL REFERENCE TO THE PROGENY *By Raymond Pearl* 675

PHYSICS.—AN IONIZATION MANOMETER . . . . *By O. E. Buckley* 683

BOTANY.—PHYSIOLOGICAL STUDIES ON RHIZOPHORA. *By Howard H. M. Bowman* 685

PHYSIOLOGY.—ON THE HYDROGEN ION CONCENTRATION OF SEA WATER, AND THE PHYSIOLOGICAL EFFECTS OF THE IONS OF SEA WATER . . *By J. F. McClendon* 689

PHYSIOLOGY.—SOME INTERRELATIONS BETWEEN DIET, GROWTH, AND THE CHEMICAL COMPOSITION OF THE BODY . . *By Lafayette B. Mendel and Sarah E. Judson* 692

CHEMISTRY.—FURTHER STUDY OF THE ATOMIC WEIGHT OF LEAD OF RADIOACTIVE ORIGIN . . . . . . *By Theodore W. Richards and Charles Wadsworth, 3d* 694

PALEONTOLOGY.—ON SOME ANOMALIES IN GEOGRAPHIC DISTRIBUTION OF PACIFIC COAST MOLLUSCA . . . . *By William Healey Dall* 700

PSYCHOLOGY.—SOME PSYCHO-PHYSIOLOGICAL PROCESSES AS AFFECTED BY ALCOHOL *By W. R. Miles* 703

ZOOLOGY.—THE INFLUENCE OF THE MARGINAL SENSE ORGANS ON METABOLIC ACTIVITY IN CASSIOPEA XAMACHANA BIGELOW . . . . *By L. R. Cary* 709

ANTHROPOLOGY.—NEW EVIDENCE IN REGARD TO THE INSTABILITY OF HUMAN TYPES *By Franz Boas* 713

CHEMISTRY.—A REVISION OF THE ATOMIC WEIGHT OF TIN . . . . *By Gregory Paul Baxter and Howard Warner Starkweather* 718

PHYSIOLOGY.—FURTHER STUDIES OF NERVE CONDUCTION IN CASSIOPEA . . . . *By Alfred Goldsborough Mayer* 721

PALEONTOLOGY.—THE EARLIEST FRESH-WATER ARTHROPODS. *By Charles Schuchert* 726

ZOOLOGY.—OBSERVATIONS UPON TROPICAL FISHES AND INFERENCES FROM THEIR ADAPTIVE COLORATION . . . . *By W. H. Longley* 733

NOTICES OF BIOGRAPHICAL MEMOIRS 737

NATIONAL RESEARCH COUNCIL, REPORT OF THE SECOND MEETING OF THE COUNCIL. 738

———, REPORT OF MEETINGS OF THE EXECUTIVE COMMITTEE 738

———, SCIENTIFIC RESEARCH FOR NATIONAL DEFENSE AS ILLUSTRATED BY THE PROBLEMS OF AERONAUTICS . . . . *By Lieut. Col. George O. Squier, U. S. A.* 740

RESEARCH GRANTS FROM THE TRUST FUNDS OF THE ACADEMY 743

REPORT OF THE AUTUMN MEETING 744

INDEX 751

Barker

SMITHSONIAN INSTITUTION LIBRARIES
3 9088 01299 9413